Design and Analysis of Algorithms
Part I: Divide and Conquer
Lecture 4: Heapsort and Lower Bound for Sorting


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Outline

- Introduction to Part I
- Heapsort Problem
  - Priority Queues
  - (Binary) Heap
  - Heapsort
- Lower Bound for Comparison-based Sorting
  - Objective
  - Decision Tree Model

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Introduction to Part I

- In Part I, we will illustrate Divide-and-Conquer using several examples:
  - Maximum Contiguous Subarray (最大子数组)
  - Counting Inversions (逆序计数)
  - Polynomial Multiplication (多项式乘法)
  - QuickSort and Partition (快速排序与划分)
  - Lower Bound for Sorting (基于比较的排序下界)

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Priority Queue: Motivating Example

3 jobs have been submitted to a printer in the order A, B, C. Consider the printing pool at this moment.

Sizes: Job A — 100 pages

Job B — 10 pages

Job C — 1 page

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Average finish time with FIFO service:

$$(100 + 110 + 111)\ /\ 3\ =\ 107\ time\ units$$

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Average finish time with FIFO service:

$$(100 + 110 + 111) \,/\, 3 \;=\; 107\; time\ units$$

Average finish time for shortest-job-first service:

$$(1 + 11 + 111) \,/\, 3 \;=\; 41\; time\ units$$

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- The elements in the queue are printing jobs, each with the associated number of pages that serves as its priority
- Processing the shortest job first corresponds to extracting the smallest element from the queue
- Insert new printing jobs as they arrive

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A queue is capable of supporting two operations: Insert and Extract-Min?

Priority Queue

Priority queue is an abstract data structure that supports two operations

- Insert: inserts the new element into the queue
- Extract-Min: removes and returns the smallest element from the queue

Possible Implementations

- Unsorted list + a pointer to the smallest element
  - Insert in $O(1)$ time
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Question

Is there any data structure that supports both these priority queue operations in $O(\log n)$ time?

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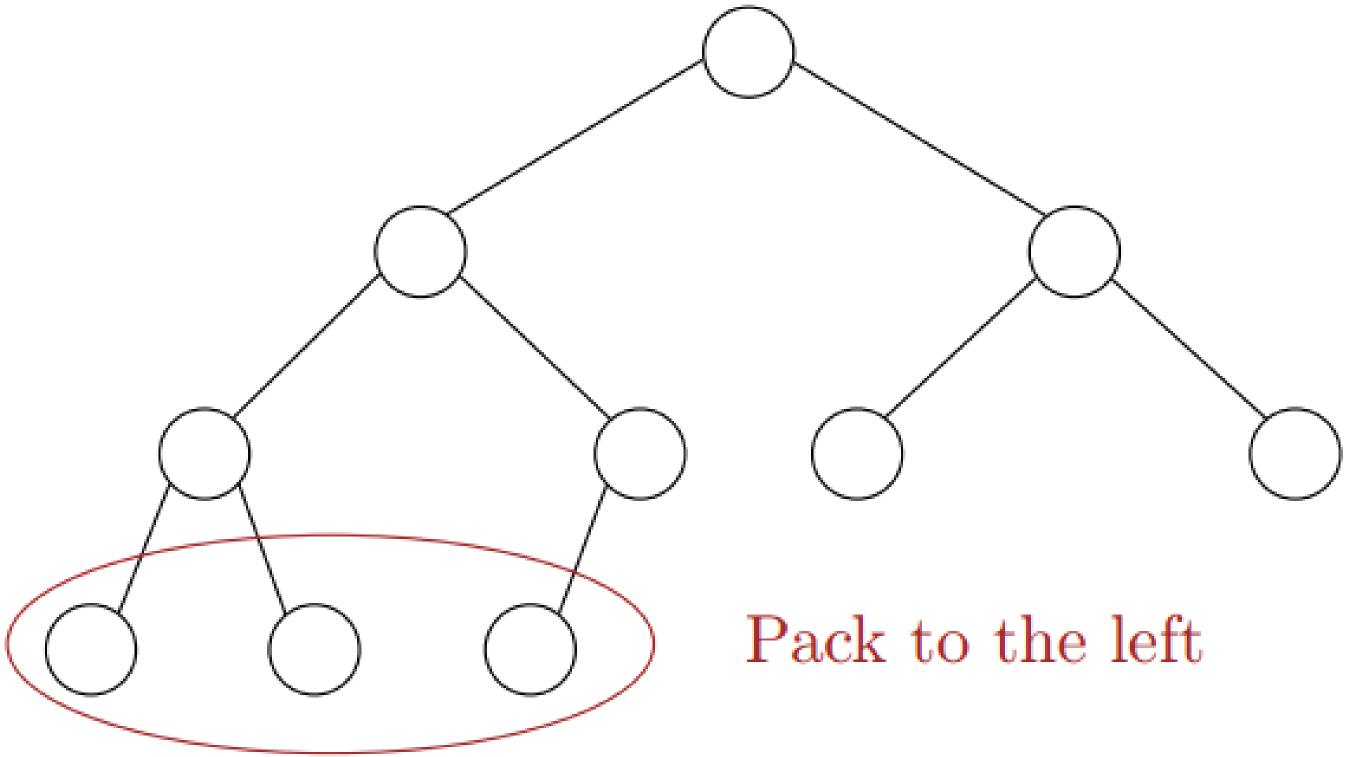
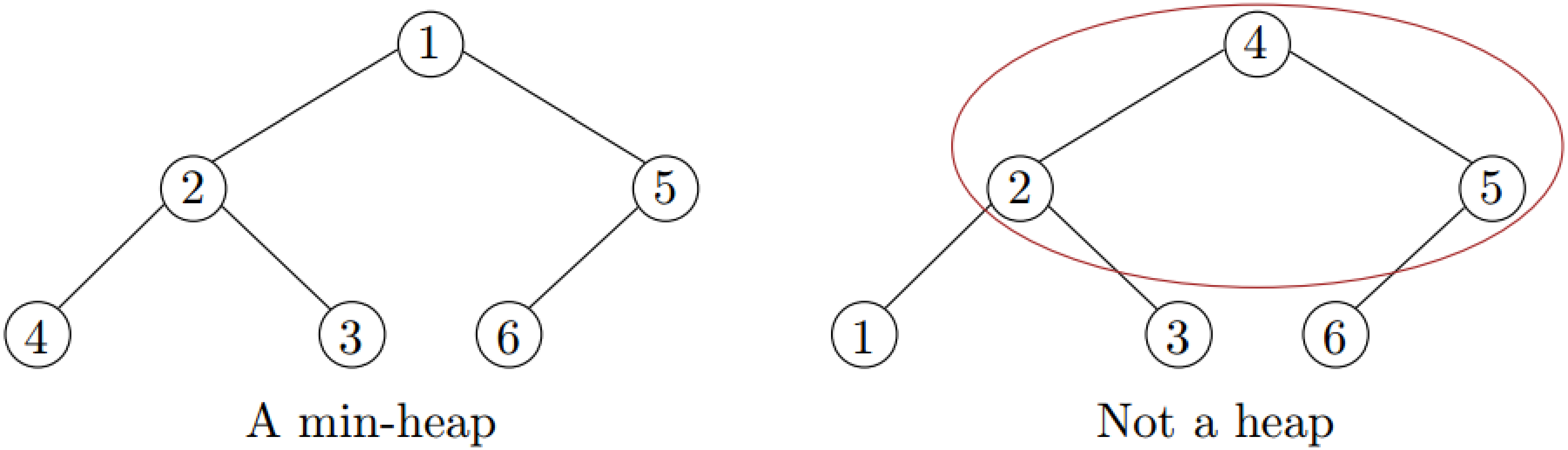
(Binary) Heap



Heaps are "almost complete binary trees"

- All levels are full except possibly the lowest level.
- If the lowest level is not full, then nodes must be packed to the left.

Heap-order Property



Heap-order property (*Min-heap*):

The value of a node is at least the value of its parent.

A[Parent(i)] ≤ A[i]

Heap Properties

- If the heap-order property is maintained, heaps support the following operations efficiently (assume there are *n* elements in the heap)
  - Insert in O(log *n*) time
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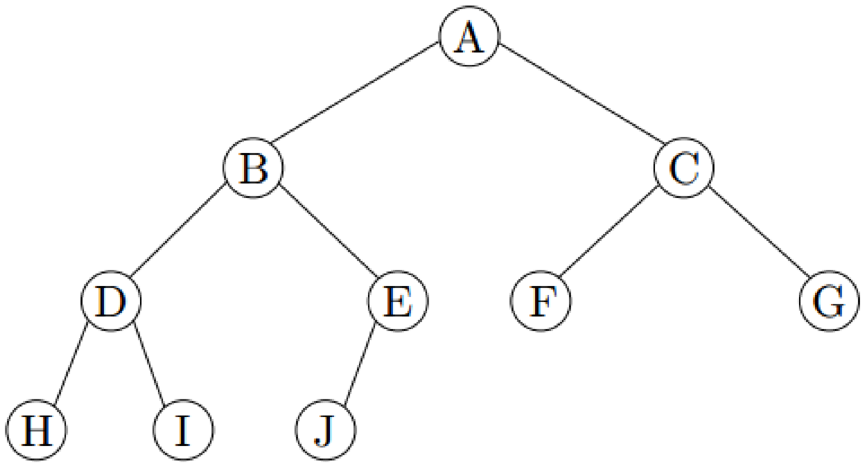
  - A heap of height *h* has between 2^h to $2^{h+1}-1$ nodes. Thus, an n-element heap has height $\Theta(\log n)$.
  - The structure is so regular, it can be represented in an array and no links are necessary !

Array Implementation of Heap

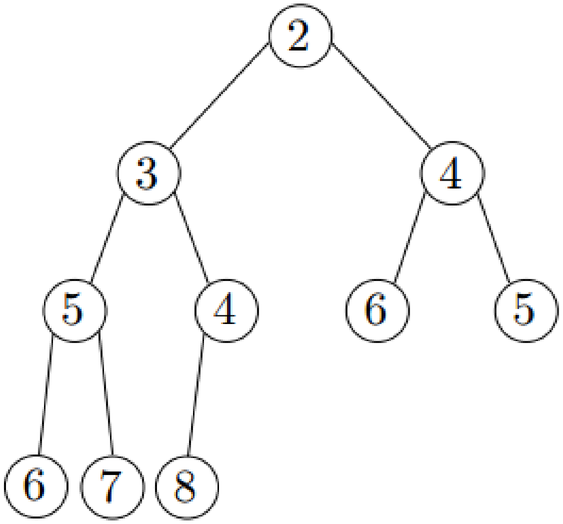


1	2	3	4	5	6	7	8	9	10
A	B	C	D	E	F	G	H	I	J

- The root is in array position 1.
- For any element in array position i,
  - The left child is in position 2i.
  - The right child is in position 2i+1.
  - The parent is in position $\lfloor i/2 \rfloor$.
- We will draw the heaps as trees, with the understanding that an actual implementation will use simple arrays.

Insertion

- Add the new element to the next available position at the lowest level
- Restore the min-heap property if violated
  - General strategy is percolate up (or bubble up): if the parent of the element is larger than the element, then interchange the parent with child.



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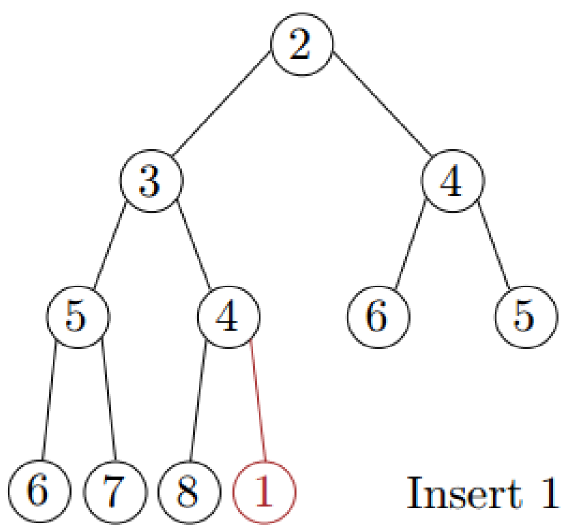

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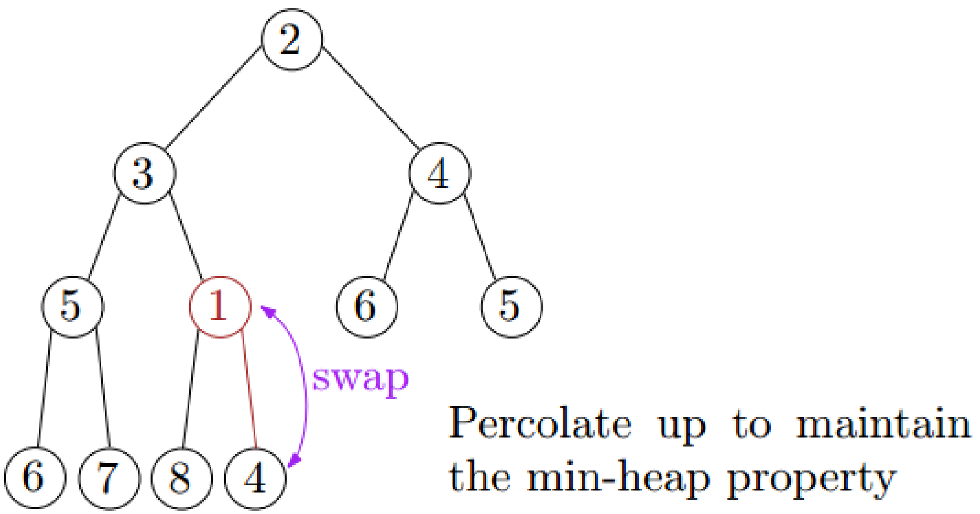

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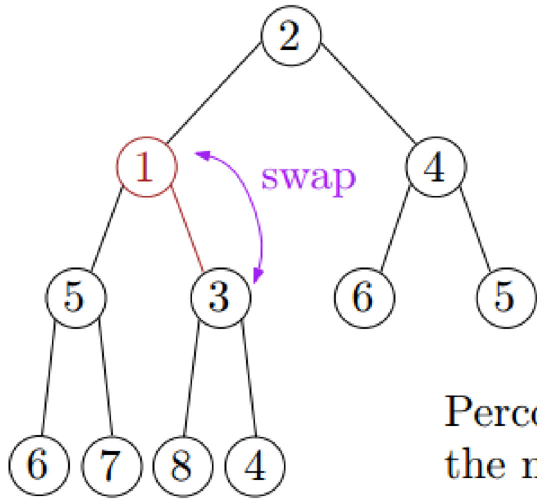

Percolate up to maintain the min-heap property

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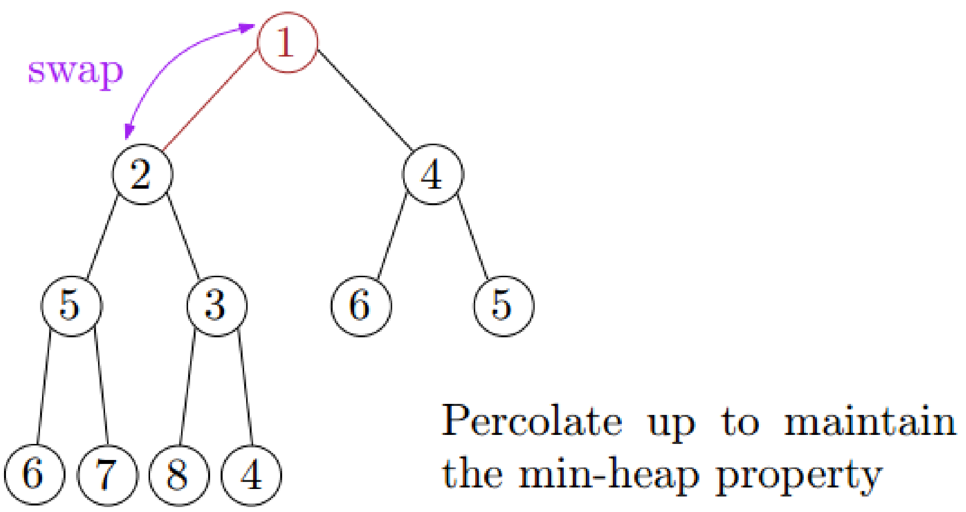

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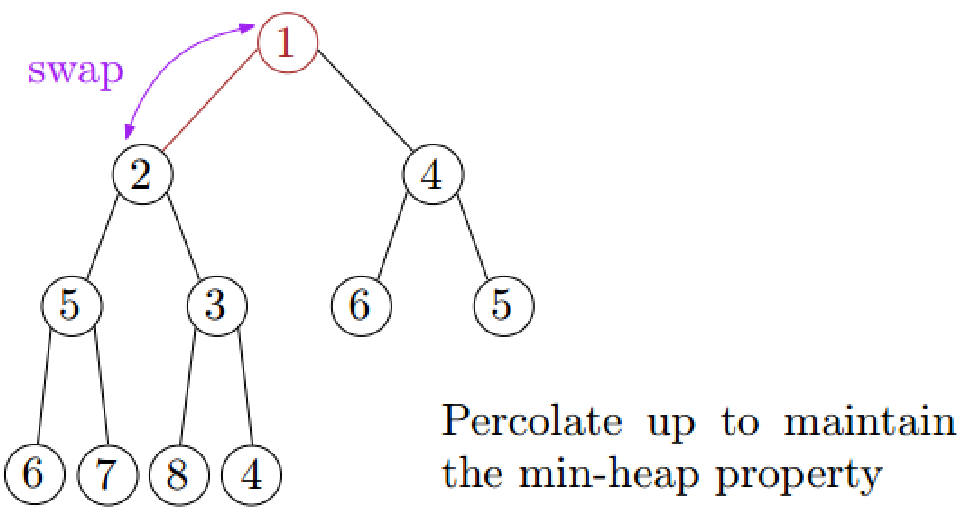

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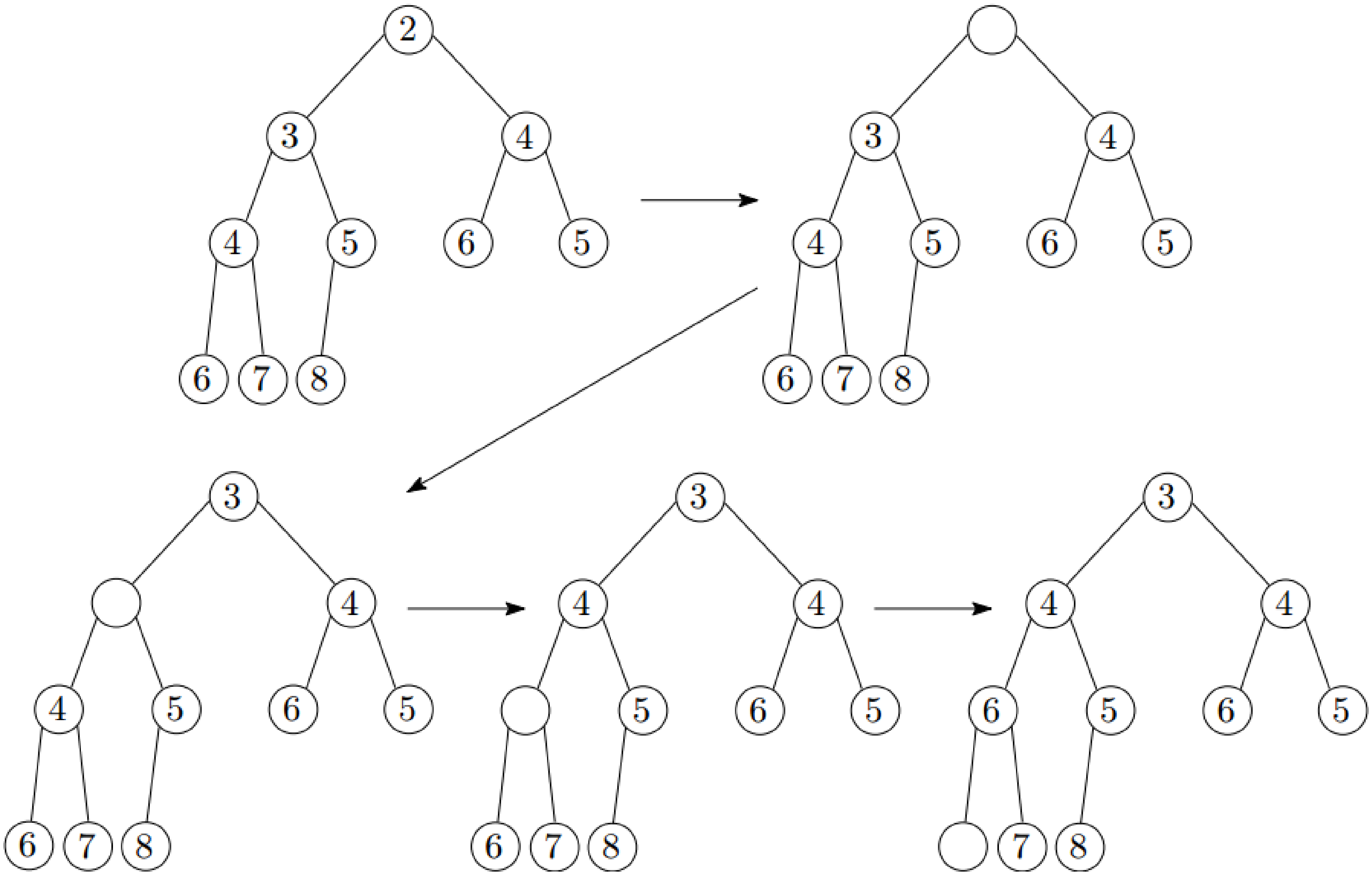
Extract-Min: First Attempt



