10.2 Vectors

Definition

A vector is a quantity that has both maginitudes and direction. Magnitude is the length of a vector, while direction is the "arrow tip" of a vector.

Equivalent Vectors

For two vectors to be considered equivalent, they must have the same length and direction, $\vec{v} = \vec{v}$.

Zero Vector

A vector with a length of 0 and no specific direction, $\vec{0}$.

Vector Addition

The addition of the two vectors, $\vec{AB} + \vec{BC}$ can be represented by a vector \vec{AC} .

Triangle Law

Two vectors (\vec{u}, \vec{v}) where it creates the initial point of a vector is at the endpoint of other vector. The addition of these two vectors, $\vec{u} + \vec{v}$ can be represented as a straight line connected from the initial point of the feeding vector to the endpoint of the nonfeeding vector.

Parallelogram Law

Imagine two vectors, (\vec{u}, \vec{v}) connected by the same initial point, creating some sort of triangle. Parallelogram law is where the triangle is copied and mirrored across in a way to create a parallelogram. The sum of these two vectors can be represented as a diagonal line cutting through the parallelogram.

Scalar Multiplication

A vector can be scaled by a quantity. For example, a scalar multiplication of 2, increases the vector's size by 2x. While a scalar multiplication of -1 reverses the direction of the vector.

Vector Subtraction

If \vec{u} and \vec{v} are vectors, then $\vec{u} - \vec{v} = \vec{u} + (\vec{-v})$ This can be represented by drawing $\vec{-v}$, which is mirroring \vec{v} in the opposite direction. Then, we draw $\vec{-v}$ again but from the endpoint of \vec{u} . From there, we can finish the Parallelogram Law by drawing \vec{u} from the endpoint of the first $\vec{-v}$ to the endpoint of the second $\vec{-v}$. Then $\vec{u} - \vec{v}$ can be represented as a diagonal line cutting across the parallelogram.

Components

 $\vec{a} = \langle a_1, a_2, a_3 \rangle$ are components of \vec{a}

$$\vec{AB}=< x_2-x_1, y_2-y_1>$$
 If \vec{AB} is a vector with initial print $\vec{AB}=< x_2-x_1, y_2-y_1, z_2-z_1>$

$$\vec{AB} = <-2-2, 1-(-3), 1-4> = \boxed{<-4, 4-3>}$$

Magnitude (or Length) of a Vector

The length of the 2-dimensional vector, $\vec{a} = \langle a_1, a_2 \rangle$ is $\vec{a} = \sqrt{(a_1)^2 + (a_2)^2}$. The length of the 3-dimensional vector, $\vec{a} = \langle a_1, a_2, a_3 \rangle$ is $\vec{a} = \sqrt{(a_1)^2 + (a_2)^2}$

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Vector Addition and Scalar Math (cont.)

If $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$, then

1)
$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$$

2)
$$\vec{a} = \langle ca_1, ca_2 \text{ for any scalar c} \rangle$$

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$\mathbf{Ex} \ \mathbf{2}$

If
$$\vec{a} = <4, 0, 3 > \text{ and } -2, 1, 5 <_{1,2},_3 >$$

$$|\vec{a}| = \sqrt{4^2 + 0^2 + 3^2} = \sqrt{25} = \boxed{5}$$

Properties of Vectors

If \vec{a}, \vec{b} and \vec{c} are vectors and c and d are scalars, then

$$1) \quad \vec{a} + \vec{b} = \vec{b} + \vec{a}$$

2)
$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$

4)
$$\vec{a} + \vec{0} = \vec{a}$$

5)
$$\vec{a} + \vec{-a} = \vec{0}$$

6)
$$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$$

$$7) \quad (c+d)\vec{a} = c\vec{a} + d\vec{a}$$

8)
$$(cd)\vec{a} = c(d\vec{a})$$

9)
$$1 \cdot \vec{a} = \vec{a}$$

Standard Basis Vectors

$$\vec{i} = <1, 0, 0 > \vec{j} = <0, 1, 0 > , \vec{k} = <0, 0, 1 >$$

If
$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$
, then

$$\vec{a} = \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0, 0, a_3 \rangle$$

$$= a_1 < 1, 0, 0 > +a_2 < 0, 1, 0 > +a_3 < 0, 0, 1 >$$

$$= a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

$$= a_1 \vec{i} + a_2 \vec{i} + a_3 \vec{k}$$

Ex 3 If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 4\vec{i} + 7\vec{k}$, express the vector $2\vec{a} + 3\vec{b}$ in terms of \vec{i}, \vec{j} , and \vec{k} .

$$2(\vec{i}+2\vec{j}-3\vec{k})+3(4\vec{i}+7\vec{k})$$

$$\vec{i} + 4\vec{j} - 6\vec{k} + 12\vec{i} + 21\vec{k}$$

$$\boxed{14\vec{i} + 4\vec{j} + 15\vec{k}}$$

Unit Vector

A unit vector is a unit whose length is 1. Note that $\vec{a} \neq \vec{0}$, then the unit vector \vec{u} that has the same direction as \vec{a} is $\vec{u} = \frac{\vec{a}}{|\vec{a}|}$

Find the unit vector in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$,