An Optical Reservoir Computing Design based on Two-Dimensional Quantum Walk

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Outline

- **Preface: Quantum Walks, Entanglement and Interference**
- Introduction: Reservoir Computing & Extreme Learning Machine
- Method: Two-Dimensional Quantum Walk Reservoir Computing (2D-QWRC)
  - Architecture and Simulations
  - Evaluation Results
- Conclusions and Perspectives

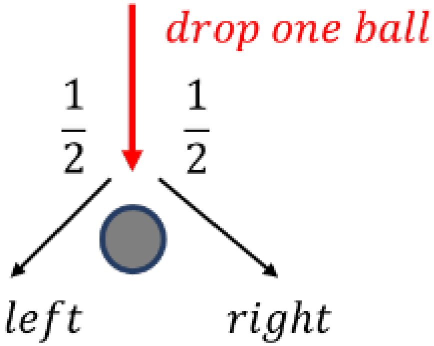
Quantum Walks

Questions ahead

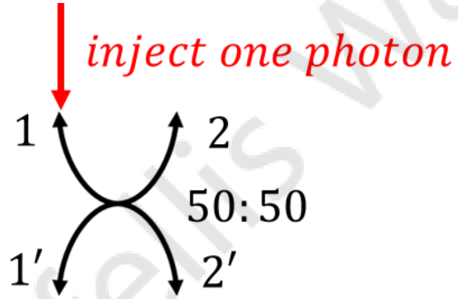
- **Why** Quantum?
- **How** to Introduce Quantum Benefits into Computational Architectures?
- **What** Kind of Quantum Properties Shall We Exploit?

Classical Random Walk

Galton Board



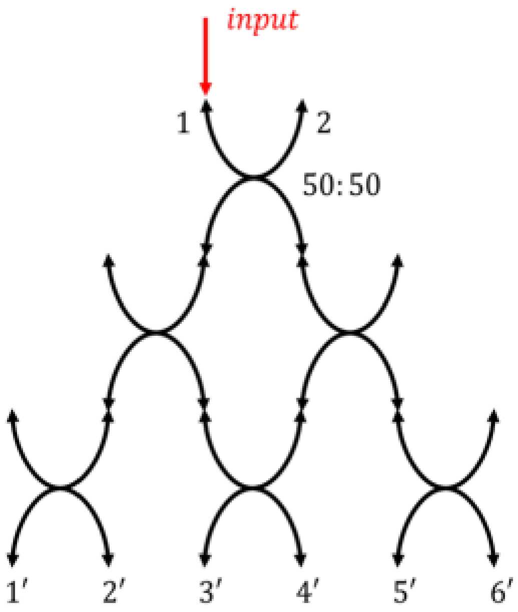
- spatial
- fixed possibility
- discrete



- can walk in **different dimension & quantities**
- **adjustable** possibility
- **discrete/continuous** walks
- **quantum entanglement & interference**

…

Quantum Walk



Splitter Array

Quantum Walks

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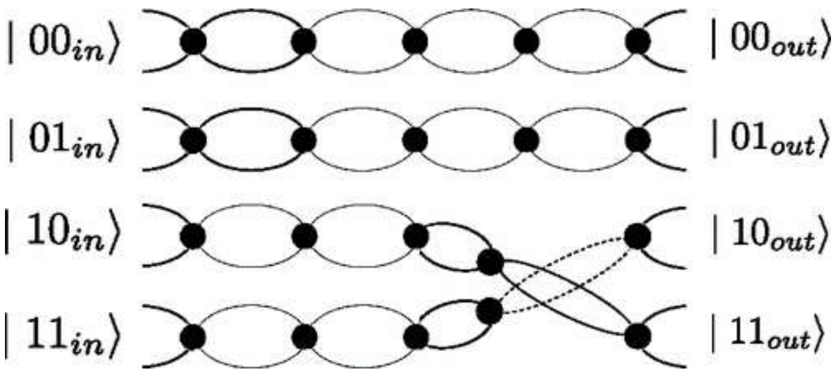
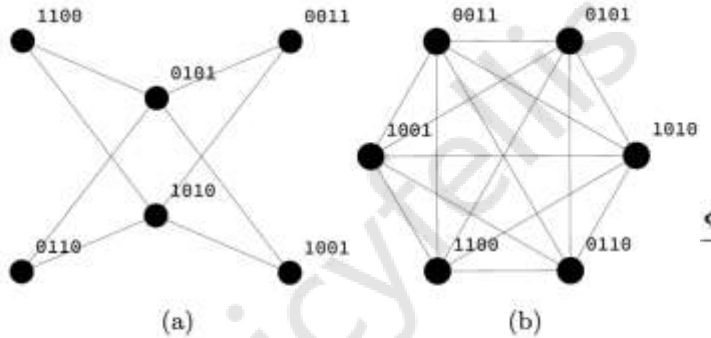
Quantum Walk (QW): Powerful and Convenient Physical Platform for Quantum Algorithms

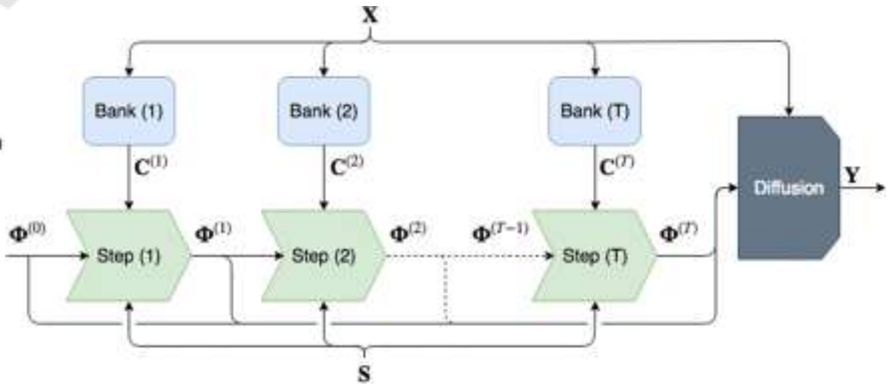
I. Multiple **quantum computing algorithms** realized through Quantum Walks



[1] universal quantum computation



[2] combinatorial optimization algorithms



[3] machine learning

[1] N. B. Lovett, et al., Universal quantum computation using the discrete-time quantum walk, Phys. Rev. A, vol. 81, p. 042330, Apr. 2010.
[2] S. Marsh, et al., Combinatorial optimization via highly efficient quantum walks, Phys. Rev. Res., vol. 2, p. 023302, June 2020.
[3] S. Dernbach, et al., Quantum walk neural networks with feature dependent coins, Applied Network Science, vol. 4, no. 1, p. 76, Sept. 2019.

