

**NANYANG  
TECHNOLOGICAL  
UNIVERSITY**  
**SINGAPORE**

## **Experiment 4: Implicit Surfaces and Solids**

CZ2003 Computer Graphics and Visualization

**SS3**

Name	Matric Number
Pang Yu Shao	U17216 <u>80</u> D

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING  
NANYANG TECHNOLOGICAL UNIVERSITY  
SINGAPORE  
23rd March 2021

## CONTENTS

Contents	1
1 Defining Surfaces Implicitly	2
1.1 Plane passing through three defined co-ordinates	2
1.2 Half-sphere with defined radius	3
1.3 Cylindrical surface with defined height and radius	4
1.4 Two-side conical surface with defined radius and height	5
2 Building Complex Shape with set-theoretic operations	6
3 Colouring Shape with an implicit colour function.	7

## 1 DEFINING SURFACES IMPLICITLY

### 1.1 Plane passing through three defined co-ordinates

For this exercise, a plane has to be defined such that it passes through three points:  $(N, M, 0)$ ,  $(0, M, N)$  and  $(N, 0, M)$

Since  $N = 8$  and  $M = 10$ , the three points are:  $A(8, 10, 0)$ ,  $B(0, 10, 8)$  and  $C(8, 0, 10)$ .

The implicit function can be defined by:  $f(x, y, z) = a(x - x_0) + b(y - y_0) + c(z - z_0)$

To obtain the unknowns  $a$ ,  $b$  and  $c$ , a normal vector perpendicular to the plane has to be obtained.

$$\vec{V}_1 = \vec{AB} = \langle 0 - 8, 10 - 10, 8 - 0 \rangle$$

$$\vec{V}_1 = \langle -8, 0, 8 \rangle$$

$$\vec{V}_2 = \vec{AC} = \langle 8 - 8, 0 - 10, 10 - 0 \rangle$$

$$\vec{V}_2 = \langle 0, -10, 10 \rangle$$

$$\vec{V}_1 \times \vec{V}_2 = \langle (0 * 10) - (8 * -10), (8 * 0) - (-8 * 10), (-8 * -10) - (0 * 0) \rangle$$

$$\vec{V}_1 \times \vec{V}_2 = \langle 80, 80, 80 \rangle$$

Therefore,  $f(x, y, z) = 80(x - x_0) + 80(y - y_0) + 80(z - z_0)$

Using point A for  $x_0$ ,  $y_0$  and  $z_0$ ,

$$f(x, y, z) = 80(x - 8) + 80(y - 10) + 80(z)$$

Since  $f(x, y, z) = 0$ ,

$$f(x, y, z) = x - 8 + y - 10 + z = 0$$

$$f(x, y, z) = x + y + z - 18 = 0$$

As the plane defines an endless plane the bounding box is defined such that the three defined points are minimally contained within it, hence the bounding box is defined as:

$$BB_{pos} = (8/2, 10/2, 10/2) = (4, 5, 5), BB_{size} = (8, 10, 10)$$

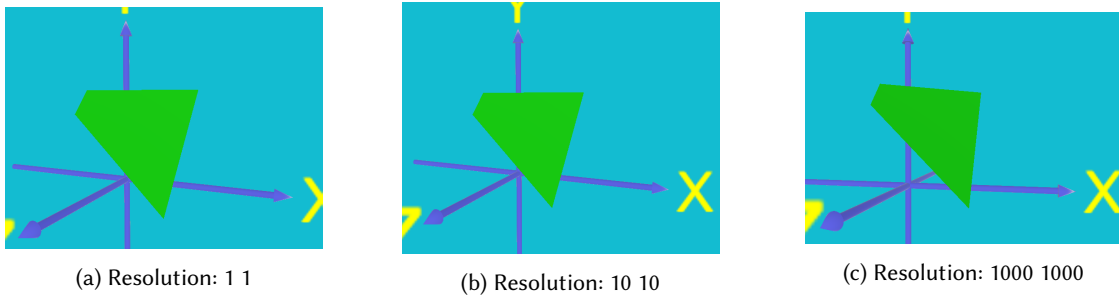


Fig. 1. Plots of the Plane defined in "1a.Func" with differing resolutions

As seen in Fig. 1 above, a sampling resolution of **1** for  $x, y$  and  $z$  is sufficient for rendering the plane and utilizing a higher resolution would produce the exact same rendering.

