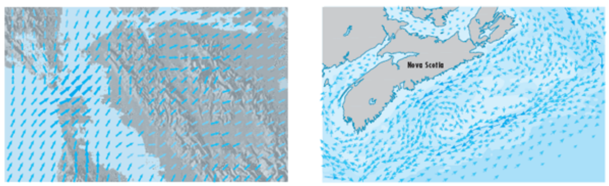
13.1 Vector Fields

In Figure 1(A), the vectors are air velocity vectors indicating the wind speed and direction at points $10m$ above the surface elevation in the San Francisco Bay Area at $6:00$ pm on March 1, 2010.

THe largest arrows indicate the greatest wind speeds at the the time occured. At every point in the air, there is a wind velocity vector. This is an example of a velocity vector field.



(a) San Francisco Bay wind patterns (b) Ocean currents off the coast of Nova Scotia

Figure 1(B) is another example of a velocity vector field.

There are other types of vector fields, namely a force field, where each point in a region is associated with a force vector.

To generalize, a vector field is a function whose domain is a set of points in $\mathbb{R}^2$ or $\mathbb{R}^3$ and whose range is a set of vectors in v_2 or v_3.

Def
Let D be a set in $\mathbb{R}^2$ (a plane region). A vector field on $\mathbb{R}^2$ is a function F that assigns to each point (x, y) in D a two-dimesnional vector $F(x, y)$.

Since $F(x, y)$ is a two-dimensional vector, it can be written in terms of its component functions P & Q as follows

$$F(x, y) = P(x, y)i + Q(x, y)j =< P(x, y), Q(x, y) >$$

Or, in its shorthanded form,

$$F = Pi + QJ$$

Def
Let E be a subset of $\mathbb{R}^3$. A vector field on $\mathbb{R}^3$ is a function F that assigns to each point $x, \vec{y}, z$ in E a three-dimensional vector $F(x, y, z)$.

Since $F(x, y, z)$ is a three-dimensional vector, it can be represented in terms of its component functions P, Q & R as

$$F(x, y, z) = P(x, y, z)i + Q(x, y, z) + R(x, y, z)k$$

or, for short,

$$F = Pi + Qj + Rk$$

Just like with vector functions, we can define the continuity of vector fields and show that F is continuous if and only if its component functions $P, Q,$ & R are continuous.

A point (x, y, z) is sometimes identified with its position vector $x = x, \vec{y}, z$ and $F(x)$ is written in place of $F(x, y, z)$. Then F becomes a function that assigns a vector $F(x)$ to a vector x.