Quantum Walks

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- **Why** Quantum?
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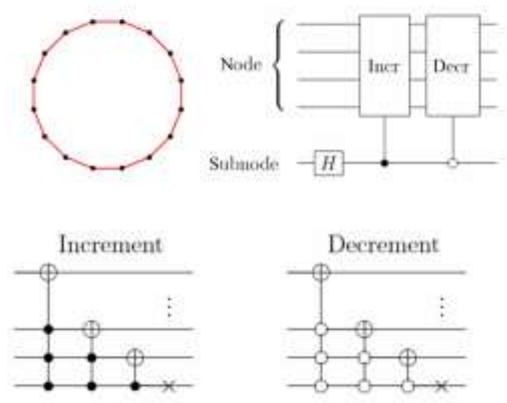
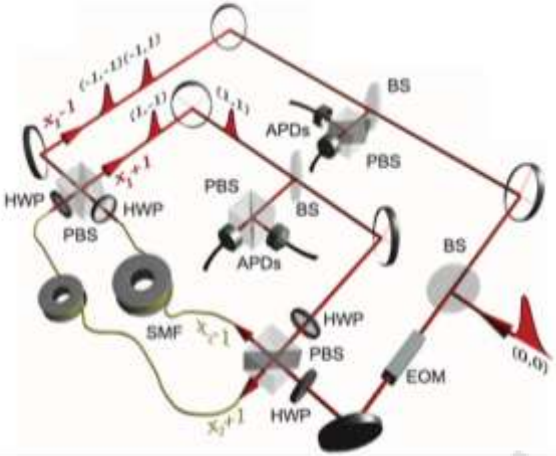
Quantum Walk (QW): Powerful and Convenient Physical Platform for Quantum Algorithms

II. Discrete & Continuous QWs of different dimensions implemented on various **physical systems**

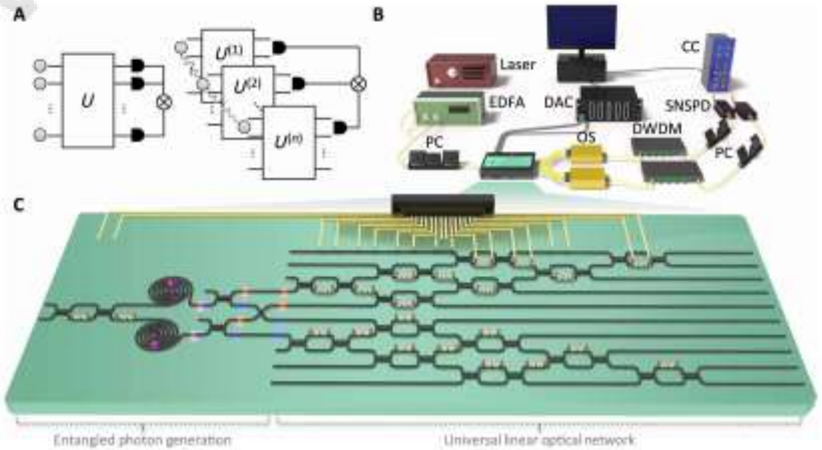


[4] quantum circuit



[5] laser optics

[6] superconducting material



[7] integrated photonics

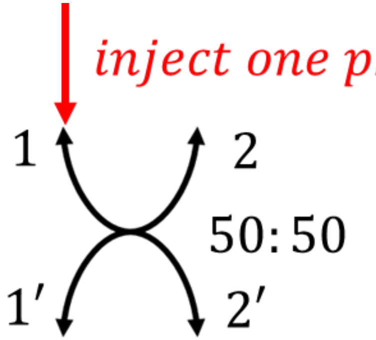
[4] Douglas BL, et al., Efficient quantum circuit implementation of quantum walks. Phys Rev A.2009;79(5):Article 052335.
[5] A. Schreiber, et al., A 2D Quantum Walk Simulation of Two-Particle Dynamics, Science, vol. 336, no. 6077, pp. 55–58, 2012.
[6] Gong M, et al., Quantum walks on a programmable two-dimensional 62-qubit superconducting processor. Science. 2021;372(6545):948–952.
[7] Qiang X, et al., Implementing graph-theoretic quantum algorithms on a silicon photonic quantum walk processor. Sci Adv. 2021;7(9):Article eabb8375.

Classical Random Walk & Quantum Walk

Toy example: Galton Board-like random walk and its quantum version

A. 2-indistinguishable photons injected **when entanglement emerges**

inject one photon



$$|\psi\rangle_{in} = \hat{a}_1^\dagger|0\rangle$$

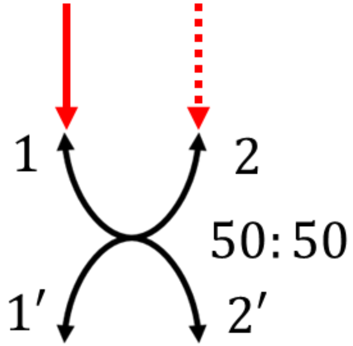
$$\begin{pmatrix}\hat{a}_{1'}^\dagger\\ \hat{a}_{2'}^\dagger\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{pmatrix}$$

$$|\psi\rangle_{out} = \frac{1}{\sqrt{2}}(\hat{a}_{1'}^\dagger + i\hat{a}_{2'}^\dagger)|0\rangle$$

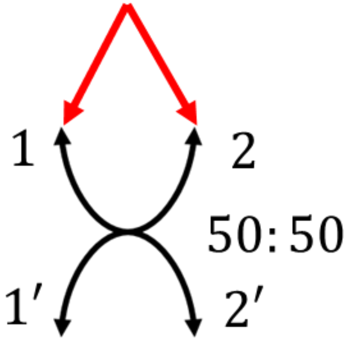
connect two independent states

$$|\psi\rangle_{combination} = |\psi_1\rangle_{in}\otimes|\psi_2\rangle_{in}$$

two independent injection



a single photon enters simultaneously

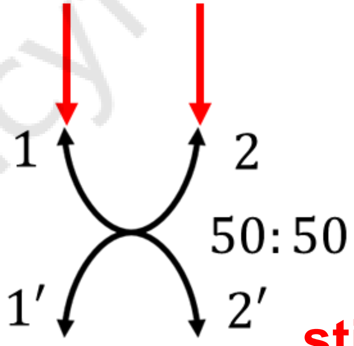


superposition

$$|\psi\rangle_{in} = \frac{1}{\sqrt{2}}(\hat{a}_1^\dagger + \hat{a}_2^\dagger)|0\rangle$$

$$|\psi\rangle_{out} = \frac{1+i}{2}(\hat{a}_1^\dagger + \hat{a}_2^\dagger)|0\rangle$$

indistinguishable photons



$$|\psi\rangle_{in} = \hat{a}_1^\dagger\,\hat{a}_2^\dagger|0\rangle$$

$$|\psi\rangle_{out} = \frac{i}{2}((\hat{a}_{1'}^\dagger)^2 + (\hat{a}_{2'}^\dagger)^2)|0\rangle$$

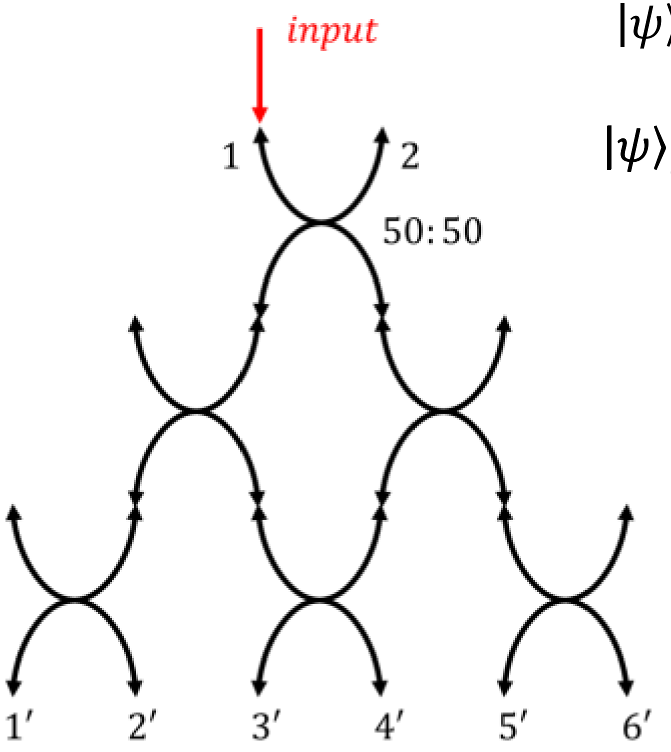
"path-entangled two-photon state"

stick indistinguishable states together

[8] Markus Gräfe, et al., 2020 J. Phys. B: At. Mol. Opt. Phys. **53** 073001.

