CZ2003: Tutorial 5

Due on February 16, 2021 at 10:30am

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Problem 1

Using rotational sweeping **clockwise**, define by parametric functions $x(u,v)$, $y(u,v)$, $z(u,v)$, $u,v \in [0,1]$ the surface displayed in Figure Q1.

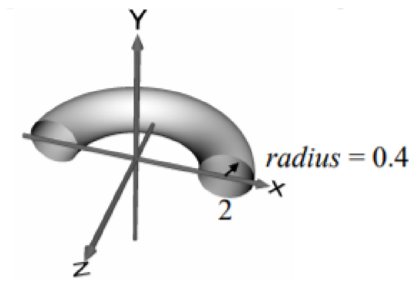
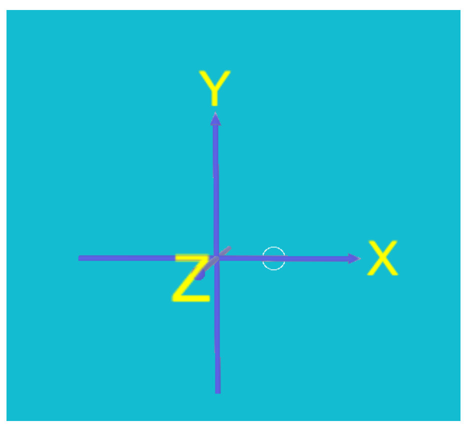
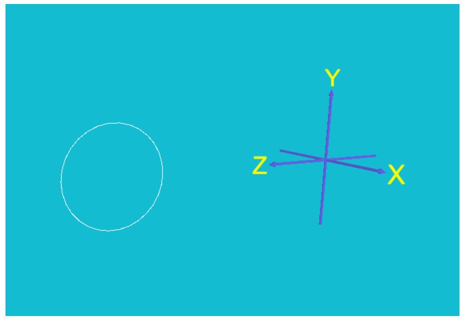


Figure Q1

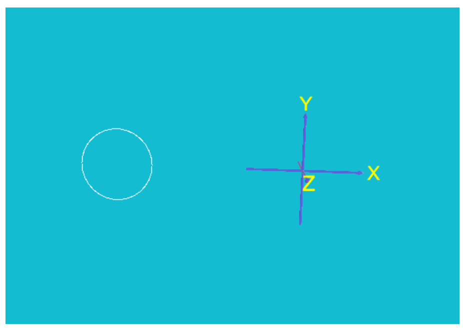
Solution
First, get the curve to be swept, curve is a circle with radius **0.4**, with origin at **(2,0,0)**.
This curve can be parametrically defined as:
$x(u,v) = 2 + 0.4cos(2\pi u)$
$x(u,v) = 0.4sin(2\pi u)$
$z(u,v) = 0$



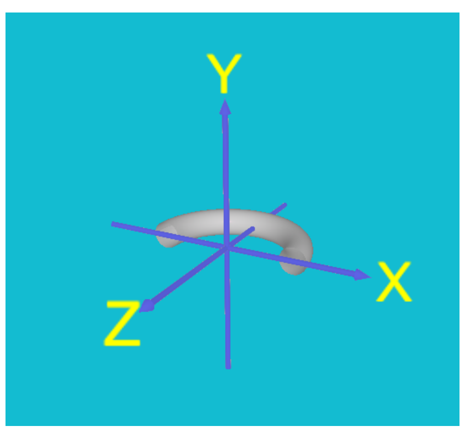
Next, to sweep the curve along the X-Z plane, we copy the defined function $x(u,v)$ to $z(u,v)$ and multiply the function of x by $sin(0)$ and the function of z by $cos(0)$, and we obtain the following functions and curve:
$x(u,v) = (2 + 0.4cos(2\pi u))sin(0)$
$x(u,v) = 0.4sin(2\pi u)$
$z(u,v) = (2 + 0.4cos(2\pi u))cos(0)$



We can see from the figure above that the curve is lying on the Z-axis, however it needs to begin from the negative X-axis and swept clockwise, In order to get the correct starting location for sweeping to begin, we introduce an offset of $-\frac{\pi}{\mathbf{2}}$ to the sin/cos functions:
$x(u, v) = (2 + 0.4cos(2\pi u))sin(-\frac{\pi}{2})$
$x(u, v) = 0.4sin(2\pi u)$
$z(u, v) = (2 + 0.4cos(2\pi u))cos(-\frac{\pi}{2})$



Lastly, we perform the sweeping to produce the surface. This is done by introducing the variable **v** into the sin/cos functions, multiplied by $-\pi$ for the clockwise sweeping from negative x-axis to positive x-axis.
$\mathbf{x(u, v) = (2 + 0.4cos(2\pi u))sin(-\frac{\pi}{2} - \pi v)}$
$\mathbf{x(u, v) = 0.4sin(2\pi u)}$
$\mathbf{z(u, v) = (2 + 0.4cos(2\pi u))cos(-\frac{\pi}{2} - \pi v)}$
$u, v \in [0, 1]$



Problem 2

Write parametric equations $x(u,v)$, $y(u,v)$, $z(u,v)$, $u,v \in [0,1]$ defining the surface created by sweeping (**clockwise** rotation by $3\pi/2$ and vertical displacement by **-2**) of the curve which is defined in polar coordinates by $r = 0.5sin(4\alpha), \alpha \in [0, 2\pi]$ (Figure Q2).

