CZ2003: Tutorial 4

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Problem 1

Using an equation in intercepts, write an implicit equation of the plane which intersects the Cartesian coordinate axes X, Y and Z at three points with coordinates $P_1 = (1, 0, 0)$, $P_2 = (0, 3, 0)$, $P_3 = (0, 0, 6)$, respectively.

Solution
Implicit equation in intercepts:
$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
a = 1, b = 3, c = 6

$\therefore \mathbf{f(x, y, z) = \frac{x}{1} + \frac{y}{3} + \frac{z}{6} - 1 = 0}$

Problem 2

Write an implicit equation of a plane which passes through the point with Cartesian coordinates $(1, 2, -3)$ while being orthogonal to the straight line defined by $x = u + 2$, $y = u - 1$, $z = 3u + 1$, $u \in (-\infty, \infty)$

Solution
Normal vector to plane:
$\vec{N} = [A\ B\ C]$
$\vec{N} = [1\ 1\ 3]$

Point on plane:
$r_o = (x_o, y_o, z_o)$
$r_o = (1, 2, -3)$

Equation of a Plane (Point - Normal Form):
$A(x - x_0) + B(y - y_o) + C(z - z_0) = 0$

$\therefore \mathbf{f(x, y, z) = (x - 1) + (y - 2) + 3(z + 3) = 0}$

Problem 3

Propose how to define parametrically with functions $x(u, v)$, $y(u, v)$, $z(u, v)$ a plane passing through points with coordinates $(-3, 0, 0)$, $(0, 2, 0)$, $(0, 0, 4)$.

Solution

To find a parametric equation of a plane with 3 points:

$P_1 + u(P_2 - P_1) + v(P_3 - P_1), \quad u, v \in (-\infty, \infty)$

$x(u, v) = x_1 + u(x_2 - x_1) + v(x_3 - x_1)$
$y(u, v) = y_1 + u(y_2 - y_1) + v(y_3 - y_1)$
$z(u, v) = z_1 + u(z_2 - z_1) + v(z_3 - z_1)$

$x(u, v) = -3 + u(0 + 3) + v(0 + 3)$
$y(u, v) = 0 + u(2 - 0) + v(0 - 0)$
$z(u, v) = 0 + u(0 - 0) + v(4 - 0)$

$\mathbf{x(u, v) = -3 + 3u + 3v}$
$\mathbf{y(u, v) = 2u}$
$\mathbf{z(u, v) = 4v}$
$u, v \in (-\infty, \infty)$

Problem 4

A bilinear surface is defined by four points $P1 = (-1, 1, -1)$, $P2 = (1, 0, -1)$, $P3 = (-1, 0, 1)$ and $P4 = (1, 0.5, 1)$ and two parametric coordinates $u \in [0, 1]$ and $v \in [0, 1]$, as illustrated in Figure Q3.

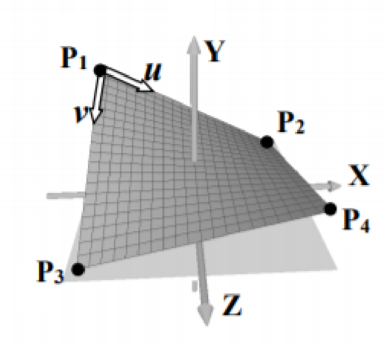

Figure Q3

(a) Write parametric equations defining the bilinear surface.

(b) What are the coordinates of the point with the parametric coordinates 0.2, 0.4?

Solution

a)
Bilinear Surface Parametric Representation:
$P = P1 + u(P2 - P1) + v(P3 - P1 + u(P4 - P3 - (P2 - P1))), \quad u, v \in [0, 1]$

$x(u, v) = x_1 + u(x_2 - x_1) + v(x_3 - x_1 + u(x_4 - x_3 - (x_2 - x_1)))$
$y(u, v) = y_1 + u(y_2 - y_1) + v(y_3 - y_1 + u(y_4 - y_3 - (y_2 - y_1)))$
$z(u, v) = z_1 + u(z_2 - z_1) + v(z_3 - z_1 + u(z_4 - z_3 - (z_2 - z_1)))$

$x(u, v) = -1 + u(1 - (-1)) + v(-1 - (-1) + u(1 - (-1) - (1 - (-1))))$
$\mathbf{x(u, v) = 2u - 1} \quad u, v \in [0, 1]$

$y(u, v) = 1 + u(0 - 1) + v(0 - 1 + u(0.5 - 0 - (0 - 1)))$
$\mathbf{y(u, v) = 1 - u - v + 1.5uv} \quad u, v \in [0, 1]$

$z(u, v) = -1 + u(-1 - (-1)) + v(1 - (-1) + u(1 - 1 - (-1 - (-1))))$
$\mathbf{z(u, v) = 2v - 1} \quad u, v \in [0, 1]$

b)
$u = 0.2, v = 0.4$

$$\begin{aligned} x(0.2, 0.4) &= 2(0.2) - 1 \\ &= -0.6 \\ y(0.2, 0.4) &= 1 - 0.2 - 0.4 + 1.5 * 0.2 * 0.4 \\ &= 0.52 \\ z(0.2, 0.4) &= 2(0.4) - 1 \\ &= -0.2 \end{aligned}$$

$$\therefore P(0.2, 0.4) = (\mathbf{-0.6}, \mathbf{0.52}, \mathbf{-0.2})$$

Problem 5

Write parametric equations $x(u,v),\ y(u,v),\ z(u,v),\ u,v \in [0,1]$ defining a triangular polygon which is bounded by the three segments defined by:

$x = 1 + 2u \quad y = 1 + u \quad z = 1 - u \quad u \in [0,1]$
$x = 3 - u \quad y = 2 + u \quad z = 4u \quad u \in [0,1]$
$x = 2 - u \quad y = 3 - 2u \quad z = 4 - 3u \quad u \in [0,1]$

Finding vertices, let u = 0:
$P1 = (1,1,1)$
$P2 = (3,2,0)$
$P3 = (2,3,4)$

Bilinear Surface Parametric Representation:
$P = P1 + u(P2 - P1) + v(P3 - P1 + u(P4 - P3 - (P2 - P1))), \quad u, v \in [0,1]$
For triangle, let P3 = P4:
$P = P1 + u(P2 - P1) + v(P3 - P1 + u(P3 - P3 - (P2 - P1)))$
$P = P1 + u(P2 - P1) + v(P3 - P1 + u(P1 - P2))$

$x(u,v) = 1 + u(3-1) + v(2 - 1 + u(1-3))$
$\mathbf{x(u,v) = 1 + 2u + v(1 - 2u)}$

$y(u,v) = 1 + u(2-1) + v(3 - 1 + u(1-2))$
$\mathbf{y(u,v) = 1 + u + v(2 - u)}$

$z(u,v) = 1 + u(0-1) + v(4 - 1 + u(1-0))$
$\mathbf{z(u,v) = 1 - u + v(3 + u)}$

$\mathbf{u, v \in [0,1]}$