Classical Random Walk & Quantum Walk

B. 3-step Galton Board-like walk **when quantum interference emerges**



Photons

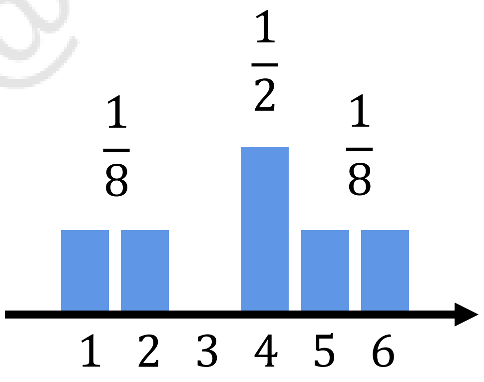
$$|\psi\rangle_{in} = \hat{a}_1^\dagger|0\rangle$$

Destructive interference

$$|\psi\rangle_{out} = \frac{1}{2\sqrt{2}}\left(-\hat{a}_1^\dagger + i\hat{a}_2^\dagger + \hat{a}_3^\dagger - \hat{a}_3^\dagger + 2i\hat{a}_4^\dagger - \hat{a}_5^\dagger - i\hat{a}_6^\dagger\right)|0\rangle$$

$$= \frac{1}{2\sqrt{2}}\left(-\hat{a}_1^\dagger + i\hat{a}_2^\dagger + 2i\hat{a}_4^\dagger - \hat{a}_5^\dagger - i\hat{a}_6^\dagger\right)|0\rangle$$

Average Photon Number



To avoid interference:

Introduce random phase noise θ_1 & θ_2 in each step

$$\begin{pmatrix} \hat{a}_{1'}^\dagger \\ \hat{a}_{2'}^\dagger \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} \hat{a}_1^\dagger e^{i\theta_1} \\ \hat{a}_2^\dagger e^{i\theta_2} \end{pmatrix}$$

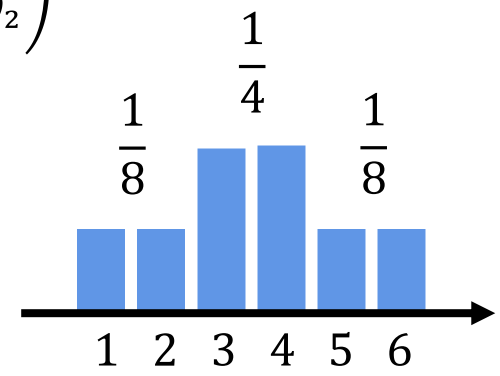
Classical

Balls

$$P(k') = \binom{k}{3}\left(\frac{1}{2}\right)^{k+1} \quad (k \in \boldsymbol{N})$$

Binomial distribution

Probability



[8] Markus Gräfe, et al., 2020 J. Phys. B: At. Mol. Opt. Phys. **53** 073001.

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Reservoir Computing & Extreme Learning Machine

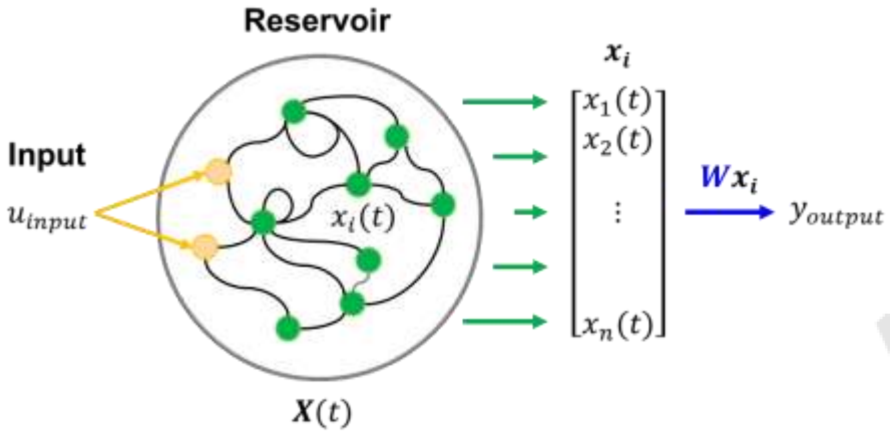
- **Given**: Training Input signal $\boldsymbol{u_{train}}(t)$, target output $\boldsymbol{y_{train}}(t)$
- **Target**: **a filter $\mathcal{F}$:** input $\boldsymbol{u_{train}}(t)$, generates an output signal $\boldsymbol{\hat{y}_{train}}(t) \to$ target $\boldsymbol{y_{train}}(t)$

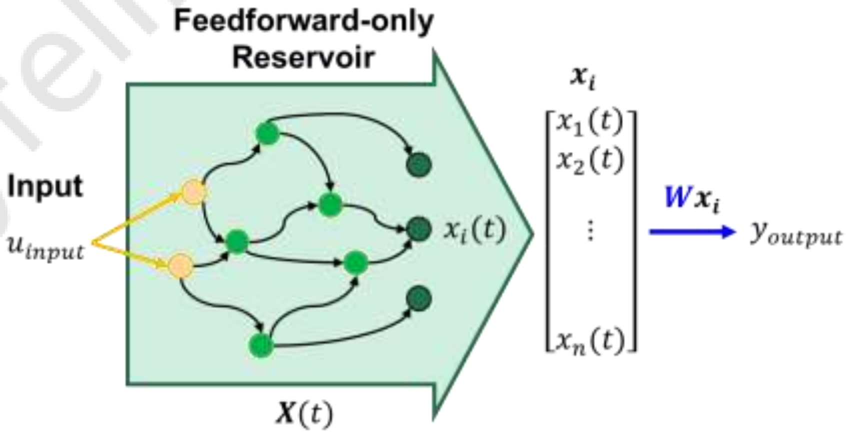
- **Approach**
  - Step1: Prepare reservoir $\boldsymbol{X}(t)$, observable variables $x_i(t)$
  - Step2: Drive reservoir by input $\boldsymbol{u_{train}}(t)$, get the internal variables $\boldsymbol{x_i^{train}}(t)$
  - Step3: Find a readout vector $\boldsymbol{W}$: $\boldsymbol{W}\boldsymbol{x_i^{train}}(t) = \boldsymbol{\hat{y}_{train}}(t) \sim \boldsymbol{y_{train}}(t)$

Reservoir Computing (RC) $\ni$ **Extreme Learning Machine (ELM)**

Interconnected nodes with nonlinearity

Forward-propagation only



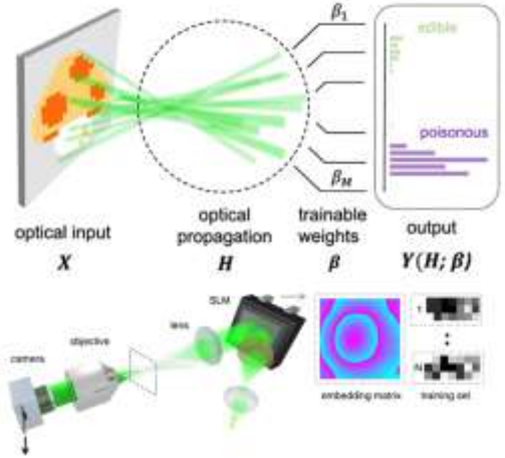


[9] Ingo Fischer, et al., Reservoir Computing Theory, Physical Implementations, and Applications Book, 2021.
[10] Gauthier. et al. Next generation reservoir computing. Nat Commun **12**, 5564, 2021.

