11.1 Functions of Several Variables

Definition

A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the domain of f and its range is the set of values f takes on, that is

$$\{f(x, y) | (x, y) \in D\}$$

At often times, $z = f(x, y)$ is written to make explicit the value taken on by f at the general point (x, y). So x & y are independent variables and z is the dependent variable.

A function of two variables is just a function whose domain is a subset of $\mathbb{R}^2$ and whose range is a subset of $\mathbb{R}$. If a function f is given by a formula and no domain is specified, then the domain of f is the set $\{(x, y) | x, y \in \mathbb{R}\}$.

Ex 1

Find the domains of the following functions and evaluate $f(3, 2)$.

A) $f(x, y) = \frac{\sqrt{x+y+1}}{x-1}$

B) $f(x, y) = x \ln(y^2 - x)$

Ex 1A

$$f(x, y) = \frac{\sqrt{x + y + 1}}{x - 1}$$

$$f(3, 2) = \frac{\sqrt{6}}{2}$$

$$D = \{(x, y) | x + y + 1 \geq 0, x \neq 1\}$$

Ex 1B

$$f(x, y) = x \ln(y^2 - x)$$

$$f(3, 2) = 0$$

$$D = \{(x, y) | y^2 - x > 0\}$$

Ex 2

Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

$$D = \{(x, y) | 9 - x^2 - y^2 \geq 0\} \qquad R = \{z | z = \sqrt{9 - x^2 - y^2}, (x, y) \in D\}$$