## 1.2 Half-sphere with defined radius

To define the lower half of a sphere, we first define a full sphere. A sphere of radius 10 can be defined with the following implicit function:

$$10^2 - x^2 - y^2 - z^2 \geq 0$$

To obtain a half sphere, a bounding box is defined to only render the bottom half of the sphere. Therefore, the bounding box is defined to be centered at:  $BB_{pos} = (0, 0, -5)$

Bounding box size:  $BB_{size} = (20, 20, 10)$

By plotting the implicit function on Shape Explorer with varying resolutions, the following figures were obtained:

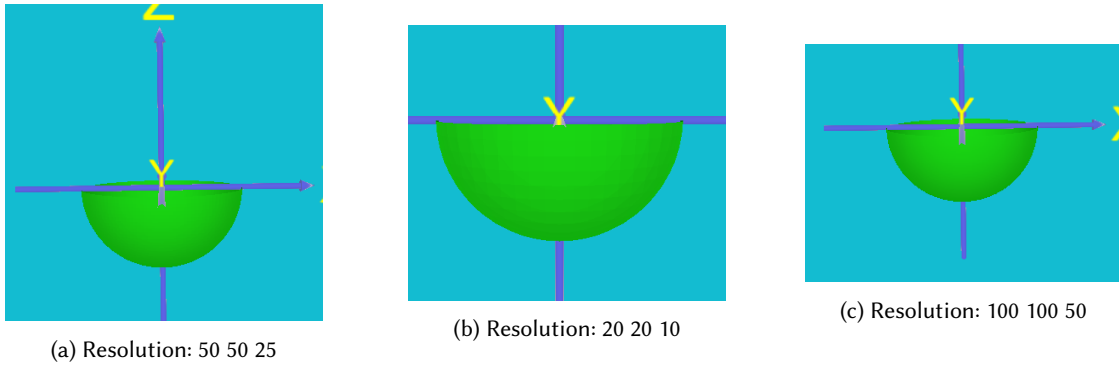


Fig. 2. Plots of the Half-Sphere defined in "**1b.Func**" with differing resolutions

As seen in Fig. 2 above, a sampling resolution of **50, 50, 25** for x, y and z respectively is optimal resolution for rendering the half-sphere. When the resolution is reduced to 20, 20, 10, the surface of the half-sphere is not smooth and the polygons that make up the surface can be seen. When the resolution is increased to 100, 100, 50, there is no visible difference in the rendered object. However, the rendering time is significantly increased.

### 1.3 Cylindrical surface with defined height and radius

To define the a cylindrical surface implicitly, we first define a circle implicitly. A circle of radius 10 can be defined on the x-y plane with the following implicit function:

$$10^2 - x^2 - y^2 \geq 0$$

The above implicit function defines a cylindrical surface for the entire Z-axis. Therefore, to render it such that it only spans from Z=-8 to Z=10, the bounding box can be defined to be centered at:  $BB_{pos} = (0, 0, -1)$

Bounding box size:  $BB_{size} = (20, 20, 18)$

By plotting the implicit function on Shape Explorer with varying resolutions, the following figures were obtained:

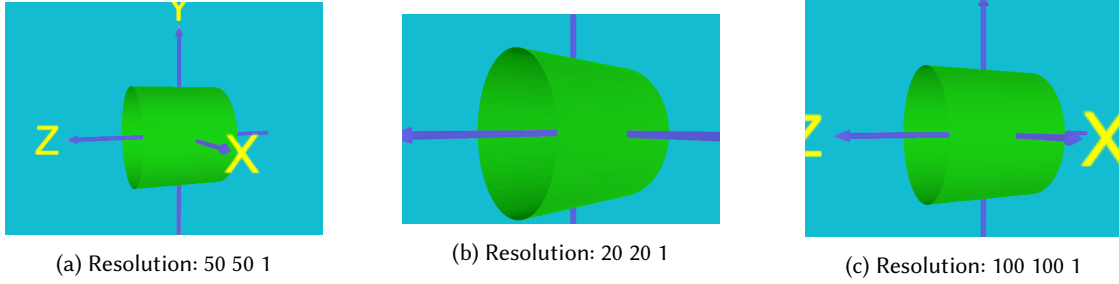


Fig. 3. Plots of the Cylindrical Surface defined in "**1c.Func**" with differing resolutions