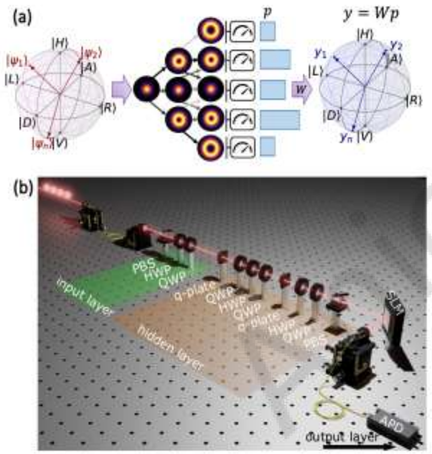
Existing Works & Limitations

Comparison		[11] PELM	[12] QELM *empirical platform*	[13] Boson Sampler ELM
Reservoir Dynamics		Classical: Coherent wave propagation in free space	Discrete quantum walk on bulk optics	Photons propagate through interferometer matrices
Reservoir Size	**Input channels**	28×28 (for MNIST)	2	28×28 (for MNIST)
	Output channels	~4000 even more	5	$\frac{M!}{(N!(M-N)!)}$ (N~2; M~40)
Application Example		Classification/Regression (Abalone & MNIST)	Estimating expectation values of the Pauli matrices	Classification (MNIST & k-MNIST)

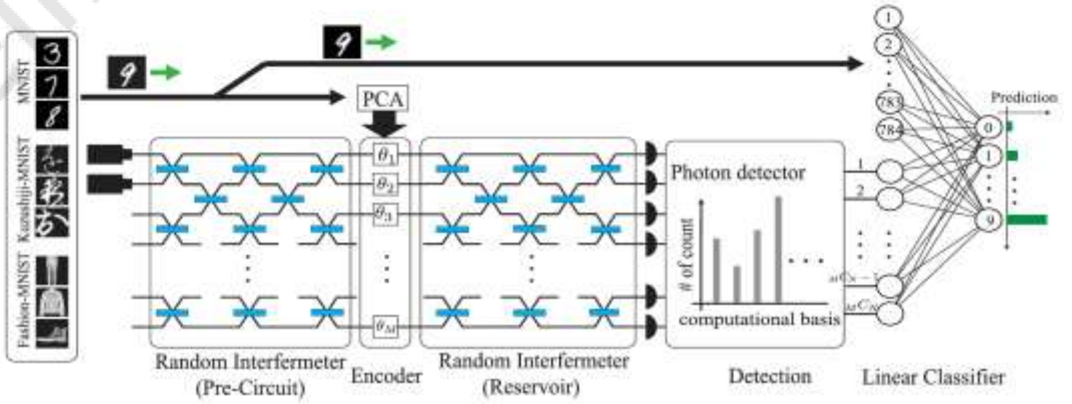
[11]



[12]



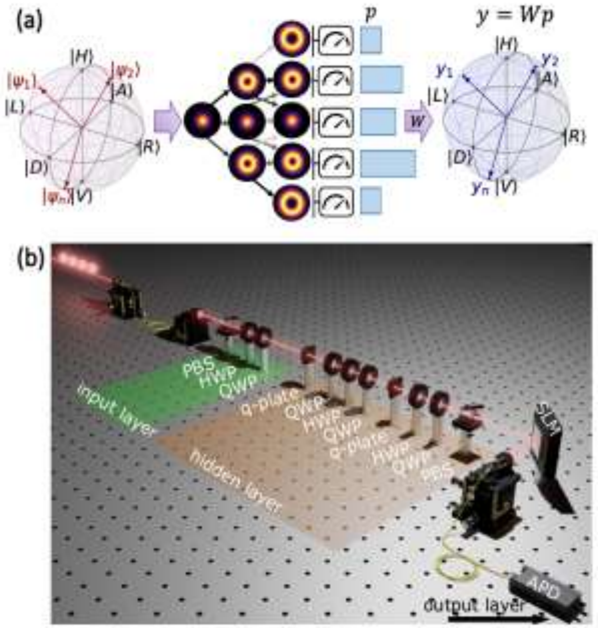
[13]



[11] Davide Pierangeli, et al., Photonic extreme learning machine by free-space optical propagation, Photon. Res. 9, 1446-1454, 2021
[12] A. Suprano, et al., Experimental Property Reconstruction in a Photonic Quantum Extreme Learning Machine, Phys. Rev. Lett., vol. 132, p. 160802, 2024.
[13] A. Sakurai, et al., Quantum optical reservoir computing powered by boson sampling, Optica Quantum, vol. 3, pp. 238–245, 05 2025.

Existing Works & Limitations

[12] QW-based ELM (QELM)



Limited Quantum Benefits, Application Scenarios and Cascade Ability!!

Limitations

- Weak quantum features
- One-dimensional quantum walk can be exactly mapped onto EM wave phenomena [14]

- Weak nonlinearity & simple intrinsic dynamic
- Nonlinearity comes from encoding layer & detection process, QW does not introduce nonlinearity [15]

- Bulk optical components

Counterplay

- Multi-dimensional QW
- **Interference** between different possibility distribution (coin DOFs) emerges

- Entangled States in QW
- **Entanglement** stick states in different dimensions together, enhancing **interconnected dynamics**

- Potentially compatibility with integrated quantum optics

[14] A. Peruzzo et al., Quantum walks of correlated photons, Science, vol. 329, no. 5998, pp. 1500–1503, 2010, doi: 10.1126/science.1193515.
[15] L. Innocenti et al., Potential and limitations of quantum extreme learning machines, Communications Physics, vol. 6, no. 1, pp. 118, May 2023, doi: 10.1038/s42005-023-01233-w.

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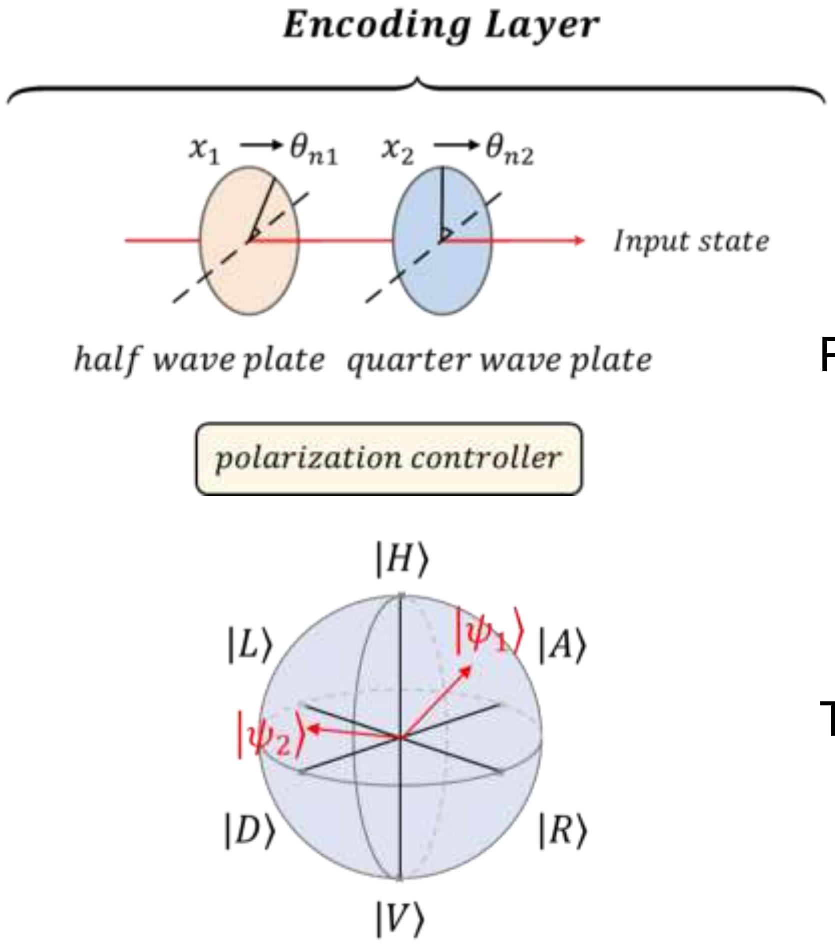
Two-Dimensional Quantum Walk Reservoir Computing (2D-QWRC)

