CZ2003: Tutorial 10

Due on March 30, 2021 at 10:30am

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Problem 1

Propose an animation model in implicit representation, which defines the movement of a unit sphere along a 3D helical curve in a uniform speed along the z-direction (see Figure Q1). The helical curve has radius 30, period 10, and 15 full rotations, as shown below. The animation sequence should have 100 frames.

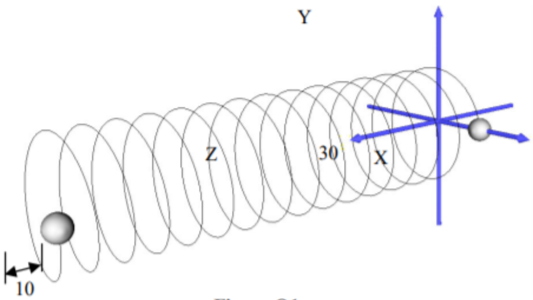


Figure Q1

Solution

Unit Sphere centered at (x_c, y_c, z_y): $f(x,y,z) = 1 - (x - x_c)^2 - (y - y_c)^2 - (z - z_c)^2$
Step 1: Represent the path by parametric equations:
$x_c(\tau) = 30cos(30\pi\tau)$
$y_c(\tau) = 30sin(30\pi\tau)$
$z_c(\tau) = 15 * 10 * \tau = 150\tau$

Step 2: Link path to object, represent object with parametric/implicit function:
$f(x,y,z,\tau) = 1 - (x - x_c(\tau))^2 - (y - y_c(\tau))^2 - (z - z_c(\tau))^2$

Step 3: Control speed of animation (uniform speed) by defining $\tau = f(k)$:
$\tau = f(k) = \frac{k-1}{100-1}$

$x_c(k) = 30cos(30\pi\frac{k-1}{100-1})$
$y_c(k) = 30sin(30\pi\frac{k-1}{100-1})$
$z_c(k) = 150\frac{k-1}{100-1}$

$f(x,y,z,k) = 1 - (x - x_c(k))^2 - (y - y_c(k))^2 - (z - z_c(k))^2$

$f(x,y,z,k) = 1 - (x - 30cos(30\pi\frac{k-1}{100-1}))^2 - (y - 30sin(30\pi\frac{k-1}{100-1}))^2 - (z - 150\frac{k-1}{100-1})^2$

where k is the frame index, $1 \leq k \leq 100$

Problem 2

With reference to Figure Q2, propose parametric functions defining animated transformation of the 2D polygon labelled as “A” into the 2D disk labelled as “B”. The animation takes 256 frames and involves deceleration.

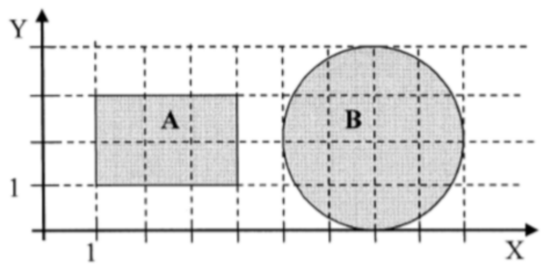


Figure Q2

Solution

First, define A and B parametrically:

$x_A(u, v) = 1 + 3u$

$y_A(u, v) = 1 + 2v$

$u, v \in [0, 1]$

$x_B(u, v) = 7 + 2vcos(2\pi u)$

$y_B(u, v) = 2 + 2vsin(2\pi u)$

$u, v \in [0, 1]$

Next, add animation:

$x(u, v, \tau) = (1 - \tau)x_A(u, v) + \tau x_B(u, v)$

$y(u, v, \tau) = (1 - \tau)y_A(u, v) + \tau y_B(u, v)$

$u, v, \tau \in [0, 1]$

Lastly, control speed of animation by defining τ (deccelerating):

$\tau = f(k) = sin(\frac{\pi}{2}\frac{k-1}{256-1})$

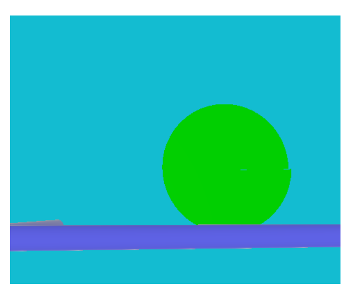
Therefore,

$x(u, v, k) = (1 - sin(\frac{\pi}{2}\frac{k-1}{256-1}))(1 + 3u) + sin(\frac{\pi}{2}\frac{k-1}{256-1})(7 + 2vcos(2\pi u))$

$y(u, v, k) = (1 - sin(\frac{\pi}{2}\frac{k-1}{256-1}))(1 + 2v) + sin(\frac{\pi}{2}\frac{k-1}{256-1})(2 + 2vsin(2\pi u))$

$u, v \in [0, 1]$, where k is the frame index, $1 \leq k \leq 256$





Problem 3

Propose a mathematical model that implements morphing which transforms a solid unit sphere centered at the origin into a solid cylinder parallel to the Z-axis, center at the point (1,1,3) and height of 6. The morphing sequence has 200 frames and involves acceleration. The frame index starts at 1

Solution
Let A be the solid sphere and B be the solid cylinder
First, define A and B parametrically:
$x_A(u, v, w) = wcos(2\pi u)cos(\pi v)$
$y_A(u, v, w) = wcos(2\pi u)sin(\pi v)$
$z_A(u, v, w) = wsin(2\pi u)$
$u, v, w \in [0, 1]$

(Assuming radius of cylinder is 1)
$x_B(u, v, w) = 1 + vcos(2\pi u)$
$y_B(u, v, w) = 1 + vsin(2\pi u)$
$z_B(u, v, w) = 6w$
$u, v, w \in [0, 1]$

Next, add animation:
$x(u, v, w, \tau) = (1 - \tau)x_A(u, v, w) + \tau x_B(u, v, w)$
$y(u, v, w, \tau) = (1 - \tau)y_A(u, v, w) + \tau y_B(u, v, w)$
$z(u, v, w, \tau) = (1 - \tau)z_A(u, v, w) + \tau z_B(u, v, w)$
$u, v, w, \tau \in [0, 1]$

Lastly, control speed of animation by defining τ (accelerating):
$\tau = f(k) = 1 - cos(\frac{\pi}{2}\frac{k-1}{200-1})$

Therefore,
$x(u, v, w, k) = (1 - (1 - cos(\frac{\pi}{2}\frac{k-1}{200-1})))(wcos(2\pi u)cos(\pi v)) + (1 - cos(\frac{\pi}{2}\frac{k-1}{200-1}))(1 + vcos(2\pi u))$
$y(u, v, w, k) = (1 - (1 - cos(\frac{\pi}{2}\frac{k-1}{200-1})))(wcos(2\pi u)sin(\pi v)) + (1 - cos(\frac{\pi}{2}\frac{k-1}{200-1}))(1 + vsin(2\pi u))$
$z(u, v, w, k) = (1 - (1 - cos(\frac{\pi}{2}\frac{k-1}{200-1})))(wsin(2\pi u)) + (1 - cos(\frac{\pi}{2}\frac{k-1}{200-1}))(6w))$
$u, v, w \in [0, 1]$, where k is the frame index, $1 \leq k \leq 200$