When a basis $\mathcal{B}$ is chosen for an n-dimensional vector space V, the associated coordinate mapping onto $\mathbb{R}^n$ provides a coordinate system for V. Each x in V is identified uniquely bt its $\mathcal{B}$ coordinate vector $[x]_{\mathcal{B}}$.

Ex 1

Consider two bases $\mathcal{B} = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ for a vector space V, such that

$$b_1 = 4c_1 + c_2 \qquad b_2 = -6c_1 + c_2$$

Suppouse that

$$x = 3b_1 + b_2$$

That is, suppouse $[x]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Find $[x]_{C}$.

Apply the coordinate mapping determined by C.

$$[x]_C = [3b_1 + b_2]_C$$
$$[x]_C = \left[[b_1]_C \quad [b_2]_C \right] \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$$[x]_C = \begin{bmatrix} 4 & -6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6 \\ 4 \end{bmatrix}}$$

Theorem 5

Let $\mathcal{B} = \{b_1, ..., b_n\}$ & $C = \{c_1, ..., c_n\}$ be bases for a vector space V. There is a unique $n \times n$ matrix P sich that

$$[x]_C = P_{C \leftarrow B} [x]_B$$

and $P_{C \leftarrow B}$ is given by $P_{C \leftarrow P} = \left[[b_1]_C \quad [b_2]_C \quad \cdots \quad [b_n]_C \right]$