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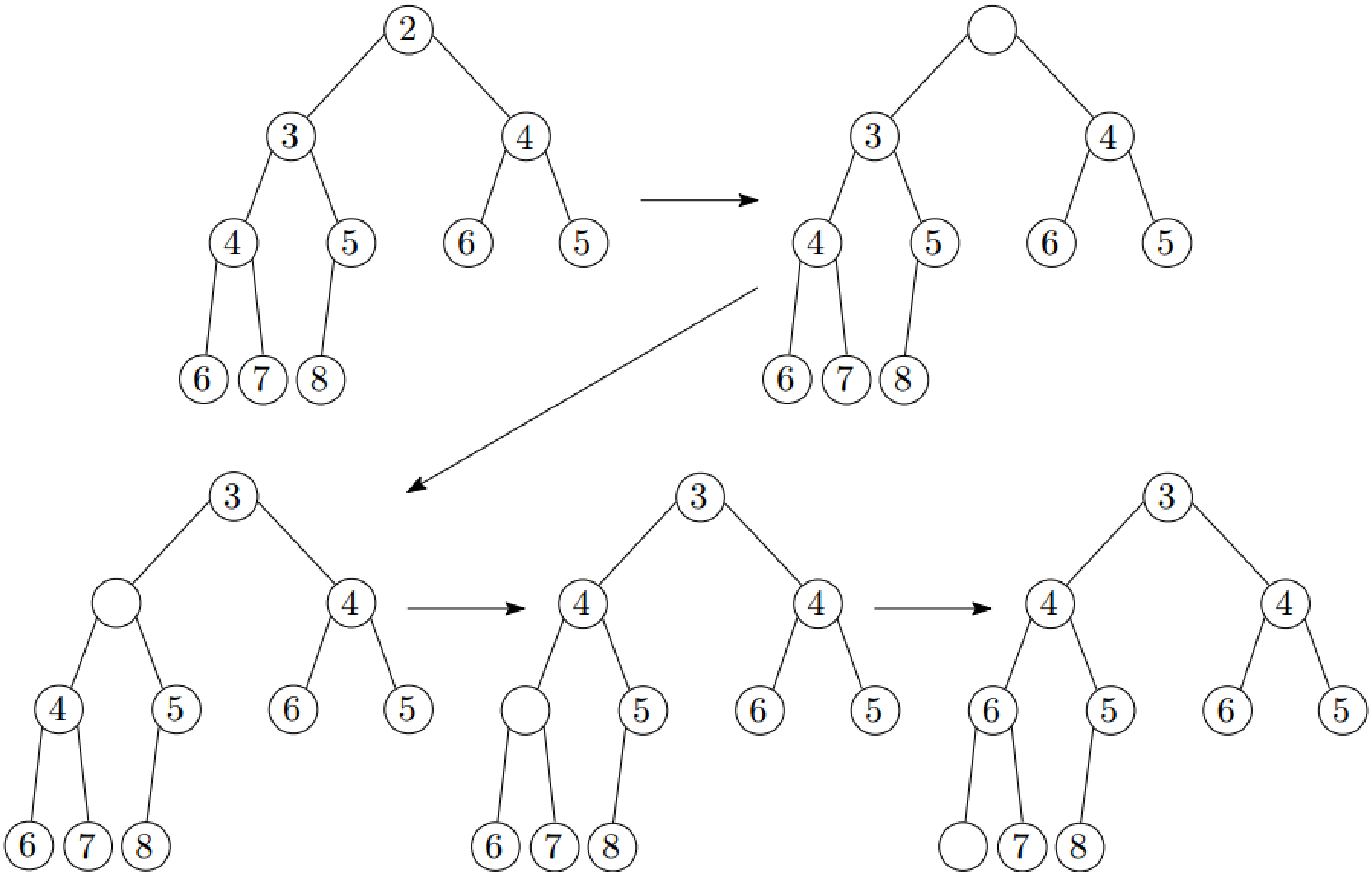

Min-heap property preserved, but completeness not preserved!

Extract-Min

- Copy the last element to the root (i.e., overwrite the minimum element stored there)
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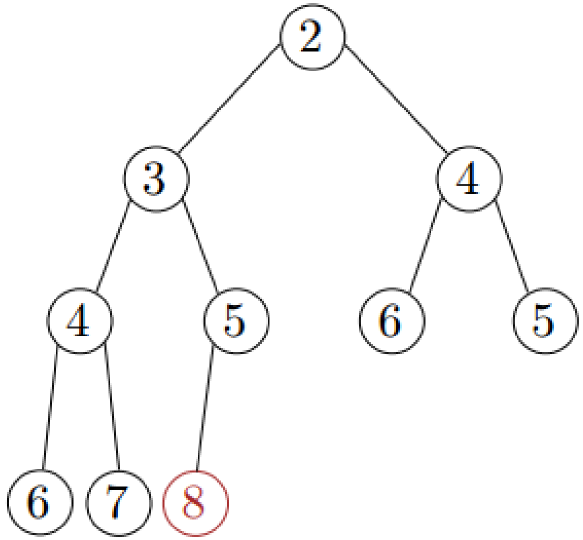

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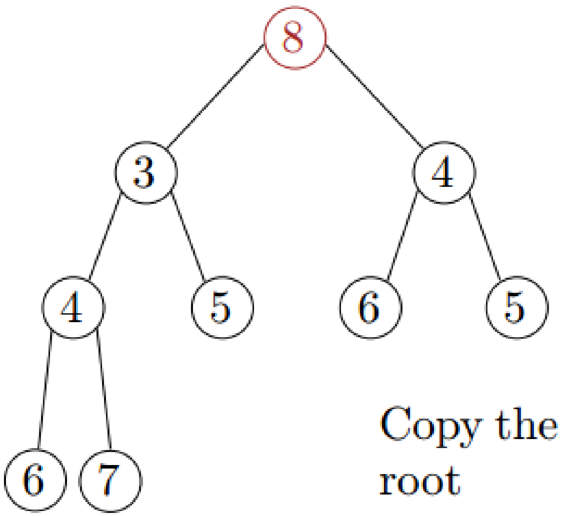

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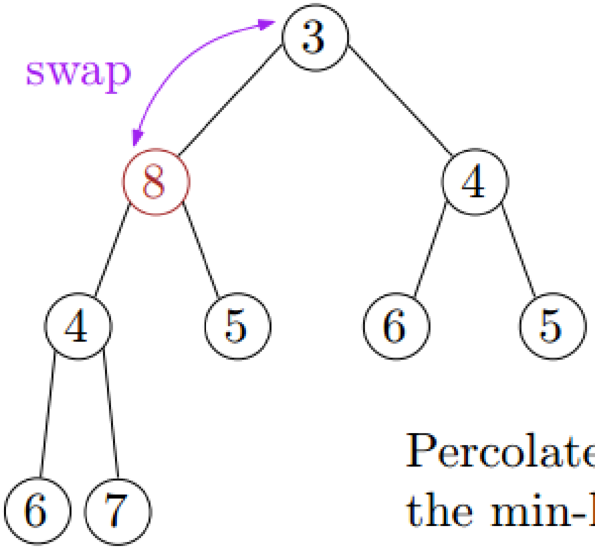

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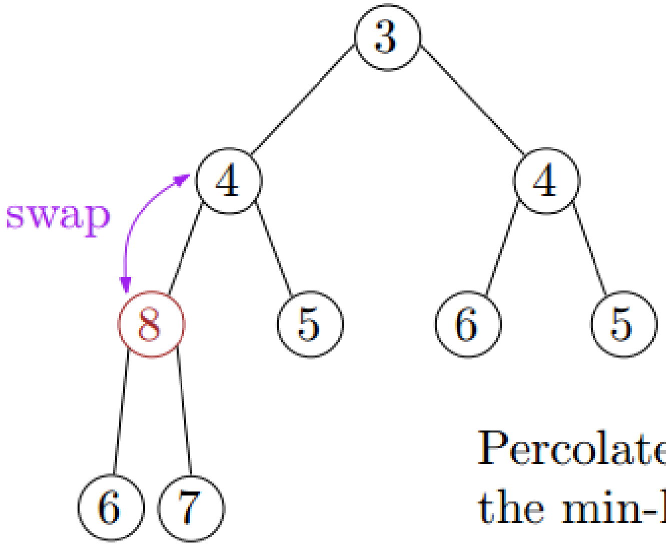

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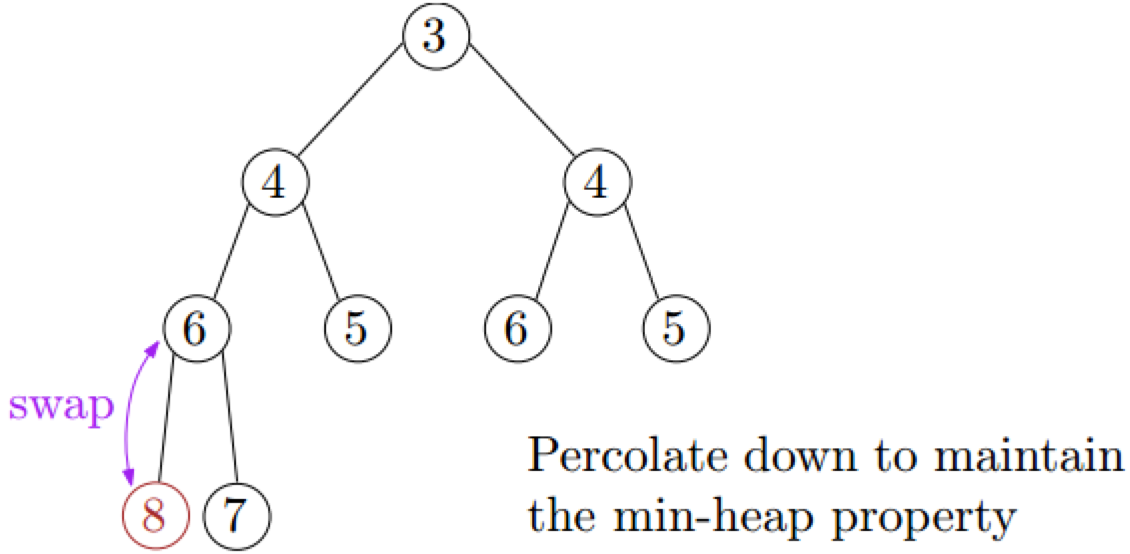

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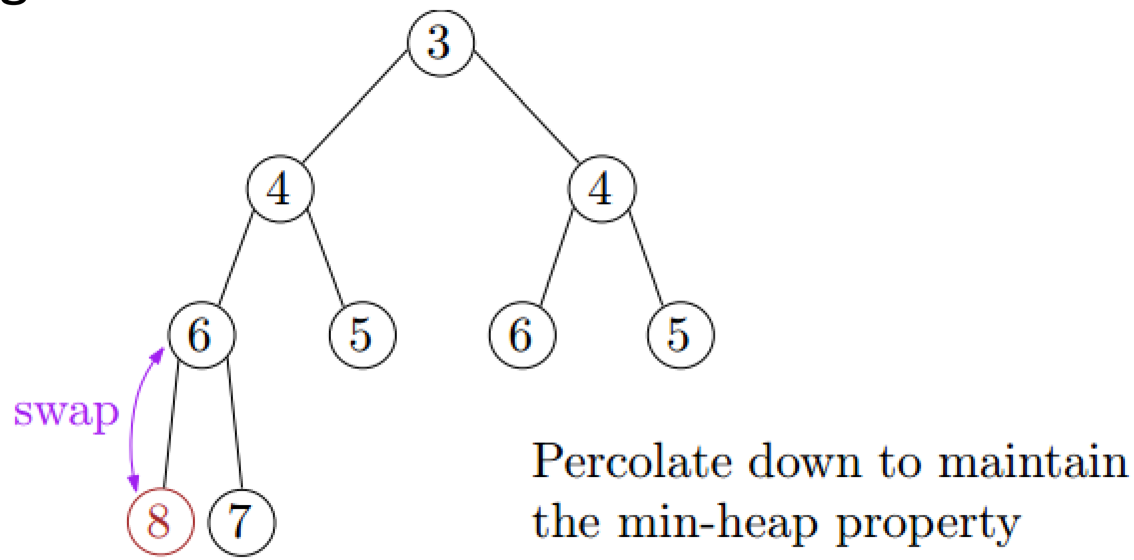

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- Total time complexity: O(n log n)

Heapsort - Example

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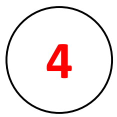

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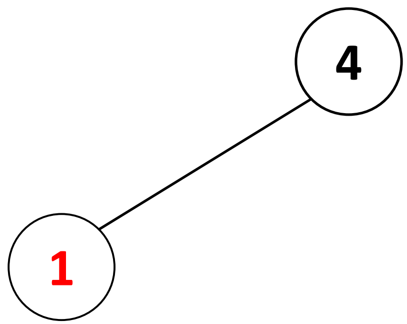

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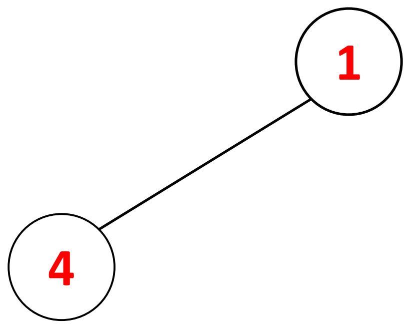

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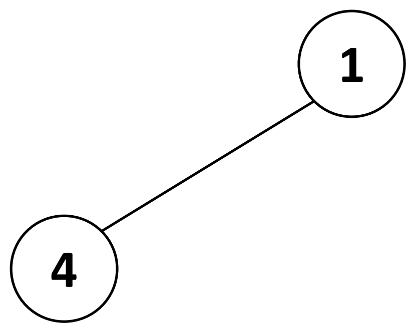

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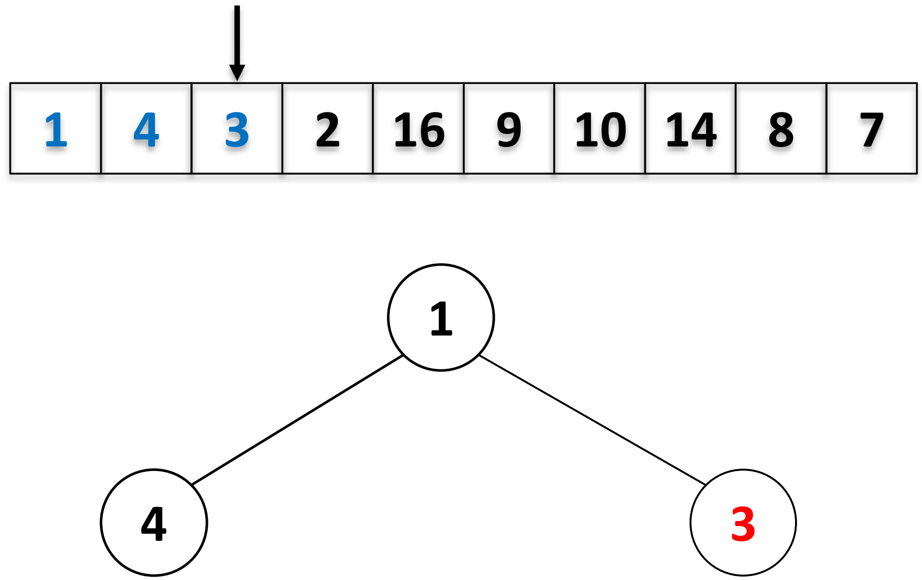

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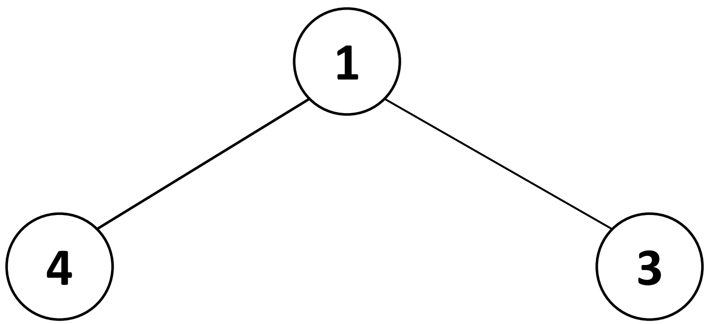

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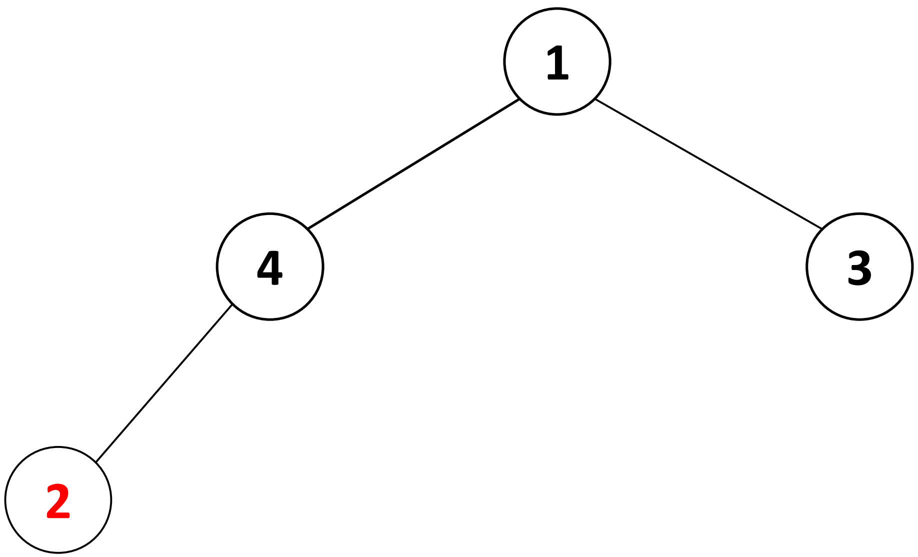

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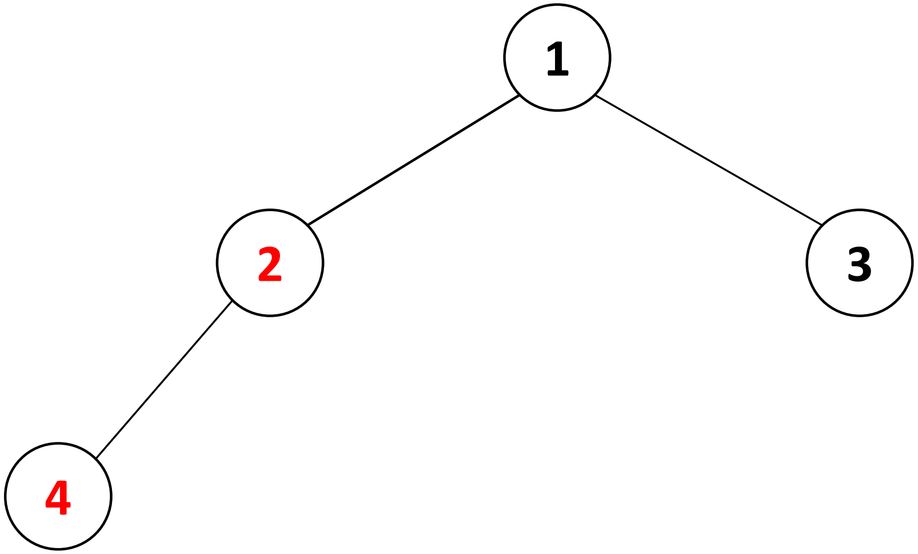

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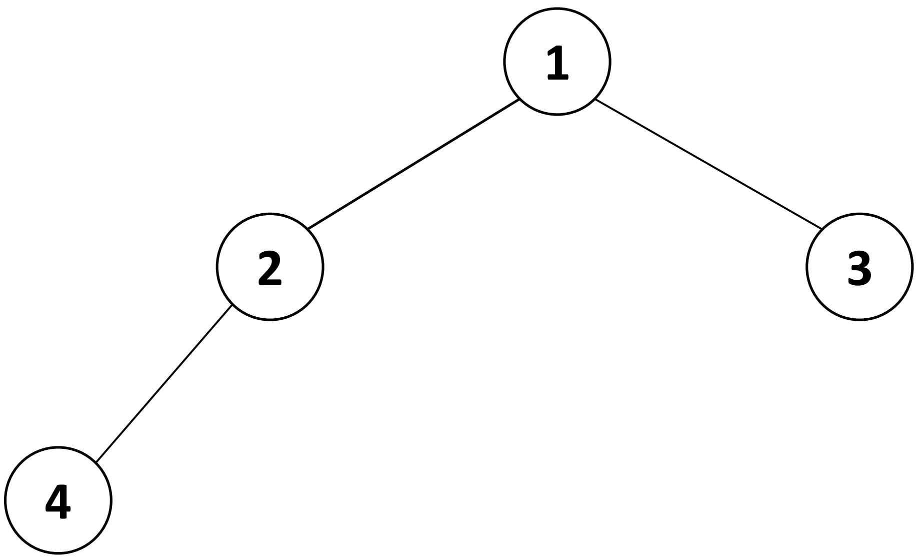

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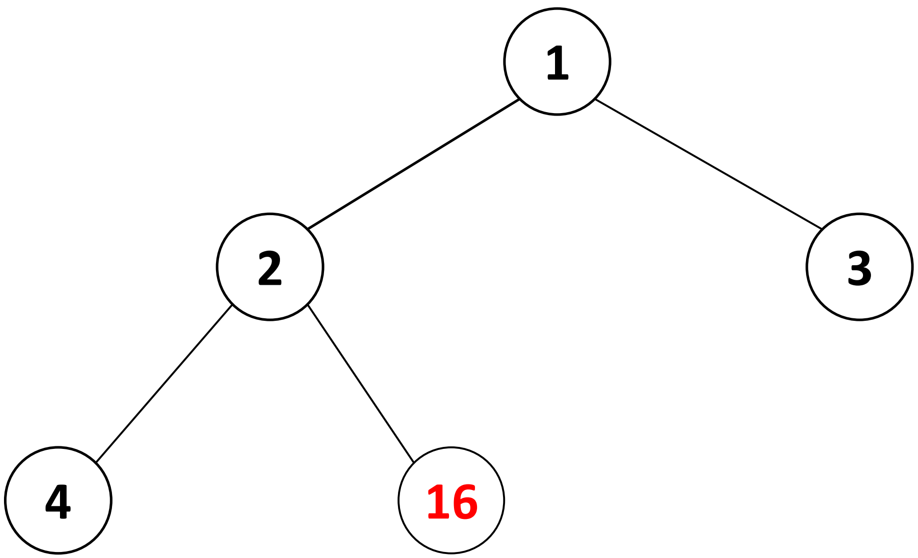