I: Encoding Layer encodes data to polarization states



- Input data: x_1 & x_2
- Normalize: $x_{max} \rightarrow \pi$; coefficient $m = \frac{\pi}{x_{max}}$
- Linear mapping: $\theta_1 = mx_1, \theta_2 = mx_2$

Polarization state:

$$|\psi\rangle_{in} = \boldsymbol{G_{QWP} G_{HWP}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

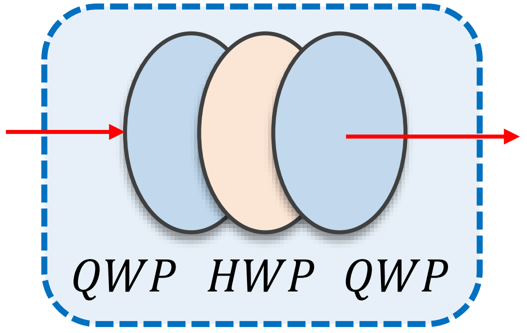
$$\boldsymbol{G_{Waveplate}} = \begin{pmatrix} cos\theta & sin\theta \\ sin\theta & -cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & exp(i\delta) \end{pmatrix} \begin{pmatrix} cos\theta & sin\theta \\ sin\theta & -cos\theta \end{pmatrix}$$

Two dimensional QW: **connect two state by tensor product**

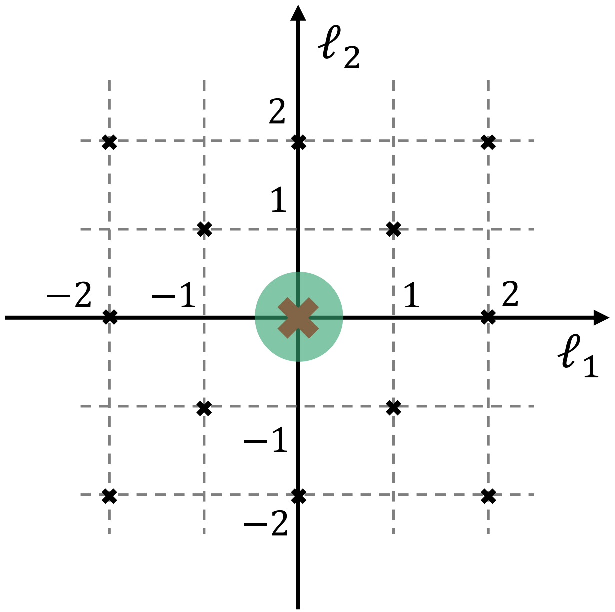
$$|\psi\rangle_{input} = |\psi\rangle_{in} \otimes |\psi\rangle_{in}$$

Two-Dimensional Quantum Walk Reservoir Computing (2D-QWRC)

II: Reservoir Layer performs 2-step quantum walks with entanglement



$coin$



- **Entangle two dimensions**: entanglement matrices

$$\hat{C}_{total} = (\hat{C}_1 \otimes \hat{C}_2)\hat{C}_{entangle}$$

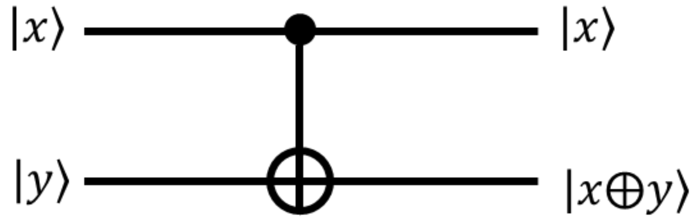
- Prepare two-dimensional **Coins**: polarization manipulation

$$\hat{C} = \boldsymbol{G}_{QWP}\boldsymbol{G}_{HWP}\boldsymbol{G}_{QWP} = \begin{pmatrix} e^{-i(\zeta-\phi)}cos\eta & e^{i(\zeta+\phi)}sin\eta \\ -e^{-i(\zeta+\phi)}sin\eta & e^{i(\zeta-\phi)}cos\eta \end{pmatrix} \qquad (\eta = \zeta + \phi - 2\theta)$$

$$\hat{C}_{com} = \hat{C}_1 \otimes \hat{C}_2$$

$$\hat{C}_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

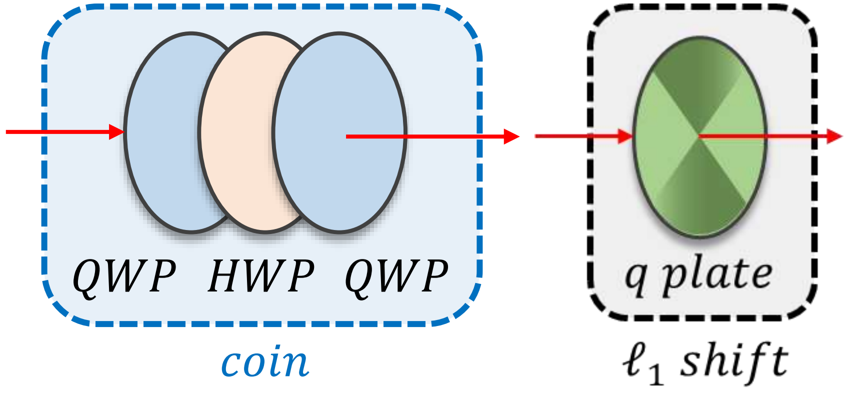
$$\hat{C}_{upper\_triangle} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



A typical two-qubit CNOT gate

Two-Dimensional Quantum Walk Reservoir Computing (2D-QWRC)

