10.1 3-Dimensional Coordinate Systems

Introduction
In 2d, we are able to move along two axises.

$$\mathbb{R}^2 = \{\ (x, y)\ |\ x, y \in \mathbb{R}\}$$

While in 3D, we can move along three axises.

$$\mathbb{R}^3 = \{\ (x, y, z)\ |\ x, y, z \in \mathbb{R}\}$$

The 3 planes divide space into 8 parts called octants. In $\mathbb{R}^2$, the graph of an equation in x & y is a curve. While in $\mathbb{R}^3$, the equation of an equation in x, y & z is a surface.

Distance Formula in 3 Dimensions

$$|P_1, P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Distance between Points $P_1\ (x_1, y_1, z_1)$ & $P_2(x_2, y_2, z_2)$

Ex 2
Calculate the distance between the points P(2,-1,3) and Q(1,-3,5)

$$(x_1, y_1, z_1) = (2, -1, 3)$$

$$(x_2, y_2, z_2) = (1, -3, 5)$$

$$|P\ Q| = \sqrt{(1-2)^2 + (-3-(-1))^2 + (5-3)^2} \to 3$$

Equation of a Sphere
An equation of a sphere with center C (h,k,e) and radius r is $(x-h)^2 + (y-k)^2 + (z-e)^2 = r^2$

Ex 3
Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is the equation of a sphere and find its center and radius

$$(x^2 + 4x) + (y^2 - 6y) + (z^2 + 2z) = -6$$

Completing the square

$$(\frac{4}{2})^2 = 4, (\frac{-6}{2})^2 = 9, (\frac{2}{2})^2 = 1$$

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) + (z^2 + 2z + 1) = -6 + 4 + 9 + 1$$

$$(x+2)^2 + (y-3)^2 + (z+1)^2 = 8$$

Ex 4
What region is $\mathcal{R}$ represented by the following inequalities?
$1 \leq a^2 + y^2 + z^2 \leq 4,\ z \leq 0$
Region that lies between (or on) the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ and beneath (or on) the xy plane