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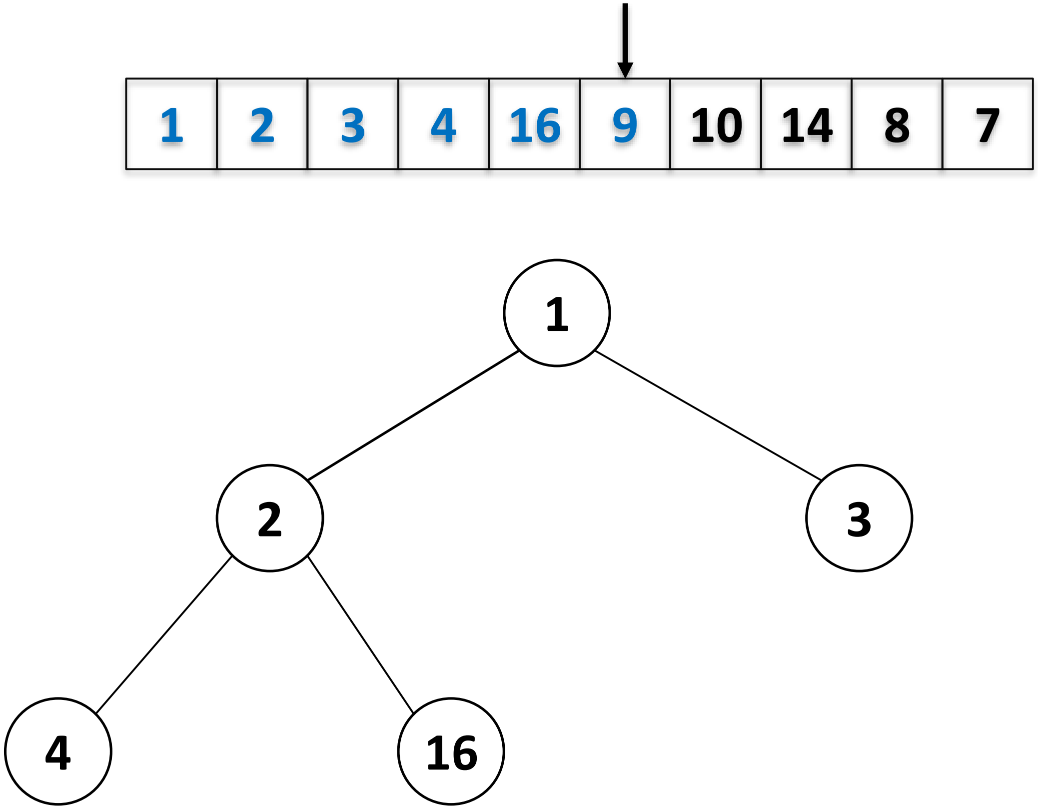

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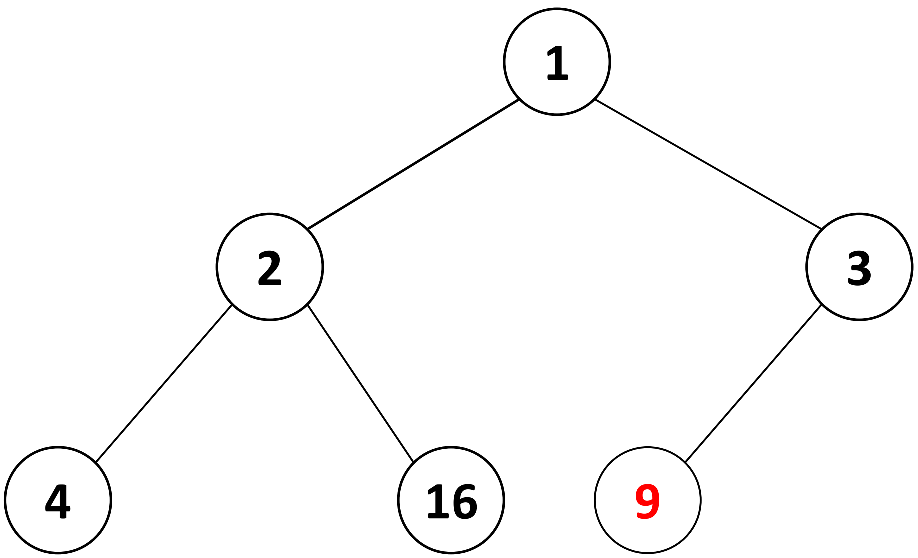

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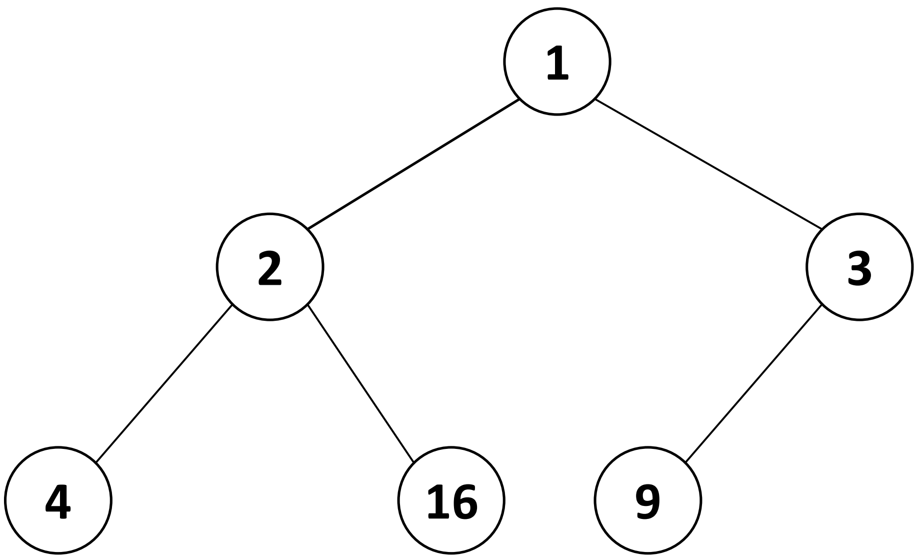

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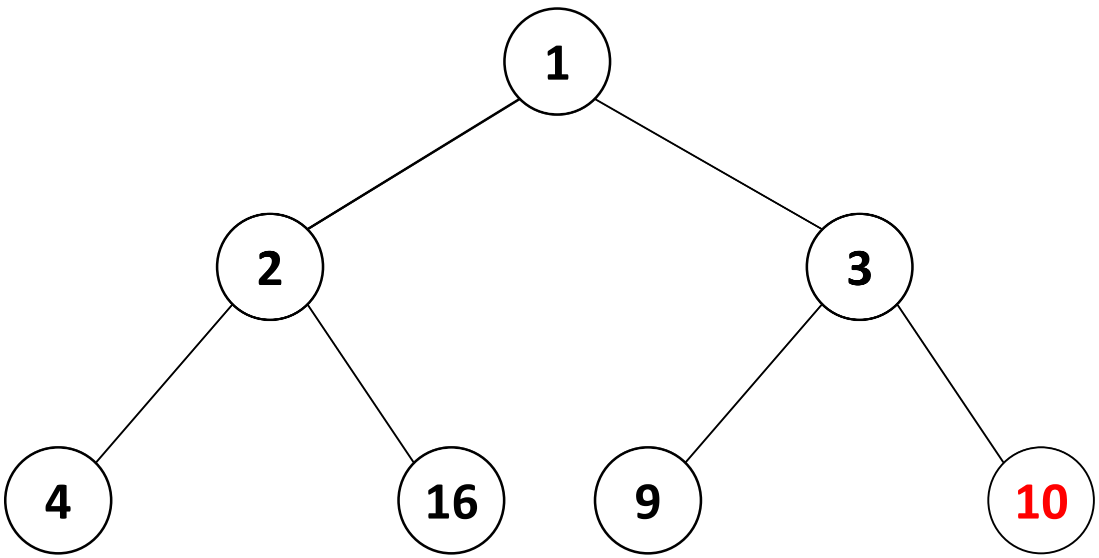

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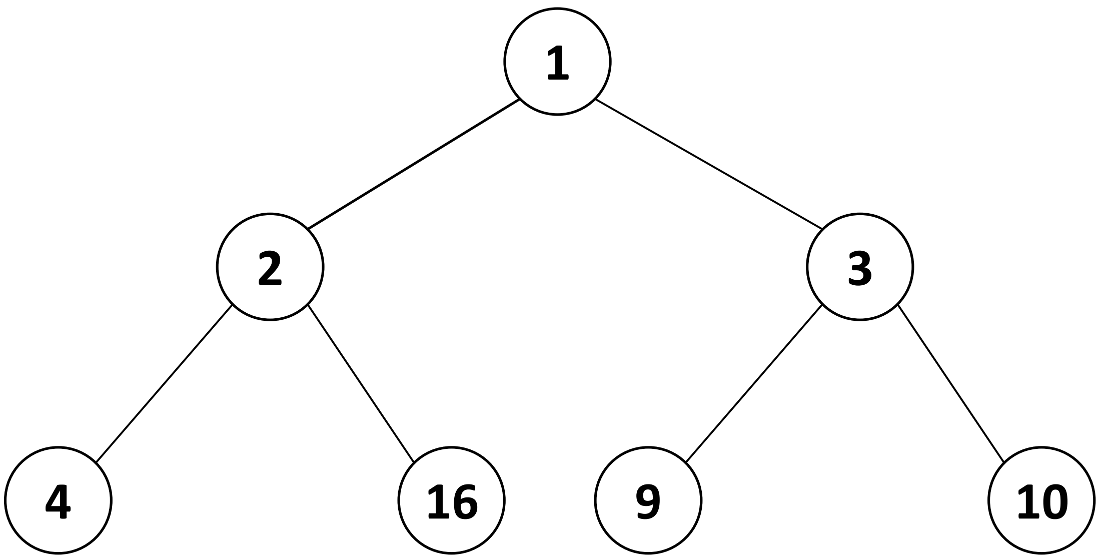

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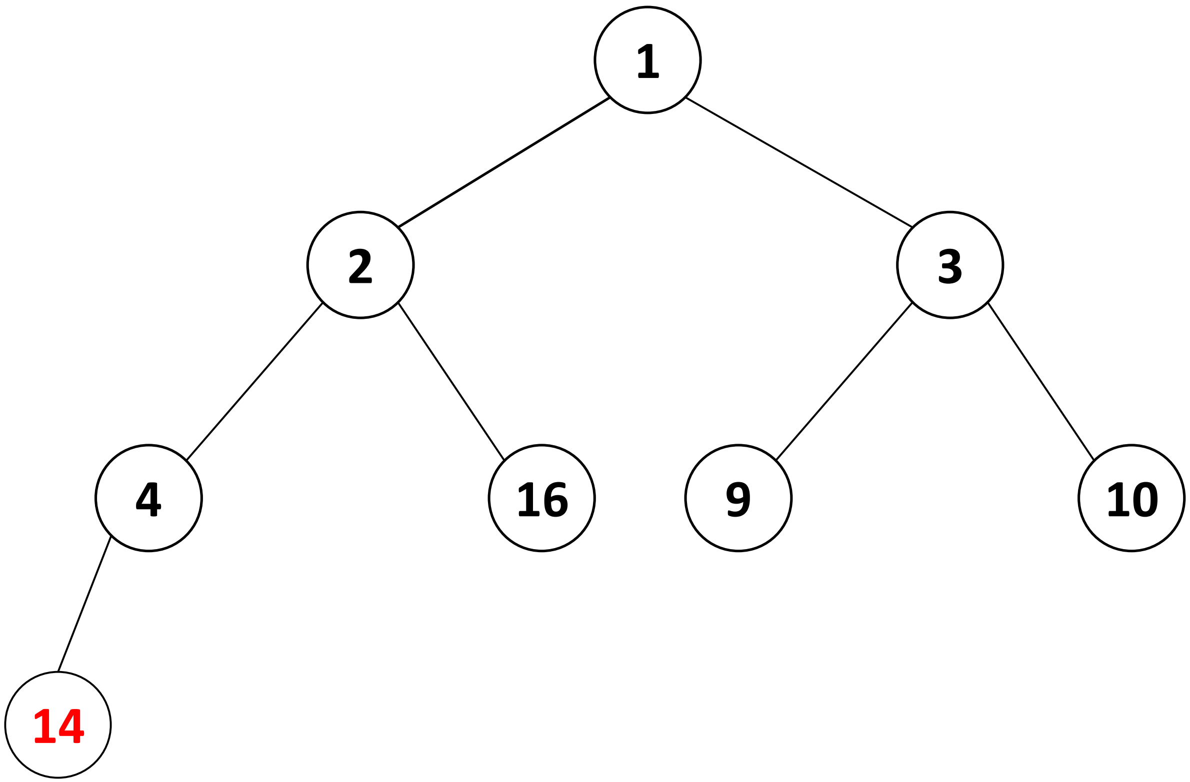

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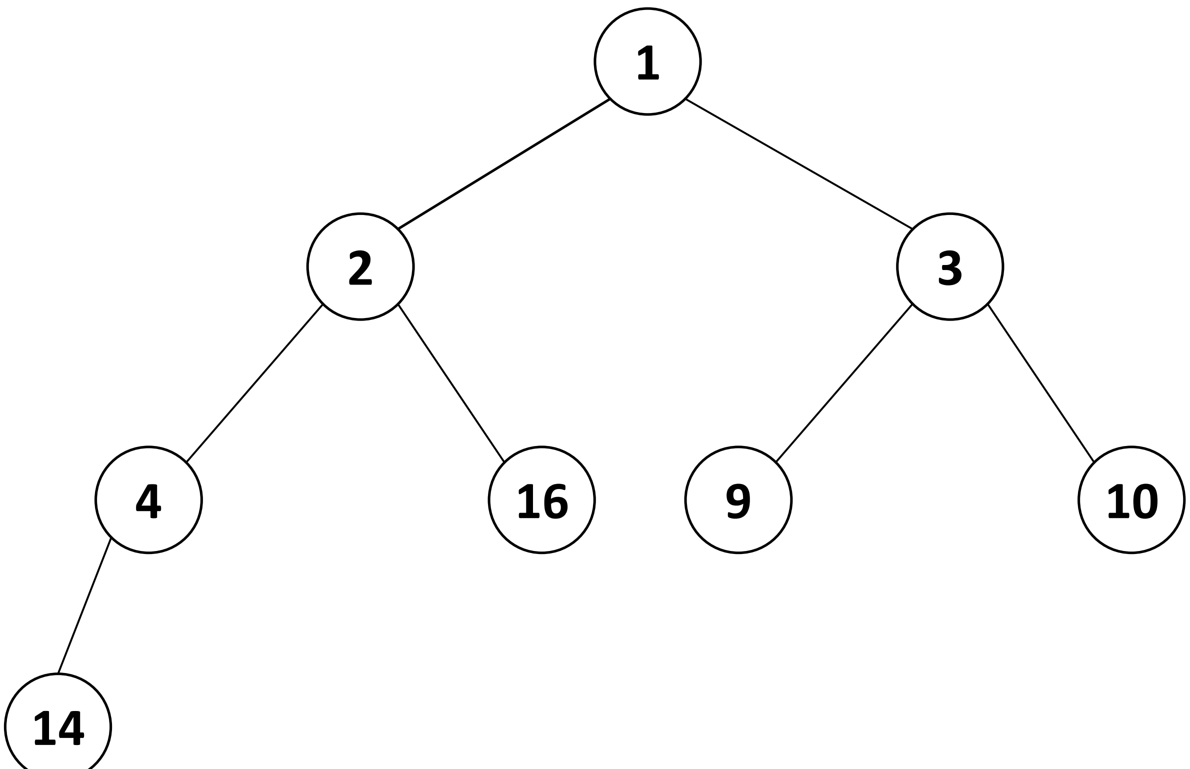

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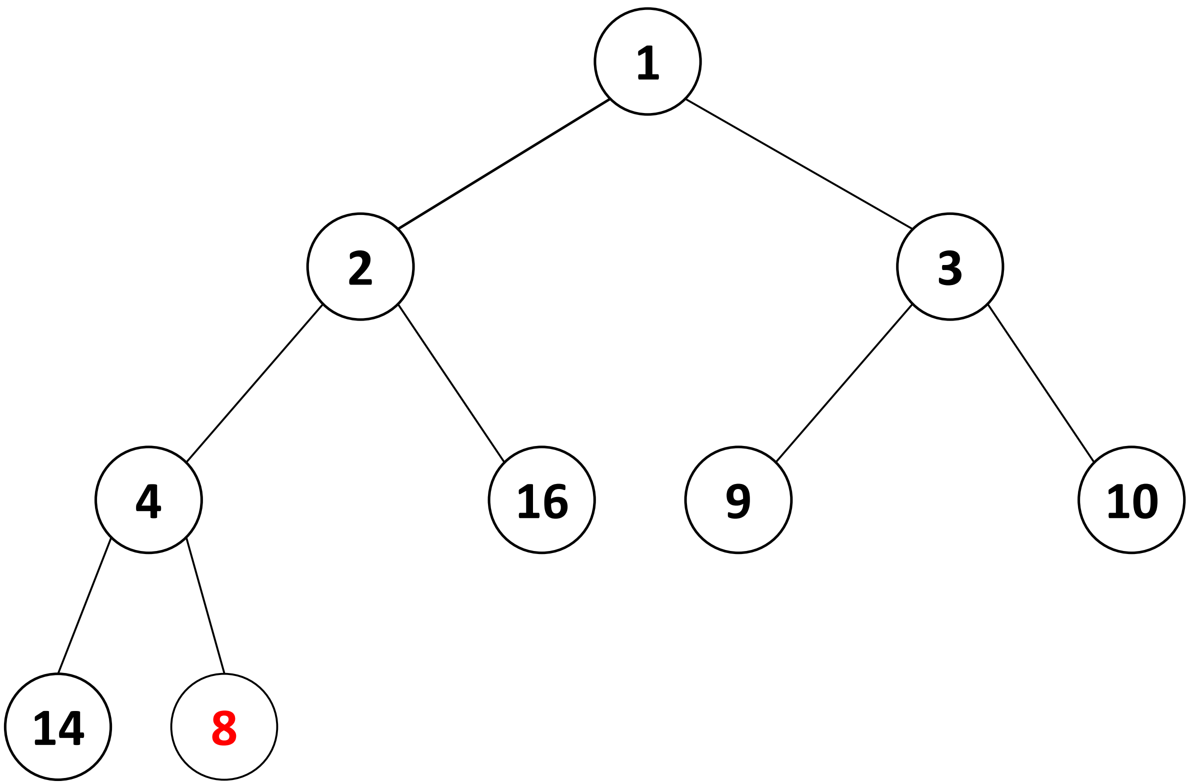

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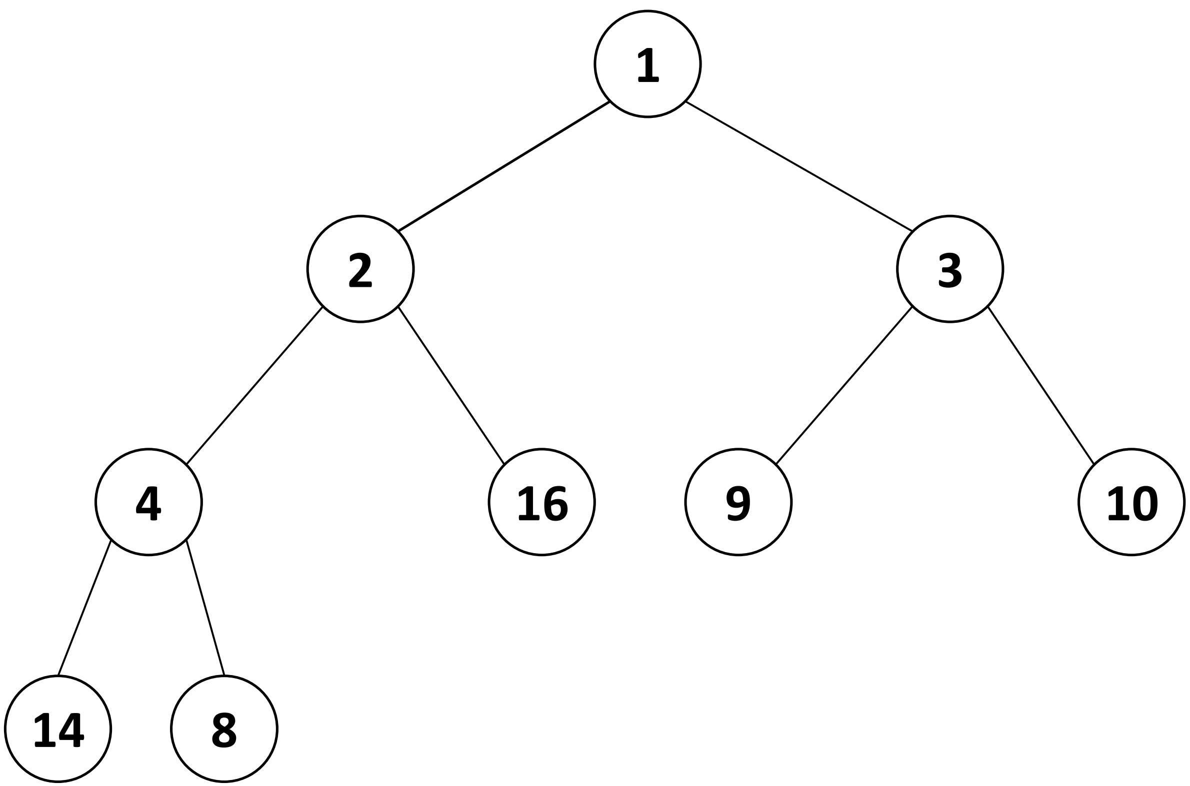