II: Reservoir Layer performs 2-step quantum walks with entanglement

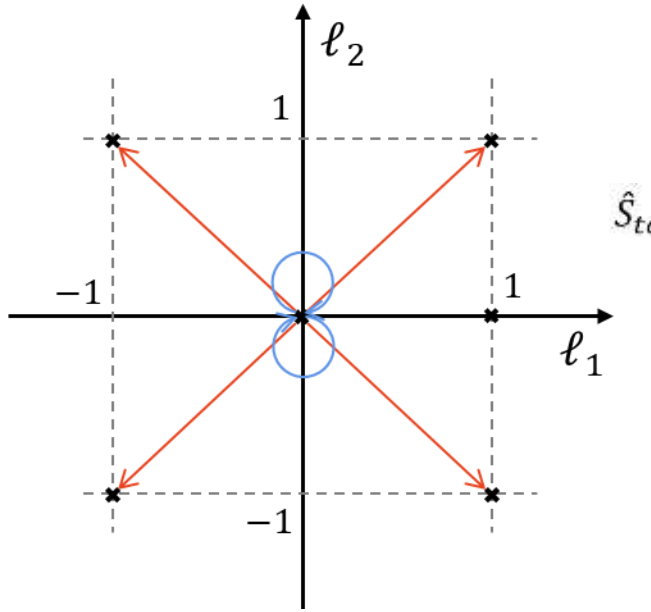


- **Shifts**: **Walk and interfere in OAM space**

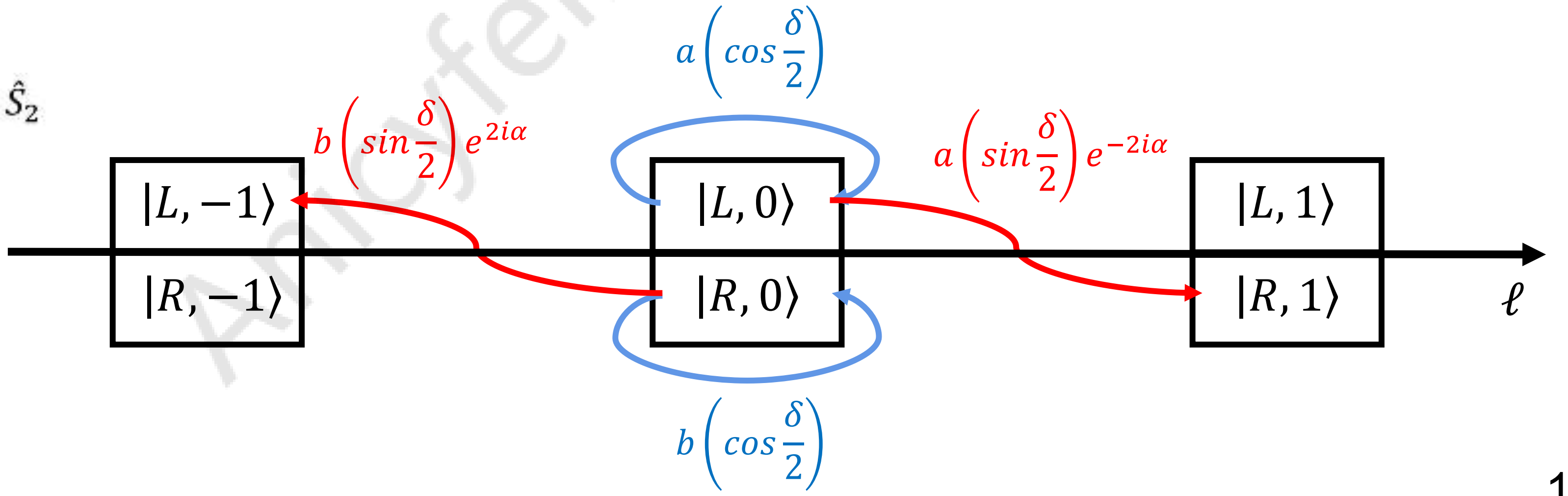
$$\hat{S} = \sum_{\ell} \Big\{ \cos\frac{\delta}{2}\left(|L,\ell\rangle\langle L,\ell| + |R,\ell\rangle\langle R,\ell|\right) + \sin\frac{\delta}{2}\left(e^{2i\alpha}|L,\ell\rangle\langle R,\ell+1| + e^{-2i\alpha}|R,\ell\rangle\langle L,\ell-1|\right) \Big\}$$

- **Polarization states** determine the walking possibilities (average photon number) to different **OAM states**

$$|\psi\rangle_{in} = a|L\rangle + b|R\rangle\ (at\ OAM\ value\ \ell_1 = 0))$$



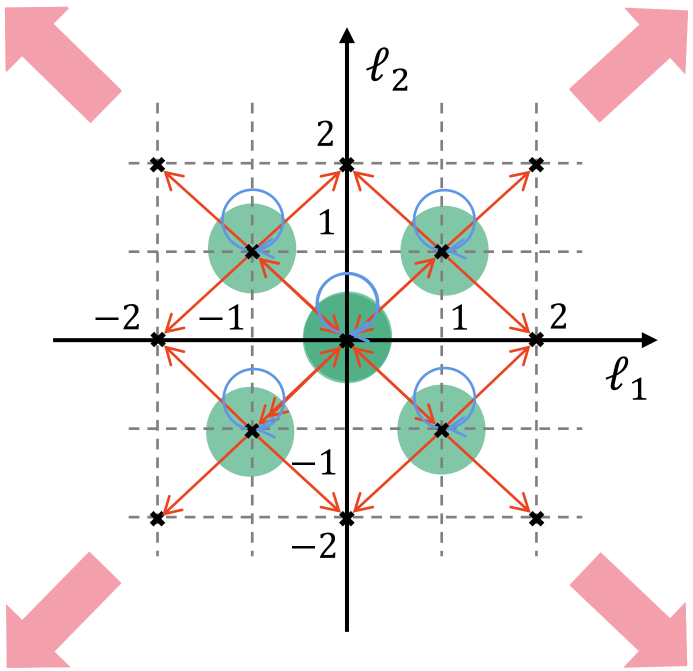
$$\hat{S}_{total} = \hat{S}_1 \otimes \hat{S}_2$$



Two-Dimensional Quantum Walk Reservoir Computing (2D-QWRC)

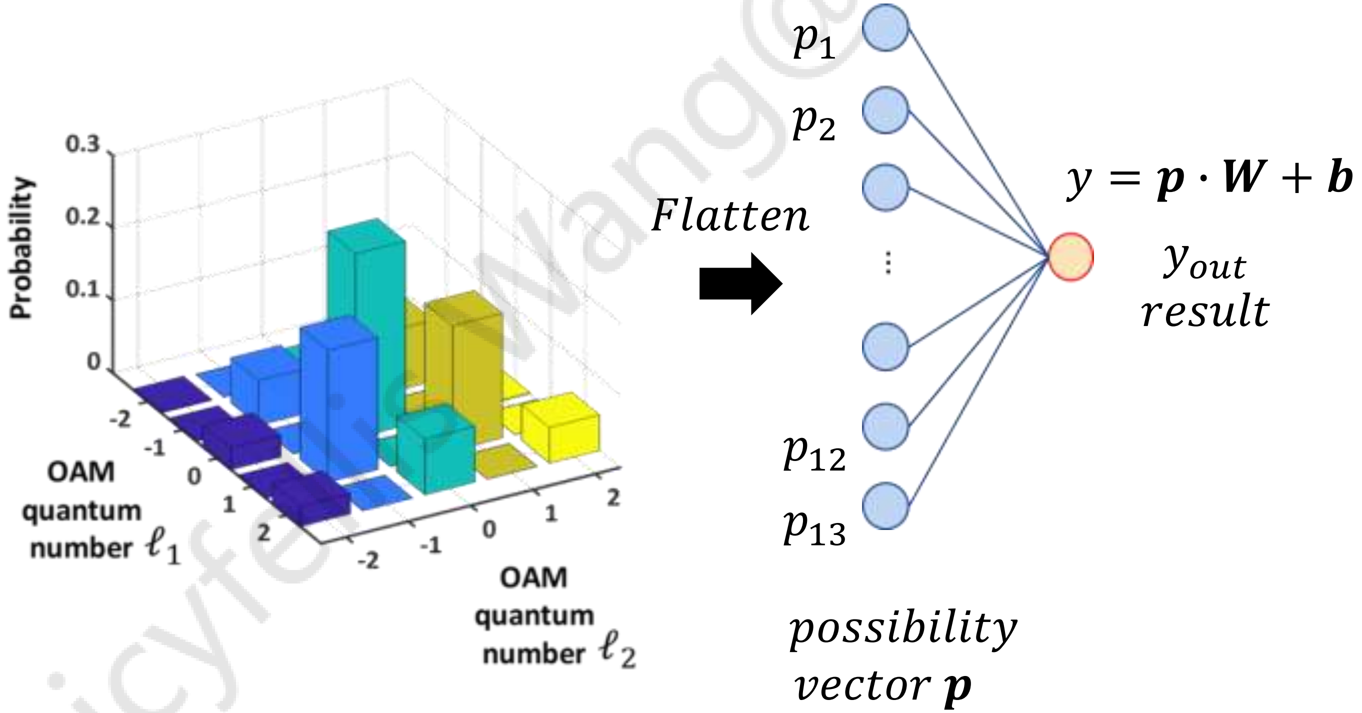
Multi-step: Coin operators & Shift operators act alternately

$$\hat{S}'_{total}\hat{C}'_{total}\,\hat{S}_{total}\hat{C}_{total}|\psi\rangle_{input}$$



: where entanglement and interference join the game

III: Output Layer optimizes weights and generates the final result



Optimize weight vector $\boldsymbol{W}$ conveniently with **damped least square** (also called Levenberg-Marquardt algorithm)

Evaluation Results

I: Function fitting task

- **Training dataset**: points sampled from original functions, with interval of 0.002
- **Models**: To keep weight vector sizes in output layer the same (13 parameters)
  - 2D-QWRC: 2-step 2D quantum walk
  - 2D-QWRC with entanglement: 2-step 2D quantum walk with CNOT entangle matrices
  - QELM: 6-step 1D quantum walk
- **Objective/Evaluation Function**: **MSE** between original functions and predicted functions

