**Microscopic study of factors affecting**

**liquefaction strength during anisotropic consolidation**

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**ABSTRACT**

This study employs the discrete element method (DEM) to investigate the effects of stress anisotropy on liquefaction resistance of sand soils through undrained cyclic shear simulations. A combined servo mechanism replicates undrained conditions and stress states in hollow cylinder apparatus (HCA) tests. The influence of lateral to vertical stress ratios (), as well as the stress path for specimen preparation, on the soil's response to cyclic loading are examined across a wide range of values from 0.33 to 3.33. Results demonstrate that increasing stress anisotropy reduces liquefaction resistance and the stress path, whether through initial isotropic consolidation followed by linear anisotropic consolidation (IC-AC) or initial isotropic consolidation followed by constant- triaxial shear (IC-TS), does not significantly affect liquefaction resistance. The factors that influence liquefaction resistance are attributed to changes in both macroscopic relative density and microscopic quantities, such as coordination number () and fabric anisotropy (). Anisotropic consolidation states with <1.0 or >1.0 produce different morphologies of contact density, affecting fabric anisotropy and liquefaction resistance.

Keywords: liquefaction, hollow torsional shear test, undrained cyclic shear, discrete element method

**1. Introduction**

The phenomenon of liquefaction, which occurs with the loss of soil strength due to the buildup of pore water pressure during an earthquake, has been recognized for its potential to cause severe damage to infrastructure and buildings (Ishihara and Koga, 1981; Seed and Idriss, 1967). Triaxial tests have been extensively conducted to elucidate the mechanisms of liquefaction, where saturated specimens were subjected to cyclic loading under undrained conditions until liquefaction was triggered. The influence of factors such as cyclic stress ratio (CSR), relative density, as well as confining pressure on the resistance to liquefaction was examined (Hyodo et al., 1991; Seed and Lee, 1966; Silver et al., 1976; Toki et al., 1986; Yoshimi et al., 1984). However, vertically propagating shear waves in the ground apply gradually varying shear stress on soil elements, leading to a continuous rotation of principal stress axes (Arthur et al., 2009; Arthur et al., 1980; Ishihara and Yasuda, 1975; Ishihara and Towhata, 1983; Yamashita and Toki, 1993).

Alternative testing methods, such as the hollow torsional shear test, apply shear forces to specimens, allowing for continuous variation of principal stress axes and thereby addressing the limitations of the triaxial test. Ishihara and Yasuda (1975) pioneered the utilization of hollow torsional cylindrical apparatus (HCA) by subjecting the hollow cylindrical samples to irregular wave loading, studying the disparities compared to triaxial shear tests. Tatsuoka et al. (1986) performed both triaxial and torsional tests on specimens prepared using different methods and found that the results were inconsistent between the triaxial and torsional tests. Torsional and triaxial shear tests conducted by Yamashita and Toki (1993) and employed by Oka et al. (1999) to enhance the constitutive model for liquefiable sands also demonstrated that method of testing with torsional or triaxial shear, influences the results of liquefaction resistance. These studies highlight the significance of experimental methods, such as HCA tests, in liquefaction analyses and aroused great interest in numerical replication of these tests.

Soils under a natural state generally display various ratios of lateral to vertical effective stress, denoted as . The impact of values on liquefaction strength frequently garners attention, yet the corresponding findings remain controversial. Ishihara and Takatsu (1979) observed that the liquefaction strength of Fuji River sand does not exhibit a notable dependency on the initial stress state with different values. Similar results were also obtained in the laboratory tests conducted by Yamashita and Toki (1993). On the other hand, the hollow torsional experiments conducted by Georgiannou and Konstadinou (2014) indicated that isotropically consolidated (IC) specimens demonstrate higher liquefaction resistance for loose sands than anisotropically consolidated (AC) specimens. By contrast, that pattern does not hold in dense states, where increasing relative density reverses the trend. Additionally, Vargas et al. (2020) concluded from similar laboratory tests on Ottawa sand with relative densities ranging from 50% to 80% that AC specimens with a of 0.5 showed a liquefaction strength approximately 20% higher than IC specimens. The experimental conclusions regarding the influence of on liquefaction resistance have been debated for decades,underscoring the necessity of elucidating effects on liquefaction resistance through alternative means. Additionally, the previous studies mentioned above focus primarily on a narrow range of initial states, typically involving values of 0.5, 1.0, and 2.0, without exploring a wider range of .

The discrete element method (DEM) (Cundall and Strack, 1979) simulation provides an insight into granular material and offers advantages by eliminating concerns related to variations in initial states caused by sample preparation, making it a desirable numerical method to study the cause of changes in liquefaction resistance. Numerous examples utilizing DEM exist for undrained cyclic shear tests to find explanations of microscopic factors affecting liquefaction resistance. Huang et al. (2018) conducted undrained shear tests on triaxial specimens, trying to relate monotonic and cyclic behaviors. Yang et al. (2021) performed undrained simple shear tests and studied the influence of multi-directional shear stress on liquefaction resistance. Jiang et al. (2021) applied various forms of strain waves to specimens, investigating their impact on liquefaction resistance. Morimoto et al. (2021) examined the impact of pre-shearing on the liquefaction resistance using DEM simulation of undrained triaxial cyclic shear tests. Xie et al. (2023), as well as Yang and Huang (2023) explored the effect of liquefaction history-induced fabric on liquefaction resistance by conducting reliquefication simulation. Zhang et al. (2023) arranged ellipsoidal clumped pebbles and applied both vertical and horizontal shear loading in to discuss the influence of inherent fabric anisotropy on liquefaction resistance. Some of these studies included triaxial specimens, which do not account for principal stress axis rotation. Others utilized virtual periodic boundaries or cubic rigid box, which are difficult to implement in real-world scenarios. Replicating HCA tests through simulation provides a meaningful connection between numerical and experimental methods.

