

1.3 Vector Equations

What is Linear Algebra?

Linear equations in n unknown x_1, \dots, x_n

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = k$$

System of Linear Equations

m linear equations in n unknowns x_1, \dots, x_n .

$$\begin{aligned} \{a_1x_1 + a_2x_2 + \dots + a_nx_n &= b_1 \rightarrow Ax = b \\ \{a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

$$Ax = b$$
$$\begin{bmatrix} a_1 & \dots & a_n \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

2 Dimensional Matrix

$$\mathbb{R}^2, \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Operations on Vectors

Vector Addition

$$\vec{u} + \vec{v}$$

Scalar Multiplication

$$c \cdot \vec{u}$$

Linear Combinations *

Given vectors $v_1, v_2, v_p \in \mathbb{R}^n$ and scalars c_1, c_2, \dots, c_p . The vector y defined by $y = c_1v_1 + c_2v_2 + \dots + c_pv_p$ is called a linear combination of the vectors v_1, \dots, v_p with weights c_1, \dots, c_p .

An example of linear combination being applied would be RGB colors. Where v_1, v_2 & v_3 are red, green, & blue. While the scalar values c_1, c_2 , & c_3 would be the magnitude of those colors.

Least Squares

Span v_1, \dots, v_p is the set of all linear combinations of v_1, v_2, \dots, v_p .

$$v_1, v_2, v_3 =$$