Lecture 4

A & B are inverétible

$(AB)\cdot(B^{-1}A^{-1})=I \rightarrow A(B\cdot B^{-1})A^{-1}=A\cdot I\cdot A^{-1}=AA^{-1}=I$

transpose? → $\boxed{\text{将 A矩阵的行变为}A^T\text{的列}}$

$AA^{-1}=I \xrightarrow{\text{Transpose}} \boxed{(A^{-1})^T}(A)^T=I$

↳ the inverse of A^T

2×2 matrix

$$\underset{E_{21}}{\begin{bmatrix}1&0\\-4&1\end{bmatrix}}\underset{A}{\begin{bmatrix}2&1\\8&7\end{bmatrix}}=\underset{U\text{(上三角阵)}}{\begin{bmatrix}2&1\\0&3\end{bmatrix}} \rightarrow \text{to get } A=LU$$

$L=E_{21}^{-1}$

$$\begin{bmatrix}2&1\\8&7\end{bmatrix}=\begin{bmatrix}1&0\\4&1\end{bmatrix}\begin{bmatrix}2&1\\0&3\end{bmatrix}\ (LU)$$

pivots

$$=\begin{bmatrix}1&0\\4&1\end{bmatrix}\begin{bmatrix}2&0\\0&3\end{bmatrix}\begin{bmatrix}1&1/2\\0&1\end{bmatrix}\ (LDU)$$

三角阵

$$\begin{bmatrix}2&0\\0&3\end{bmatrix}\begin{bmatrix}1&1/2\\0&1\end{bmatrix}=\begin{bmatrix}2&0\\0&3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}2&0\\0&3\end{bmatrix}\begin{bmatrix}1/2\\1\end{bmatrix}$$

$$=\begin{bmatrix}2&0\\0&3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\qquad \begin{matrix}2\times 1/2+0\times 1\\0\times 1/2+3\times 1\end{matrix}$$

$$=\begin{bmatrix}2\times1+0\times0\\0\times1+3\times0\end{bmatrix}$$

3×3

$E_{32}E_{31}E_{21}A=U$ (no row exchanges)

$A=\underbrace{E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}}_{L}U$

$$\underset{E_{32}}{\begin{bmatrix}1&0&0\\0&1&0\\0&-5&1\end{bmatrix}}\underset{E_{21}}{\begin{bmatrix}1&0&0\\-2&1&0\\0&0&1\end{bmatrix}}\ (\text{此处假设}E_{31}\text{为}I)$$

$$=\begin{bmatrix}1&0&0\\-2&1&0\\10&-5&1\end{bmatrix}=E$$

inverses

$$\underset{E_{21}^{-1}}{\begin{bmatrix}1&0&0\\2&1&0\\0&0&1\end{bmatrix}}\underset{E_{32}^{-1}}{\begin{bmatrix}1&0&0\\0&1&0\\0&5&1\end{bmatrix}}=\begin{bmatrix}1&0&0\\2&1&0\\0&5&1\end{bmatrix}=L$$

$A = LU$

If no row exchanges, multipliers go directly into L

How many operations on $n \times n$ matrix A?

$$\underbrace{n^2+(n-1)^2+\cdots+3^2+2^2+1^2}_{\int_1^n x^2\,dx} \approx \frac{1}{3}n^3 \quad (\text{cost on } A)$$

$$n^2 \quad (\text{cost on } B)$$

Permutations

$3 \times 3 \rightarrow 6$ P's

$\rightarrow P^{-1} = P^T$