As seen in Fig. 3 above, a sampling resolution of **50, 50, 1** for x, y and z respectively is optimal resolution for rendering the cylindrical surface. As the shape does not have any curvature along the z-axis, a resolution of 1 is sufficient for the z-axis.

When the resolution is reduced to 20 for both x and y, the surface of the cylinder is not smooth and the interpolations could be seen. When the resolution is increased to 100 for x and y there is no observable difference in the rendered object than that when the resolution is 50.

#### 1.4 Two-side conical surface with defined radius and height

A cone can be defined using the following implicit function:

$(x^2 + y^2)/(c^2) - z^2 \geq 0$ , where  $c = r/h$ , which is the ratio of radius to height at some distance from the vertex.

Since the cone needs to have a radius of 8(M) at distance 1 from its apex,  $c = 8$ .

Therefore, the implicit function to define the cone is:

$$(x^2 + y^2)/64 - z^2 \geq 0$$

To render the cone such that it spans from  $Z = -1$  to  $Z = 1$ , the bounding box can be defined to be centered at the origin,  $BB_{pos} = (0, 0, 0)$  with a bounding box size:  $BB_{size} = (16, 16, 2)$

By plotting the implicit function on Shape Explorer with varying resolutions, the following figures were obtained:

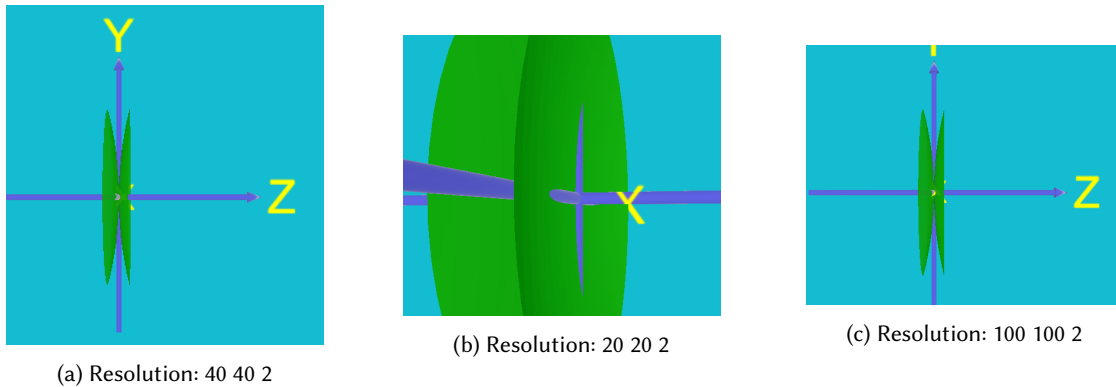
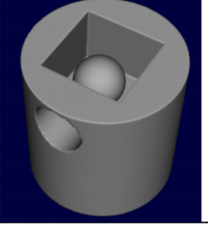


Fig. 4. Plots of the two-sided cone defined in "1d.Func" with differing resolutions

When experimenting with the resolutions, it was found that a resolution of 2 for  $Z$  is sufficient for rendering the cone shape. The best resolution obtained for rendering the shape was 40 for  $x$  and  $y$ . When the resolution for  $x$  and  $y$  is decreased to 20, the interpolations could be seen on the surface of the cone when zoomed in (as seen in the figure above). When the resolution was increased instead to 100, no visible difference could be seen between that and when the resolution was 40.

