

10.4 The Cross Product

Determinant of Order 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Ex 1

$$\begin{bmatrix} 2 & 1 \\ -6 & 4 \end{bmatrix} = 2(4) - 1(-6) = \boxed{14}$$

Determinant of Order 3

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Ex 2

$$1 \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ -5 & 4 & 2 \end{bmatrix} - 2 \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix} + (-1) \begin{bmatrix} 3 & 0 \\ -5 & 4 \end{bmatrix} \rightarrow \boxed{-38}$$

Definition

If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, then the cross product of \vec{a} & \vec{b} is the vector.

$$\vec{a} \cdot \vec{b} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

Ex 3

If $\vec{a} = \langle 1, 3, 4 \rangle$ & $\vec{b} = \langle 2, 7, -5 \rangle$ find $\vec{a} \cdot \vec{b}$.

$$\vec{a} \cdot \vec{b} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{bmatrix}$$

$$\vec{i} \begin{bmatrix} 3 & 4 \\ 7 & -5 \end{bmatrix} - \vec{j} \begin{bmatrix} 1 & 4 \\ 2 & -5 \end{bmatrix} + \vec{k} \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

$$\vec{i}(-15 - 28) - \vec{j}(-5 - 8) + \vec{k}(7 - 6)$$

$$\boxed{-43\vec{i} + 13\vec{j} + \vec{k}}$$

Example 4

Show that if $\vec{a} = \langle a_1, a_2, a_3 \rangle$, then $\vec{a} \cdot \vec{a} = 0$

Theorem

The vector $\vec{a} \cdot \vec{b}$ is orthogonal to both vectors \vec{a} & \vec{b} .

Proof

Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$ & $\vec{b} = \langle b_1, b_2, b_3 \rangle$. Now,

$$\begin{aligned} \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} &= \vec{i} \begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix} - \vec{j} \begin{bmatrix} a_1 & a_3 \\ b_1 & b_3 \end{bmatrix} + \vec{k} \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \\ &= \vec{i}(a_2b_3 - a_3b_2) - \vec{j}(a_1b_3 - a_3b_1) + \vec{k}(a_1b_2 - a_2b_1) \end{aligned}$$