## Lecture 5

Permutations P: execute row exchanges / identity matrix with reordered rows

# of possible reorderings ( $n \times n$ permutations) : $n!$

$P^T P = I$

Transpose

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

General : $(A^T)_{ij} = A_{ji}$

Symmetrix Matrices : $A^T = A$

example

$$\begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 9 \\ 7 & 9 & 4 \end{bmatrix}$$

good property. so how to get one?

$R^T R$ is always symmetric.

why?

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 11 & 7 \\ 11 & 13 & 11 \\ 7 & 11 & 17 \end{bmatrix}$$

Take transpose

$(R^T R)^T = R^T R^{TT} = R^T R$

## Vector Spaces

→ 对向量进行加&数乘，得到的组合

vector space 对线性运算封闭

e.g. $R^2$ : all 2-D real vectors $\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \pi \\ e \end{bmatrix} \ldots$

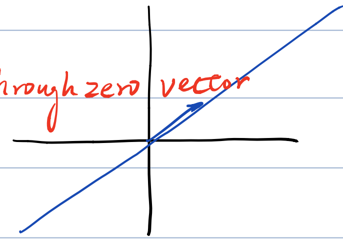
: "x-y plane"

$R^3$ : all 3-D real vectors (all vectors with 3 real components)

$R^n$ = all vectors with n real components

## Subspace

e.g: a line in $R^2$ through zero vector



Subspace of $R^2$:

① all of $R^2$

② any line through $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ vector

③ zero vector only

Subspace of $R^3$:

① $R^3$

② plane $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

③ line $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

④ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$$

columns in $R^3$
all their linear combination form a subspace
called column space $C(A)$