cs274ahw5

March 4, 2020

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import math
        import pandas as pd
```

```
In [2]: from numpy import asarray as arr
        from numpy import asmatrix as mat
        from numpy import atleast_2d as twod
```

0.0.1 Prob 1 Set up

$$J_i(\beta) = \begin{cases} y^{(i)} \log(f(x^{(i)} \cdot \beta)) & \text{if } y^{(i)} = 1 \\ (1 - y^{(i)}) \log(1 - f(x^{(i)} \cdot \beta)) & \text{if } y^{(i)} = 0 \end{cases}$$

$$\sigma(r) = (1 + exp(-r))^{-1}$$

$$J_i(\beta) = y^{(i)} \log(f(x^{(i)} \cdot \beta)) + (1 - y^{(i)}) log(1 - f(x^{(i)} \cdot \beta))$$

Thus, its gradient is:

$$\nabla J_{i,j}(\beta) = \frac{y^{(i)} f'(x^{(i)} \cdot \beta) \cdot}{f(x^{(i)} \cdot \beta)} - \frac{(1 - y^{(i)}) \cdot f'(x^{(i)} \cdot \beta)}{1 - f(x^{(i)} \cdot \beta)}$$

$$f'(x^{(i)} \cdot \beta) = f(x^{(i)} \cdot \beta) \cdot (1 - f(x^{(i)} \cdot \beta)) \cdot (x^{(i)} \cdot \beta)'$$

$$f'(x^{(i)} \cdot \beta) = f(x^{(i)} \cdot \beta) \cdot (1 - f(x^{(i)} \cdot \beta)) \cdot x^{(i,j)}$$

$$\nabla J_{i,j}(\beta) = \frac{y^{(i)} f'(x^{(i)} \cdot \beta)}{f(x^{(i)} \cdot \beta)} - \frac{(1 - y^{(i)}) \cdot f'(x^{(i)} \cdot \beta)}{1 - f(x^{(i)} \cdot \beta)}$$

$$\nabla J_{i,j}(\beta) = \frac{y^{(i)} \cdot x^{(i,j)} \cdot f(x^{(i)} \cdot \beta) \cdot (1 - f(x^{(i)} \cdot \beta))}{f(x^{(i)} \cdot \beta)} - \frac{(1 - y^{(i)}) \cdot x^{(i,j)} \cdot f(x^{(i)} \cdot \beta) \cdot (1 - f(x^{(i)} \cdot \beta)}{1 - f(x^{(i)} \cdot \beta)}$$

$$\nabla J_{i,j}(\beta) = y^{(i)} \cdot (1 - f(x^{(i)} \cdot \beta)) \cdot x^{(i,j)} - (1 - y^{(i)}) \cdot f(x^{(i)} \cdot \beta) \cdot x^{(i,j)}$$

$$\nabla J_{i,j}(\beta) = y^{(i)} \cdot (1) \cdot x^{(i,j)} - (1) \cdot f(x^{(i)} \cdot \beta) \cdot x^{(i,j)}$$

$$\nabla J_{i,j}(\beta) = (y^{(i)} - f(x^{(i)} \cdot \beta)) \cdot x^{(i,j)}$$

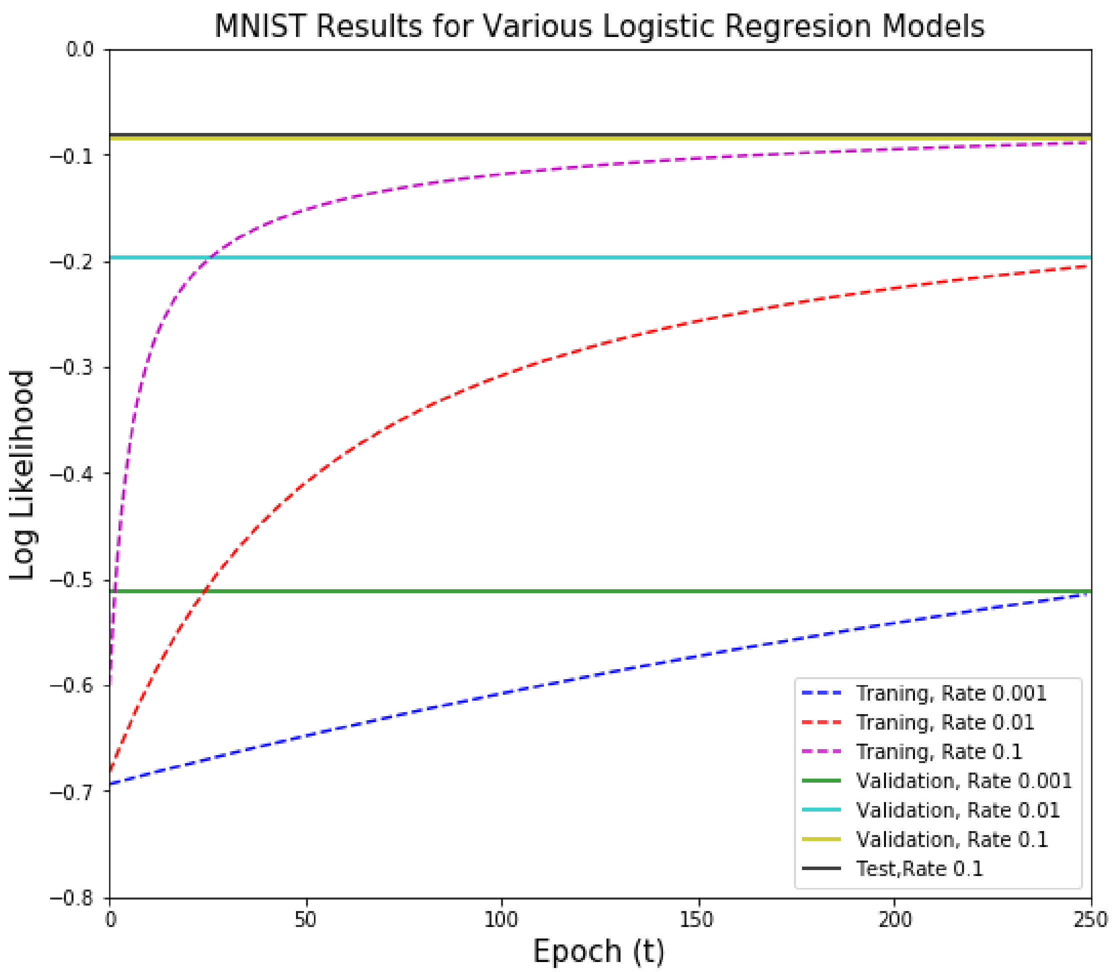
$$\nabla l_j(\beta) = \frac{1}{N}\sum_{i=1}^{N}(y^{(i)} - f(x^{(i)} \cdot \beta)) \cdot x^{(i,j)}$$

log likelihood grad[j] = $np.dot((Y - f(X, beta)).T, X[:, j]).T/N$
log likelihood grad = $np.dot((Y - f(X, beta)).T, X).T/N$

