11.7 Maximum and Minimum Values

A function of two variables has a local maximum at (a, b) if $f(x, y) \leq f(a, b)$ when (x, y) is near (a, b). In the opposite case when $f(x, y) ge f(a, b)$, then $f(a, b)$ is a local minimum.

If the inequalities hold for all points (x, y) in the domain of f or $\{(x, y)|(x, y) \in D\}$, then f has an absolute maximum or minimum at (a, b).

Theorem 12.2
If f has a local maximum or minimum at (a, b) and the first-order partial derivatives of f exist there, then $f_x(a, b)$ & $f_y(a, b) = 0$.

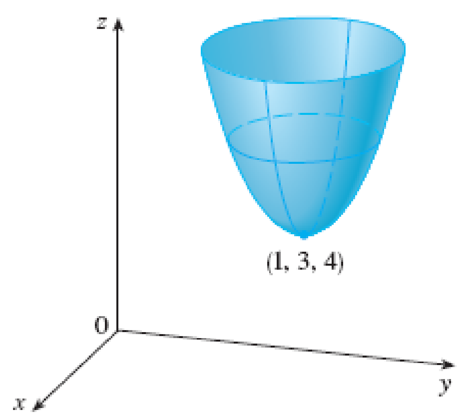
Ex 1
Let $f(x, y) = x^2 + y^2 - 2x - 6y + 14$. Then

$$f_x = 2x - 2 \qquad f_y(x, y) = 2y - 6$$

These partial derivatives are equal to 0 at the point $(1, 3)$. So the only critical point is $(1, 3)$. By completing the square, we find that

$$f(x, y) = 4 + (x - 1)^2 + (y - 3)^2$$

Since $(x - 1)^2 \geq 0$ & $(y - 3)^2 \geq 0$, we have $f(x, y) \geq 4$ for all values of x & y, therefore $f(x, y) \geq 4$ for all values of x & y. Then not only is $f(1, 3) = 4$ is a local minimum, it is also an absolute minimum. This can be verified by the graph below.



Ex 2
Find the extreme values of $f(x, y) = y^2 - x^2$.

$$f_x = -2x \qquad f_y = 2y$$

The only critical point is $(0, 0)$. Notice that the points on the x axis where $y = 0$.

$$f(x, y) = -x^2 < 0 \ (x \neq 0)$$

The same can be said for points on the y axis where $x = 0$

$$f(x, y) = y^2 > 0 \ (x \neq 0)$$

Since every disk with the center $(0, 0)$ contains points where f takes on both positive and negative values. Then $f(0, 0) = 0$ can't be an extreme value so f does not have any extremas.