CZ2003: Tutorial 6

Due on February 23, 2021 at 10:30am

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Problem 1

Define the three-dimensional solid object displayed in Figure Q1

1. by functions $x(u,v,w)$, $y(u,v,w)$, $z(u,v,w)$, $u \in [0,1]$, $v \in [0,1]$
2. by functions $f(x,y,z) \geq 0$

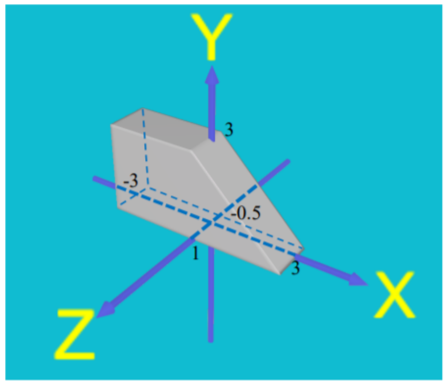


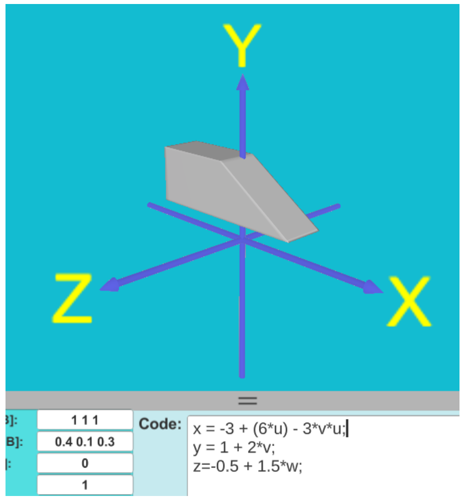
Figure Q1

Solution
Part 1:
Draw 2D surface,
Using bilinear representation: $P = P1 + u(P2 - P1) + v(P3 - P1 + u(P4 - P3–(P2 - P1)))$
$P1 = (-3, 1, 0)$, $P2 = (3, 1, 0)$, $P3 = (-3, 3, 0)$, $P4 = (0, 3, 0)$,

$x(u,v) = -3 + u(3 - (-3)) + v(-3 - (-3) + u(0 - (-3)–(3 - (-3))))$
$x(u,v) = -3 + 6u - 3vu$

$y(u,v) = 1 + u(1 - 1) + v(3 - 1 + u(3 - 3 - (1 - 1)))$
$y(u,v) = 1 + 2v$

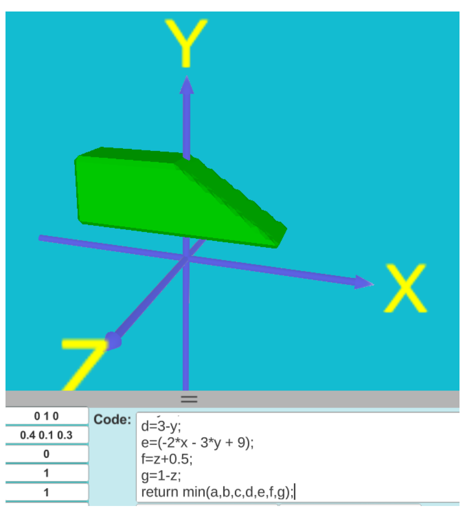
To get solid, sweep z with third parameter from -0.5 to 1:
$x(u,v,w) = -3 + 6u - 3vu$
$y(u,v,w) = 1 + 2v$
$z(u,v,w) = -0.5 + 1.5w$
$u, v, w \in [0,1]$



Part 2:
Components to form solid:

1. $x \geq -3$, or $x + 3 \geq 0$
2. $x \leq 3$, or $3 - x \geq 0$
3. $y \geq 1$, or $y - 1 \geq 0$
4. $y \leq 3$, or $3 - y \geq 0$
5. $\frac{x}{-3/(-2/3)} + \frac{y}{3} - 1 \geq 0$, or $2x + 3y - 9 \geq 0$
6. $z \geq -0.5$, or $z + 0.5 \geq 0$
7. $z \leq 1$, or $1 - z \geq 0$

$\therefore f(x, y, z) : min(x + 3, 3 - x, 3 - y, 2x + 3y - 9, z + 0.5, 1 - z) \geq 0$



Problem 2

A curve displayed in Figure Q2 (left) is defined in polar coordinates r and a by the function $r = 1.2sin(2\alpha - 0.5\pi)$, $\alpha \in [0, 2\pi]$. Propose parametric functions $x(u, v)$, $y(u, v)$, $u, v \in [0, 1]$ defining the 2D solid shape located in the XY Cartesian coordinates system as it is displayed in Figure Q2 (right).