```
In [3]: def f(X,beta):
            return 1/(1+np.exp(-X.dot(beta)))

        def dfdx(X,beta):
            # f(x,beta)/x = f(x,beta)(1  f(x,beta))*z/x
            return f(X,beta)*(1-f(X,beta))*beta

        def loglike(X,beta,Y):
            N,M = X.shape
            like = [None]*N
            for i in range(N):
                like[i] = np.log(f(X[i,:],beta)) if Y[i] else np.log(1-f(X[i,:],beta))
            return np.mean(like)

        def loglike_grad(X,beta,Y):
            N,M = X.shape
            like_grad_beta = [None]*M
            like_grad_beta = np.dot((Y-f(X,beta)).T,X).T/N
            return arr(like_grad_beta)

        def err_rate(X,beta,Y):
            N,M = X.shape
            predict = np.zeros(Y.shape)
            predict[f(X,beta)>0.5]=1
            return np.mean(predict!=Y)

        def train(X,beta,Y,alpha,stopEpochs=250):
            log_like = arr([None]*stopEpochs)
            for i in range(stopEpochs):
                beta += alpha*loglike_grad(X,beta,Y)
                log_like[i] = loglike(X,beta,Y)
            return beta,log_like
        def get_matrix(X,parameter_list,stopEpochs):
            beta_matrix = np.zeros((Xtr.shape[1],len(parameter_list)))
            loglike_matrix = np.zeros((stopEpochs,len(parameter_list)))
            return beta_matrix,loglike_matrix
        def load_data_pairs(type_str):
            return pd.read_csv("./mnist_2s_and_6s/"+type_str+"_x.csv").values,pd.read_csv("./m
```

0.0.2 Prob 1.1.1

```
In [4]: Xtr,Ytr = load_data_pairs("train")
        Xva,Yva = load_data_pairs("valid")
        Xte,Yte = load_data_pairs("test")

In [5]: Xtr = np.hstack([Xtr,np.ones((Xtr.shape[0],1))])
        Xva = np.hstack([Xva,np.ones((Xva.shape[0],1))])
        Xte = np.hstack([Xte,np.ones((Xte.shape[0],1))])

In [6]: learning_rate = [1e-3,1e-2,1e-1]
        stopEpochs = 250
        beta_matrix,loglike_matrix = get_matrix(Xtr,learning_rate,stopEpochs)
        for k,rate in enumerate(learning_rate):
            beta = np.random.normal(scale=.001,size=(Xtr.shape[1],1))
            alpha = learning_rate[k]
            beta,log_like = train(Xtr,beta,Ytr,alpha,stopEpochs)
            beta_matrix[:,k] = beta.T.flatten()
            loglike_matrix[:,k] = log_like.T.flatten()


In [7]: validation_loglike = np.zeros((3,1))
        for i in range(3):
            validation_loglike[i] = loglike(Xva,beta_matrix[:,i],Yva)
        test_loglike = loglike(Xte,beta_matrix[:,np.argmax(validation_loglike)],Yte)

In [8]: fig,ax = plt.subplots(1,1,figsize=(9,8))
        ax.plot(range(stopEpochs),loglike_matrix[:,0],'b--')
        ax.hlines(validation_loglike[0], 0, 250, colors='g')
        ax.plot(range(stopEpochs),loglike_matrix[:,1],'r--')
        ax.hlines(validation_loglike[1], 0, 250, colors='c')
        ax.plot(range(stopEpochs),loglike_matrix[:,2],'m--')
        ax.hlines(validation_loglike[2], 0, 250, colors='y')
        ax.hlines(test_loglike, 0, 250, colors = 'k')
        ax.legend(["Traning, Rate 0.001",
                   "Traning, Rate 0.01",
                   "Traning, Rate 0.1",
                   "Validation, Rate 0.001",
                   "Validation, Rate 0.01",
                   "Validation, Rate 0.1",
                  "Test,Rate 0.1"])
        ax.set_title("MNIST Results for Various Logistic Regresion Models",fontsize=15)
        plt.xlabel("Epoch (t)",fontsize=15)
        plt.ylabel("Log Likelihood",fontsize=15)
        plt.xlim((0,250))
        plt.ylim((-0.8,0.0))

Out[8]: (-0.8, 0.0)
```



0.0.3 Prob 1.1.2

In [9]:
```
Training_Error_list = [None]*3
Validation_Error_list = [None]*3
Test_Error_list = [None]*3
Training_like_list = [None]*3
Validation_like_list = [None]*3
Test_like_list = [None]*3
for i in range(3):
    Training_Error_list[i]  = err_rate(Xtr,beta_matrix[:,i],Ytr)
    Validation_Error_list[i]= err_rate(Xva,beta_matrix[:,i],Yva)
    Test_Error_list[i]      = err_rate(Xte,beta_matrix[:,i],Yte)
    Training_like_list[i]   = loglike(Xtr,beta_matrix[:,i],Ytr)
    Validation_like_list[i] = loglike(Xva,beta_matrix[:,i],Yva)
    Test_like_list[i]       = loglike(Xte,beta_matrix[:,i],Yte)
```

In [10]:
```
import pandas as pd
import os
```

```
new_dataframe = pd.DataFrame(
    {
        "Training Error" : Training_Error_list,
        "Validation Error":Validation_Error_list,
        "Test Error":Test_Error_list
    },
    index = ['Rate 0.001','Rate 0.01','Rate 0.1']
)
new_dataframe
```

Out[10]:

	Training Error	Validation Error	Test Error
Rate 0.001	0.055984	0.044782	0.043342
Rate 0.01	0.047005	0.038868	0.034987
Rate 0.1	0.027147	0.024081	0.022454

In [11]:
```
new_dataframe = pd.DataFrame(
    {
        "Training likelihood" : Training_like_list,
        "Validation likelihood":Validation_like_list,
        "Test likelihood":Test_like_list
    },
    index = ['Rate 0.001','Rate 0.01','Rate 0.1']
)
new_dataframe
```

Out[11]:

	Training likelihood	Validation likelihood	Test likelihood
Rate 0.001	-0.514495	-0.510975	-0.505024
Rate 0.01	-0.205158	-0.197636	-0.188133
Rate 0.1	-0.088650	-0.085469	-0.080531

0.0.4 Prob 1.1.3

In [ ]:

0.0.5 Prob 1.2.1

In [12]:
```
from random import sample
```

In [15]:
```
import copy
def loglike_grad_stoch(X,beta,Y,ind):
    sample_stoch = ind
    # print(sample_stoch)
    like_grad_beta = [None]*X.shape[1]
    like_grad_beta = np.dot((Y[sample_stoch]-f(X[sample_stoch],beta)).T,X[sample_stoc
    return arr(like_grad_beta)

def train_stoch(X,beta,Y,alpha,M,stopEpochs):
    N,P = X.shape
    t_ext = np.int(np.floor(stopEpochs*N/M))
```

```
    log_like = arr([None]*stopEpochs)
    j = 0
    res = [np.int(np.floor((t_ext)/stopEpochs)),np.int(np.ceil((t_ext)/stopEpochs))]
    if stopEpochs*res[1]>t_ext:
        res[1] =res[0]

    arry = np.arange(N)
    np.random.shuffle(arry)
    # arry_matrix = copy.copy(arry[0:np.int(res[1]*M)]).reshape(res[1],M)
    new_array = np.split(arry[0:np.int(res[1]*M)],res[1])

    for i in range(t_ext):
        ind = new_array[np.int(i%(res[1]))]
        # print(ind)
        beta += alpha*loglike_grad_stoch(X,beta,Y,ind)
        if i%np.int(res[1]) ==0 or i == np.int(t_ext-1):
            log_like[j] = loglike(X,beta,Y)
            if j<stopEpochs-1:
                j +=1
    return beta,log_like
```

In [ ]:

In [16]:
```
batch_size = [1e+3,1e+2,1e+1]
stopEpochs = 250
beta_matrix2,loglike_matrix2 = get_matrix(Xtr,batch_size,stopEpochs)
for k,M in enumerate(batch_size):
    endl = np.int(np.floor(stopEpochs*Xtr.shape[0]/M))
    print(endl)
    beta = np.random.normal(scale=.001,size=(Xtr.shape[1],1))
    alpha = 0.01
    beta,log_like = train_stoch(Xtr,beta,Ytr,alpha,M,stopEpochs)
    beta_matrix2[:,k] = beta.T.flatten()
    loglike_matrix2[:,k] = log_like.T.flatten()
```

