1.7 Linear Independence

Given vectors $\{v_1, v_2, ...v_p\} \in \mathbb{R}^n$. We will solve the system of equations $x_1 v_2 + x_2 v_2 + ... + x_p v_p = 0$. $v_1 v_1, ..., x_p v_p$ are vectors in $\mathbb{R}^n$ that we will be solving for and eventually give us values for $x_1, x_2, ..., x_p$.

Two Cases

    1) If $x_1 = 0, x_2 = 0, ..., x_p = 0$ then we say that the set $\{v_1, v_2, ..., v_p\}$ is linearly independent.

    2) If $x_1, ..., x_p$ are not all zeros, then the set $\{v_1, v_2, ..., v_p\}$ is linearly dependent.

Ex 1 p60

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

1A. Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent. 2B. If possible find a linear dependence relation among v_1, v_2, v_3.

    A) $x_1 v_1 + x_2 v_2 + x_3 v_3 = 0,$ $v_1, ...v_p$ are vectors in $\mathbb{R}^3$.

$$\begin{bmatrix} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1, \ x_2 = \text{pivot}, \ x_3 = \text{free}$$

$$x_3 = 1, \ x_2 = -1, \ x_1 = -4x_2 - 2x_3 \rightarrow 2$$

$(2, -1, 1)$ is a solution. So the set $\{v_1, v_2, v_3\}$ is linearly dependent.

    B) Linear Dependence Relation

$$2v_1 - v_2 + v_3 = 0$$

Ex 2 p61

Determine if the columns of a matrix, A are linearly independent.

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -2 \\ 5 & 8 & 0 \end{bmatrix}$$

$x_1 c_1 + x_2 c_2 + x_3 c_3 = 0,$ is what we are solving for where c_n represents a column.

$$x_1 \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Ax = 0$$

$$\begin{bmatrix} 0 & 1 & 4 & 0 \\ 1 & 2 & -1 & 0 \\ 5 & 8 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 13 & 0 \end{bmatrix}$$

$x_3 = 0, \ x_2 = 0, \ x_1 = 0,$ linearly independent