# Lecture 3

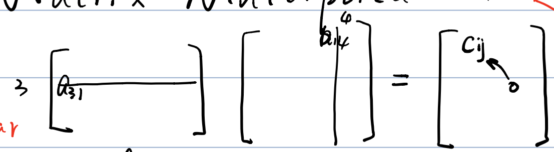
本质上矩阵的代数运算就是
线性方程组的运算

## Matrix Multiplication

遵循结合律、分配律、x交换律

$\boxed{\text{C row, column}}$

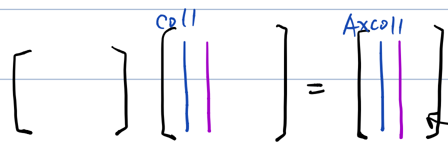
① Regular Way



$A(m\times n)$ $B(n\times p)$ $C = AB\,(m\times p)$

$$C_{34} = (\text{row 3 of } A)\cdot(\text{column 4 of } B)$$
$$= a_{31}\cdot b_{14} + a_{32}\cdot b_{24} + \cdots +$$
$$= \sum_{k=1}^{n} a_{3k}\, b_{k4}$$

② Column Way

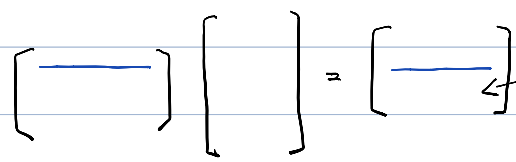


$A(m\times n)$ $B(n\times p)$ $C(m\times p)$

(matrix) x (vector)

columns of C are combinations of columns of A

③ Row Way



rows of C are combination of rows of B

④ column of A x row of B
$(m\times 1)$ $(1\times p)$

$$\underset{A}{\begin{bmatrix}2\\3\\4\end{bmatrix}}\underset{B}{\begin{bmatrix}1 & 6\end{bmatrix}} = \begin{bmatrix}2 & 12\\3 & 18\\4 & 24\end{bmatrix}$$

multiples of row B

multiples of column A

AB = Sum of (col of A) x (rows of B)

$$\begin{bmatrix}2 & 7\\3 & 8\\4 & 9\end{bmatrix}\begin{bmatrix}1 & 6\\0 & 0\end{bmatrix} = \begin{bmatrix}2\\3\\4\end{bmatrix}\begin{bmatrix}1 & 6\end{bmatrix} + \begin{bmatrix}7\\8\\9\end{bmatrix}\begin{bmatrix}0 & 0\end{bmatrix} = \begin{bmatrix}2 & 12\\3 & 18\\4 & 24\end{bmatrix}$$

## Block Multiplication

$A_1B_1 + A_2B_3$

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

A B

## Inverses (Square matrices)

单位矩阵

$$A^{-1}A = I = AA^{-1}$$

if this matrix exist, A is called "invertible" "non-singular"

## Singular Case (No inverse)

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

I can find a vector x, with $Ax = 0$ ($x$ is non-zero)

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For noniversible/singular matrix. Some combinations of their columns gives the zero column.

$$\underset{A}{\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}} \underset{A^{-1}}{\begin{bmatrix} a & c \\ b & d \end{bmatrix}} = \underset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$A \times$ column j of $A^{-1}$ = column j of $I$

## Gauss-Jordan (solve two equations at once)

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

augemented matrix

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$$

A I

using elimination to make this identity

inverse will show up here

$$\downarrow$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \end{array}\right.$$

$$\left[\begin{array}{cc|cc} 0 & 1 & -2 & 1 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{cc} \left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right] \\ I \qquad\quad A^{-1} \end{array}$$

How to see it?

$E[A\ \ I] = [I\ \ ?] \to [I\ \ A^{-1}]$

$\uparrow$

overall elimination matrix

$EA = I$ tells us $E = A^{-1}$

单位矩阵性质： $AI_n = A$ 且 $I_nB = B$