```
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236675
```

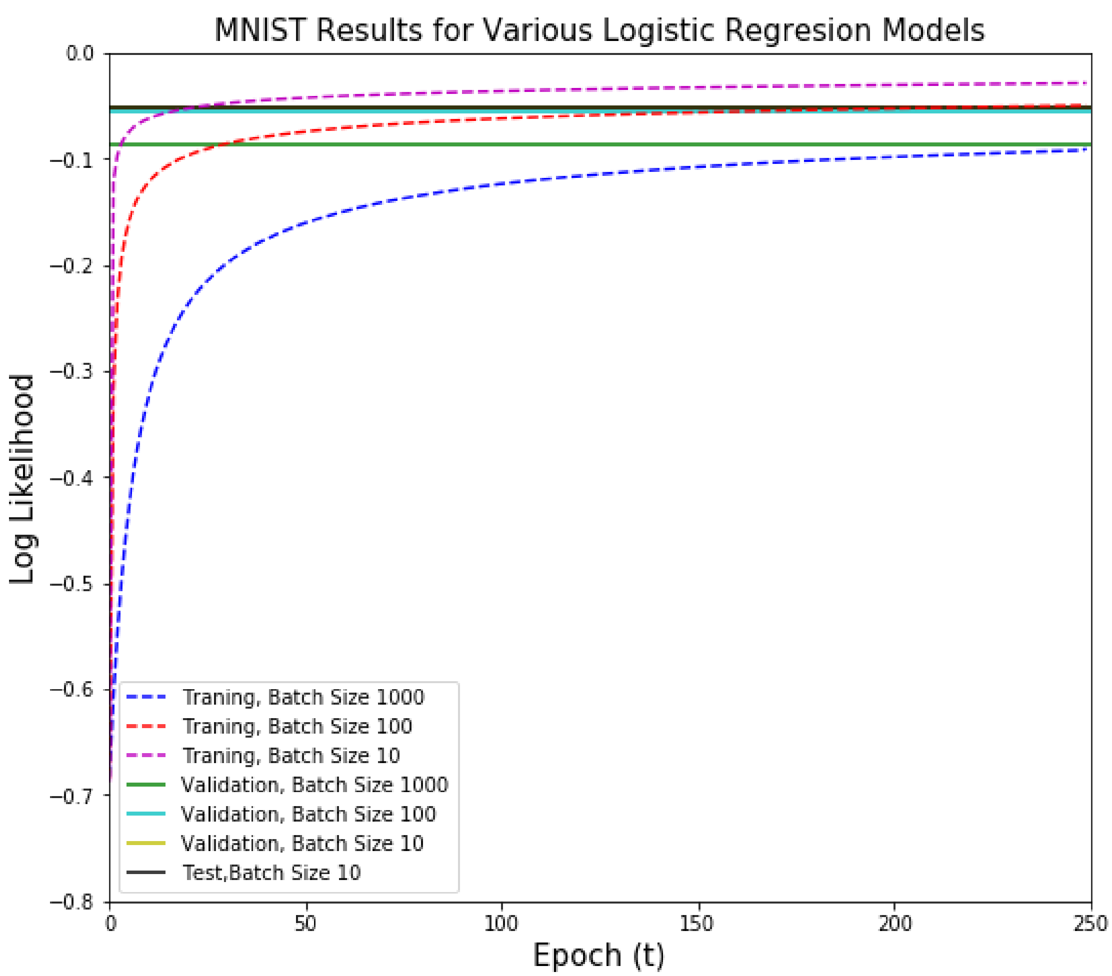
In [17]:
```
validation_loglike = np.zeros((3,1))
for i in range(3):
    validation_loglike[i] = loglike(Xva,beta_matrix2[:,i],Yva)
test_loglike = loglike(Xte,beta_matrix2[:,np.argmax(validation_loglike)],Yte)
```

In [ ]:

In [18]:
```
fig,ax = plt.subplots(1,1,figsize=(9,8))
ax.plot(range(stopEpochs),loglike_matrix2[0:stopEpochs,0],'b--')
```

```
ax.hlines(validation_loglike[0], 0, stopEpochs, colors='g')
ax.plot(range(stopEpochs),loglike_matrix2[0:stopEpochs,1],'r--')
ax.hlines(validation_loglike[1], 0, stopEpochs, colors='c')
ax.plot(range(stopEpochs),loglike_matrix2[0:stopEpochs,2],'m--')
ax.hlines(validation_loglike[2], 0, stopEpochs, colors='y')
ax.hlines(test_loglike, 0, stopEpochs, colors = 'k')
ax.legend(["Traning, Batch Size 1000",
           "Traning, Batch Size 100",
           "Traning, Batch Size 10",
           "Validation, Batch Size 1000",
           "Validation, Batch Size 100",
           "Validation, Batch Size 10",
           "Test,Batch Size 10"])
ax.set_title("MNIST Results for Various Logistic Regresion Models",fontsize=15)
plt.xlabel("Epoch (t)",fontsize=15)
plt.ylabel("Log Likelihood",fontsize=15)
plt.xlim((0,250))
plt.ylim((-0.8,0.0))
```

```
Out[18]: (-0.8, 0.0)
```



0.0.6 Prob 1.2.1

In [19]:
```
Training_Error_list = [None]*3
Validation_Error_list = [None]*3
Test_Error_list = [None]*3
Training_like_list = [None]*3
Validation_like_list = [None]*3
Test_like_list = [None]*3
for i in range(3):
    Training_Error_list[i]  = err_rate(Xtr,beta_matrix2[:,i],Ytr)
    Validation_Error_list[i]= err_rate(Xva,beta_matrix2[:,i],Yva)
    Test_Error_list[i]      = err_rate(Xte,beta_matrix2[:,i],Yte)
    Training_like_list[i]   = loglike(Xtr,beta_matrix2[:,i],Ytr)
    Validation_like_list[i] = loglike(Xva,beta_matrix2[:,i],Yva)
    Test_like_list[i]       = loglike(Xte,beta_matrix2[:,i],Yte)
```

In [20]:
```
import pandas as pd
import os
new_dataframe = pd.DataFrame(
    {
        "Training Error" : Training_Error_list,
        "Validation Error":Validation_Error_list,
        "Test Error":Test_Error_list
    },
    index = ['Batch Size 1000','Batch Size 100','Batch Size 10']
)
new_dataframe
```

Out[20]:

	Training Error	Validation Error	Test Error
Batch Size 1000	0.027358	0.023659	0.022454
Batch Size 100	0.015316	0.016054	0.015666
Batch Size 10	0.008662	0.015209	0.017232

In [21]:
```
new_dataframe = pd.DataFrame(
    {
        "Training Log Likelihood" : Training_like_list,
        "Validation Log Likelihood":Validation_like_list,
        "Test Log Likelihood":Test_like_list
    },
    index = ['Batch Size 1000','Batch Size 100','Batch Size 10']
)
new_dataframe
```

Out[21]:

	Training Log Likelihood	Validation Log Likelihood \
Batch Size 1000	-0.090410	-0.086988
Batch Size 100	-0.049727	-0.056108

