Experiment 1: Parametric Curves

CZ2003 Computer Graphics and Visualization

SS3

Name	Matric Number
Pang Yu Shao	U1721680D

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY
SINGAPORE

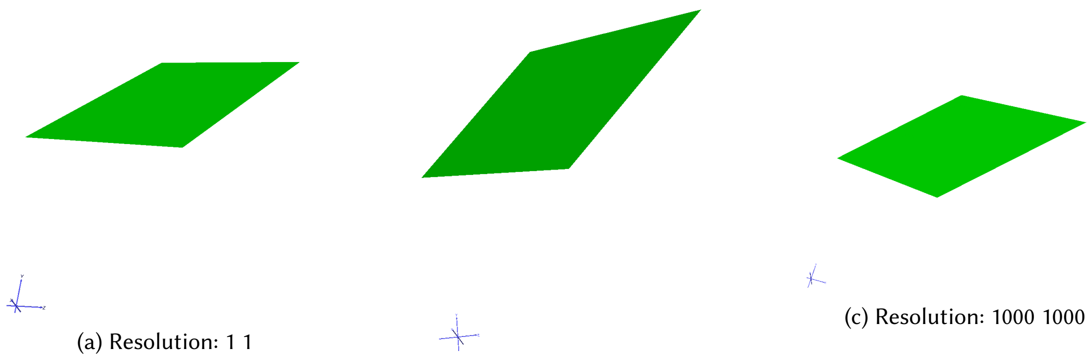
2nd Febraury 2021

Contents

1 DEFINING SURFACES PARAMETRICALLY

1.1 Plane Passing Through Three Defined Points

To define the plane parametrically, we can use the following formula: $P = P1 + u(P2 - P1) + v(P3 - P1)$
Therefore, with the 3 points $(N, M, 0),\ (0, M, N),\ (N, 0, M)$, we get:
$x(u, v) = N - Nu = \mathbf{8 - 8u}$
$y(u, v) = M + Mv = \mathbf{10 + 10v}$
$z(u, v) = Nu + Mv = \mathbf{8u + 10v}$
$\mathbf{u, v \in [0, 1]}$



(a) Resolution: 1 1

(b) Resolution: 10 10

(c) Resolution: 1000 1000

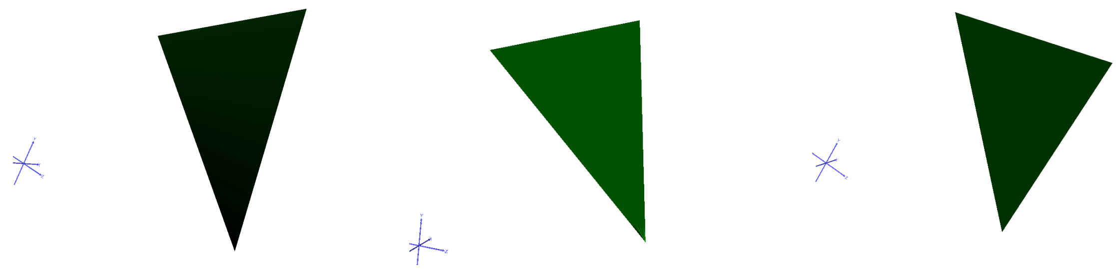
Fig. 1. Plots of the plane defined in "**1a.wrl**" with differing resolutions

As seen in Fig. 1 above, a sampling resolution of **1** for both u and v is sufficient for drawing the plane as it has no curvature and having a higher resolution would produce the exact same drawing.

1.2 Triangular Polygon with Three Defined Vertices

To define the Triangular Polygon, we use the formula for defining Bilinear Surface Parametrically, and we set two of the points to be the same point, essentially resulting in a Triangular polygon.