## 2 BUILDING COMPLEX SHAPE WITH SET-THEORETIC OPERATIONS

The complex shape that is to be made is the following:

10	$((Cylinder_1 \setminus Box_1) \setminus Cylinder_2) \cup Sphere$	
----	---	--

Therefore, the following functions are defined to obtain the components of the complex shape:

$$\begin{aligned}
 Cylinder_1 &\mapsto \min((5^2 - x^2 - y^2), -z, 10 + z) \\
 Box_1 &\mapsto \min(3 - x, x + 3, 3 - y, y + 3, -z, 6 + z) \\
 Cylinder_2 &\mapsto \min((2.5^2 - y^2 - (z + 3)^2), -x, 5 + x) \\
 Sphere &\mapsto 1.5^2 - x^2 - y^2 - (z + 3)^2
 \end{aligned}$$

Thus, the complex shape can be defined with:

$$Shape \mapsto \max(\min(\min(Cylinder_1, -Box_1), -Cylinder_2) Sphere)$$

To render this complex shape, the bounding box size and position is set to wrap tightly over the outer and larger cylinder:  $BB_{pos} = (0, 0, -5)$ ,  $BB_{size} = (10, 10, 10)$

By rendering the shape in Shape Explorer and by varying the resolution, the following figures were obtained:

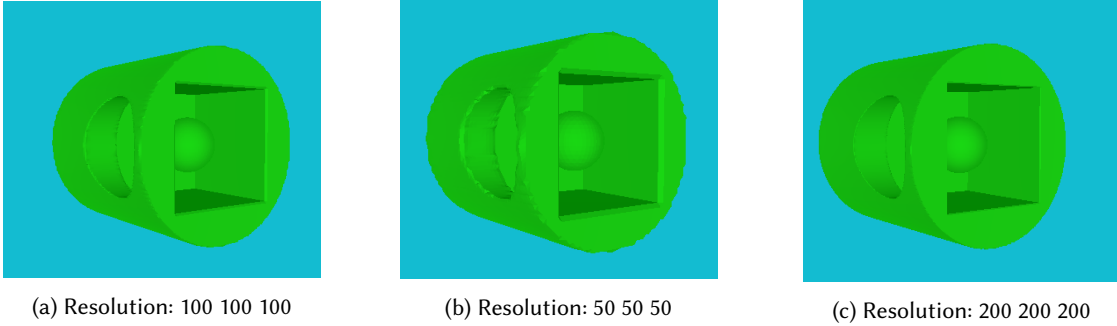


Fig. 5. Plots of the Solid defined in "2.Func" with differing resolutions

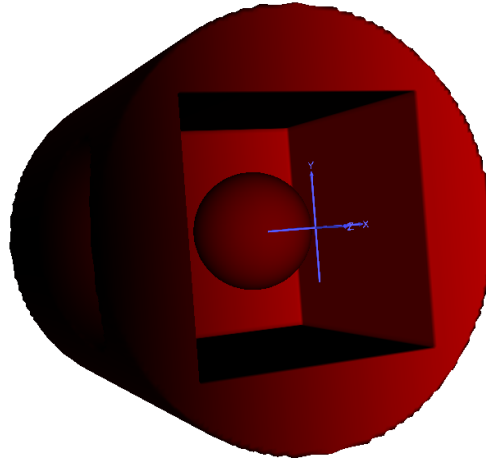
The sampling resolution for  $x$ ,  $y$  and  $z$  was chosen to be **100** as it was able to render the shape without the polyline interpolations being obvious. When the sampling resolution was reduced to 50, these interpolations are visible when zoomed in. If the sampling resolution is increased to 200, there is no visible difference between the original figure which was rendered with a resolution of 100.

### 3 COLOURING SHAPE WITH AN IMPLICIT COLOUR FUNCTION.

For this exercise, the tanh function was applied to the colouring function. After scaling the function properly such that the values displayed is located within  $[0,1]$ , the following function was obtained:  $r = \tanh((u + 5)/10)$ .

The bounding box size was increased slightly to (10, 10, 10.2) due to some rendering artifacts that was only encountered in FVRML.

The following coloured object was obtained.



(a) Resolution: 100 100 100

Fig. 6. Rendering of the coloured Solid defined in "3.wrl"

As the shape is the exact same as the one done in exercise 2, the best sampling resolution would be the same. Therefore a sampling resolution of **100, 100, 100** for x, y and z respectively was chosen.