```
Batch Size 10                  -0.028701                  -0.051667
```

```
                 Test Log Likelihood
Batch Size 1000            -0.081827
Batch Size 100             -0.054370
Batch Size 10              -0.051401
```

0.0.7 Prob 1.2.3

In [ ]:

0.0.8 Prob 2 setup

In [22]:
```python
def get_matrix2(X,parameter_list,stopEpochs):
    beta_matrix = np.zeros((Xtr.shape[1],len(parameter_list)))
    loglike_matrix = np.zeros((stopEpochs+1,len(parameter_list)))
    return beta_matrix,loglike_matrix

def train_stoch_RM(X,beta,Y,alpha0,M,stopEpochs):
    N,P = X.shape
    alpha = alpha0
    t_ext = np.int(np.floor(stopEpochs*N/M))
    log_like = arr([None]*stopEpochs)
    j = 0
    res = [np.int(np.floor((t_ext)/stopEpochs)),np.int(np.ceil((t_ext)/stopEpochs))]
    arry = np.arange(N)
    np.random.shuffle(arry)
    new_array = np.split(arry[0:np.int(res[0]*M)],res[0])
    for i in range(t_ext):
        ind = new_array[np.int(i%(res[0]))]
        alpha = alpha0/(i+1)
        beta += alpha*loglike_grad_stoch(X,beta,Y,ind)
        if i%np.int(res[1]) ==0 or i == np.int(t_ext-1):
            log_like[j] = loglike(X,beta,Y)
            if j<stopEpochs-1:
                j +=1
    return beta,log_like
```

0.0.9 Prob 2.1.1 Robbins-Monro

In [23]:
```python
batch_size = [200]
stopEpochs = 50
beta_matrix2,loglike_matrix2 = get_matrix2(Xtr,batch_size,stopEpochs)
beta = np.random.normal(scale=.001,size=(Xtr.shape[1],1))
loglike_matrix2[:,0] = loglike(Xtr,beta,Ytr)

for k,M in enumerate(batch_size):
    endl = np.int(np.floor(stopEpochs*Xtr.shape[0]/M))
```

```
            print(endl)
            alpha = 0.1
            beta,log_like = train_stoch_RM(Xtr,beta,Ytr,alpha,M,stopEpochs)
            beta_matrix2[:,k] = beta.T.flatten()
            loglike_matrix2[1:,k] = log_like.T.flatten()
```

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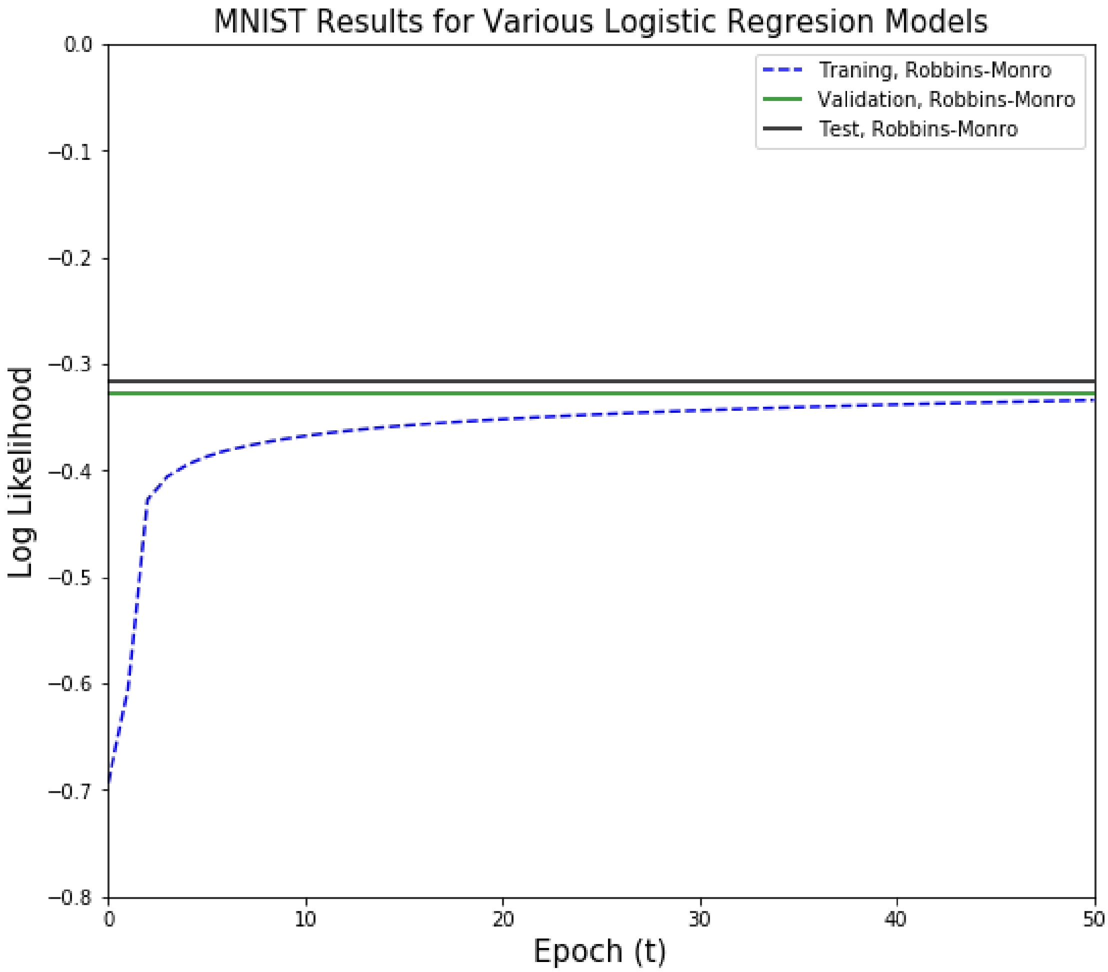
```

```
In [24]: validation_loglike = np.zeros((1,1))
         for i in range(1):
             validation_loglike[i] = loglike(Xva,beta_matrix2[:,i],Yva)
         test_loglike = loglike(Xte,beta_matrix2[:,np.argmax(validation_loglike)],Yte)
         fig,ax = plt.subplots(1,1,figsize=(9,8))
         ax.plot(range(0,stopEpochs+1),loglike_matrix2,'b--')
         ax.hlines(validation_loglike, 0, stopEpochs, colors='g')
         ax.hlines(test_loglike, 0, stopEpochs, colors = 'k')
         ax.legend(["Traning, Robbins-Monro",
                    "Validation, Robbins-Monro",
                    "Test, Robbins-Monro"])
         ax.set_title("MNIST Results for Various Logistic Regresion Models",fontsize=15)
         plt.xlabel("Epoch (t)",fontsize=15)
         plt.ylabel("Log Likelihood",fontsize=15)
         plt.xlim((0,50))
         plt.ylim((-0.8,0.0))
```

```
Out[24]: (-0.8, 0.0)
```




```
In [25]: import copy
         Training_Error_list = [None]*1
         Validation_Error_list = [None]*1
         Test_Error_list = [None]*1
         Training_like_list = [None]*1
         Validation_like_list = [None]*1
         Test_like_list = [None]*1
         for i in range(1):
             Training_Error_list[i]   = err_rate(Xtr,beta_matrix2[:,i],Ytr)
             Validation_Error_list[i]= err_rate(Xva,beta_matrix2[:,i],Yva)
             Test_Error_list[i]       = err_rate(Xte,beta_matrix2[:,i],Yte)
             Training_like_list[i]    = loglike(Xtr,beta_matrix2[:,i],Ytr)
             Validation_like_list[i]  = loglike(Xva,beta_matrix2[:,i],Yva)
             Test_like_list[i]        = loglike(Xte,beta_matrix2[:,i],Yte)

         loglike_matrix_rm = copy.copy(loglike_matrix2)
```

```
Training_Error_rm  = err_rate(Xtr,beta_matrix2[:,i],Ytr)
Validation_Error_rm= err_rate(Xva,beta_matrix2[:,i],Yva)
Test_Error_rm      = err_rate(Xte,beta_matrix2[:,i],Yte)
Training_like_rm   = loglike(Xtr,beta_matrix2[:,i],Ytr)
Validation_like_rm = loglike(Xva,beta_matrix2[:,i],Yva)
Test_like_rm       = loglike(Xte,beta_matrix2[:,i],Yte)
```

