4.2

Nul SPaces, Column Spaces, Row Spaces and Linear Transformations
Given an $m \times n$ matrix $A = (a_{ij})$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Def
Null Space of A is $NulA = \{x \in \mathbb{R}^n LAx = 0\}$. The Column Space of A is $ColA = \text{Span}\{a_1, a_2, ..., a_n\}$. The Row Space of A is $RowA = \text{Span}\{r_1, r_2, ...r_m\}$.

Ex 3
Find a spanning set for the null space of the matrix. As in solve $Ax = 0$

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & 4 \end{bmatrix}$$

Solution, find all x that satisfies $Ax = 0$

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 & 0 \\ 1 & -2 & 2 & 3 & -1 & 0 \\ 2 & -4 & 5 & 8 & 4 & -0 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x must be in $\mathbb{R}^n$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_2 + x_4 - 3x_5 \\ x_2 \\ -2x_4 + 2x_5 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Ex 4

let $w = \{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in \mathbb{R}\}$. Find a matrix A, such that $w = ColA$.

$$w = \{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in \mathbb{R}\} = \{a \begin{bmatrix} 6 \\ 1 \\ -7 \end{bmatrix} + b \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} : a, b \in \mathbb{R}\} = \text{Span}\{ \begin{bmatrix} 6 \\ 1 \\ -7 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \} = ColA, \text{ where } A = \begin{bmatrix} 6 & -1 \\ 1 & 1 \\ -7 & 0 \end{bmatrix}$$