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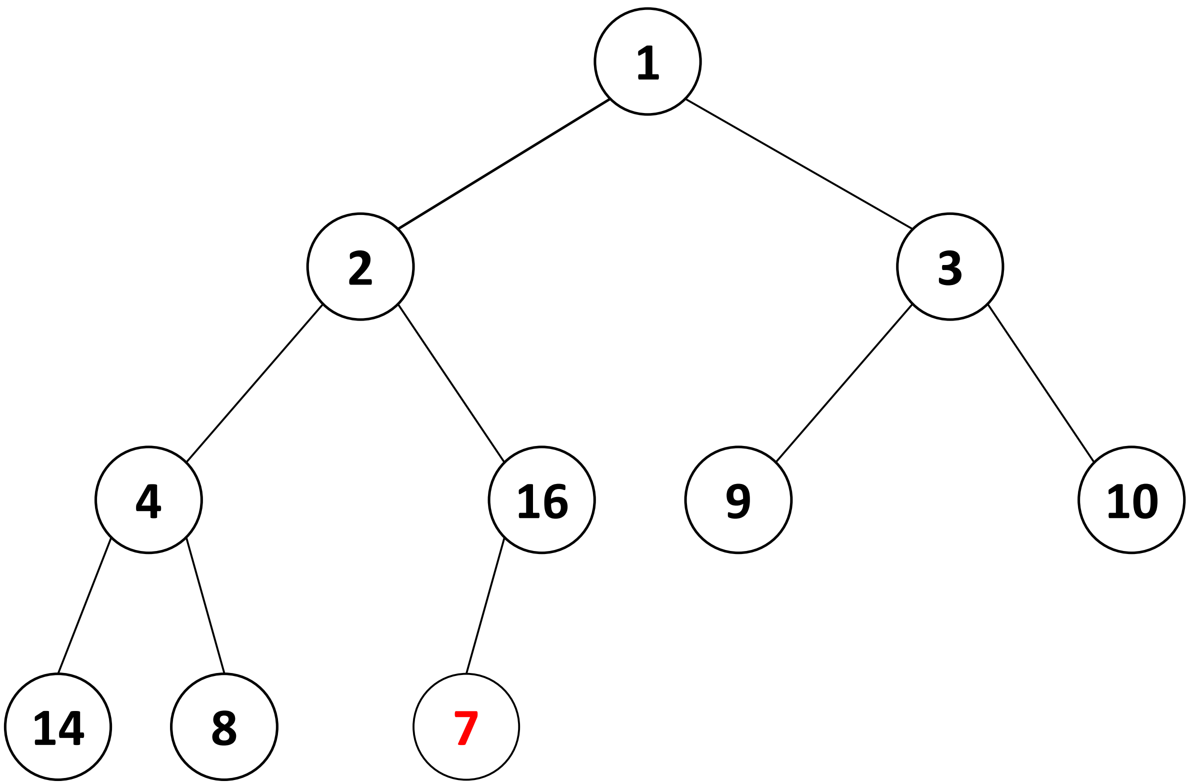

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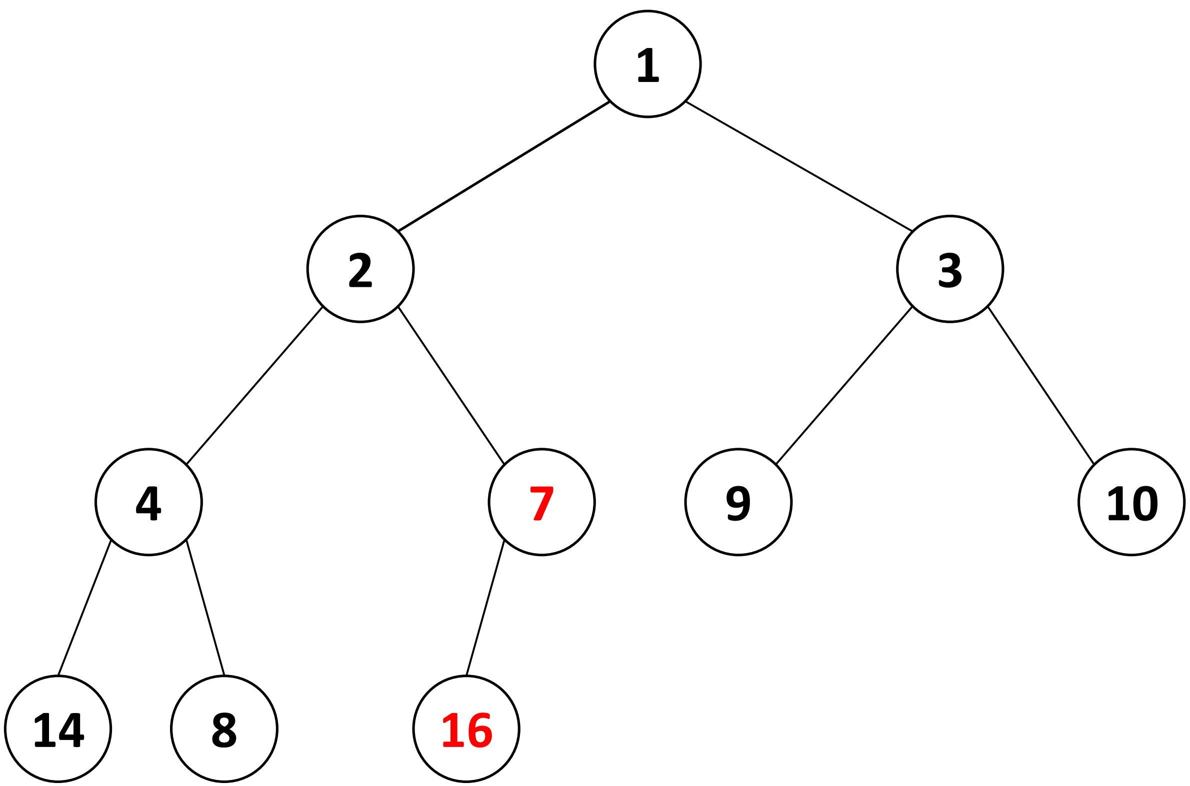
1	2	3	4	7	9	10	14	8	16



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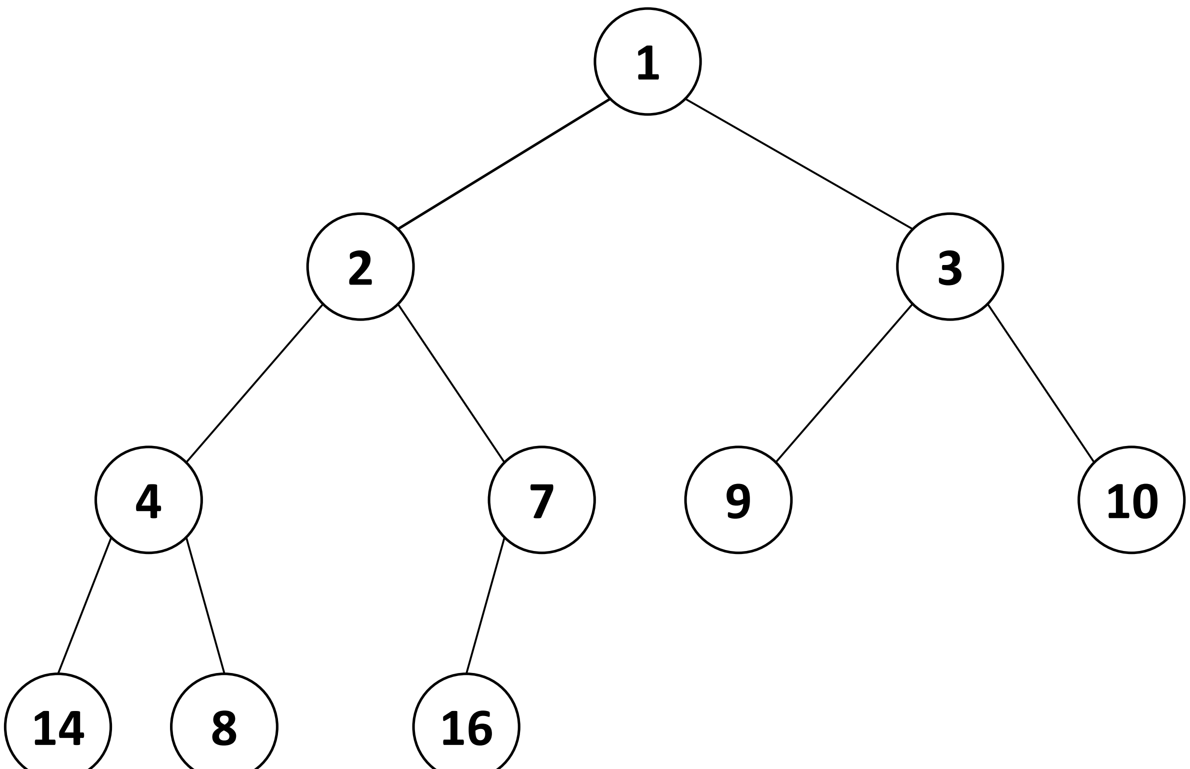
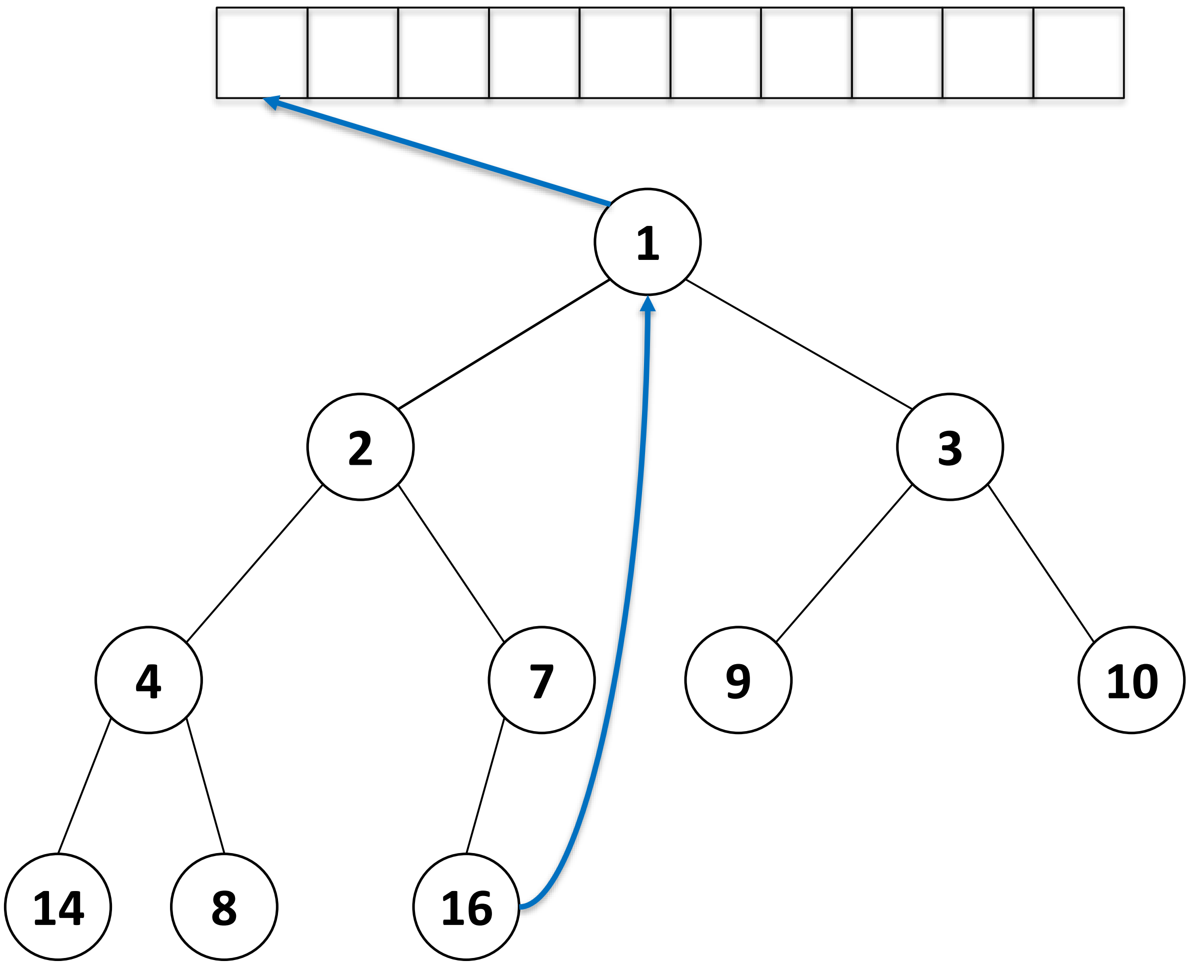


Heapsort - Example

- Perform n Extract-Min operations



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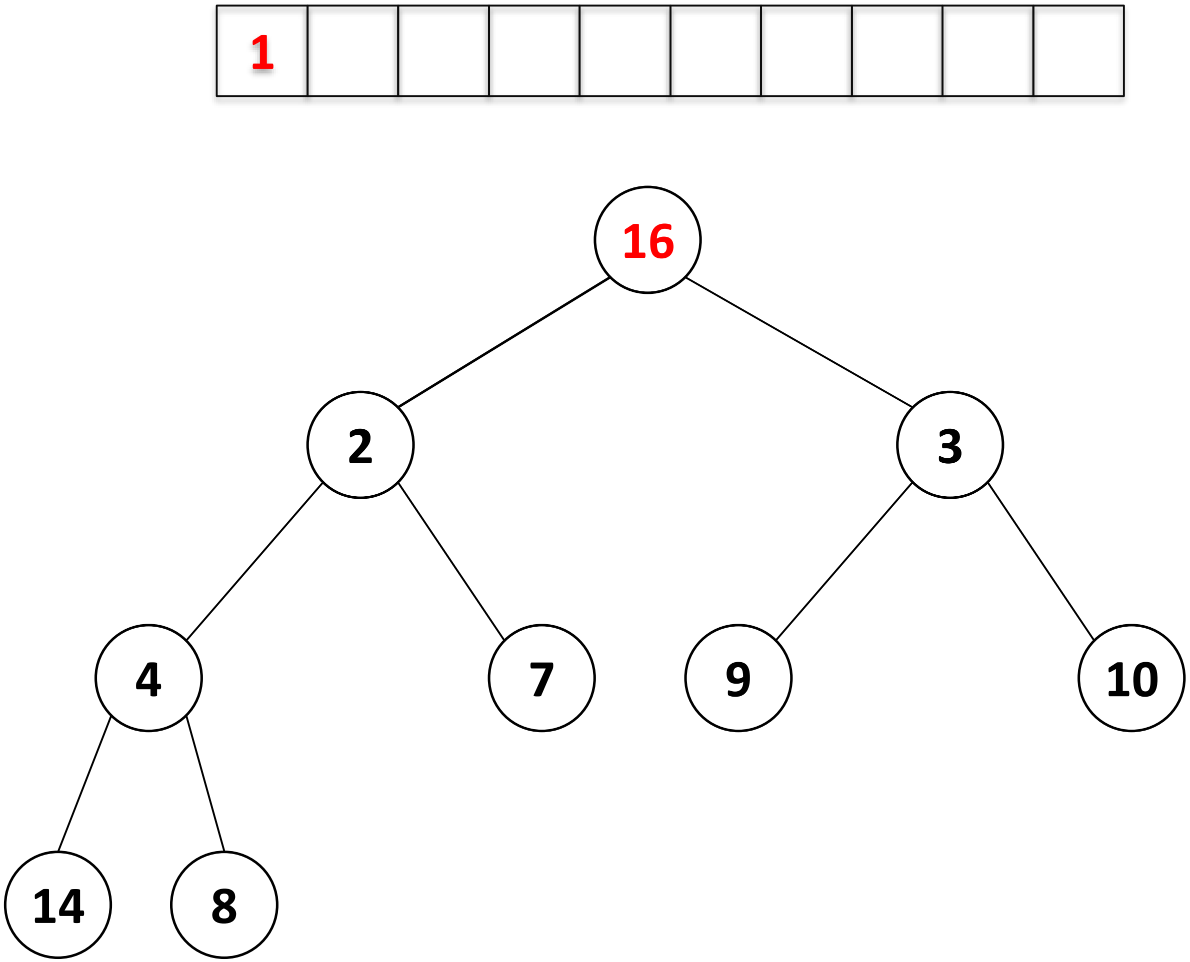

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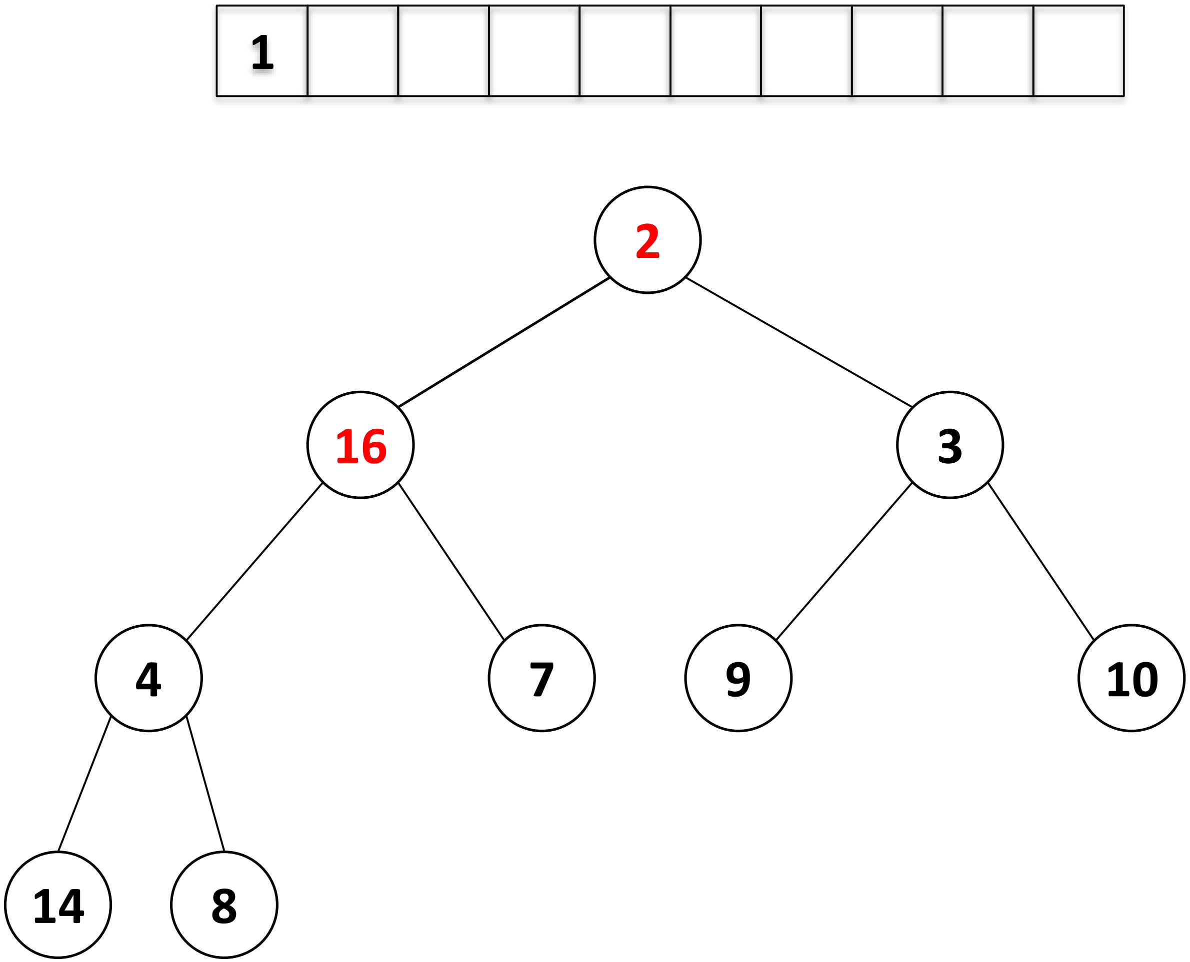

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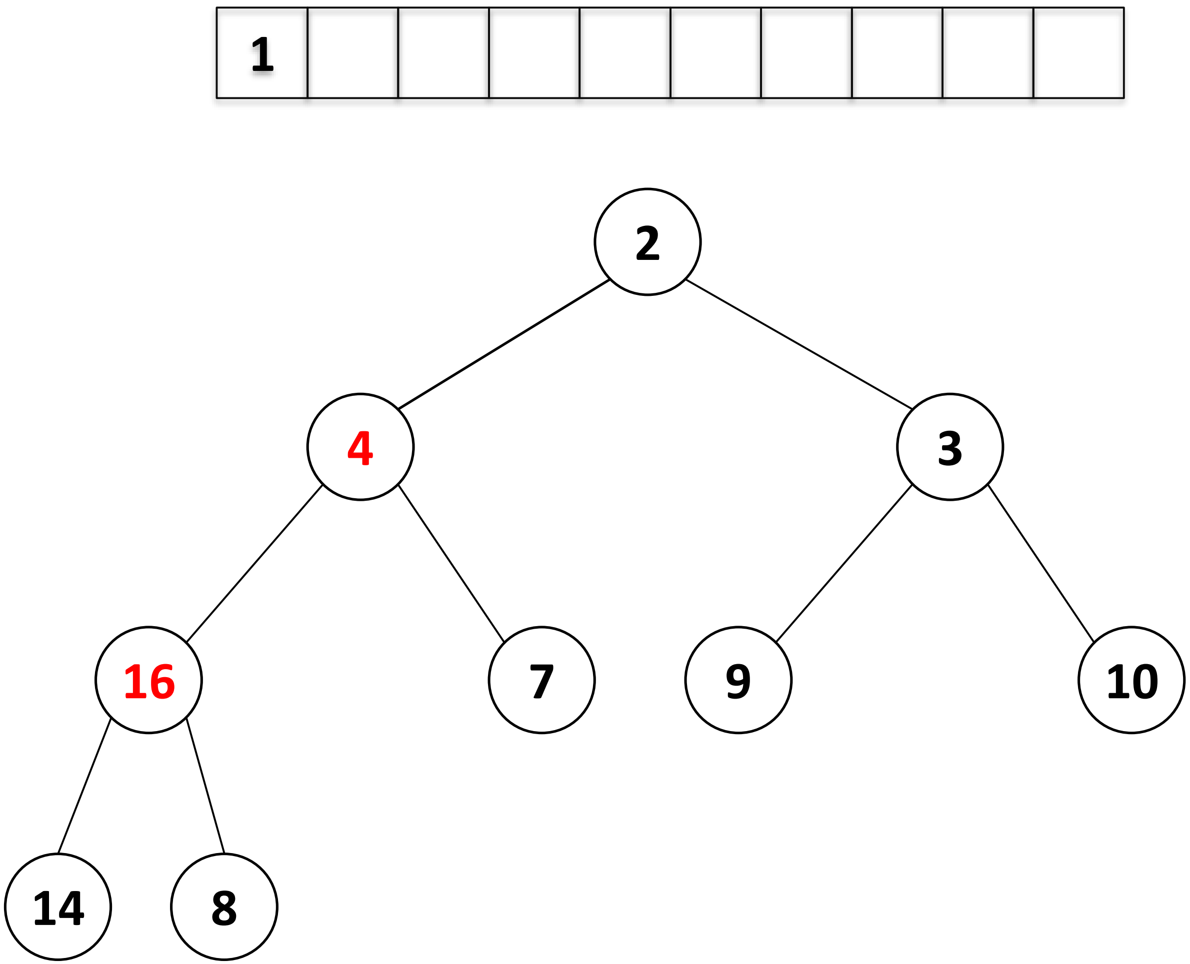