In [26]:
```
import pandas as pd
import os
new_dataframe = pd.DataFrame(
    {
        "Training Error" : Training_Error_list,
        "Validation Error":Validation_Error_list,
        "Test Error":Test_Error_list
    },
    index = ['Robbins-Monro']
)
new_dataframe
```

Out[26]:
```
               Training Error  Validation Error  Test Error
Robbins-Monro        0.052287          0.041825    0.038642
```

In [27]:
```
new_dataframe = pd.DataFrame(
    {
        "Training Log Likelihood" : Training_like_list,
        "Validation Log Likelihood":Validation_like_list,
        "Test Log Likelihood":Test_like_list
    },
    index = ['Robbins-Monro']
)
new_dataframe
```

Out[27]:
```
               Training Log Likelihood  Validation Log Likelihood  \
Robbins-Monro                -0.334296                  -0.327885

               Test Log Likelihood
Robbins-Monro            -0.317392
```

0.0.10 2.1.2 AdaM

In [28]:
```
def get_AdaM_update(alpha_0,grad,adam_values,b1=.95,b2=.99,e=1e-8):
    adam_values['t'] +=1
    # update mean
    adam_values['mean']=b1*adam_values['mean']+(1-b1)*grad
    m_hat = adam_values['mean']/(1-b1**adam_values['t'])
    #update variance
    adam_values['var']=b2*adam_values['var']+(1-b2)*grad**2
    v_hat = adam_values['var']/(1-b2**adam_values['t'])
    return alpha_0 * m_hat/(np.sqrt(v_hat)+e)
```

```
def train_stoch_Adam(X,beta,Y,alpha0,M,stopEpochs):
    N,P = X.shape
    alpha = alpha0
    t_ext = np.int(np.floor(stopEpochs*N/M))
    log_like = arr([None]*stopEpochs)
    j = 0
    res = [np.int(np.floor((t_ext)/stopEpochs)),np.int(np.ceil((t_ext)/stopEpochs))]
    arry = np.arange(N)
    np.random.shuffle(arry)
    new_array = np.split(arry[0:np.int(res[0]*M)],res[0])
    for i in range(t_ext):
        ind = new_array[np.int(i%(res[0]))]
        alpha = alpha0/(i+1)
        beta_grad = loglike_grad_stoch(Xtr,beta,Ytr,ind)
        beta_update = get_AdaM_update(alpha_0,beta_grad,adam_values)
        beta+=beta_update
        if i%np.int(res[1]) ==0 or i == np.int(t_ext-1):
            log_like[j] = loglike(X,beta,Y)
            if j<stopEpochs-1:
                j +=1
    return beta,log_like
```

In [29]:
```
batch_size = [200]
stopEpochs = 50
beta_matrix2,loglike_matrix2 = get_matrix2(Xtr,batch_size,stopEpochs)
beta = np.random.normal(scale=.001,size=(Xtr.shape[1],1))
loglike_matrix2[:,0] = loglike(Xtr,beta,Ytr)
adam_values = {'mean':np.zeros(beta.shape),'var':np.zeros(beta.shape),'t':0}
for k,M in enumerate(batch_size):
    endl = np.int(np.floor(stopEpochs*Xtr.shape[0]/M))
    print(endl)
    alpha_0 = 0.001
    beta,log_like = train_stoch_Adam(Xtr,beta,Ytr,alpha_0,M,stopEpochs)
    beta_matrix2[:,k] = beta.T.flatten()
    loglike_matrix2[1:,k] = log_like.T.flatten()
```

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```
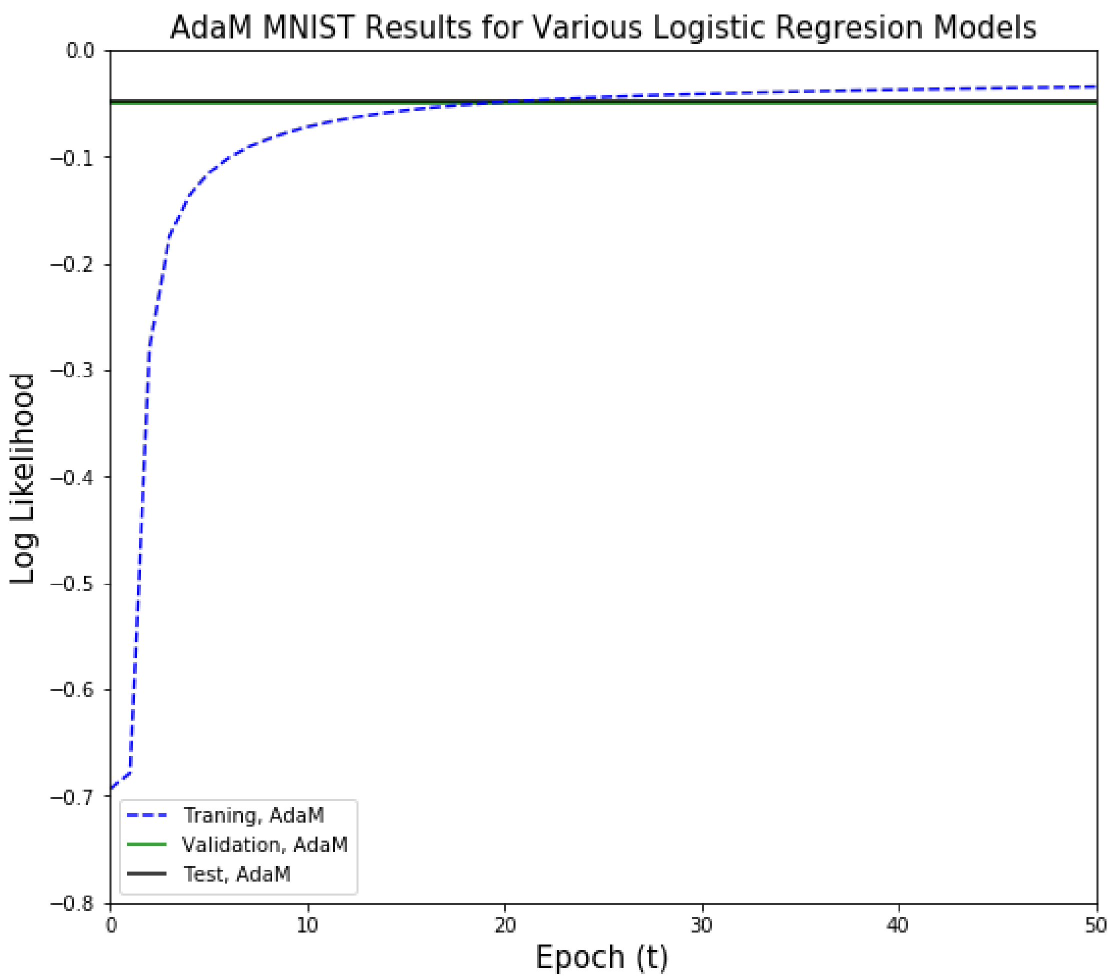

In [ ]:

In [30]:
```
import copy
validation_loglike = np.zeros((1,1))
for i in range(1):
    validation_loglike[i] = loglike(Xva,beta_matrix2,Yva)
test_loglike = loglike(Xte,beta_matrix2,Yte)
fig,ax = plt.subplots(1,1,figsize=(9,8))
ax.plot(range(0,stopEpochs+1),loglike_matrix2,'b--')
```

```
ax.hlines(validation_loglike, 0, stopEpochs, colors='g')
ax.hlines(test_loglike, 0, stopEpochs, colors = 'k')
ax.legend(["Traning, AdaM",
           "Validation, AdaM",
           "Test, AdaM"])
ax.set_title("AdaM MNIST Results for Various Logistic Regresion Models",fontsize=15)
plt.xlabel("Epoch (t)",fontsize=15)
plt.ylabel("Log Likelihood",fontsize=15)
plt.xlim((0,50))
plt.ylim((-0.8,0.0))
```

Out[30]: (-0.8, 0.0)




```
In [31]: Training_Error_list = [None]*1
         Validation_Error_list = [None]*1
         Test_Error_list = [None]*1
         Training_like_list = [None]*1
```

```
Validation_like_list = [None]*1
Test_like_list = [None]*1
for i in range(1):
    Training_Error_list[i]  = err_rate(Xtr,beta_matrix2[:,i],Ytr)
    Validation_Error_list[i]= err_rate(Xva,beta_matrix2[:,i],Yva)
    Test_Error_list[i]      = err_rate(Xte,beta_matrix2[:,i],Yte)
    Training_like_list[i]   = loglike(Xtr,beta_matrix2[:,i],Ytr)
    Validation_like_list[i] = loglike(Xva,beta_matrix2[:,i],Yva)
    Test_like_list[i]       = loglike(Xte,beta_matrix2[:,i],Yte)


loglike_matrix_am = copy.copy(loglike_matrix2)

Training_Error_am  = err_rate(Xtr,beta_matrix2[:,i],Ytr)
Validation_Error_am= err_rate(Xva,beta_matrix2[:,i],Yva)
Test_Error_am      = err_rate(Xte,beta_matrix2[:,i],Yte)
Training_like_am   = loglike(Xtr,beta_matrix2[:,i],Ytr)
Validation_like_am= loglike(Xva,beta_matrix2[:,i],Yva)
Test_like_am       = loglike(Xte,beta_matrix2[:,i],Yte)
```

In [32]:
```
import pandas as pd
import os
new_dataframe = pd.DataFrame(
    {
        "Training Error" : Training_Error_list,
        "Validation Error":Validation_Error_list,
        "Test Error":Test_Error_list
    },
    index = ['AdaM']
)
new_dataframe
```

Out[32]:

	Training Error	Validation Error	Test Error
AdaM	0.010246	0.013942	0.014099

In [33]:
```
new_dataframe = pd.DataFrame(
    {
        "Training Log Likelihood" : Training_like_list,
        "Validation Log Likelihood":Validation_like_list,
        "Test Log Likelihood":Test_like_list
    },
    index = ['Adam']
)
new_dataframe
```

Out[33]:

	Training Log Likelihood	Validation Log Likelihood	Test Log Likelihood
Adam	-0.034397	-0.049313	-0.047404

0.0.11 Prob 2.1.3 Newton-Raphson

```
In [34]: import numpy as np
         import matplotlib.pyplot as plt
         import math
         import pandas as pd
         from numpy import asarray as arr
         from numpy import asmatrix as mat
         from numpy import atleast_2d as twod
         def f(X,beta):
             return 1/(1+np.exp(-X.dot(beta)))

         def dfdx(X,beta):
             # f(x,beta)/x = f(x,beta)(1  f(x,beta))*z/x
             return f(X,beta)*(1-f(X,beta))*beta

         def loglike(X,beta,Y):
             N,M = X.shape
             like = [None]*N
             for i in range(N):
                 like[i] = np.log(f(X[i,:],beta)) if Y[i] else np.log(1-f(X[i,:],beta))
             return np.mean(like)

         def loglike_grad(X,beta,Y):
             N,M = X.shape
             like_grad_beta = [None]*M
             like_grad_beta = np.dot((Y-f(X,beta)).T,X).T/N
             return arr(like_grad_beta)

         def err_rate(X,beta,Y):
             N,M = X.shape
             predict = np.zeros(Y.shape)
             predict[f(X,beta)>0.5]=1
             return np.mean(predict!=Y)

         def train(X,beta,Y,alpha,stopEpochs=250):
             log_like = arr([None]*stopEpochs)
             for i in range(stopEpochs):
                 beta += alpha*loglike_grad(X,beta,Y)
                 log_like[i] = loglike(X,beta,Y)
             return beta,log_like
         def get_matrix(X,parameter_list,stopEpochs):
             beta_matrix = np.zeros((Xtr.shape[1],len(parameter_list)))
             loglike_matrix = np.zeros((stopEpochs,len(parameter_list)))
             return beta_matrix,loglike_matrix
         def load_data_pairs(type_str):
             return pd.read_csv("./mnist_2s_and_6s/"+type_str+"_x.csv").values,pd.read_csv("./m
```

```
Xtr,Ytr = load_data_pairs("train")
Xva,Yva = load_data_pairs("valid")
Xte,Yte = load_data_pairs("test")
Xtr = np.hstack([Xtr,np.ones((Xtr.shape[0],1))])
Xva = np.hstack([Xva,np.ones((Xva.shape[0],1))])
Xte = np.hstack([Xte,np.ones((Xte.shape[0],1))])


def get_matrix2(X,parameter_list,stopEpochs):
    beta_matrix = np.zeros((Xtr.shape[1],len(parameter_list)))
    loglike_matrix = np.zeros((stopEpochs+1,len(parameter_list)))
    return beta_matrix,loglike_matrix


#def loglike_grad_stoch(X,beta,Y,M):
#    N,P = X.shape
#    sample_stoch = np.sort(sample(range(N),np.int(M)))
#    like_grad_beta = [None]*P
#    like_grad_beta = np.dot((Y[sample_stoch]-f(X[sample_stoch],beta)).T,X[sample_sto
#    return arr(like_grad_beta)

def train_stoch(X,beta,Y,alpha,M,stopEpochs):
    N,P = X.shape
    t_ext = np.int(np.floor(stopEpochs*N/M))
    log_like = arr([None]*stopEpochs)
    j = 0
    for i in range(t_ext):
        beta += alpha*loglike_grad_stoch(X,beta,Y,M)
        res = [np.floor((t_ext)/stopEpochs),np.ceil((t_ext)/stopEpochs)]
        if i%np.int(res[1]) ==0 or i == np.int(t_ext-1):
            log_like[j] = loglike(X,beta,Y)
            if j<stopEpochs-1:
                j +=1
    return beta,log_like

def loglike_grad_stoch_newton(X,beta,Y,sample_stoch):
    N,P = X.shape
    like_grad_beta = [None]*P
    like_grad_beta = np.dot((Y[sample_stoch]-f(X[sample_stoch],beta)).T,X[sample_stoc
    return arr(like_grad_beta)
```