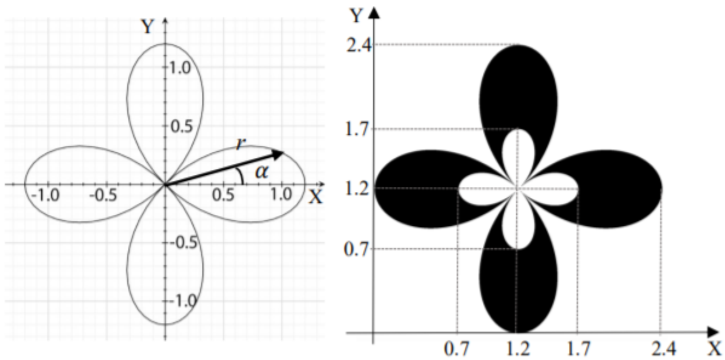


Figure Q2

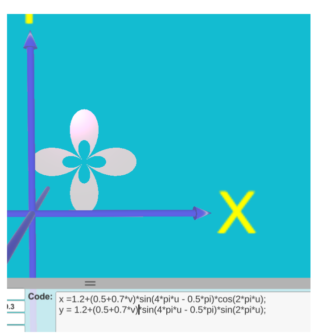
Solution

$r = 1.2sin(2\alpha - 0.5\pi)$

$x(u) = rcos(2\pi u)$
$y(u) = rsin(2\pi u)$

$x(u) = 1.2sin(4\pi u - 0.5\pi)cos(2\pi u)$
$y(u) = 1.2sin(4\pi u - 0.5\pi)sin(2\pi u)$

Offset by (1.2,1.2):
$x(u) = 1.2 + 1.2sin(4\pi u - 0.5\pi)cos(2\pi u)$
$y(u) = 1.2 + 1.2sin(4\pi u - 0.5\pi)sin(2\pi u)$
Radius changes from (0.5 to 1.2), incorporate this information using parameter **v**:
$x(u, v) = 1.2 + (0.5 + 0.7 * v)sin(4\pi u - 0.5\pi)cos(2\pi u)$
$y(u, v) = 1.2 + (0.5 + 0.7 * v)sin(4\pi u - 0.5\pi)sin(2\pi u)$
$u, v \in [0, 1]$



Problem 3

Define parametrically with functions $x(u,v,w)$, $y(u,v,w)$, $z(u,v,w)$, $u,v,w \in [0,1]$ the solid object displayed in Figure Q3. The object is created by rotational sweeping counterclockwise by $5\pi/4$ about axis Y of the sinusoidal curve followed by translational sweeping by +1.5 units parallel to axis Y.

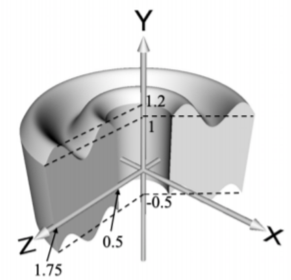


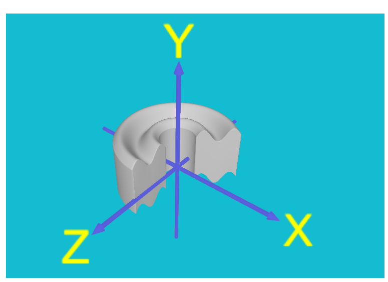
Figure Q3

Solution

1. Define the sine wave:
Amplitude: 0.2
Periods: 1.5
x: from 0.5 to 1.75
y offset: -0.5
$x(u) = 0.5 + 1.25u$
$y(u) = 0.2sin(3\pi u) - 0.5$

2. Perform counterclockwise sweeping:
$x(u,v) = (0.5 + 1.25u)sin(\frac{5\pi}{4}v - \frac{5\pi}{4})$
$y(u,v) = 0.2sin(3\pi u) - 0.5$
$z(u,v) = (0.5 + 1.25u)cos(\frac{5\pi}{4}v - \frac{5\pi}{4})$