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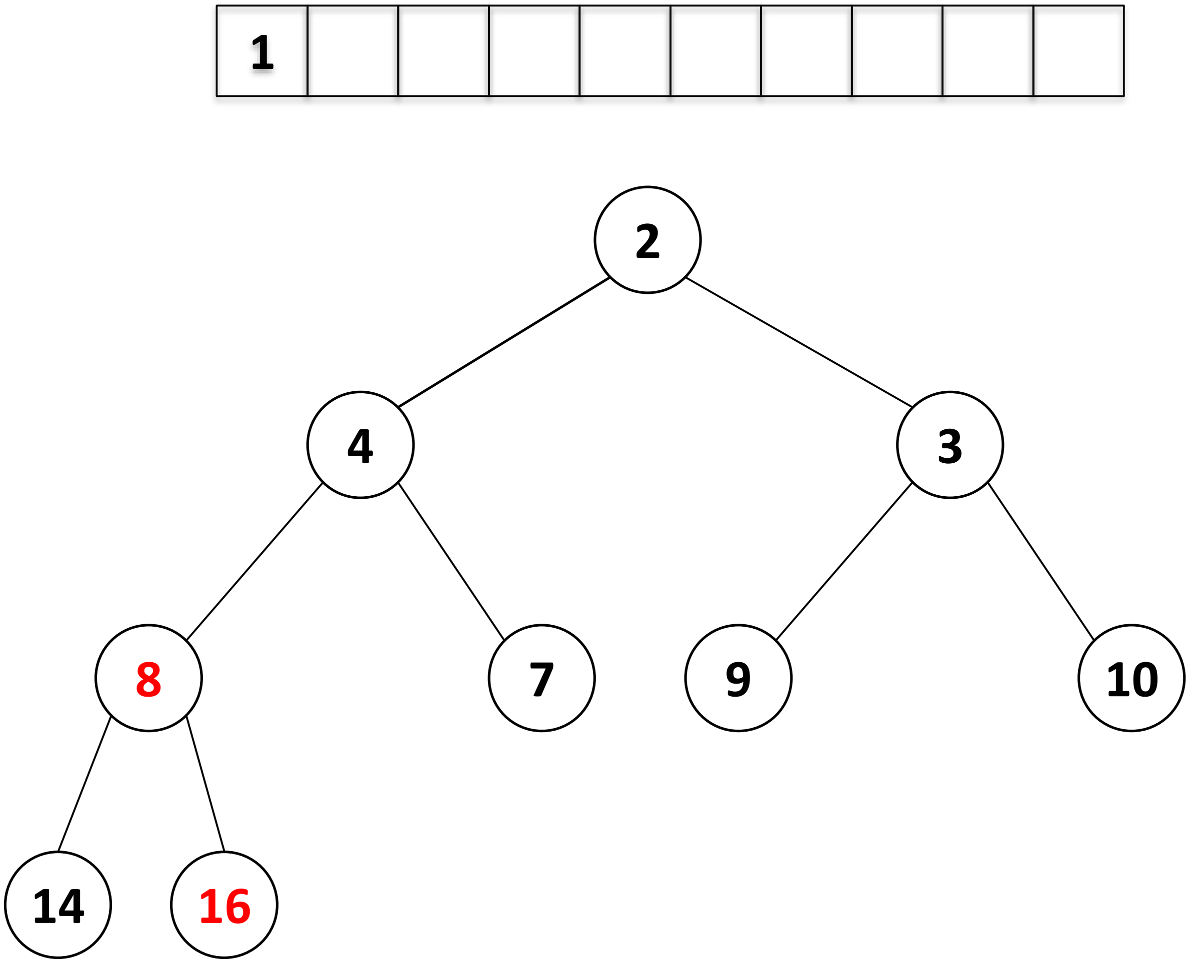

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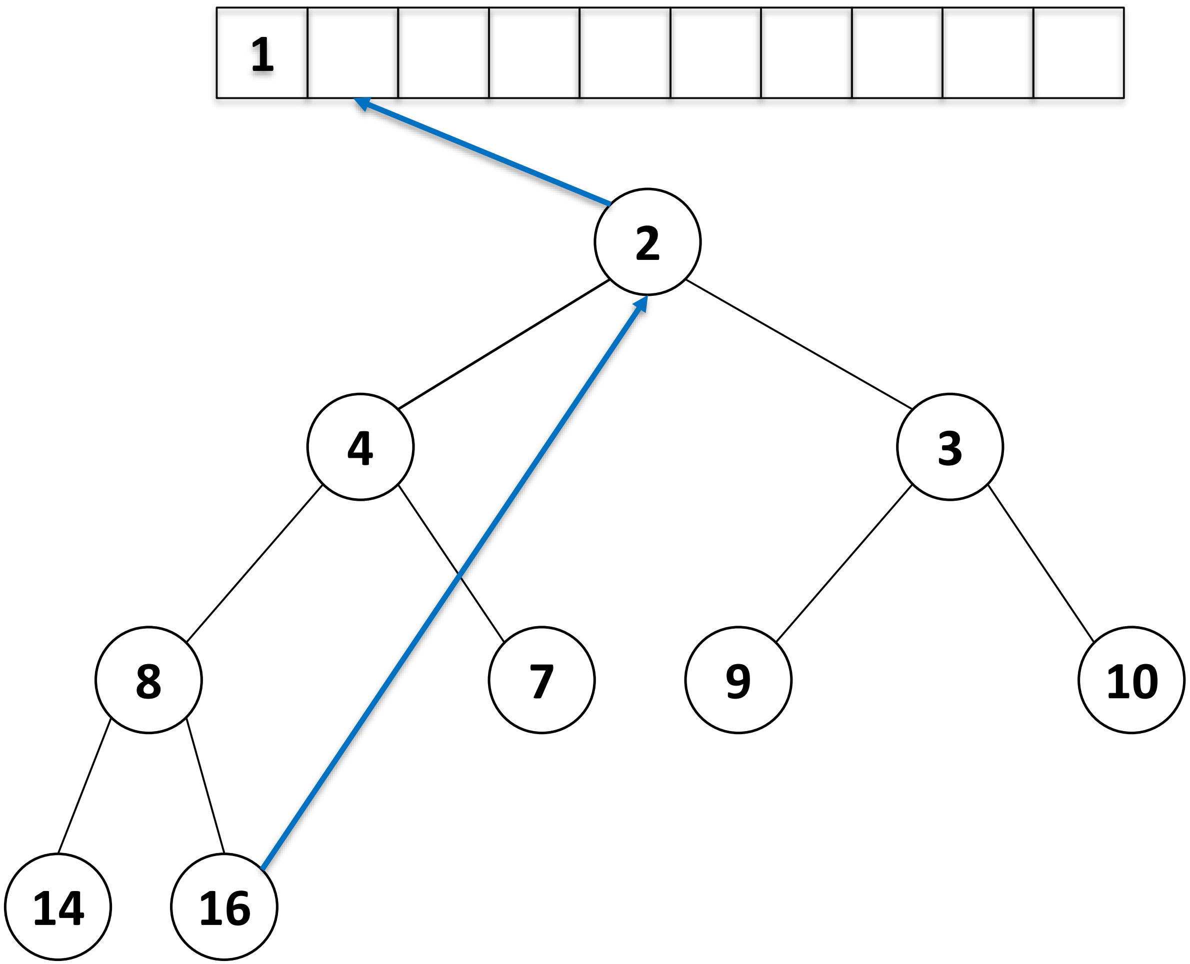
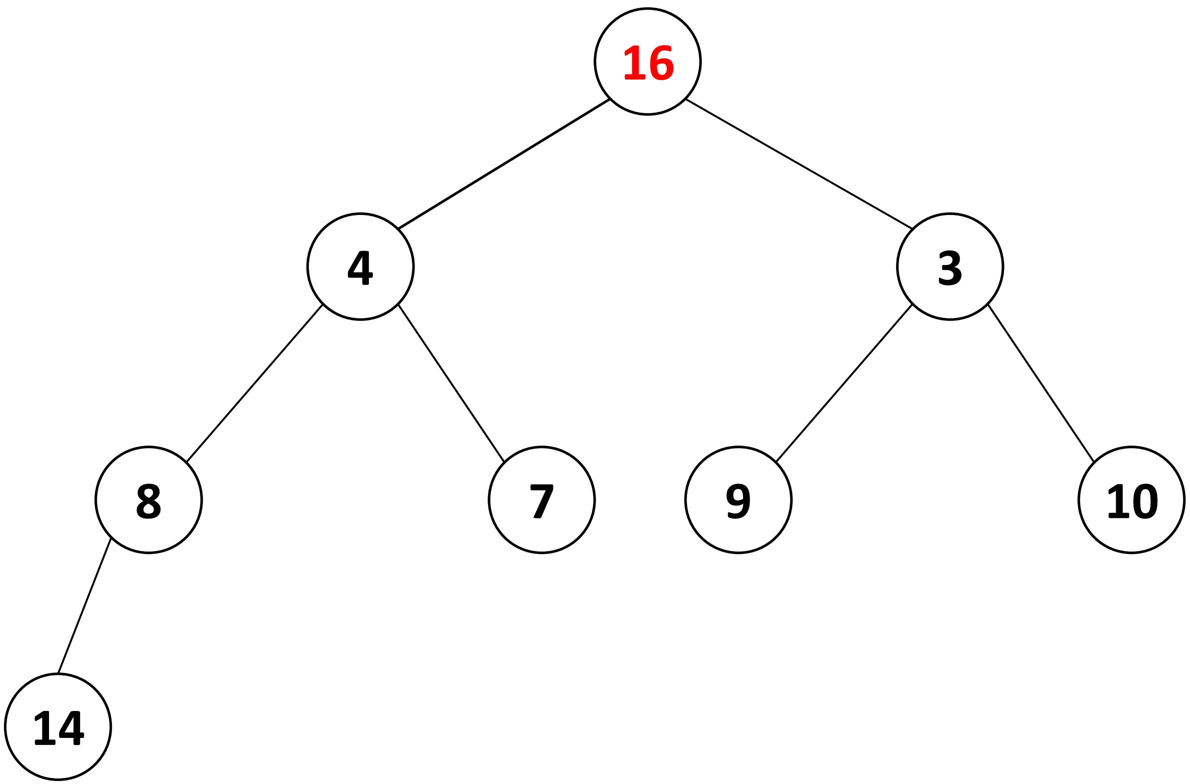


Heapsort - Example

- Perform n Extract-Min operations

1	2								



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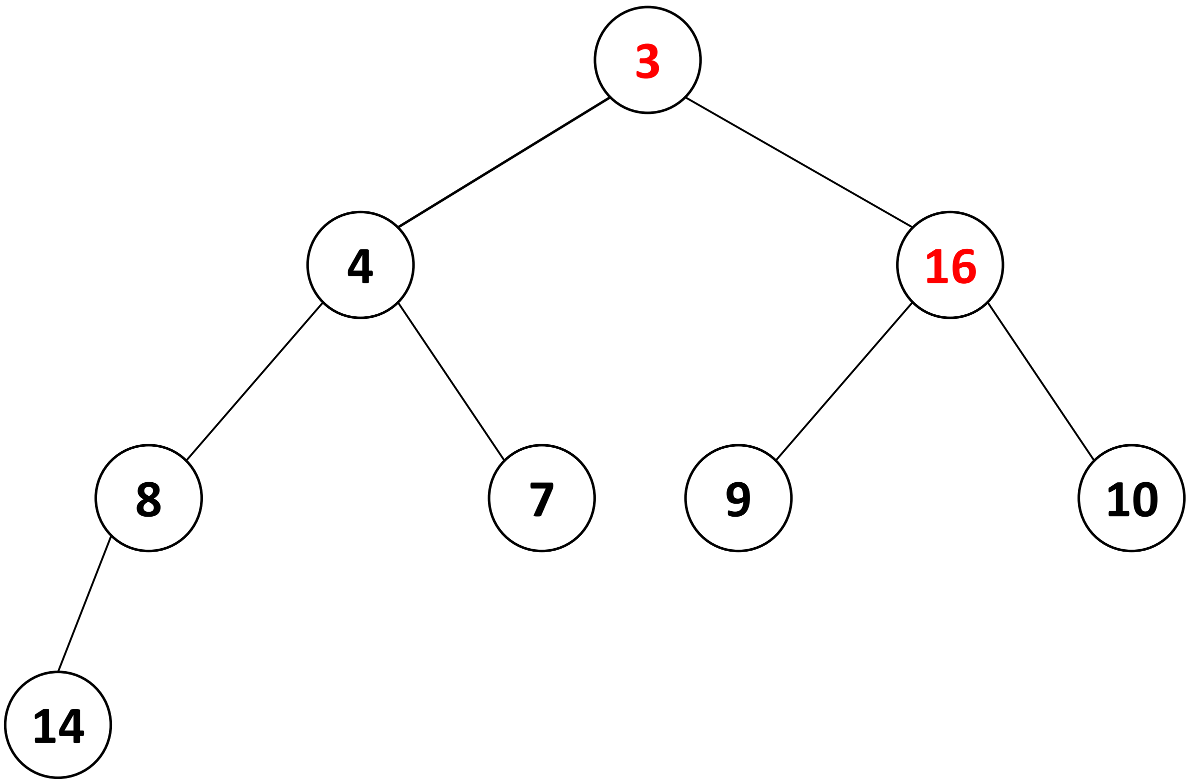

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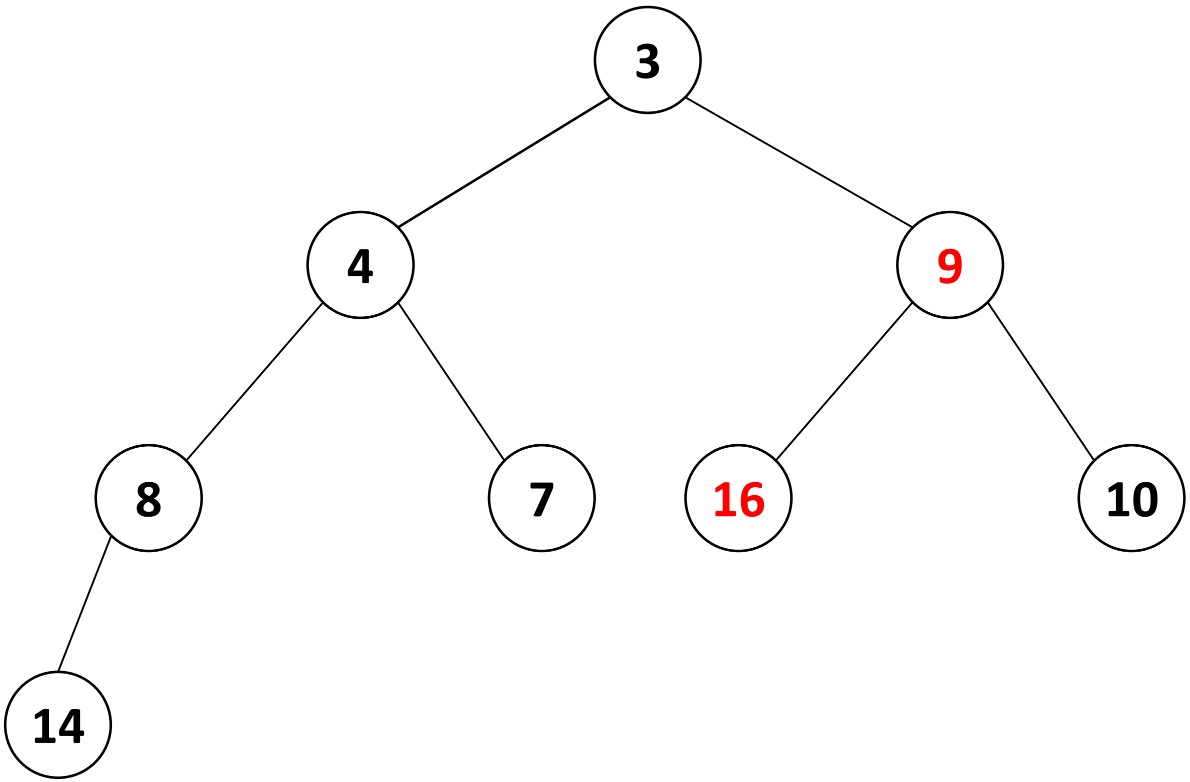

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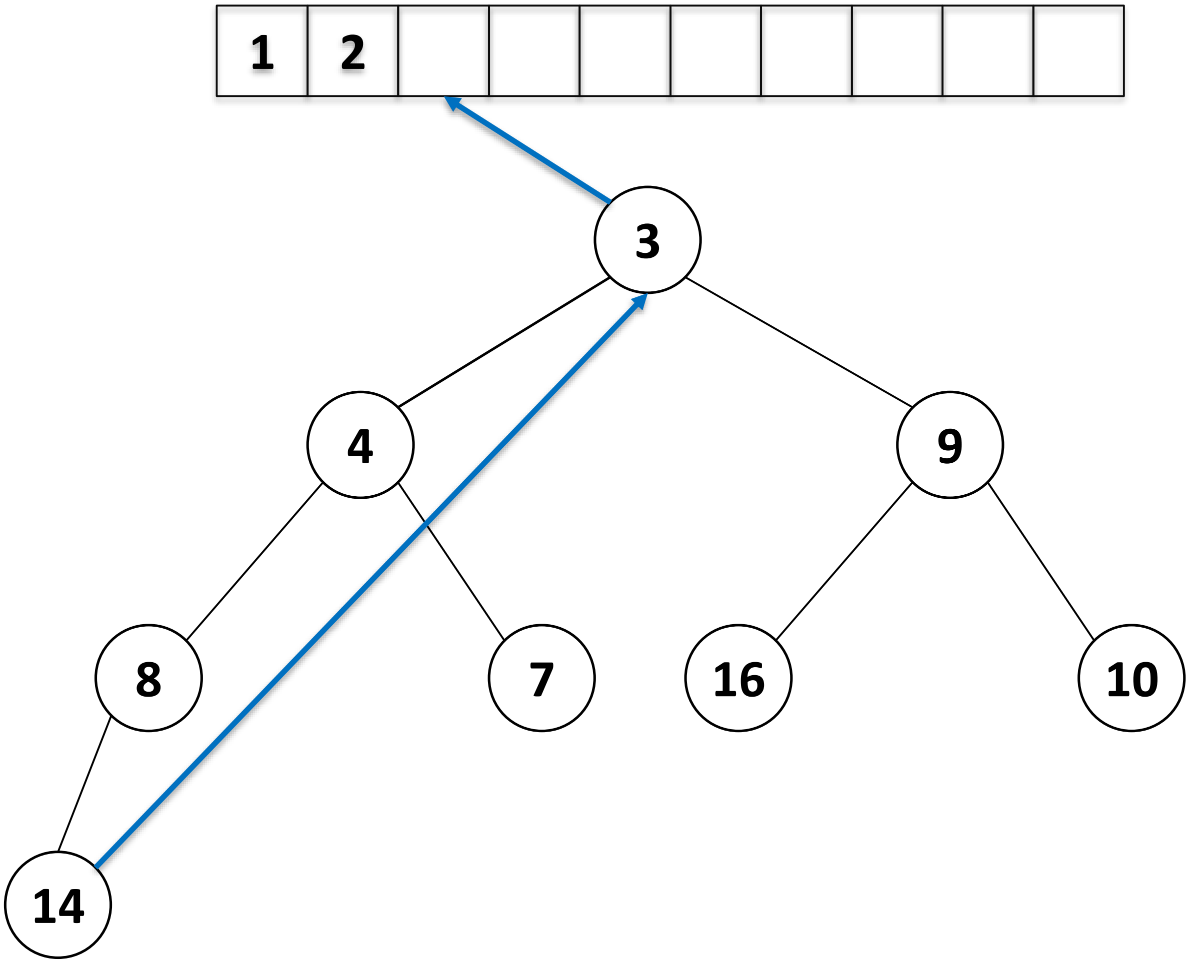