Original functions

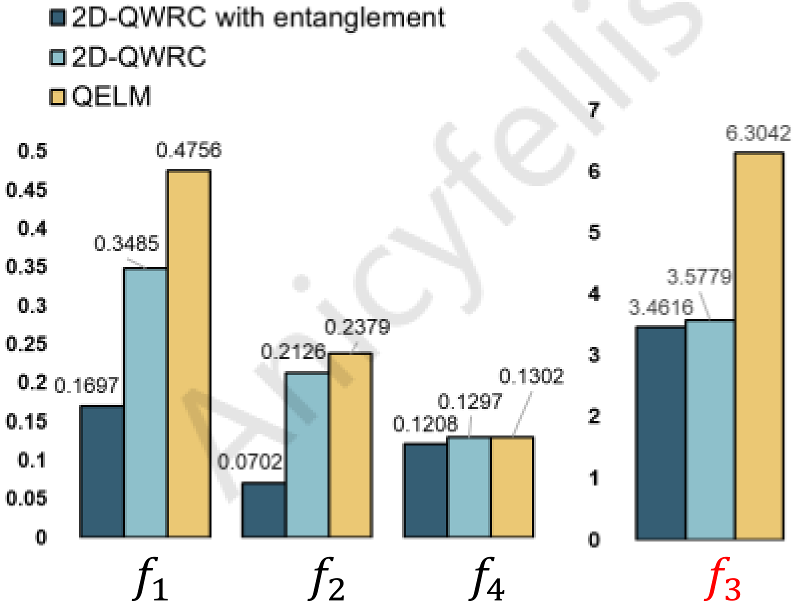
$$f_1 = \frac{\sin\pi x}{\pi x} e^{-x} + 0.01\xi \; (-3 < x < 0)$$

$$f_2 = \sin\pi x + 0.01\xi \; (0 < x < 3)$$

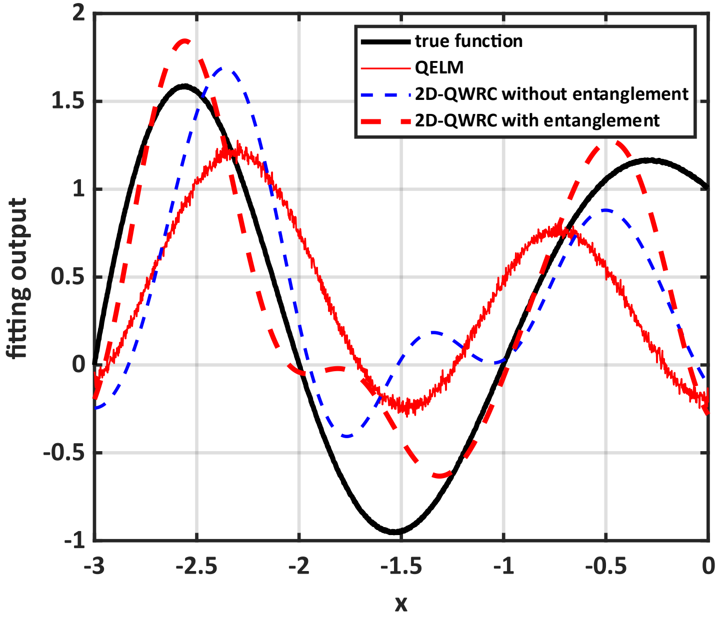
$$f_3 = e^{x} + 0.01\xi \; (0 < x < 3)$$

$$f_4 = \begin{cases} 0 & (-2 < x \le 0) \\ 1 & (0 < x < 2) \end{cases}$$

Evaluation results: MSE



Fitting results of f_1



Evaluation Results

I: Function fitting task

Original functions

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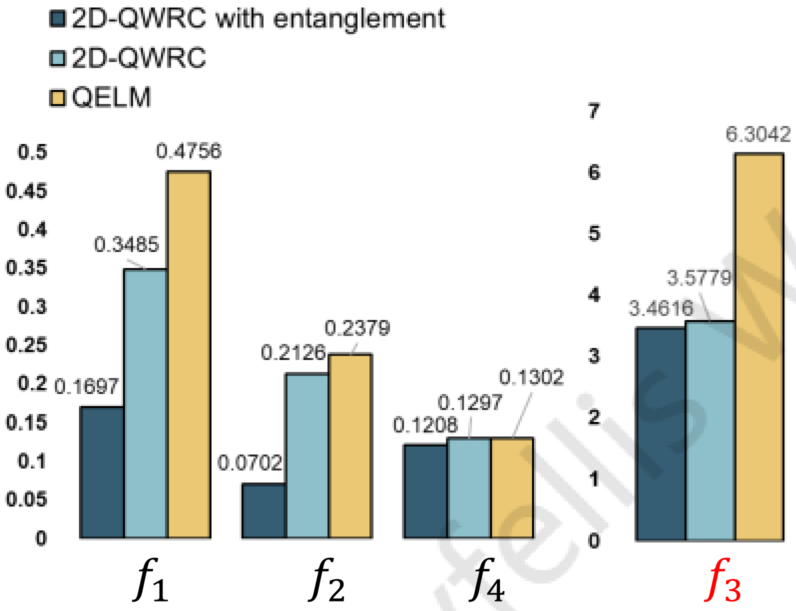
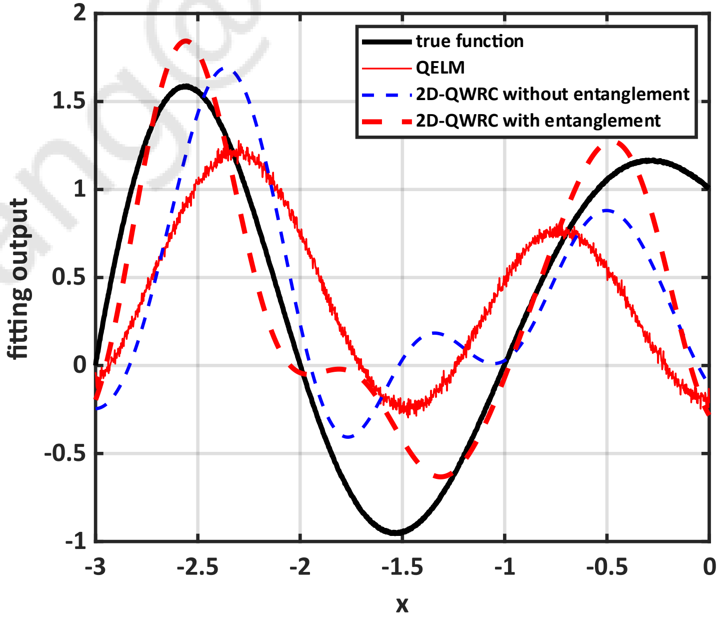
Evaluation results: MSE



