

## **CZ2003: Tutorial 2**

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## Problem 1

Give a definition of mathematical function.

### Solution

In mathematics, a function is a relation from a set of inputs to a set of possible outputs. A function maps a single input value (or argument) within the input domain to a single output value falling within the function range.

## Problem 2

What ways of defining mathematical functions do you know?

### Solution

Three ways:

- Implicit functions - a function that will equate to 0  
e.g.:  $f(x, y, z, t) = 0$ , where  $x, y, z$  are Cartesian coordinates and  $t$  is the time
- Explicit functions - a function which output is manipulated by its inputs  
e.g.:  $g = f(x, y, z, t)$
- Parametric functions - a set of explicit functions which creates values of co-ordinates from inputs consisting of variables from other domain (i.e., parametric coordinates)  
e.g.:  $x = f_x(y, v, w, t); y = f_y(u, v, w, t); z = f_z(u, v, w, t)$

## Problem 3

Given an explicit function  $y = \sin(x) + \cos(x)$ , propose how to convert it to the respective parametric functions  $x = f_1(t)$   $y = f_2(t)$ ?

### Solution

We can define the first parametric function as:

$$\mathbf{x} = \mathbf{f}_1(\mathbf{t}) = \mathbf{t}$$

The second parametric function can then be obtained as:

$$\mathbf{y} = \mathbf{f}_2(\mathbf{t}) = \mathbf{\sin(t)} + \mathbf{\cos(t)}$$

## Problem 4

- i Given parametric functions  $x = \sin^2(t)$  and  $y = \cos(t)$ , obtain the respective implicit function  $f(x, y) = 0$ .
- ii Given parametric functions  $x = 2 + 3t$  and  $y = 3 + t$ , obtain the respective implicit function  $f(x, y) = 0$ .

### Solution

#### Part i

$$\begin{aligned}y &= \cos(t) \\y^2 &= \cos^2(t) \\\therefore x + y^2 &= \sin^2(t) + \cos^2(t) \\&= 1 \\f(x, y) &= x + y^2 - 1 = 0\end{aligned}$$

#### Part ii

$$\begin{aligned}y &= 3 + t \\3y &= 9 + 3t \\3t &= 3y - 9 \\x &= 2 + (3y - 9) \\&= 3y - 7 \\x - 3y &= -7 \\f(x, y) &= x - 3y + 7 = 0\end{aligned}$$