In [35]:
```
import copy
from random import sample
def train_stoch_Newton(X,beta,Y,M,stopEpochs):
    N,P = X.shape
    t_ext = np.int(np.floor(stopEpochs*N/M))
    log_like = arr([None]*stopEpochs)
    j = 0
```

```
        res = [np.int(np.floor((t_ext)/stopEpochs)),np.int(np.ceil((t_ext)/stopEpochs))]
        arry = np.arange(N)
        np.random.shuffle(arry)
        new_array = np.split(arry[0:np.int(res[0]*M)],res[0])
        for i in range(t_ext):
            if i%100==0:
                print(i)
            ind = new_array[np.int(i%(res[0]))]
            sample_stoch = ind
            #Xnew = copy.copy(Xtr[sample_stoch,:])
            #XnewT = copy.copy(Xnew.T)
            #for k,kk in enumerate(sample_stoch):
            #    rate = f(Xtr[kk,:],beta)*(1-f(Xtr[kk,:],beta))
            #    XnewT[:,k] *= rate
            #alphat = np.dot(XnewT,Xnew)
            #try:
            #    beta_incre = np.linalg.solve(alphat, loglike_grad_stoch_newton(X,beta,Y,
            #    beta += beta_incre
            #except np.linalg.LinAlgError:
            #alpha = np.linalg.pinv(alphat)
            #beta += alpha.dot(loglike_grad_stoch(X,beta,Y,M))
            At = np.diag(arr([f(Xtr[kk,:],beta)*(1-f(Xtr[kk,:],beta)) for kk in sample_sto
            alphat = np.dot(Xtr[sample_stoch].T,At).dot(Xtr[sample_stoch])
            try:
                alpha = np.linalg.inv(alphat)
            except np.linalg.LinAlgError:
                alpha = np.linalg.pinv(alphat)
            beta += alpha.dot(loglike_grad_stoch_newton(X,beta,Y,sample_stoch))

            if i%np.int(res[1]) ==0 or i == np.int(t_ext-1):
                log_like[j] = loglike(X,beta,Y)
                if j<stopEpochs-1:
                    j +=1
        return beta,log_like

    def loglike(X,beta,Y):
        N,M = X.shape
        like = [None]*N
        for i in range(N):
            like[i] = np.log(f(X[i,:],beta)) if Y[i] else np.log(1-f(X[i,:],beta))
        return np.mean(like)
```

In [ ]:

In [36]:
```
batch_size = [200]
stopEpochs = 50
beta_matrix2,loglike_matrix2 = get_matrix2(Xtr,batch_size,stopEpochs)
beta = np.random.normal(scale=0.01,size= (Xtr.shape[1],1))
```

```
        loglike_matrix2[:,0] = loglike(Xtr,beta,Ytr)

        for k,M in enumerate(batch_size):
            endl = np.int(np.floor(stopEpochs*Xtr.shape[0]/M))
            print(endl)
            beta,log_like = train_stoch_Newton(Xtr,beta,Ytr,M,stopEpochs)
            beta_matrix2[:,k] = beta.T.flatten()
            loglike_matrix2[1:,k] = log_like.T.flatten()
```

```
2366
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
2100
2200
2300
```

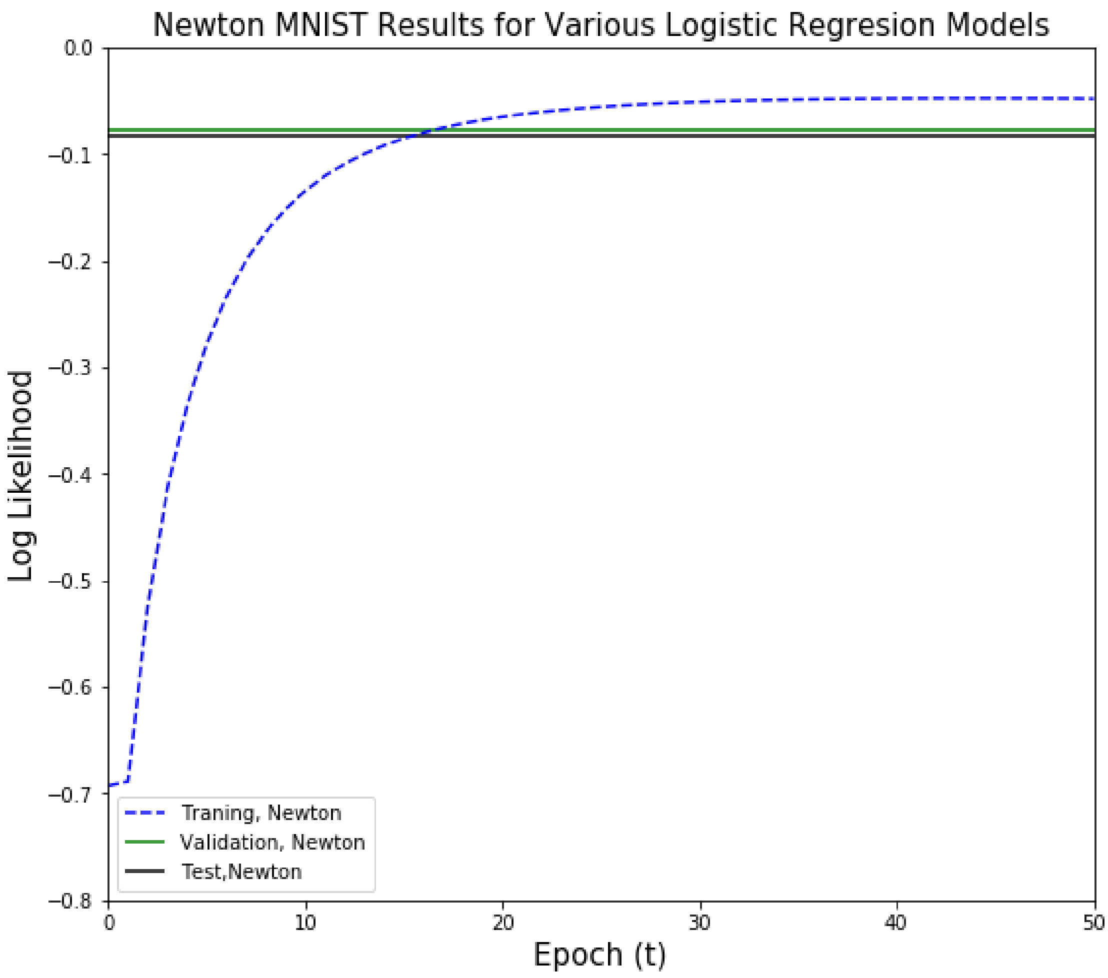
In [ ]:

In [ ]:

In [37]:
```
validation_loglike = np.zeros((1,1))
for i in range(1):
    validation_loglike[i] = loglike(Xva,beta_matrix2,Yva)
test_loglike = loglike(Xte,beta_matrix2,Yte)
fig,ax = plt.subplots(1,1,figsize=(9,8))
ax.plot(range(0,stopEpochs+1),loglike_matrix2,'b--')
ax.hlines(validation_loglike, 0, stopEpochs, colors='g')
ax.hlines(test_loglike, 0, stopEpochs, colors = 'k')
ax.legend(["Traning, Newton",
```

```
                 "Validation, Newton",
                 "Test,Newton"])
ax.set_title("Newton MNIST Results for Various Logistic Regresion Models",fontsize=15)
plt.xlabel("Epoch (t)",fontsize=15)
plt.ylabel("Log Likelihood",fontsize=15)
plt.xlim((0,50))
plt.ylim((-0.8,0.0))
```

Out[37]: (-0.8, 0.0)



In [38]:
```
Training_Error_list = [None]*1
Validation_Error_list = [None]*1
Test_Error_list = [None]*1
Training_like_list = [None]*1
Validation_like_list = [None]*1
Test_like_list = [None]*1
for i in range(1):
```

```
            Training_Error_list[i]   = err_rate(Xtr,beta_matrix2[:,i],Ytr)
            Validation_Error_list[i]= err_rate(Xva,beta_matrix2[:,i],Yva)
            Test_Error_list[i]       = err_rate(Xte,beta_matrix2[:,i],Yte)
            Training_like_list[i]    = loglike(Xtr,beta_matrix2[:,i],Ytr)
            Validation_like_list[i]  = loglike(Xva,beta_matrix2[:,i],Yva)
            Test_like_list[i]        = loglike(Xte,beta_matrix2[:,i],Yte)

        loglike_matrix_ne = copy.copy(loglike_matrix2)

        Training_Error_ne   = err_rate(Xtr,beta_matrix2[:,i],Ytr)
        Validation_Error_ne= err_rate(Xva,beta_matrix2[:,i],Yva)
        Test_Error_ne       = err_rate(Xte,beta_matrix2[:,i],Yte)
        Training_like_ne    = loglike(Xtr,beta_matrix2[:,i],Ytr)
        Validation_like_ne= loglike(Xva,beta_matrix2[:,i],Yva)
        Test_like_ne        = loglike(Xte,beta_matrix2[:,i],Yte)
```

