10.5 Equations of Line and Planes

A line in the xy plane is determined when a point on the line and the direction of the line (its slope or angle of inclination) are given. The equation of the line can then be written using point-slope form.

Vector Equation

$$\vec{r} = \vec{r}_0 + \vec{v}$$

$$\vec{r} =< x, y, z > \rightarrow \text{a point on the line}$$

$$\vec{v} = \text{parallel to line} L$$

If $\vec{v} =< a, b, c >$, then $t\vec{v} =< ta, tb, tc >$ and

$$< x, y, z >=< x_0, y_0, z_0 > + < ta, tb, tc >$$

$$< x, y, z >=< x_0 + ta, y_0 + tb, z_0 + tc >$$

Thus,

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \ \rightarrow \ \text{Parametric Equation} \\ z &= z_0 + ct \end{aligned}$$

Note
If a line L passes through the tips of position vectors $\vec{r}_0$ & $\vec{r}_1$, then the vectors $\vec{r}_1 - \vec{r}_0$ is parallel to L and

$$\vec{r} = \vec{r}_0 + t(\vec{r}_1 - \vec{r}_0)$$

$$\vec{r} = \vec{r}_0 + t\vec{r}_1 - t\vec{r}_0$$

$$\vec{r} = \vec{r}_0 - t\vec{r}_0 + t\vec{r}_1$$

$$\vec{r} = (1 - t)\vec{r}_0 + t\vec{r}_1, \ 0 \leq t \leq 1$$

The equation above is the vector equation of line segment from $\vec{r}_1 \rightarrow \vec{r}_1$.

Def
2 lines are skew if they do not intersect and are not parallel.

Ex 3
How do we know if two lines are parallel or not? We can use parametric equations as they are able to show if

2 lines are skew lines.

$$r = r_0 + tv$$

$$< x, y, z > = < x_0, y_0, z_0 > + < ta, tb, tc >$$

$$< x, y, z > = < x_0 + ta, y_0 + tb, z_0 + tc >$$

$$L_1, \qquad x = 1 + t \qquad y = -2 + 3t, \qquad z = 4 - t$$

$$L_2, \qquad x = 2s \qquad y = 3 + s \qquad z = -3 + 4s$$

$$L_1 = < 1, 3, -1 > \qquad L_2 = < 2, 1, 4 >$$

Ex 5
$\vec{v} \neq \vec{v_2}$ for some scalar c, so L_1 & L_2 are not parallel.