When a basis $\mathcal{B}$ is chosen for an n-dimensional vector space V, the associated coordinate mapping onto $\mathbb{R}^n$ provides a coordinate system for V. Each x in V is identified uniquely bt its $\mathcal{B}$ coordinate vector $\left[x\right]_{\mathcal{B}}$.

Ex 1
Consider two bases $\mathcal{B} = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ for a vector space V, such that

$$b_1 = 4c_1 + c_2 \qquad b_2 = -6c_1 + c_2$$

Suppouse that

$$x = 3b_1 + b_2$$

That is, suppouse $\left[x\right]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Find $\left[x\right]_{C}$.

Apply the coordinate mapping determined by C.

$$\left[x\right]_C = \left[3b_1 + b_2\right]_C$$
$$\left[x\right]_C = \left[\left[b_1\right]_C \quad \left[b_2\right]_C\right] \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$$\left[x\right]_C = \begin{bmatrix} 4 & -6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6 \\ 4 \end{bmatrix}}$$

Theorem 5
Let $\mathcal{B} = \{b_1, ..., b_n\}$ & $C = \{c_1, ..., c_n\}$ be bases for a vector space V. There is a unique $n \times n$ matrix P sich that

$$\left[x\right]_C = P_{C \leftarrow B} \left[x\right]_B$$

and $P_{C \leftarrow B}$ is given by $P_{C \leftarrow P} = \left[\left[b_1\right]_C \quad \left[b_2\right]_C \quad \cdots \quad \left[b_n\right]_C\right]$