

## 1.2

### Row Echelon Form

1. All non zeros are above any row of all zeros
2. Each leading entry of a row in a column is to the right of the leading entry of the row above it.
3. All entries in a column below a leading entry are zeroes

### Reduced Row Echelon Form

4. The leading entry in each nonzero row is 1.
5. Each leading 1 is the only non zero entry in its column.

### Example Of Row Echelon Form

$$A = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ -1 & 4 & 5 & -9 & -7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ 0 & 5 & 10 & -15 & -15 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 \end{bmatrix}$$

### Example 3

Matrix Form

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

Row Echelon Form

$$\begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 0 & -14 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Row Echelon Form

$$\begin{bmatrix} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

### Example 5

Determine the existence and uniqueness of the solutions to the system, find the general solution

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$

$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$

$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15$$

**Row Echelon Form**

$$\left[ \begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array} \right]$$

$$x_1 = -24 - 2x_3 + 3x_4$$

$$x_2 = -7 + 2x_3 - 2x_4$$

$$x_3 = y$$

$$x_4 = z$$

$$x_5 = 4$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \vee \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -24 \\ -7 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$