2.1 Matrix Operations

Imagine two matrixes A & B, in order for $A + B$ to be possible, they must both be $m \times n$ matrixes, meaning they have the same dimensions.

Matrix Addition

$$A,\ m \times n \qquad Bm \times n$$
$$A + B,\ m \times n$$

Matrix Multiplication

$$A,\ m \times n \qquad B, n \times p$$
$$A \cdot B,\ m \times p$$

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Compute AB where

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A,\ 2 \times 2 \qquad B,\ 2 \times 3$$

$$\begin{bmatrix} A_{r1} \cdot B_{c1} & A_{r1} \cdot B_{c1} & A_{r1} \cdot B_{c2} \\ A_{r2} \cdot B_{c1} & A_{r2} \cdot B_{c2} & A_{r2} \cdot B_{c3} \end{bmatrix} \to \begin{bmatrix} 4(2) + 3(1) & 2(3) + (3)(-2) & 2(6) + (3)(3) \\ 1(1) + (-5)(1) & 1(3) + (-5)(-2) & 1(6) + (-5)(3) \end{bmatrix}$$

$$\begin{bmatrix} 11 & 0 & 21 \\ 01 & 13 & -9 \end{bmatrix}$$

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Find the entries in the second row of AB, where

$$A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$$

$$A,\ 4 \times 3 \qquad B,\ 3 \times 2$$

$$\begin{bmatrix} A_{r1} \cdot B_{c1} & A_{r1} \cdot B_{c2} \\ A_{r2} \cdot B_{c1} & A_{r2} \cdot B_{c2} \\ A_{r3} \cdot B_{c1} & A_{r3} \cdot B_{c2} \\ A_{r4} \cdot B_{c1} & A_{r4} \cdot B_{c2} \end{bmatrix} \rightarrow \begin{bmatrix} -27 & -17 \\ 5 & 1 \\ -53 & -58 \\ 15 & 36 \end{bmatrix}$$

Power of a Matrix
If A is an $n \times n$ matrix and if k is a positive integer, then A^l deonotes the product of k copies of A.

The Transpose of a Matrix
Given an $m \times n$ matrix A, the transpose of A is the $n \times m$ matrix, denoted by A^T, whose columns are formed from the corresponding rows of A.

Ex 8

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$B = \begin{bmatrix} -5 & 2 \\ 1 & -3 \\ 0 & 4 \end{bmatrix} \qquad B^T = \begin{bmatrix} -5 & 1 & 0 \\ 2 & -3 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 5 & -2 & 7 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -3 \\ 1 & 5 \\ 1 & -2 \\ 1 & 7 \end{bmatrix}$$

Theorem
Let A & B denote matrices whose sizes are appropiate for the following sums and products.
A) $(A^T)^T = A$
B) $(A + B)^T = A^T + B^T$
C) For any scalar r, $(rA)^T = rA^2$
D) $(AB)^T = B^T A^T$

I can be treated as 1.