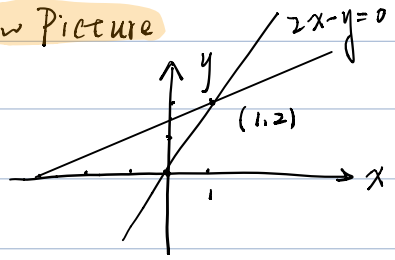
Lecture 1

$$\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases} \rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

A X b

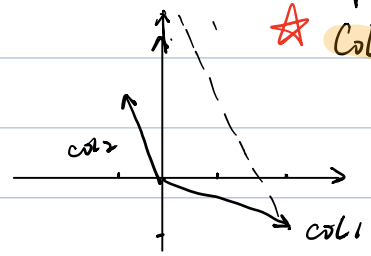
Row Picture



$$x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

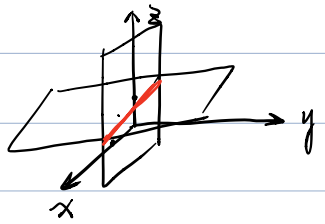
linear combination of columns

★ Column Picture



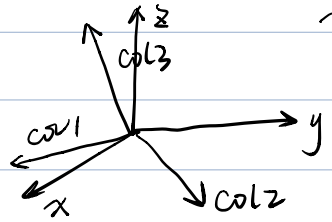
$$\begin{cases} 2x - y = 0 \\ -x + 2y - z = -1 \\ -3y + 4z = 4 \end{cases} \quad A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

Row Picture



Column Picture

$$x\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$



$x=0,\ y=0,\ z=1$

Can I solve $Ax=b$ for every b?

→ linear way: Do the linear combination of columns fill 3-D space?

How to multiply a matrix by a vector?

$Ax=b$ ① By columns

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

Ax is a combination of columns of A

② By rows

$$\begin{bmatrix} 2\times 1+5\times 2 \\ 1\times 1+3\times 2 \end{bmatrix}$$