# CZ4041: Tutorial Week 8

Due on March 4, 2021 at 8:30am

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# Problem 1

Consider a 2-dimensional dataset for two-class classification by SVM, as shown in Figure 1, where the red "square" and the blue "circle" denote the positive and negative classes respectively. Is this dataset seperable by a linear SVM classifier? If no, why? If yes, what is the decision boundary of the linear SVM? And what are the pair of parallel hyperplanes associated with the decision boundary? (No need to provide proofs)

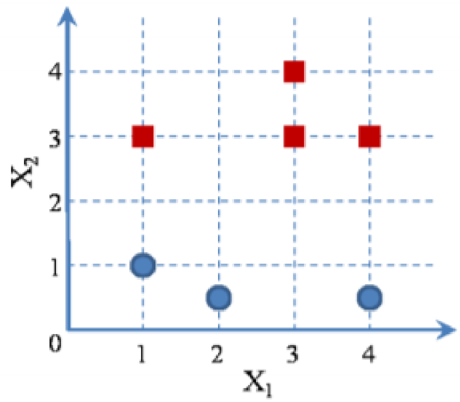


Figure 1: Dataset for Question 1.

**Solution**
Decision boundary: $X_2 = 2$

Pair of parallel hyperplanes: $X_2 = 1$ and $X_2 = 3$

# Problem 2

Please induce why the two parallel hyperplane of a decision boundary.

$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

can be written as

$$\mathbf{w} \cdot \mathbf{x} + b = k, \text{ and } \mathbf{w} \cdot \mathbf{x} + b = -k$$

where $k > 0$, respectively as shown on the 18th Slide of the lecture notes "Lecture 8a"

**Solution**
Let $x_1$ and $x_2$ be points on the two parallel hyperplanes, and $x_0$ be a point on the decision boundary that is between $x_1$ and $x_2$.

$b_{11} : \mathbf{w} \cdot \mathbf{x_1} + b = k$, where $k > 0$
$b_{12} : \mathbf{w} \cdot \mathbf{x_2} + b = k'$, where $k' < 0$
$B_1 : \mathbf{w} \cdot \mathbf{x_0} + b = 0'$

$\mathbf{w} \cdot \mathbf{x_1} + b = k$
$\mathbf{w} \cdot \mathbf{x_0} + b = 0$

$\mathbf{w} \cdot \mathbf{x_1} = k - b$
$\mathbf{w} \cdot \mathbf{x_0} = -b$

$\mathbf{w} \cdot (\mathbf{x_1} - \mathbf{x_0}) = k$
$||\mathbf{w}||_\mathbf{2} \times ||\mathbf{x_1} - \mathbf{x_0}||_\mathbf{2} \times cos(0) = k$
$||\mathbf{w}||_\mathbf{2} \times ||\mathbf{x_1} - \mathbf{x_0}||_\mathbf{2} = k$

Similarly,
$||\mathbf{w}||_\mathbf{2} \times ||\mathbf{x_2} - \mathbf{x_0}||_\mathbf{2} \times cos(180) = k'$
$||\mathbf{w}||_\mathbf{2} \times ||\mathbf{x_2} - \mathbf{x_0}||_\mathbf{2} = -k'$

Since $||x_1 - x_0||_2 = ||x_2 - x_0||_2$,
$k = -k'$, or $k' = -k$

Therefore, the two parallel hyperplanes can be written as $\mathbf{w} \cdot \mathbf{x} + b = k$, and $\mathbf{w} \cdot \mathbf{x} + b = -k$