In [39]:

```
import pandas as pd
import os
new_dataframe = pd.DataFrame(
    {
        "Training Error" : Training_Error_list,
        "Validation Error":Validation_Error_list,
        "Test Error":Test_Error_list
    },
    index = ['Newton']
)
new_dataframe
```

Out[39]:

	Training Error	Validation Error	Test Error
Newton	0.019858	0.021969	0.018799

In [40]:

```
new_dataframe = pd.DataFrame(
    {
        "Training Log Likelihood" : Training_like_list,
        "Validation Log Likelihood":Validation_like_list,
        "Test Log Likelihood":Test_like_list
    },
    index = ['Newton']
)
new_dataframe
```

Out[40]:

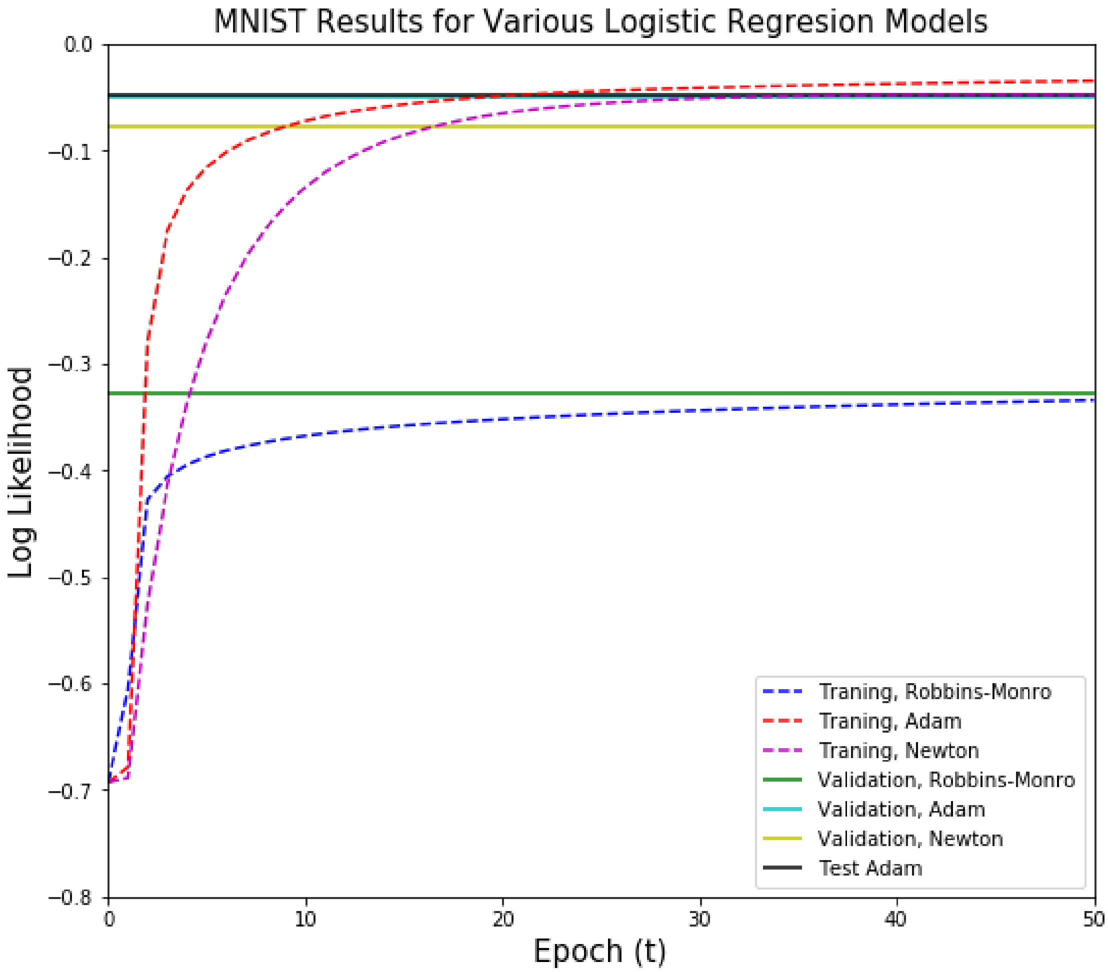
```
        Training Log Likelihood  Validation Log Likelihood  \
Newton                -0.048226                  -0.076712

        Test Log Likelihood
Newton            -0.083347
```

In [41]:

```
test_loglike = np.max([Test_like_rm,Test_like_am,Test_like_ne])
fig,ax = plt.subplots(1,1,figsize=(9,8))
```

```
ax.plot(range(0,stopEpochs+1),loglike_matrix_rm,'b--')
ax.hlines(Validation_like_rm, 0, 50, colors='g')
ax.plot(range(0,stopEpochs+1),loglike_matrix_am,'r--')
ax.hlines(Validation_like_am, 0, 50, colors='c')
ax.plot(range(0,stopEpochs+1),loglike_matrix_ne,'m--')
ax.hlines(Validation_like_ne, 0, 50, colors='y')
ax.hlines(test_loglike, 0, 50, colors = 'k')
ax.legend(["Traning, Robbins-Monro",
           "Traning, Adam",
           "Traning, Newton",
           "Validation, Robbins-Monro",
           "Validation, Adam",
           "Validation, Newton",
           "Test Adam"])
ax.set_title("MNIST Results for Various Logistic Regresion Models",fontsize=15)
plt.xlabel("Epoch (t)",fontsize=15)
plt.ylabel("Log Likelihood",fontsize=15)
plt.xlim((0,50))
plt.ylim((-0.8,0.0))
```

```
Out[41]: (-0.8, 0.0)
```



In [42]:
```
import pandas as pd
import os
new_dataframe = pd.DataFrame(
    {
        "Training Error" : [Training_Error_rm,Training_Error_am,Training_Error_ne],
        "Validation Error":[Validation_Error_rm,Validation_Error_am,Validation_Error_r
        "Test Error":[Test_Error_rm,Test_Error_am,Test_Error_ne]
    },
    index = ['Robbins-Monro','Adam','Newton']
)
new_dataframe
```

Out[42]:

	Training Error	Validation Error	Test Error
Robbins-Monro	0.052287	0.041825	0.038642
Adam	0.010246	0.013942	0.014099
Newton	0.019858	0.021969	0.018799

In [43]:
```
import pandas as pd
import os
new_dataframe = pd.DataFrame(
    {
        "Training Loglikelihood" : [Training_like_rm,Training_like_am,Training_like_ne
        "Validation Loglikelihood":[Validation_like_rm,Validation_like_am,Validation_l
        "Test Loglikelihood":[Test_like_rm,Test_like_am,Test_like_ne]
    },
    index = ['Robbins-Monro','Adam','Newton']
)
new_dataframe
```

Out[43]:
```
               Training Loglikelihood  Validation Loglikelihood  \
Robbins-Monro               -0.334296                 -0.327885
Adam                        -0.034397                 -0.049313
Newton                      -0.048226                 -0.076712

               Test Loglikelihood
Robbins-Monro           -0.317392
Adam                    -0.047404
Newton                  -0.083347
```

In [ ]:

In [ ]:

In [ ]: