10.2 Vectors

Definition
A vector is a quantity that has both maginitudes and direction. Magnitude is the length of a vector, while direction is the "arrow tip" of a vector.

Equivalent Vectors
For two vectors to be considered equivalent, they must have the same length and direction, $\vec{v} = \vec{v}$.

Zero Vector
A vector with a length of 0 and no specific direction, $\vec{0}$.

Vector Addition
The addition of the two vectors, $\vec{AB} + \vec{BC}$ can be represented by a vector $\vec{AC}$.

Triangle Law
Two vectors $(\vec{u}, \vec{v})$ where it creates the initial point of a vector is at the endpoint of other vector. The addition of these two vectors, $\vec{u} + \vec{v}$ can be represented as a straight line connected from the initial point of the feeding vector to the endpoint of the nonfeeding vector.

Parallelogram Law
Imagine two vectors, ($\vec{u}, \vec{v}$) connected by the same initial point, creating some sort of triangle. Parallelogram law is where the triangle is copied and mirrored across in a way to create a parallelogram. The sum of these two vectors can be represented as a diagonal line cutting through the parallelogram.

Scalar Multiplication
A vector can be scaled by a quantity. For example, a scalar multiplication of 2, increases the vector's size by 2x. While a scalar multiplicaiton of -1 reverses the direction of the vector.

Vector Subtraction
If $\vec{u}$ and $\vec{v}$ are vectors, then $\vec{u} - \vec{v} = \vec{u} + (\vec{-v})$ This can be represented by drawing $\vec{-v}$, which is mirroring $\vec{v}$ in the opposite direction. Then, we draw $\vec{-v}$ again but from the endpoint of $\vec{u}$. From there, we can finish the Parallelogram Law by drawing $\vec{u}$ from the endpoint of the first $\vec{-v}$ to the endpoint of the second $\vec{-v}$. Then $\vec{u} - \vec{v}$ can be represented as a diagonal line cutting across the parallelogram.

Components
$\vec{a} =< a_1, a_2, a_3 >$ are components of $\vec{a}$

$\vec{AB} =< x_2 - x_1, y_2 - y_1 >$ If $\vec{AB}$ is a vector with initial print
$\vec{AB} =< x_2 - x_1, y_2 - y_1, z_2 - z_1 >$

$\vec{AB} =< -2 - 2, 1 - (-3), 1 - 4 >= \boxed{< -4, 4 - 3 >}$

Magnitude (or Length) of a Vector
The length of the 2-dimensional vector, $\vec{a} =< a_1, a_2 >$ is $\vec{a} = \sqrt{(a_1)^2 + (a_2)^2}$. The length of the 3-dimensional vector, $\vec{a} =< a_1, a_2, a_3 >$ is $\vec{a} = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$

Vector Addition and Scalar Math (cont.)

If $\vec{a} =< a_1, a_2 >$ and $\vec{b} =< b_1, b_2 >$, then

$$\begin{aligned} &1) \quad \vec{a} + \vec{b} =< a_1 + b_1, a_2 + b_2 > \\ &2) \quad \vec{a} =< ca_1, ca_2 \text{for any scalar c} \end{aligned}$$

Ex 1
$< a_1, a_2, a_3 > + < b_1, b_2, b_3 > =< a_1 + b_1, a_2 + b_2, a_3 + b_3 >$

Ex 2
If $\vec{a} =< 4, 0, 3 >$ and $-2,\vec{1}, 5 <_{1,2},_3 >$

$|\vec{a}| = \sqrt{4^2 + 0^2 + 3^2} = \sqrt{25} = \boxed{5}$

Properties of Vectors
If $\vec{a}, \vec{b}$ and $\vec{c}$ are vectors and c and d are scalars, then

$$\begin{aligned} &1) \quad \vec{a} + \vec{b} = \vec{b} + \vec{a} \\ &2) \quad \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c} \\ &4) \quad \vec{a} + \vec{0} = \vec{a} \\ &5) \quad \vec{a} + \vec{-a} = \vec{0} \\ &6) \quad c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b} \\ &7) \quad (c + d)\vec{a} = c\vec{a} + d\vec{a} \\ &8) \quad (cd)\vec{a} = c(d\vec{a}) \\ &9) \quad 1 \cdot \vec{a} = \vec{a} \end{aligned}$$

Standard Basis Vectors
$\vec{i} =< 1, 0, 0 > \vec{j} =< 0, 1, 0 >, \vec{k} =< 0, 0, 1 >$

If $\vec{a} =< a_1, a_2, a_3 >$, then
$\vec{a} =< a_1, 0, 0 > + < 0, a_2, 0 > + < 0, 0, a_3 >$
$= a_1 < 1, 0, 0 > +a_2 < 0, 1, 0 > +a_3 < 0, 0, 1 >$
$= a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

Ex 3
If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 4\vec{i} + 7\vec{k}$, express the vector $2\vec{a} + 3\vec{b}$ in terms of $\vec{i}, \vec{j}$, and $\vec{k}$.

$$\begin{aligned} &2(\vec{i} + 2\vec{j} - 3\vec{k}) + 3(4\vec{i} + 7\vec{k}) \\ &\vec{i} + 4\vec{j} - 6\vec{k} + 12\vec{i} + 21\vec{k} \\ &\boxed{14\vec{i} + 4\vec{j} + 15\vec{k}} \end{aligned}$$

Unit Vector
A unit vector is a unit whose length is 1. Note that $\vec{a} \neq \vec{0}$, then the unit vector $\vec{u}$ that has the same direction as $\vec{a}$ is $\vec{u} = \frac{\vec{a}}{|\vec{a}|}$

Ex 4
Find the unit vector in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$,