Fitting results of f_1



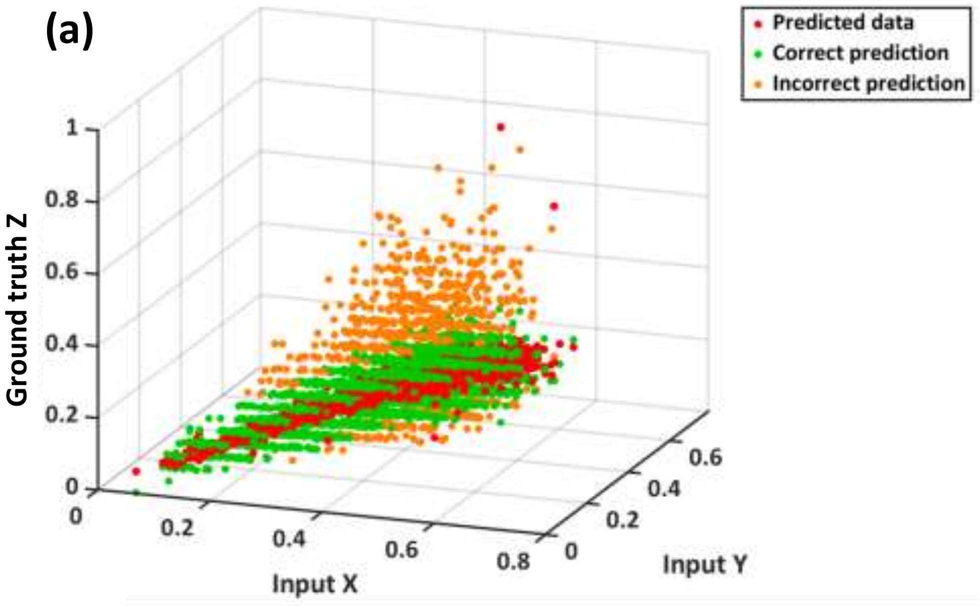
- Obvious and universal high-dimensional QW benefits
- Obvious and universal entanglement benefits
- Weak when fitting non-periodic functions ← encoding method

Evaluation Results

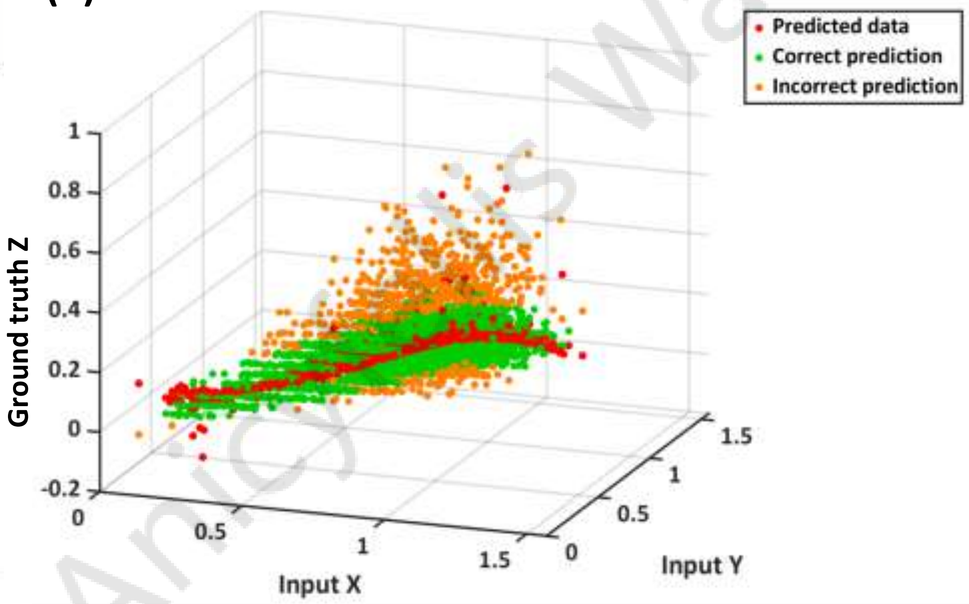
II: Abalone dataset prediction task

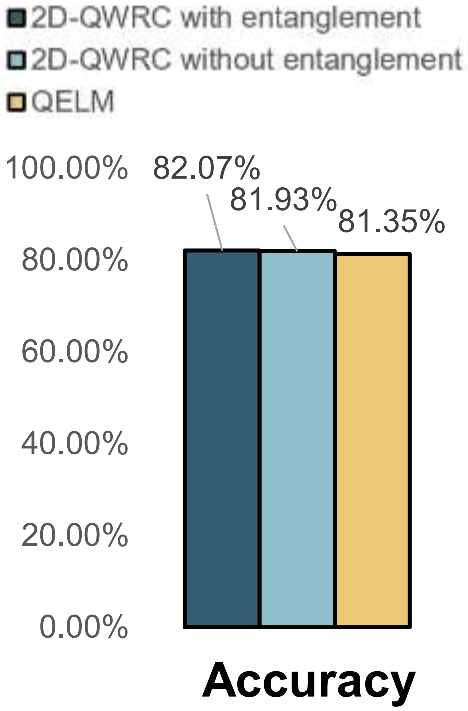
- **Training dataset**: Abalone dataset on lengths & diameters, to predict the number of rings of abalone
- **Models**: as stated formerly
- **Objective/Evaluation Function**: Accuracy, allowed error to be 0.1

QELM prediction



2D QWRC without entanglement prediction





- Prediction accuracy: potential in universal machine learning tasks
- No entanglement benefits! No high-dimensional benefits!

Outline

- Preface: Quantum Walks, Entanglement and Interference
- Introduction: Reservoir Computing & Extreme Learning Machine
- Method: Two-Dimensional Quantum Walk Reservoir Computing (2D-QWRC)
  - Architecture and Simulations
  - Evaluation Results
- **Conclusions and Perspectives**

Conclusions & Perspectives

- Presents an optical **two-dimensional quantum walk reservoir computing** model, possessing **quantum interference & entanglement**
- Demonstrates a 64.32% accuracy improvement over the QELM model in **nonlinear function approximation tasks**. Validated the **effectiveness in general machine learning tasks** of the 2D-QWRC through the abalone age prediction task, with accuracy of 82%

Towards Quantum Optical Intelligence

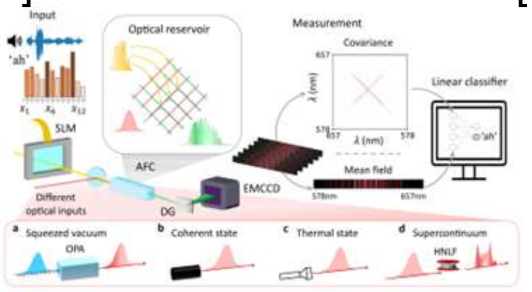
a) Intrinsic Quantum Properties of Photons

Squeeze light, Entanglement, and Correlated photons

b) Atomic, Molecular & Optics (AMO) level Compactness and Compatibility

Nanophotonics, Light-matter interactions, and Complex dynamic systems

[16]



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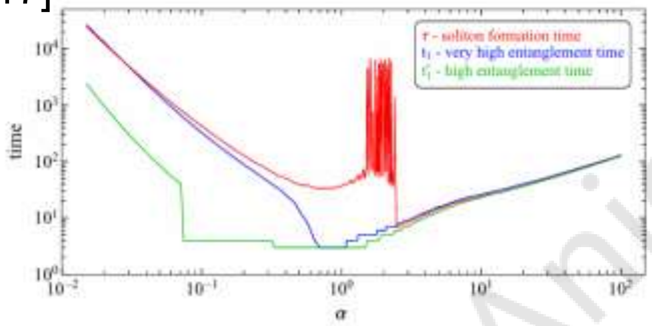

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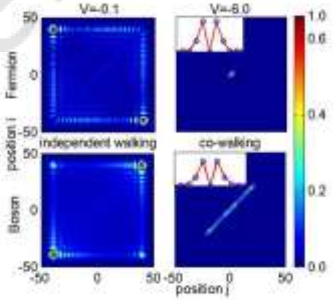

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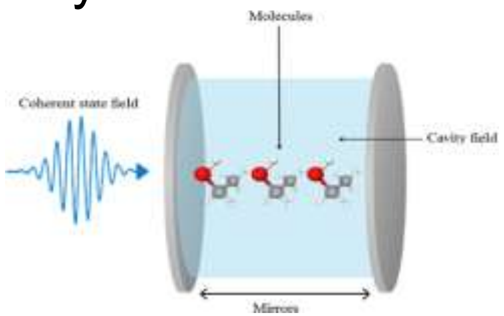

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Thanks for listening!