

1.1 Systems of Linear Equations

Linear Equation

$ax_1 + a_2x_3 + \dots + a_nx_n = b$, where x represents variables and b is given.

System of Linear Equation

$$a_{11}x_1 + a_{12}x_3 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_3 + \dots + a_{2n}x_n = b_2$$

$$a_{31}x_1 + a_{32}x_3 + \dots + a_{3n}x_n = b_3$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

Elementary Row Operations

1. Replacement - replace one row by the sum of itself and a multiple of another row.
2. Interchange - Interchange two rows
3. Multiply all entries in a row by a non zero constant

Ex 1

$$3x + 4y = 5$$

$$2x - 7y = 3$$

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -7 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 11 & 2 \\ 2 & -7 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 11 & 2 \\ 0 & -29 & -1 \end{bmatrix}$$

Ex 2 Gaussian Elimination

$$x_1 - 2x_2 + x_3 = 0$$

$$2x_2 - 8x_3 = 8$$

$$5x_1 - 5x_3 = 10$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & 5 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 10 & -10 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 10 & -10 & 10 \end{bmatrix}$$

$$\rightarrow \text{scale by } \frac{1}{3} \begin{bmatrix} 1 & 0 & -7 & 8 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 30 & -30 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -7 & 8 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$