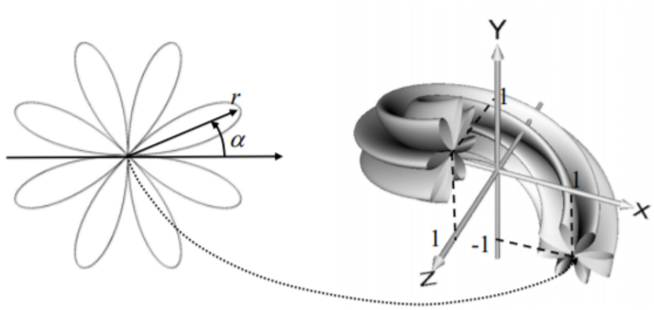
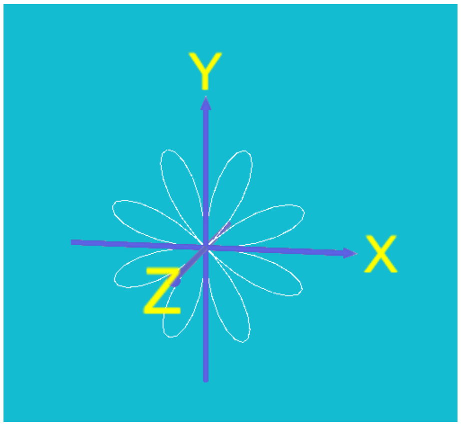


Figure Q2

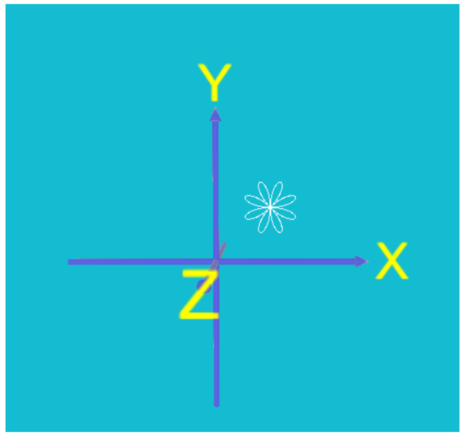
Solution
First, convert definition of curve from polar coordinates to cartesian coordinates.
$x = r\ cos(\alpha)$
$y = r\ sin(\alpha)$

$x(u,v) = 0.5sin(8\pi u)cos(2\pi u)$
$y(u,v) = 0.5sin(8\pi u)sin(2\pi u)$
$z(u,v) = 0$

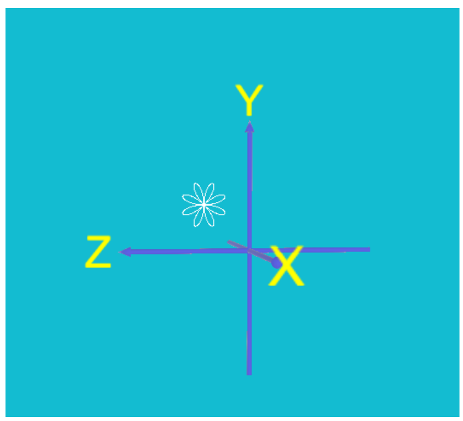


Next, we add x and y offsets of 1, which is needed for to shift the curve to the start position of the sweeping
$x(u,v) = 1 + 0.5sin(8\pi u)cos(2\pi u)$
$y(u,v) = 1 + 0.5sin(8\pi u)sin(2\pi u)$
$z(u,v) = 0$

Problem 2 continued on next page...



Next, to sweep the curve along the X-Z plane, we copy the defined function $x(u,v)$ to $z(u,v)$ and multiply the function of x by $sin(0)$ and the function of z by $cos(0)$, and we obtain the following functions and curve:
$x(u,v) = (1 + 0.5sin(8\pi u)cos(2\pi u))sin(0)$
$y(u,v) = 1 + 0.5sin(8\pi u)sin(2\pi u)$
$z(u,v) = (1 + 0.5sin(8\pi u)cos(2\pi u))cos(0)$



Lastly, to perform the clockwise sweep, we add $(-3\pi/2)v$ as the argument of the sin/cos functions of the X and Z functions. The vertical sweep component can be done by adding $-2v$ to the Y funciton. We thus end up with the following:
$\mathbf{x(u,v) = (1 + 0.5sin(8\pi u)cos(2\pi u))sin((-3\pi/2)v)}$
$\mathbf{y(u,v) = 1 + 0.5sin(8\pi u)sin(2\pi u) - 2v}$
$\mathbf{z(u,v) = (1 + 0.5sin(8\pi u)cos(2\pi u))cos((-3\pi/2)v)}$
$u, v \in [0,1]$