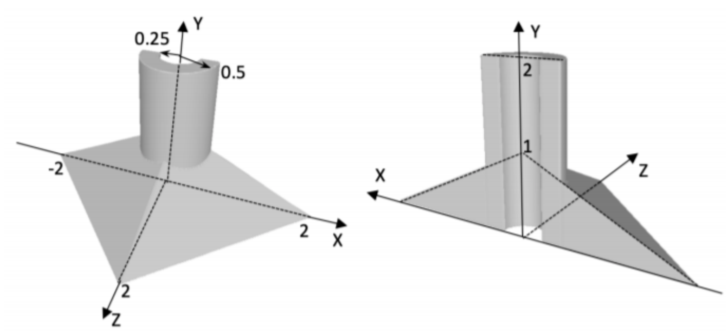
3. Perform translational sweeping:
$x(u,v,w) = (0.5 + 1.25u)sin(\frac{5\pi}{4}v - \frac{5\pi}{4})$
$y(u,v,w) = 0.2sin(3\pi u) - 0.5 + 1.5w$
$z(u,v,w) = (0.5 + 1.25u)cos(\frac{5\pi}{4}v - \frac{5\pi}{4})$
$u,v,w \in [0,1]$



Problem 4

The solid object displayed in Figure Q4 (front and back views) is constructed from a 3-sided pyramid with height 1 and a cylinder which has the height 2, the outer radius 0.5, and the inner radius 0.25.

1. Define the pyramid and the cylinder by functions $f(x, y, z) \geq 0$
2. Based on the definition obtained in part 1, define the solid object.



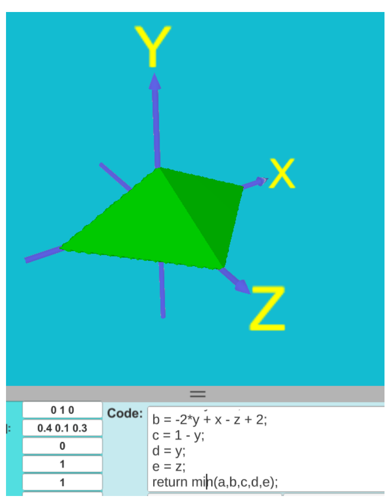
Front view Back view

Figure Q4

Solution
Part 1: Pyramid Components:

- $-(\frac{x}{2} + \frac{y}{1} + \frac{z}{2} - 1) \geq 0$, or $\mathbf{a(x, y, z) = -x - 2y - z + 2 \geq 0}$
- $-(\frac{x}{-2} + \frac{y}{1} + \frac{z}{2} - 1) \geq 0$, or $\mathbf{b(x, y, z) = -2y + x - z + 2 \geq 0}$
- $y \leq 1$, or $\mathbf{c(x, y, z) = 1 - y \geq 0}$
- $\mathbf{d(x, y, z) = y \geq 0}$
- $\mathbf{e(x, y, z) = z \geq 0}$

$\therefore \mathbf{f_{pyramid}(x, y, z) = min(a(x, y, z), b(x, y, z), c(x, y, z), d(x, y, z), e(x, y, z)) \geq 0}$



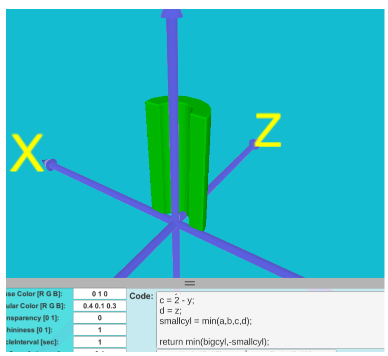
Cylinder Components:
Big Cylinder:

- $\mathbf{a(x, y, z) = 0.5^2 - x^2 - z^2 \geq 0}$
- $\mathbf{b(x, y, z) = y \geq 0}$
- $\mathbf{c(x, y, z) = 2 - y \geq 0}$
- $\mathbf{d(x, y, z) = z \geq 0}$

$\therefore \mathbf{f_{bigCyl}(x, y, z) = min(a(x, y, z), b(x, y, z), c(x, y, z), d(x, y, z)) \geq 0}$
Small Cylinder (Hollow):

- $\mathbf{a(x, y, z) = 0.25^2 - x^2 - z^2 \geq 0}$
- $\mathbf{b(x, y, z) = y \geq 0}$
- $\mathbf{c(x, y, z) = 2 - y \geq 0}$
- $\mathbf{d(x, y, z) = z \geq 0}$

$\therefore \mathbf{f_{smallCyl}(x, y, z) = min(a(x, y, z), b(x, y, z), c(x, y, z), d(x, y, z)) \geq 0}$
$\therefore \mathbf{f_{cyl}(x, y, z) = min(f_{bigCyl}(x, y, z), -f_{smallCyl}(x, y, z)) \geq 0}$



Part 2:

To get the figure, union the big cylinder and pyramid, then subtract the small cylinder.
$\mathbf{f_{final}(x, y, z) = min(max(f_{pyramid}(x, y, z), f_{bigCyl}(x, y, z)), -f_{smallCyl}(x, y, z)) \geq 0}$