

CZ4041: Tutorial Week 5

Due on February 11, 2021 at 8:30am

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11/02/2021

Problem 1

Consider the data set shown in Table 1 for a binary classification problem

Table 1: Data set for Question 1.

<i>A</i>	<i>B</i>	Class Label
M	F	+
F	T	+
T	T	+
M	F	-
M	F	-
F	F	-
N	F	-
N	T	-
T	T	-
T	F	-

1. Calculate the information gain when splitting on A and B (using multi-way split on A). Which feature would the decision tree induction algorithm choose?
2. Calculate the gain ratio when splitting on A and B (using multi-way split on A). Which feature would the decision tree induction algorithm choose?

Solution

1. Information Gain: $\Delta_{info} = Entropy(parent\ node) - Entropy(children\ nodes)$

$$\begin{aligned}
 E_p &= -(3/10)\log_2(3/10) - (7/10)\log_2(7/10) \\
 &= 0.88129
 \end{aligned}$$

$$\begin{aligned}
 E_A &= (3/10)(-(1/3)\log_2(1/3) - (2/3)\log_2(2/3)) + (2/10)(1) + (3/10)(-(1/3)\log_2(1/3) - (2/3)\log_2(2/3)) + (2/10)(0) \\
 &= 0.75097
 \end{aligned}$$

$$\begin{aligned}
 E_B &= (4/10)(1) + (6/10)(-(1/6)\log_2(1/6) - (5/6)\log_2(5/6)) \\
 &= 0.79001
 \end{aligned}$$

$$\begin{aligned}
 \Delta_{info(A)} &= 0.88129 - 0.75097 \\
 &= \mathbf{0.13032}
 \end{aligned}$$

$$\begin{aligned}
 \Delta_{info(B)} &= 0.88129 - 0.79001 \\
 &= 0.09128
 \end{aligned}$$

Therefore, the decision tree induction algorithm would choose to split on **A**.

2.

Gain Ratio: $\Delta_{InfoR} = \frac{\Delta_{info}}{SplitINFO}$ Where $SplitINFO = -\sum_{i=1}^p \frac{n_i}{n} \log_2(\frac{n_i}{n})$

$$\begin{aligned}
 SplitINFO_A &= -((3/10)\log_2(3/10) + (3/10)\log_2(3/10) + (2/10)\log_2(2/10) + (2/10)\log_2(2/10)) \\
 &= -((6/10)\log_2(3/10) + (4/10)\log_2(2/10)) \\
 &= 1.97095
 \end{aligned}$$

$$\begin{aligned}
 SplitINFO_B &= -((4/10)\log_2(4/10) + (6/10)\log_2(6/10)) \\
 &= 0.97095
 \end{aligned}$$

$$\begin{aligned}
 \Delta_{infoR(A)} &= \Delta_{info(A)} / SplitINFO_A \\
 &= 0.13032 / 1.97095 \\
 &= 0.06612
 \end{aligned}$$

$$\begin{aligned}
 \Delta_{infoR(B)} &= \Delta_{info(B)} / SplitINFO_B \\
 &= 0.09128 / 0.97095 \\
 &= \mathbf{0.09401}
 \end{aligned}$$

Therefore, the decision tree induction algorithm would choose to split on **B**.