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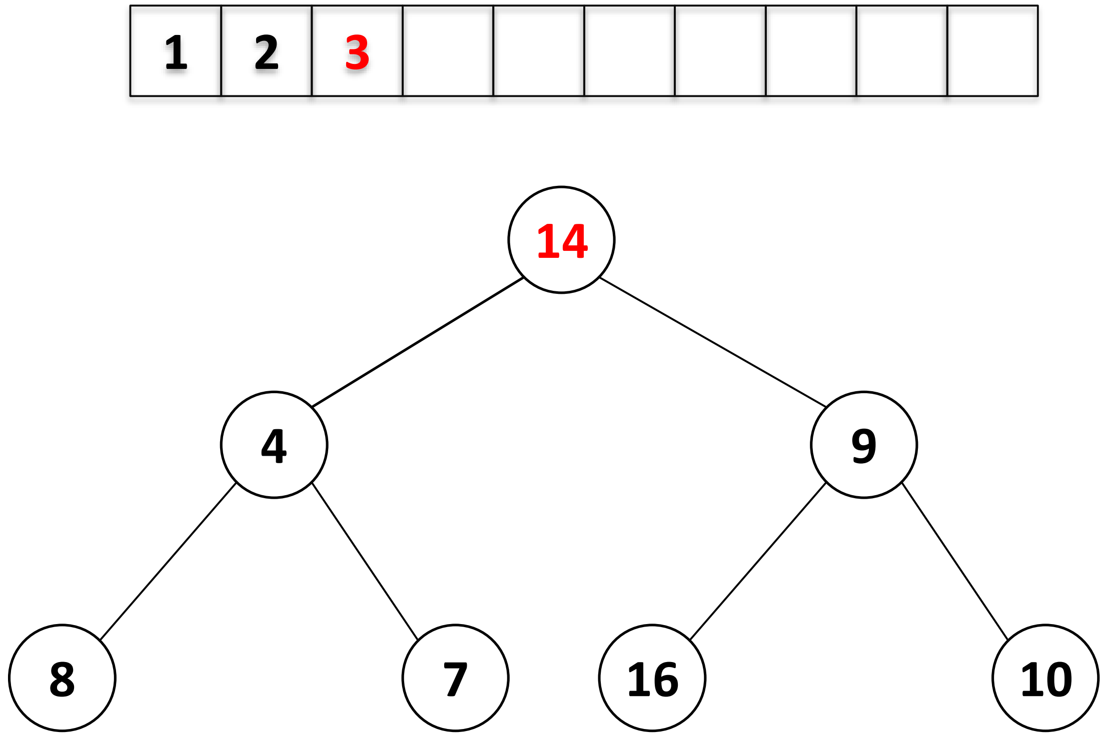
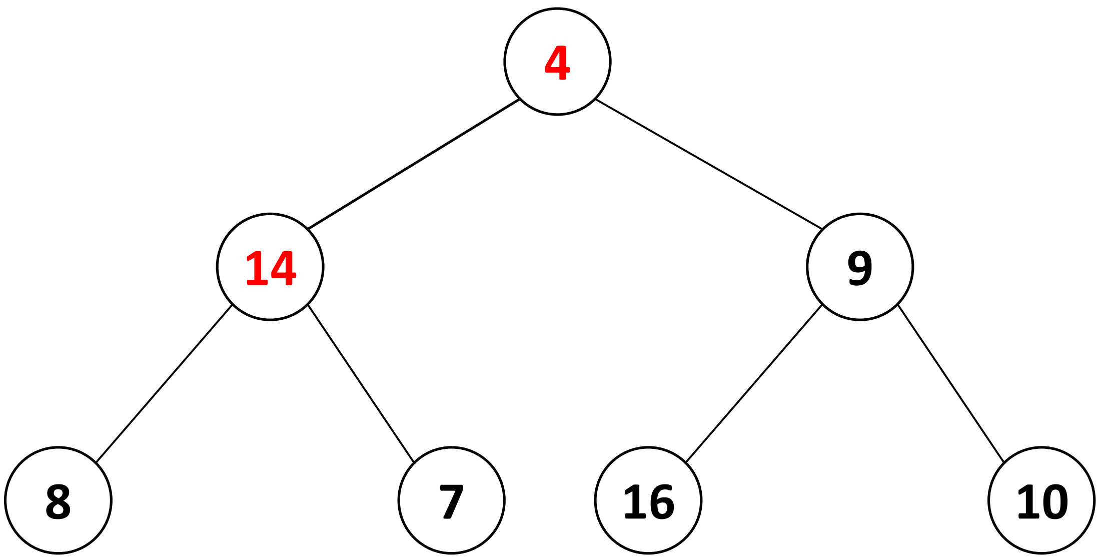


Heapsort - Example

- Perform n Extract-Min operations

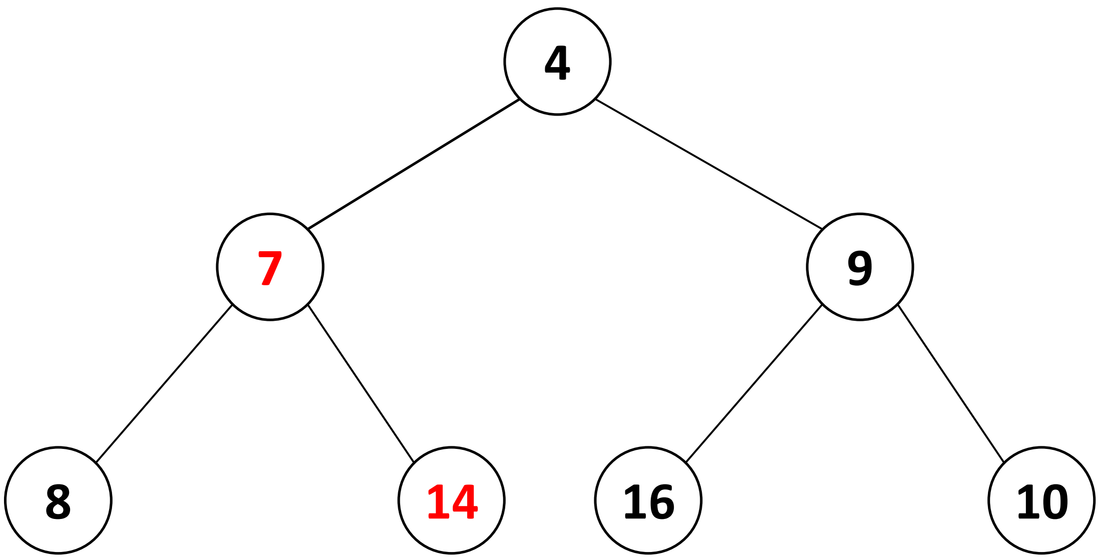
1	2	3							



Heapsort - Example

- Perform n Extract-Min operations

1	2	3							



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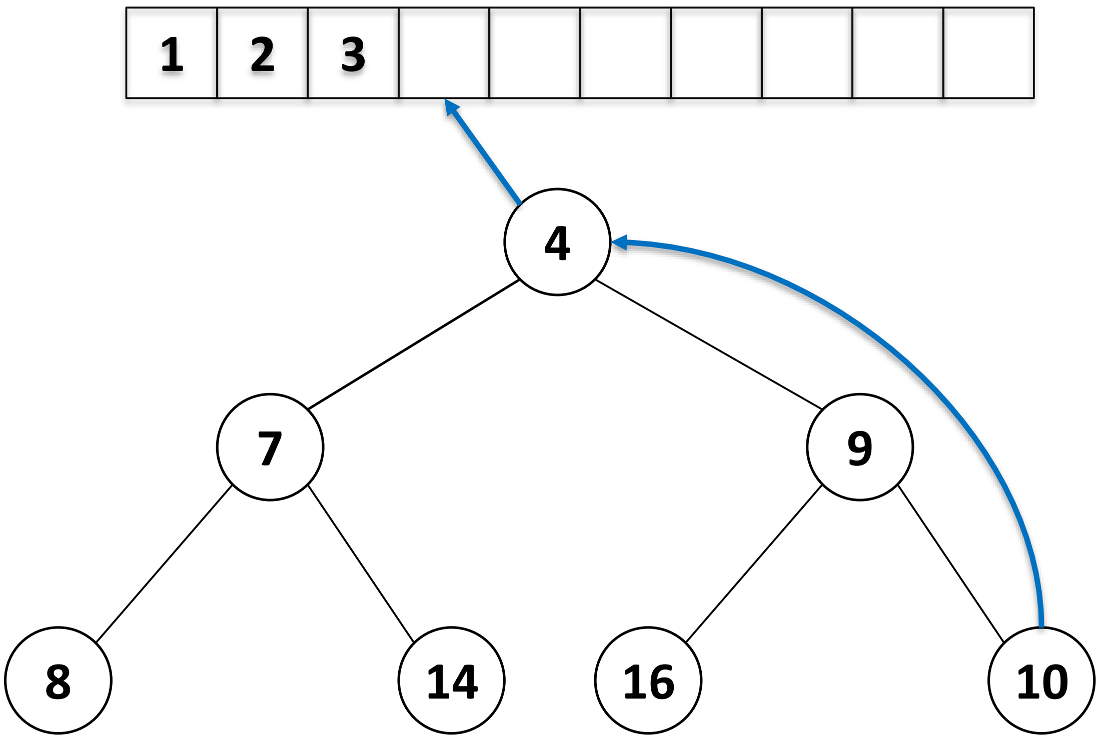

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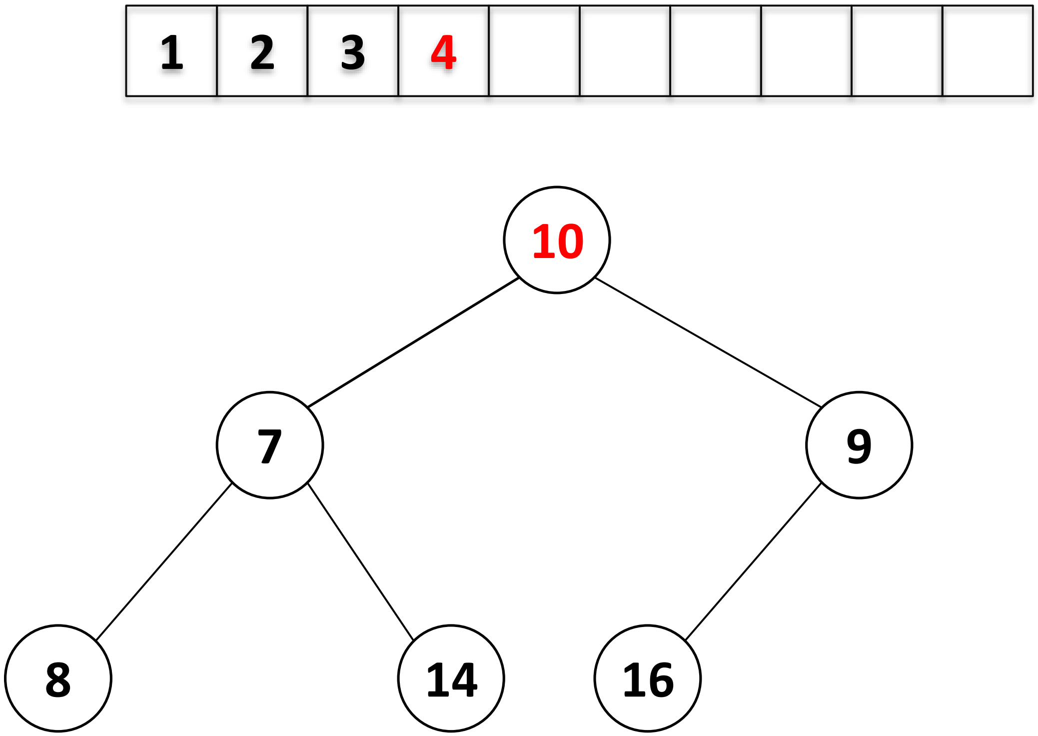
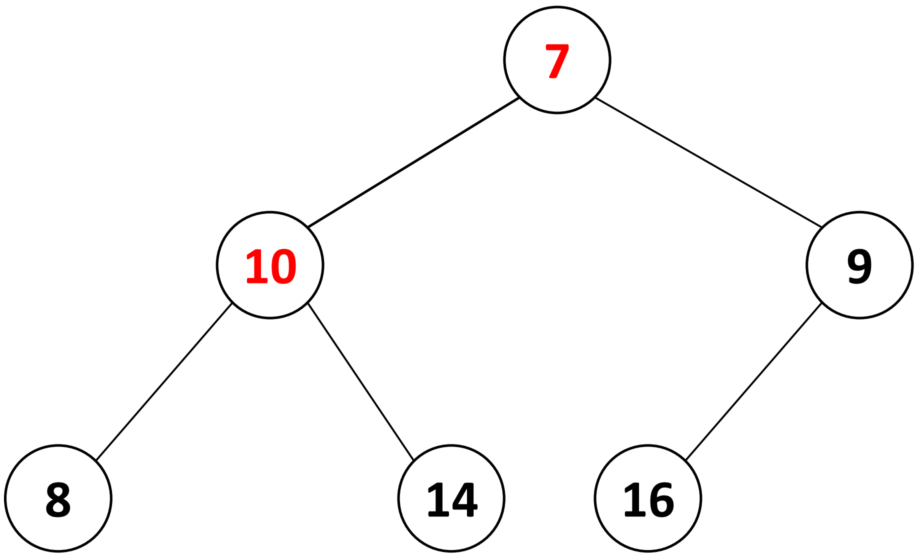


Heapsort - Example

- Perform n Extract-Min operations

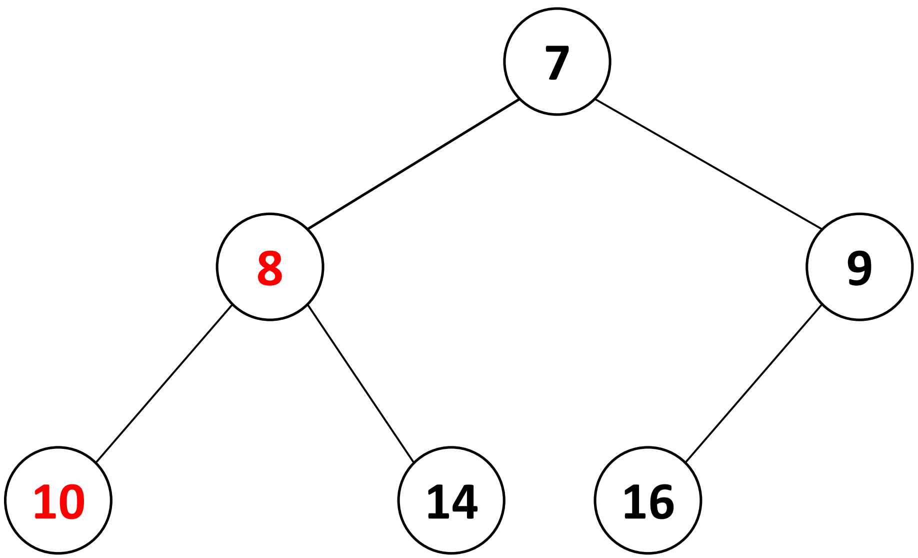
1	2	3	4						



Heapsort - Example

- Perform n Extract-Min operations

1	2	3	4						



Heapsort - Example

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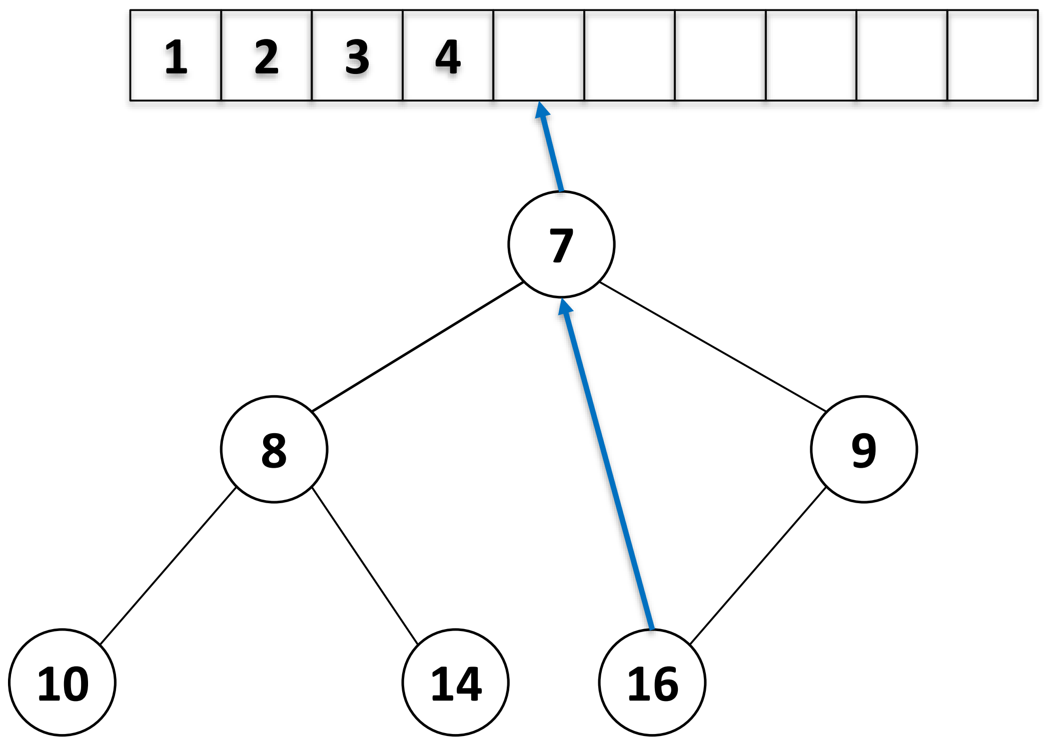
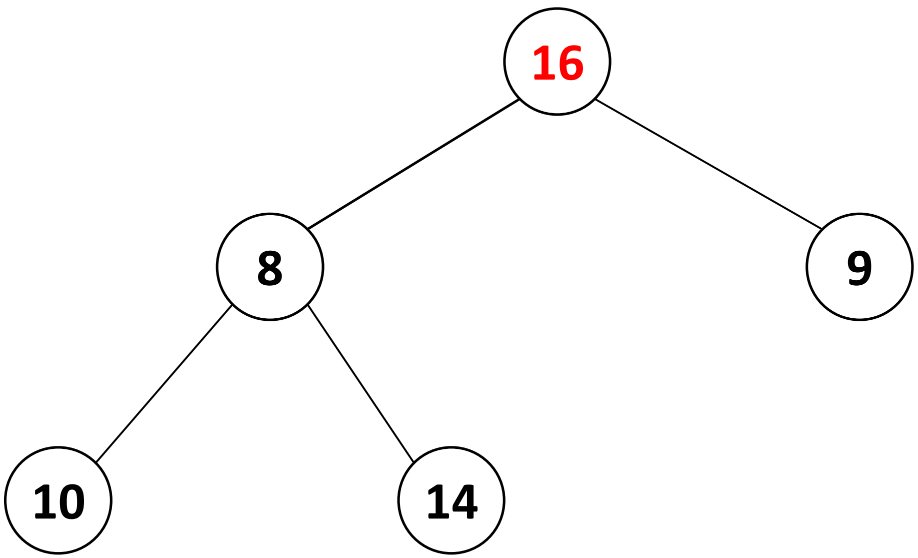


Heapsort - Example

- Perform n Extract-Min operations

1	2	3	4	7					



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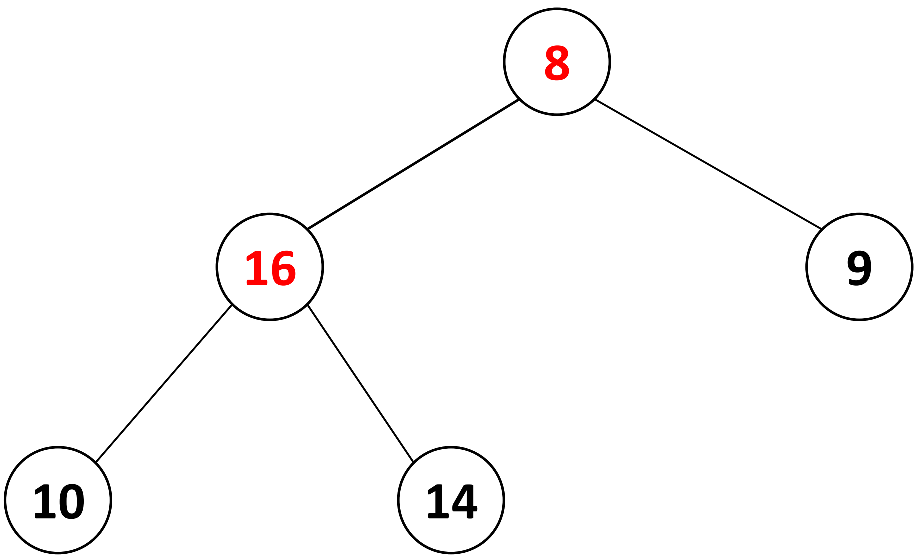

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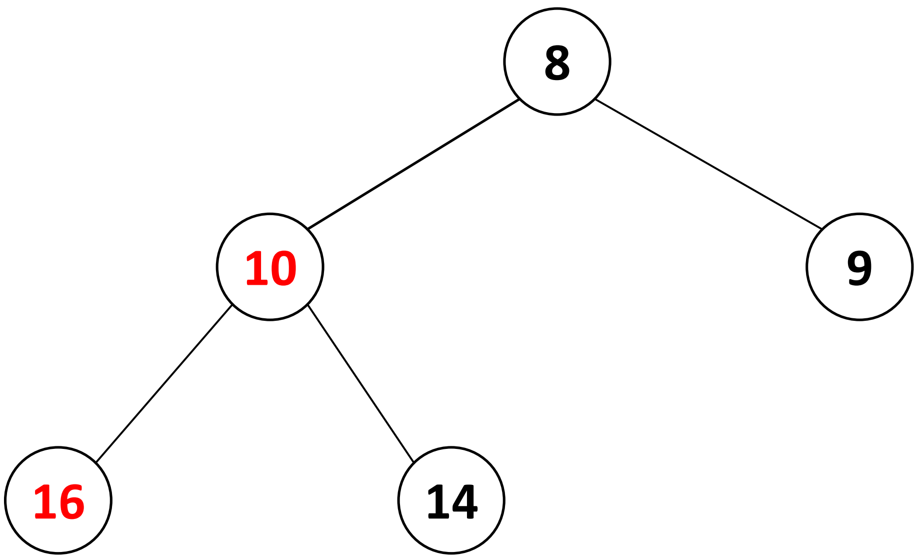

