1.5 Linear Independence

Homogeneous Linear Systems
Given vectors $\{v_1, v_2, ...v_p\} \in \mathbb{R}^n$. We will solve the system of equations $x_1v_2 + x_2v_2 + ... + x_pv_p = 0$. $v_1v_1, ..., x_pv_p$ are vectors in $\mathbb{R}^n$ that we will be solving for and eventually give us values for $x_1, x_2, ..., x_p$.

Two Cases

1) If $x_1 = 0, x_2 = 0, ..., x_p = 0$ then we say that the set$\{v_1, v_2, ..., v_p\}$ is linearly independent.

2) If $x_1, ..., x_p$ are not all zeros, then the set$\{v_1, v_2, ..., v_p\}$ is linearly dependent.

Ex 1 p60

$$v_1 = \begin{bmatrix}1\\2\\3\end{bmatrix} v_2 = \begin{bmatrix}4\\5\\6\end{bmatrix} v_3 = \begin{bmatrix}2\\1\\0\end{bmatrix}$$

1A. Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent. 2B. If possible find a linear dependence relation among v_1, v_2, v_3.

$$\text{A) } x_1v_1 + x_2v_2 + x_3v_3 = 0, \qquad v_1, ...v_p\text{are vectors in } \mathbb{R}^3.$$

$$\begin{bmatrix}1&4&2&0\\2&5&1&0\\3&6&0&0\end{bmatrix} \to \begin{bmatrix}1&4&2&0\\0&-3&3&0\\0&0&0&0\end{bmatrix} \to \begin{bmatrix}1&4&2&0\\0&1&1&0\\0&0&0&0\end{bmatrix}$$

$$x_1,\ x_2 = \text{pivot},\ x_3 = \text{free}$$

$$x_3 = 1,\ x_2 = -1,\ x_1 = -4x_2 - 2x_3 \to 2$$

$(2, -1, 1)$ is a solution. So the set $\{v_1, v_2, v_3\}$ is linearly dependent.

B) Linear Dependence Relation

$$2v_1 - v_2 + v_3 = 0$$

Ex 2 p61

Determine if the columns of a matrix, A are linearly independent.

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -2 \\ 5 & 8 & 0 \end{bmatrix}$$

$x_1c_1 + x_2c_2 + x_3c_3 = 0,$ is what we are solving for where c_n represents a column.

$$x_1 \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Ax = 0$$

$$\begin{bmatrix} 0 & 1 & 4 & 0 \\ 1 & 2 & -1 & 0 \\ 5 & 8 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 13 & 0 \end{bmatrix}$$

$$x_3 = 0,\ x_2 = 0,\ x_1 = 0,\ \text{linearly independent}$$

Due to no free variables existing. Meaning that there is only a trivial solution for the equation $Ax = 0$ thus the columns of A are linearly independent.