$P = P1 + u(P2 - P1) + v(P3 - P1 + u(P4 - P3 - (P2 - P1)))$
Let P4 = P3, we get:
$P = P1 + u(P2 - P1) + v(P3 - P1) + uv(P1 - P2)$
Therefore, with the 3 points $(N, M, 0),\ (0, M, N),\ (N, 0, M)$, we get:
$x(u, v) = N - Nu + Nuv = \mathbf{8 - 8u + 8uv}$
$y(u, v) = M - Mv = \mathbf{10 - 10v}$
$z(u, v) = Nu + Mv - Nuv = \mathbf{8u + 10v - 8uv}$
$\mathbf{u, v \in [0, 1]}$



(a) Resolution: 1 1

(b) Resolution: 10 10

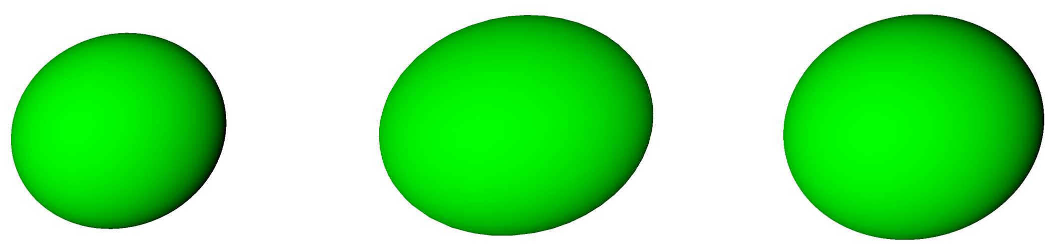
(c) Resolution: 1000 1000

Fig. 2. Plots of the Triangular Polygon defined in "**1b.wrl**" with differing resolutions

As seen in Fig. 2 above, a sampling resolution of **1** for both u and v is sufficient for drawing the triangular polygon as it has no curvature and having a higher resolution would produce the exact same drawing.

1.3 Origin-Centered Ellipsold with Defined Semi-axes

An elipsoid can be parametrically defined using the following functions:
$x = acos(u)sin(v)$
$y = bsin(u)$
$z = ccos(u)cos(v)$
Where $u \in [-\pi/2, \pi/2]$ and $v \in [-\pi, \pi]$. Therefore, we can get:
$x = Ncos(-\pi/2 + \pi u)sin(-\pi + 2\pi v) = \mathbf{8cos(-\pi/2 + \pi u)sin(-\pi + 2\pi v)}$
$y = Msin(-\pi/2 + \pi u) = \mathbf{10sin(-\pi/2 + \pi u)}$
$z = (N + M/2)cos(-\pi/2 + \pi u)cos(-\pi + 2\pi v) = \mathbf{9cos(-\pi/2 + \pi u)cos(-\pi + 2\pi v)}$
$u, v \in [0, 1]$



(a) Resolution: 50 100

(b) Resolution: 25 50

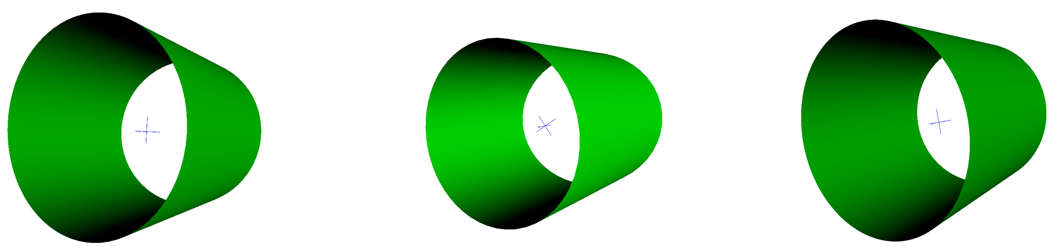
(c) Resolution: 100 200

Fig. 3. Plots of the Ellipsold defined in "**1c.wrl**" with differing resolutions

The sampling resolution for v is chosen to be 2 times larger than that of u as the coefficient of v is 2 times larger. As seen in Fig. 3 above, the best resolution obtained was **50, 100** for u and v respectively. By decreasing the resolution to 25 and 50, the interpolations could be seen when zoomed in. When the resolution was increased instead to 100 and 200, no visible difference could be seen.