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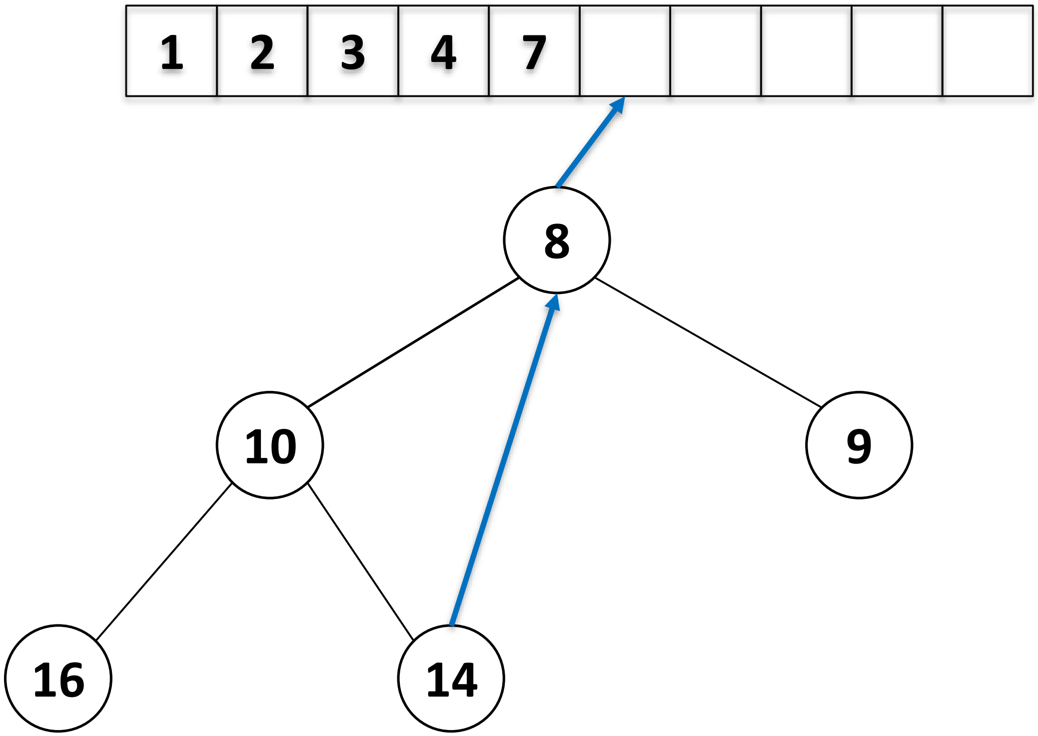
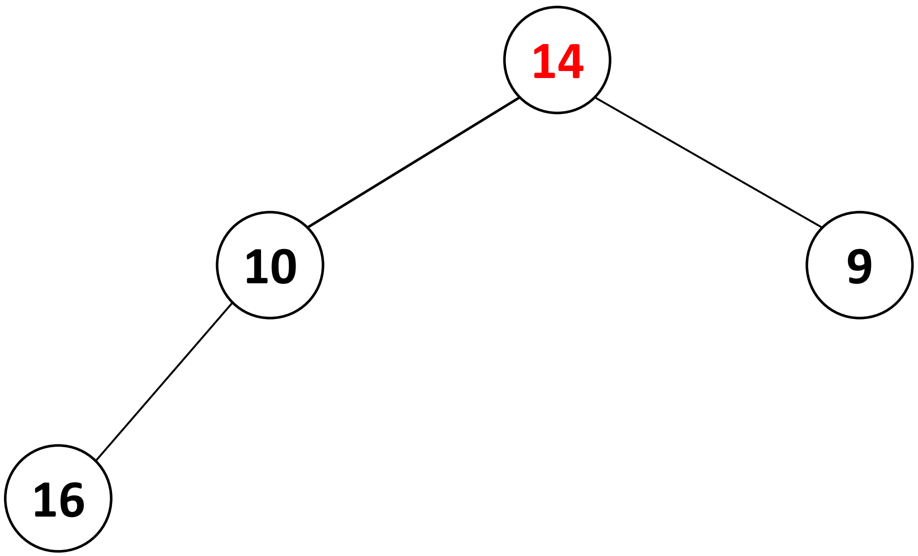


Heapsort - Example

- Perform n Extract-Min operations

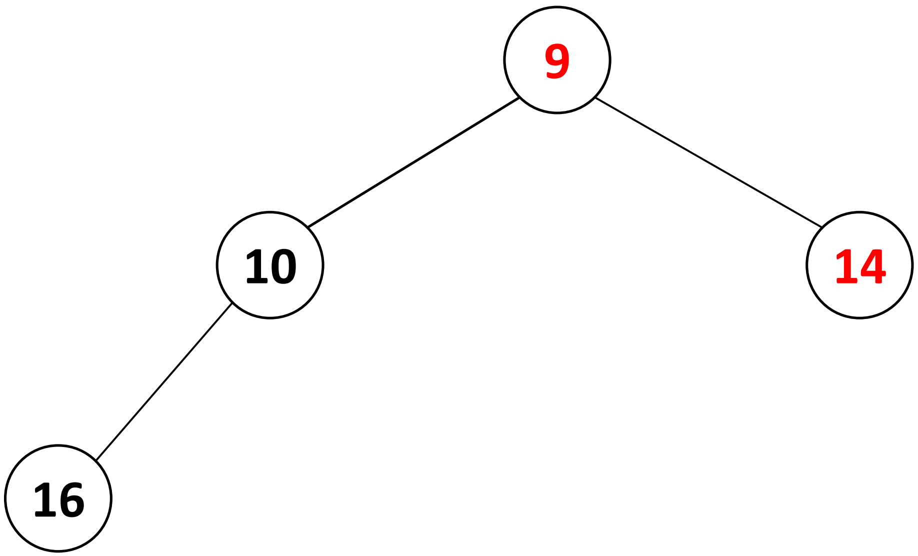
1	2	3	4	7	8				



Heapsort - Example

- Perform n Extract-Min operations

1	2	3	4	7	8				



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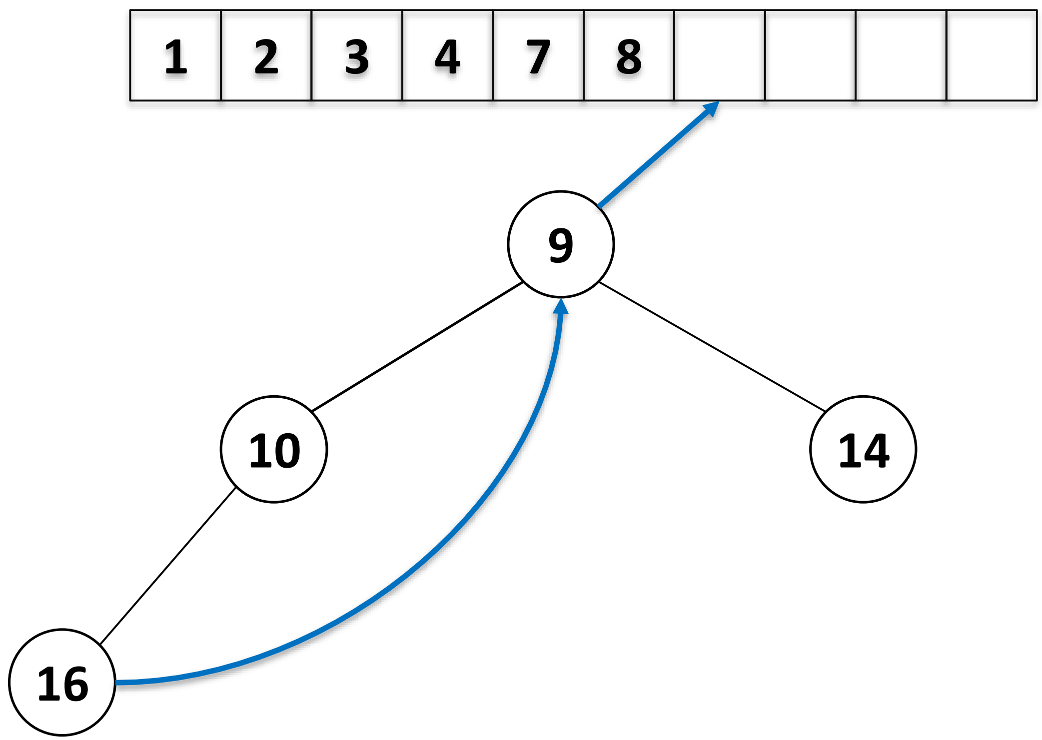
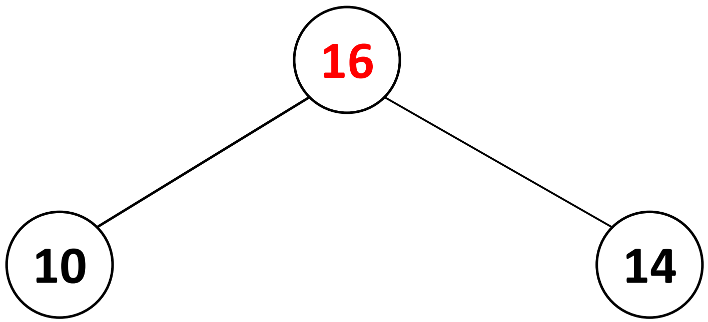


Heapsort - Example

- Perform n Extract-Min operations

1	2	3	4	7	8	9			



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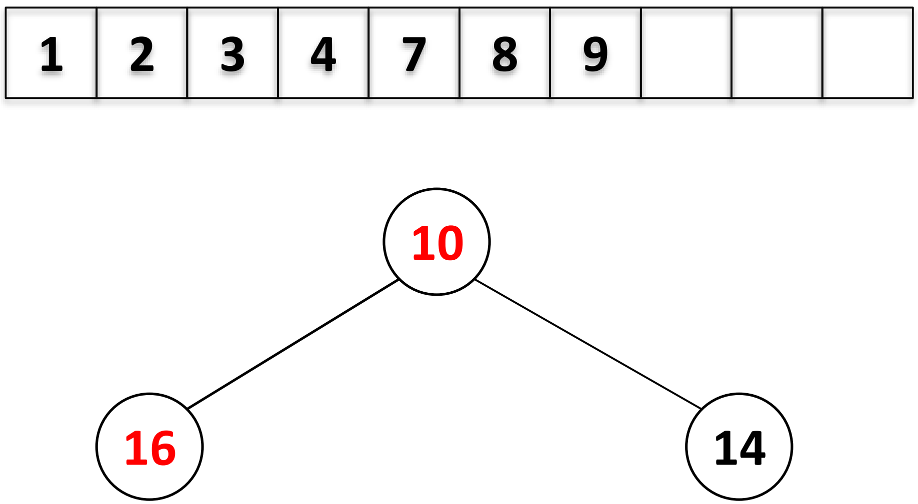

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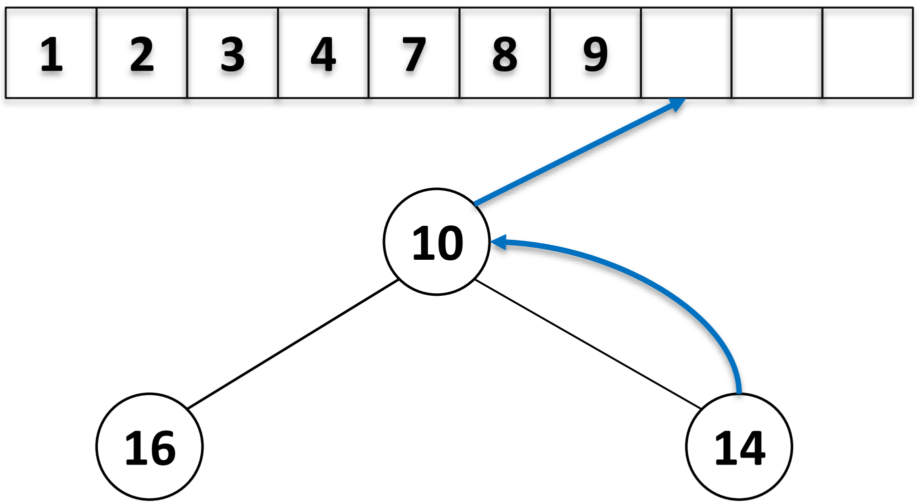
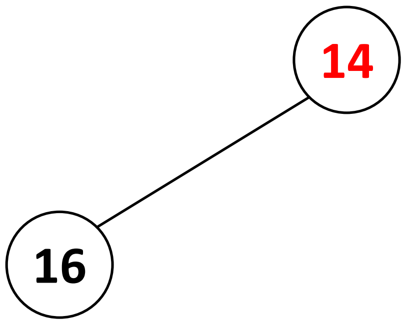


Heapsort - Example

- Perform n Extract-Min operations

1	2	3	4	7	8	9	10		



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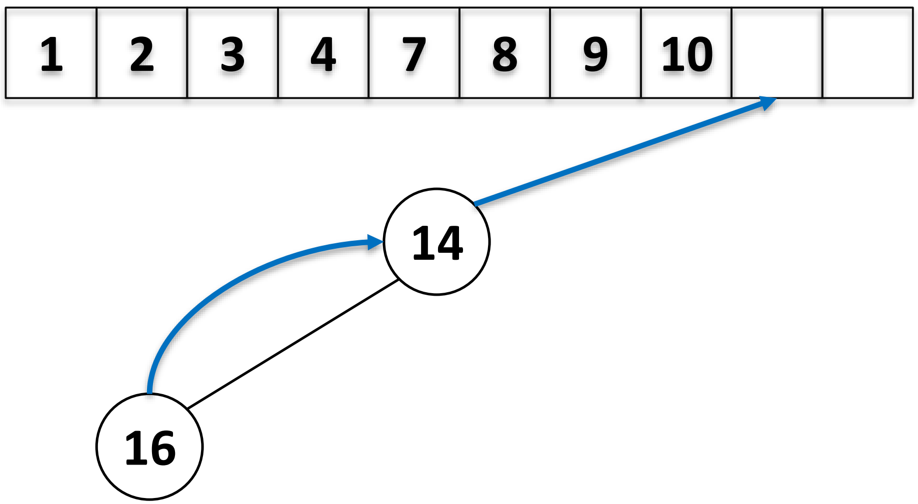

Heapsort - Example

- Perform n Extract-Min operations

1	2	3	4	7	8	9	10	14	

16

Heapsort - Example

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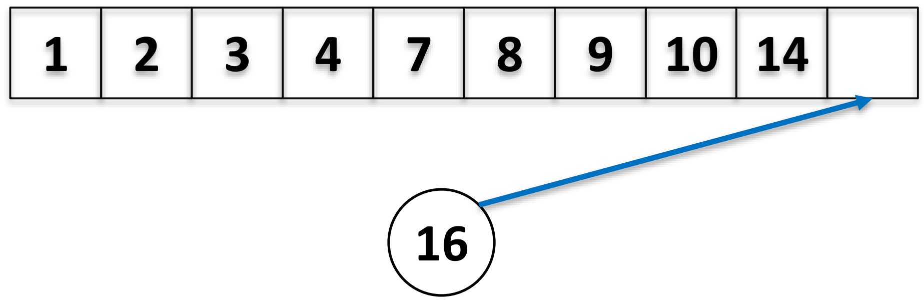

Heapsort - Example

- Perform n Extract-Min operations

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Heapsort - Example

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- If priority queues are implemented using heaps, then these two operations are supported in O(log n) time.
- Heapsort takes O(n log n) time, which is as efficient as merge sort and quicksort.

Outline

- Introduction to Part I
- Heapsort Problem
  - Priority Queues
  - (Binary) Heap
  - Heapsort
- Lower Bound for Comparison-based Sorting
  - Objective
  - Decision Tree Model

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Can we do better?

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Goal

We will prove that any comparison-based sorting algorithm has a worst-case running time $\Omega(n \log n)$.

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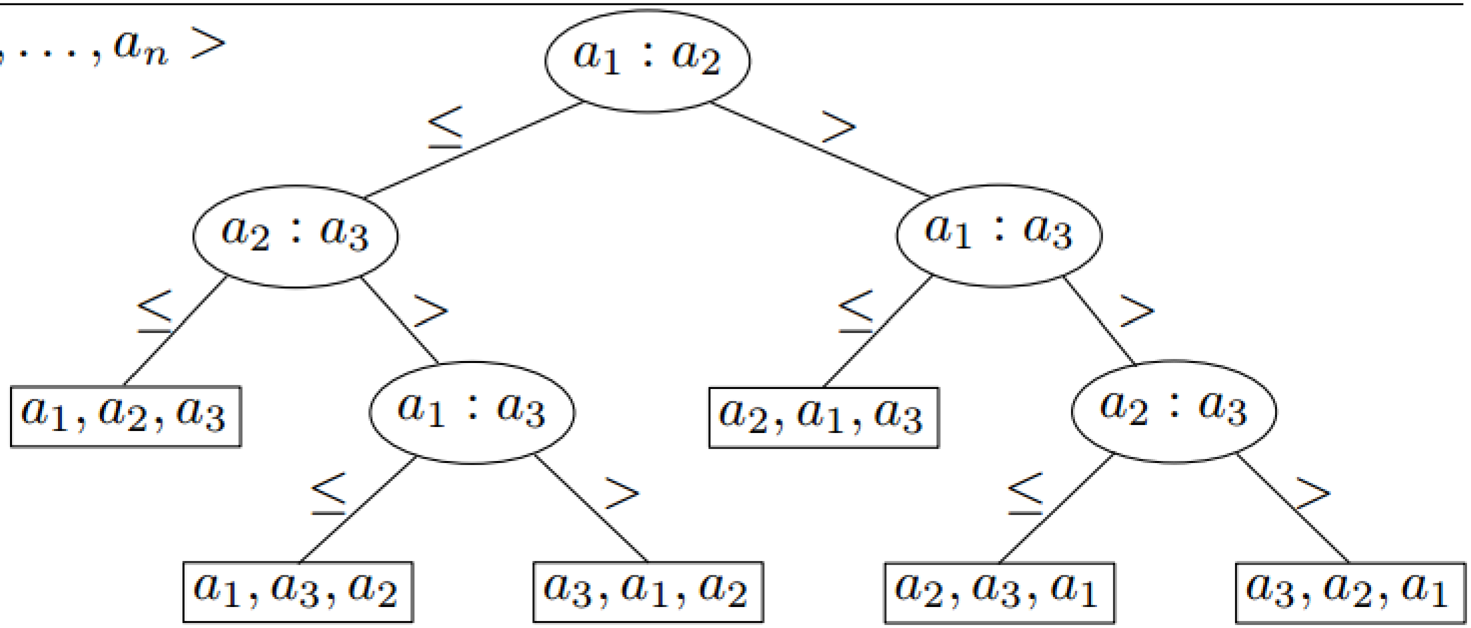
Decision-tree Model

Sort $< a_1, a_2, \ldots, a_n >$



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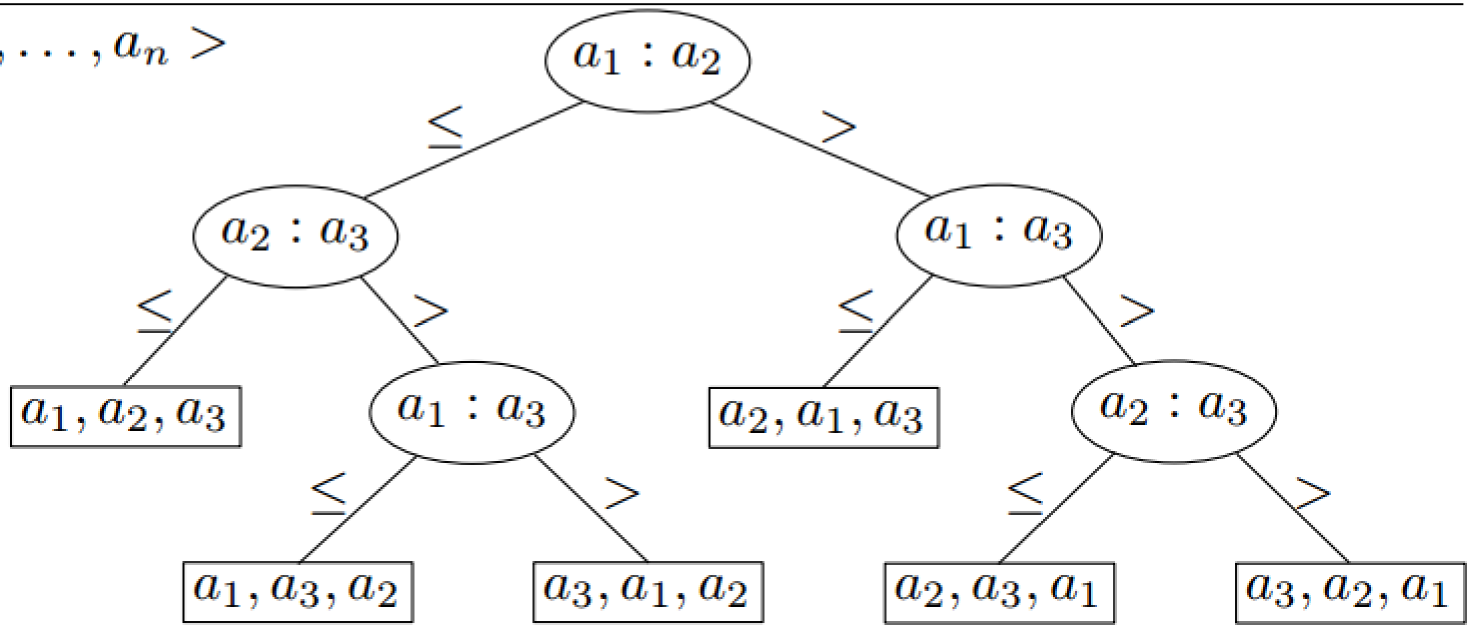

