CZ2003: Tutorial 3

Due on February 2, 2021 at 10:30am

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Problem 1

Write parametric formulas $x(u)$, $y(u)$ for the ray cast from the point with coordinates (1, 2) through the point with coordinates (4, 3). Define the domain for the parameter u.

Solution
$x(u) = x1 + u(x2 - x1)$
$x(u) = 1 + u(4 - 1)$
$\mathbf{x(u) = 1 + 3u}$

$y(u) = y1 + u(y2 - y1)$
$y(u) = 2 + u(3 - 2)$
$\mathbf{y(u) = 2 + u}$

For Rays, Domain: $u \in [0, \inf)$

Problem 2

Using an equation in intercepts, obtain an **implicit** formula $f(x, y) = 0$ for the straight line intersecting the coordinate axes X and Y at the points with coordinates (-2, 0) and (0, 3), respectively.

Solution
Intercept form:
$\frac{x}{a} + \frac{y}{b} = 1$, where
$a = -2, y = 3$

$\therefore \mathbf{f(x, y) = \frac{x}{2} - \frac{y}{3} + 1 = 0}$

Problem 3

With reference to Figure Q3, write parametric functions $x(u)$, $y(u)$, $u \in [0, 1]$ defining this spiral curve which has to be drawn clockwise from the point with coordinates $(0, 0.3)$.
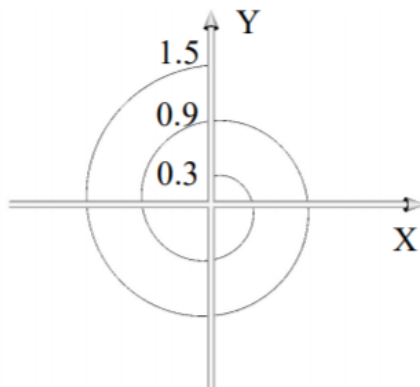

Figure Q3

Solution

Flip x and y from notes examples, such that x = sin, y = cos.
Since 2 spirals, argument of sin/cos functions will be $4\pi u$
$\mathbf{x(u) = (0.3 + 1.2u)cos(-4\pi u + \pi/2)}$
$\mathbf{y(u) = (0.3 + 1.2u)sin(-4\pi u + \pi/2)}$

Problem 4

Based on the way how polar coordinates are mapped to Cartesian, propose parametric functions $x(u)$, $y(u)$, $u \in [0, 1]$ which make the trigonometric sinusoidal curve (sine wave) follow a semicircle (half circle) with the radius of 0.75. The curve has to make 10 periodic oscillations (cycles) moving counterclockwise around the semicircle with the oscillations amplitude of ± 0.25 as shown in Figure Q4.
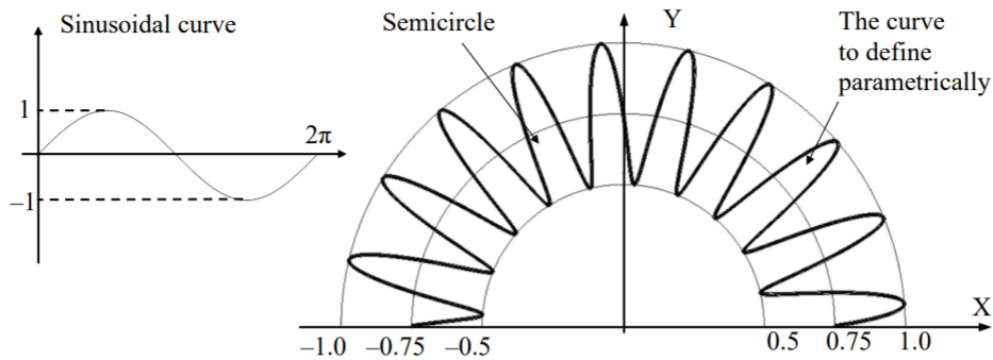

Figure Q4

Solution
10 oscilations – argument of sine wave function: $20\pi u$
Amplitude of sine wave: 0.25 (0.75 +- 0.25)

$r = 0.75 + 0.25 sin(20\pi u)$
$x = rcos(u)$
$\mathbf{x = cos(\pi u)(0.75 + 0.25sin(20\pi u))}$

$y = rsin(u)$
$\mathbf{y = sin(\pi u)(0.75 + 0.25sin(20\pi u))}$
$\mathbf{u \in [0, 1]}$