1.4 Cylindrical Surface with Defined Radius

To define a cylindrical surface parametrically, we can define it by first defining a circular curve, followed by performing translational sweeping along the Z-axis to obtain the cylindrical surface.
Therefore we can obtain the following equations and surface:
$x = Ncos(2\pi u) = \mathbf{8cos(2\pi u)}$
$y = Nsin(2\pi u) = \mathbf{8sin(2\pi u)}$
$z = -N + (M + N)v = \mathbf{-8 + 18v}$
$u, v \in [0, 1]$



(a) Resolution: 75 1

(b) Resolution: 50 1

(c) Resolution: 200 1

Fig. 4. Plots of the Cylindrical Surface defined in "**1d.wrl**" with differing resolutions

The sampling resolution for v is chosen to be 1 as the parameter is responsible for the straight line sweep. As seen in Fig. 3 above, the best resolution obtained was **75, 1** for u and v respectively. By decreasing the resolution to 50 for u, the interpolations could be seen when zoomed in. When the resolution of u was increased instead to 200, no visible difference could be seen between that and when the resolution was 75.

2 DEFINING SURFACE BY TRANSLATIONAL SWEEPING OF CURVE

We obtain the equations of the curve from lab 1:

$$\mathbf{x(u) = -10.4 + 26.4 * u}$$

$$\mathbf{y(u) = tanh(-10.4 + 26.4 * u)}$$

$$u \in [0, 1]$$

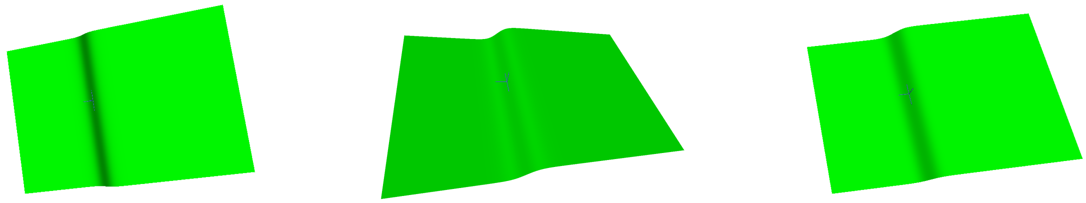
To obtain a surface by translational sweeping of the curve, we can introduce the v parameter in the function of Z. Therefore, we get:

$$\mathbf{x(u, v) = -10.4 + 26.4 * u}$$

$$\mathbf{y(u, v) = tanh(-10.4 + 26.4 * u)}$$

$$\mathbf{z(u, v) = -N + (N + M)v = -8 + 18v}$$

$$u, v \in [0, 1]$$



(a) Resolution: 200 1

(b) Resolution: 100 1

(c) Resolution: 1000 1

Fig. 5. Plots of the Cylindrical Surface defined in "**2.wrl**" with differing resolutions

Similar to the surface defined in section 1.4, the sampling resolution for v is chosen to be 1 as the parameter is responsible for the straight line sweep. As seen in Fig. 5 above, the best resolution obtained was **200, 1** for u and v respectively. By decreasing the resolution to 100 for u, the interpolations could be seen when zoomed in. When the resolution of u was increased instead to 1000, no visible difference could be seen between that and when the resolution was 200.

3 DEFINING SURFACE BY ROTATIONAL SWEEPING OF CURVE

We obtain the equations of the curve in lab 1:

$$\mathbf{x(u) = (8 - 15cos(2\pi u))cos(2\pi u)}$$
$$\mathbf{y(u) = (8 - 15cos(2\pi u))sin(2\pi u)}$$
$$u \in [0, 1]$$