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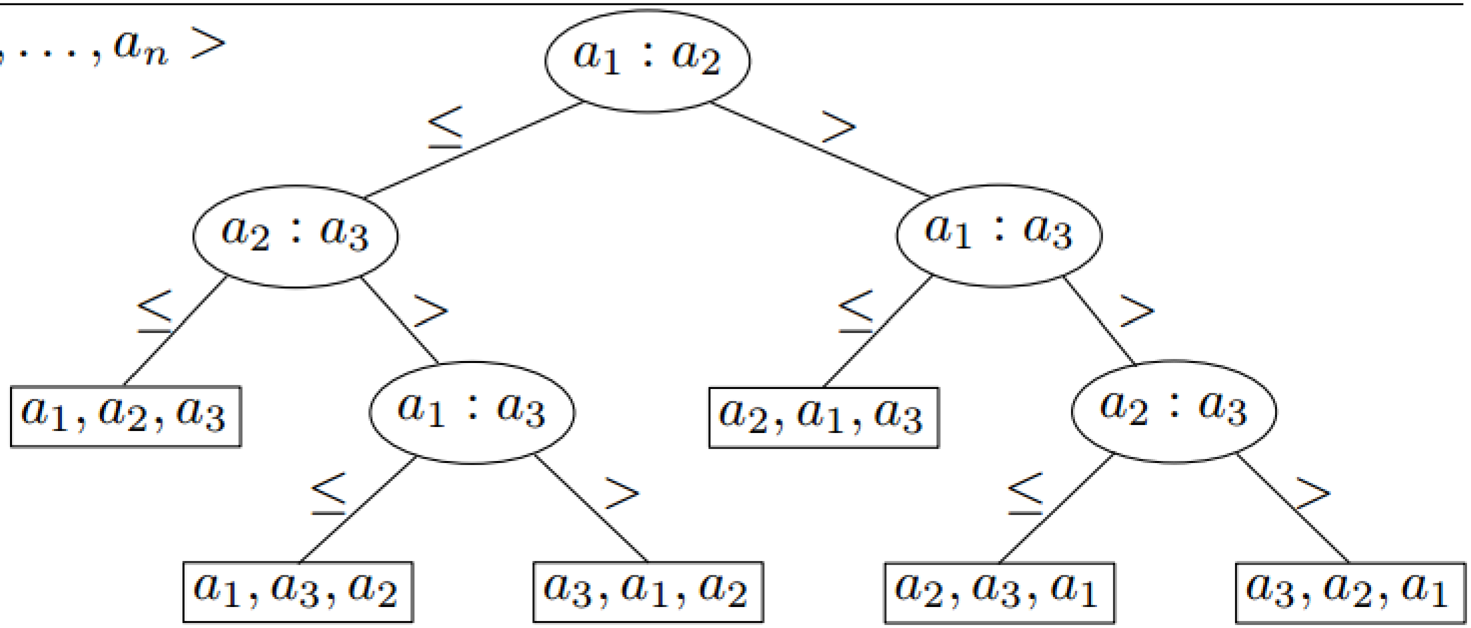

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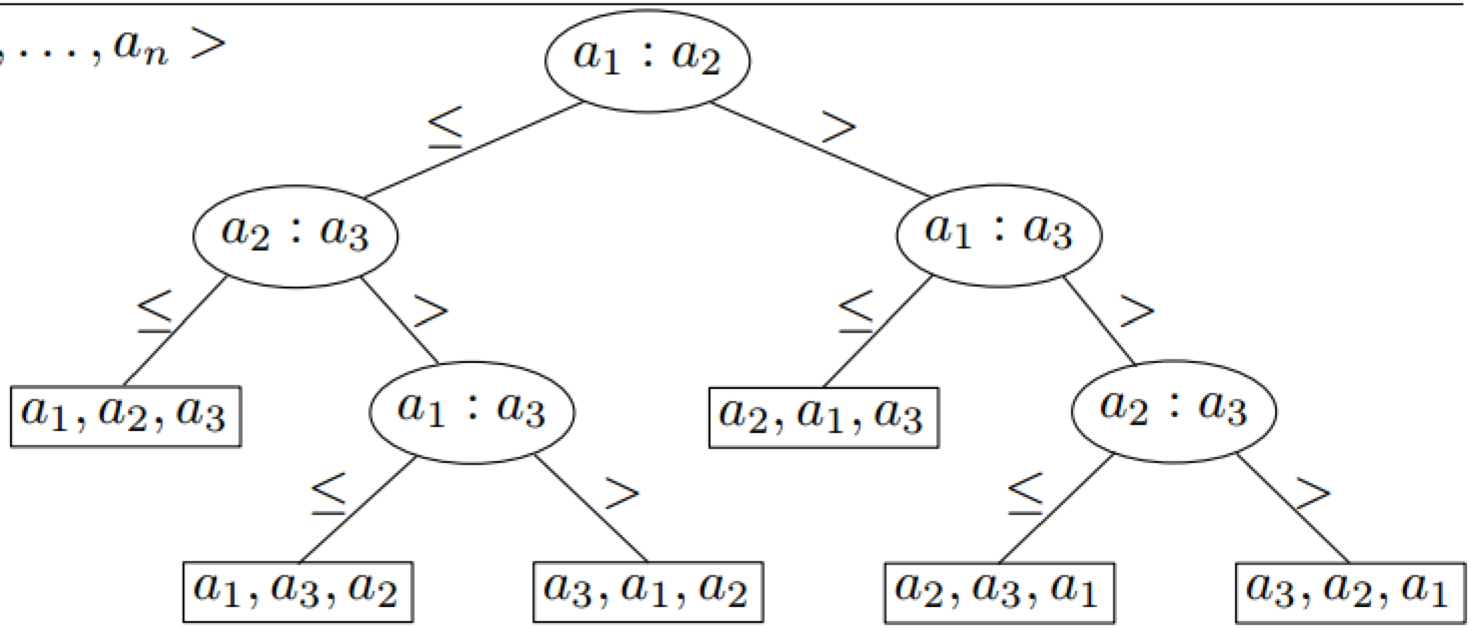

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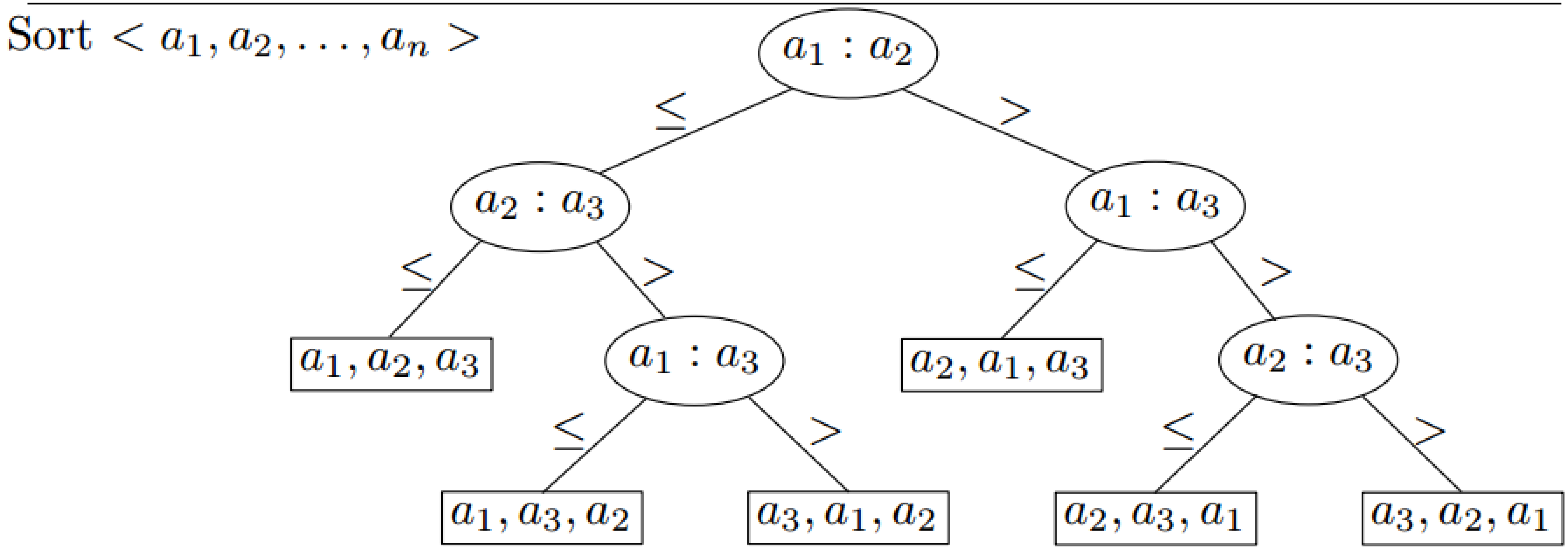

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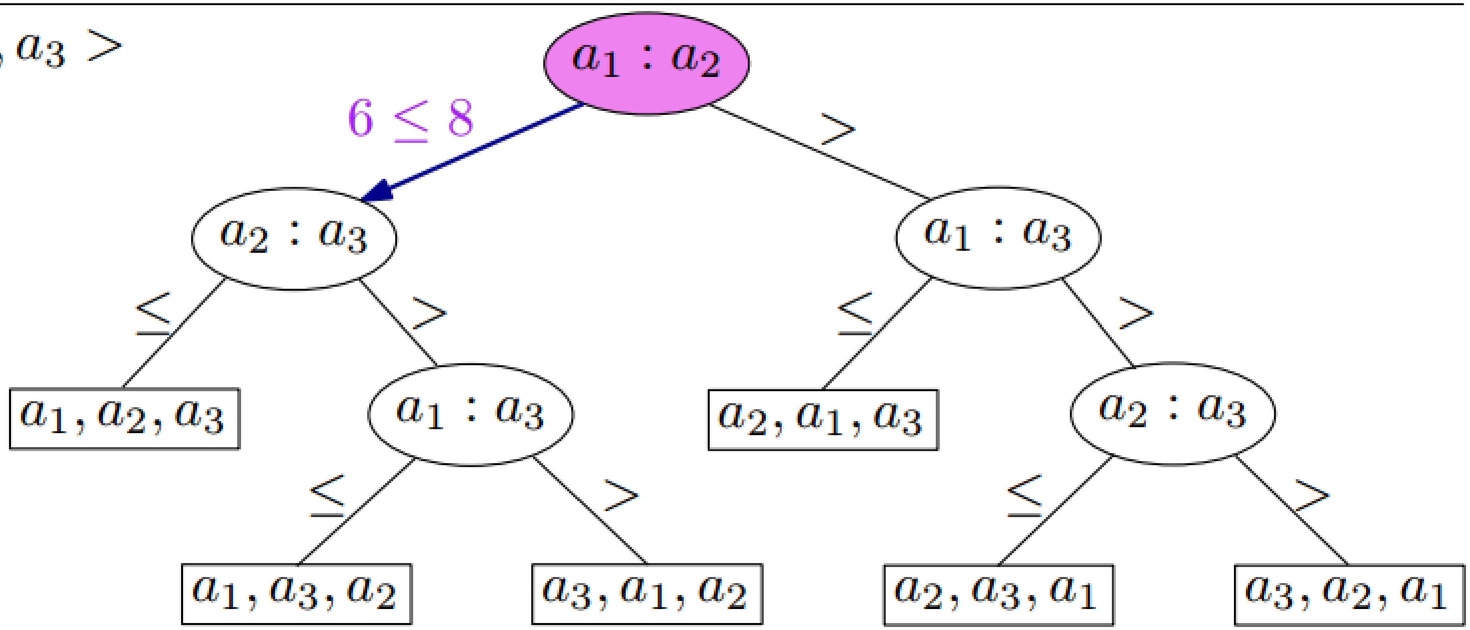

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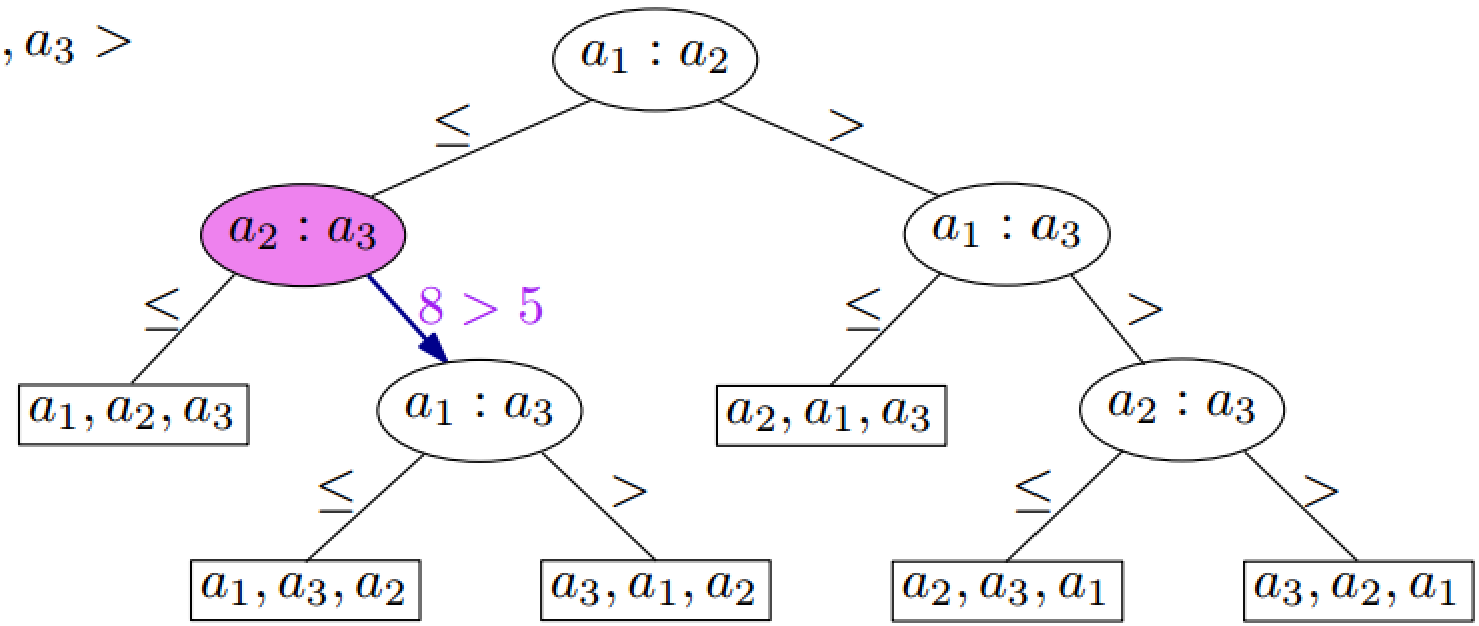

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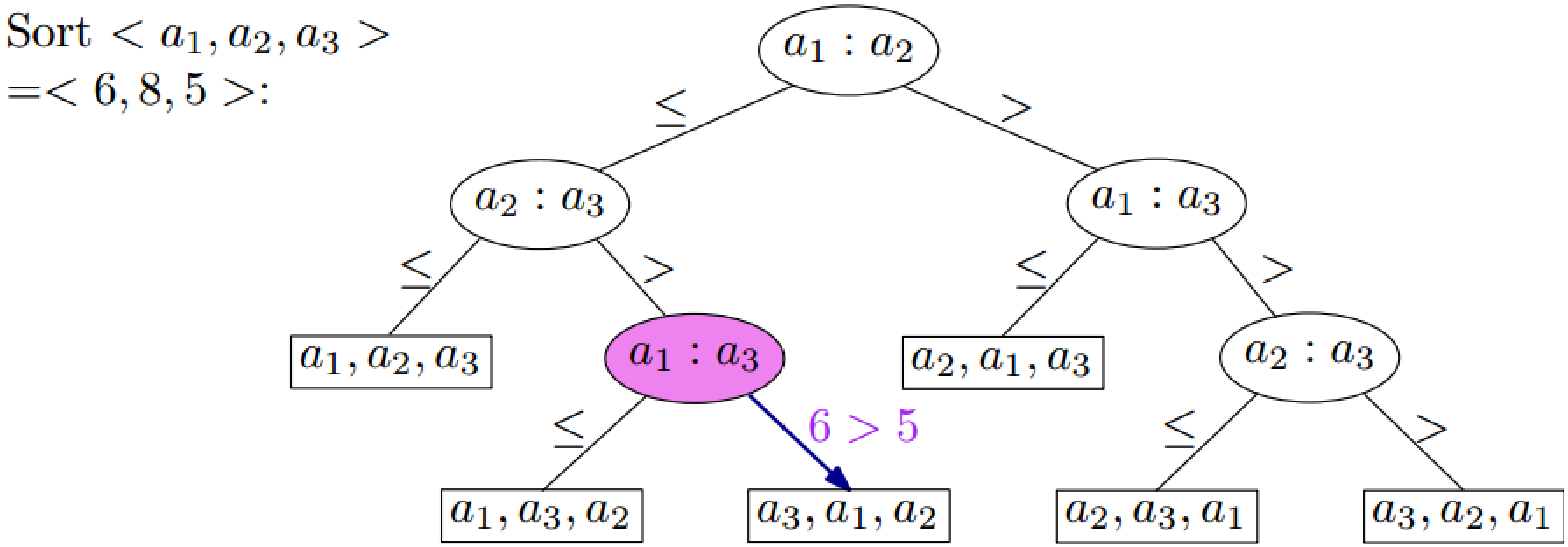

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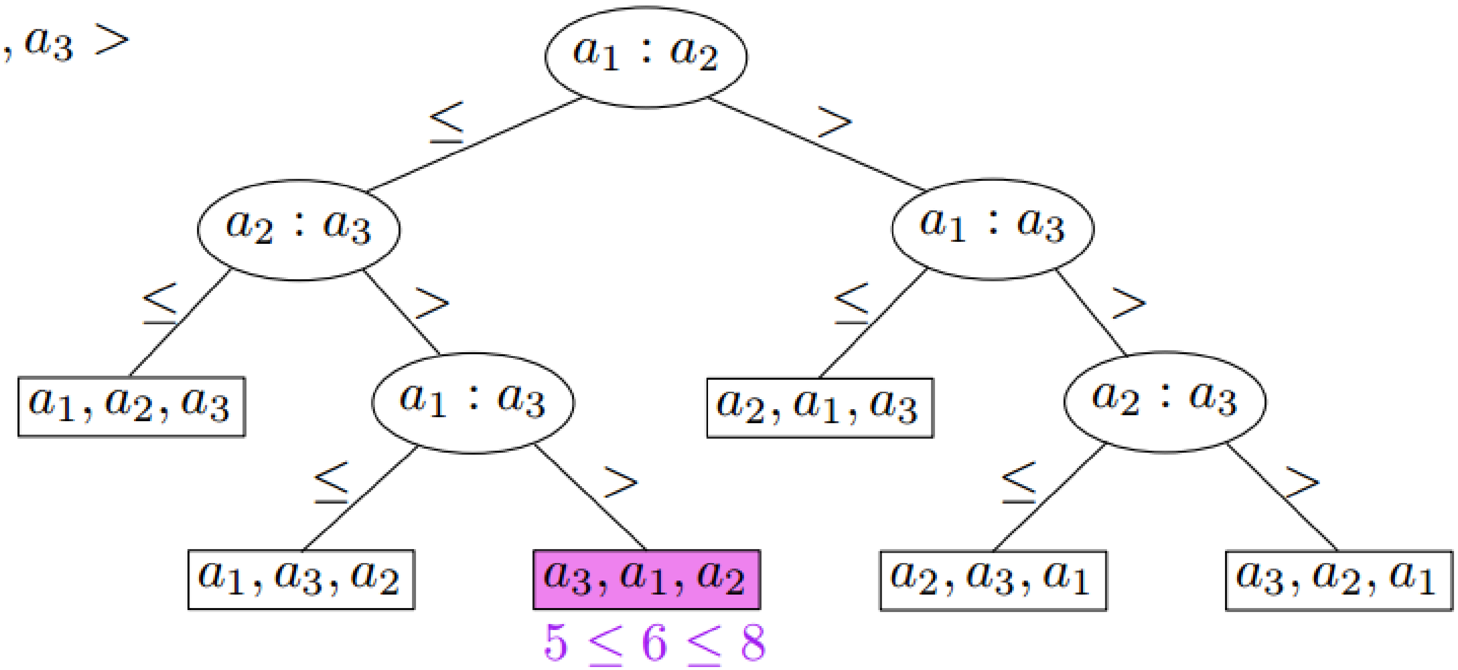

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A decision tree can model the execution of any comparison-based sorting algorithm

- One tree for each input size n

- Worst-case running time = height of tree

Lower Bound for Sorting

Theorem

Any comparison-based sorting algorithm requires $\Omega(n \log n)$ *comparisons.*

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$\square$

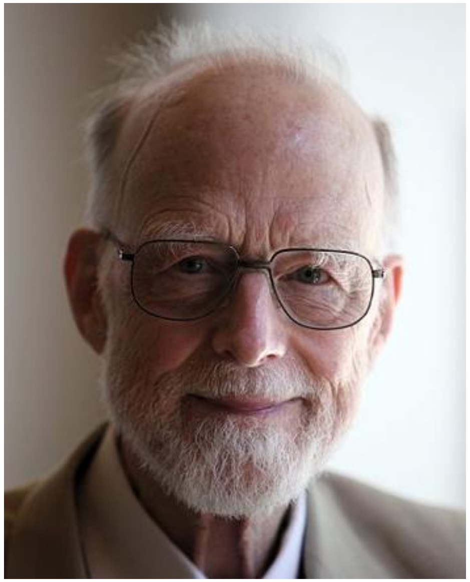
Corollary

Heapsort and merge sort are asymptotically optimal comparison-based sorting algorithms.

Summary

John von Neumann

Merge Sort Algorithm was invented in 1945



Tony Hoare

Quicksort Algorithm was invented in 1959

J. W. J. Williams

Heapsort Algorithm was invented in 1964

Which algorithm is the best in practice?


dank u
ju faleminderit
Tack
谢谢
Tak
mulţumesc
Asante
kiitos
Gracias
Salamat!
Terima kasih
Aliquam
Merci
Dankie
Obrigado
ありがとう
köszönöm
grazie
Aliquam
Go raibh maith agat
děkuji
Thank you
gam