Using DEM to replicate HCA test is relatively specialized, but still has precedents. Li et al. (2014) conducted DEM simulations of drained tests and investigated the strain localization in HCA test. Liu et al. (2021) conducted analysis of torsional shear tests under drained conditions and investigated the development of cracks at different principal stress rotation angles. Ma et al. (2024) introduced an algorithm that realizes both undrained and stress conditions in HCA test, filling a gap in HCA simulation using DEM.

To clarify the influence of the on liquefaction resistance, Yang and Taiebat's (2024) consolidated specimens with different preparation methods and conducted undrained cyclic shear tests. They found that both preparation protocols and influence liquefaction resistance. As the relative density increased, the difference in liquefaction resistance narrowed gradually for IC and AC states. Otsubo et al. (2022) employed to induce inherent fabric anisotropy in specimen under a low stress condition and then consolidated it to the target and examined its effects on liquefaction resistance. A specimen with higher anisotropy has weaker stiffness in its minor direction and resulted a lower liquefaction resistance. Still, values discussed in the mentioned numerical studies lie in a limited range. For instance, comparison between =1.0 and =0.5 (Yang and Taiebat's, 2024) or from =0.75 to =1.35 (Otsubo et al., 2022), and exploration beyond these thresholds is lacking.

The stress paths for specimen preparation often entail linearly increasing and to the state with target in both experimental (Vargas et al., 2020) and numerical (Yang and Taiebat, 2024) tests, thereby intriguing further investigation into the effects of stress paths by incorporating other consolidation stress paths. This study demonstrates DEM analysis of cyclic undrained HCA tests (Ma et al., 2024) and explores the effects of on liquefaction resistance. Specimens are prepared with two different stress paths in consolidation and subjected to an extensive range of cyclic shear stress ratios. By examining macroscopic and microscopic responses such as fabric evolution, this study aims to provide evidence that elucidates how stress anisotropy influences liquefaction resistance.

**2. DEM simulation setup**

**2.1. Specimen preparation**

Itasca PFC3D (Itasca Consulting Group, Inc., 2021) was employed to implement DEM simulations of undrained cyclic torsional shear test. Unlike the periodic boundaries commonly used in element tests, the HCA employs two cylinders, upper and lower planes, as well as six blades to provide torsional force, closely approximating the boundary conditions of HCA (Ishihara and Yasuda, 1975; Vargas et al., 2020; Li et al, 2014; Liu et al, 2021). As shown in Fig. 1(a), two rigid cylindrical walls with inner diameter of 6 cm and outer diameter of 10 cm are positioned coaxially and vertically, with the upper and lower planes placed 10 cm apart, resembling the geometric dimensions of laboratory tests (Vargas et al., 2020).

This research employed a uniform particle size distribution ranging from 1.5 mm to 3.0 mm, and utilizes a rolling resistance contact model to mimic the non-spherical effects of sand particles. The specific parameters of the contact model are listed in Table 1. To ensure similarity with laboratory methods, the particles were initially generated in the upper part of the apparatus and then allowed to flow downward under the influence of gravity, forming the specimen as shown in Fig. 1(a). The stress calculations of HCA are summarized according to the formulas provided in Table 2.

The typical approach of applying shear force to cuboidal (Wei et al., 2020; Yang et al., 2021; Banerjee et al., 2023) or cylindrical (Li et al., 2014; Liu et al., 2021) elements involves selecting and regulating the flexibility of the particles on both sides of the specimen. However, this method for providing torque on HCA specimens raises concerns, as the selected and constrained particles interfere with the cylinders’ expansion or contraction, affecting the measurement of radial stresses. Therefore, using wall elements that solely interact with particles to apply shear loads is preferred. The gravity was removed to achieve an elementary state and then, as shown in Fig. 1(b), vertically arranged torsional blades consisting of six wall elements were inserted into the specimen.

Soils not merely encounter diverse levels of , but also frequently undergo complex stress histories, potentially influencing the liquefaction resistance. For instance, Pan et al. (2019) observed that the stress history of triaxial extension-unloading can enhance liquefaction resistance compared to IC specimen, whereas the triaxial compression-unloading history yields a contrary effect. To investigate the impact of stress path on the liquefaction resistance, two protocols employing distinct stress paths were utilized to prepare specimens. To achieve the desired target stress levels, a servo mechanism (Itasca Consulting Group, 2021; Ma et al., 2024) is employed to manipulate the position of vertices of wall elements throughout the consolidation process.

In the first approach, the specimen was initially compressed to a target void ratio of 0.808 under a friction coefficient of 0.1. Subsequently, the friction coefficient was reset to 0.5, followed by anisotropic consolidation from a state with =10.0kPa and =1.0 to =133.33kPa and the target (IC-AC protocol), as shown in Fig. 2 (a) and Fig. 2(b). Notably, during the AC process with an increasing , evolves from 1.0 to the corresponding target . Ten cases of specimens with values ranging from 0.33 to 3.33 were obtained.

The other approach adopted a constant- triaxial shear method, where triaxial compression (<1.0) and extension (>1.0) shear tests were performed on the specimen with =133.33kPa and =1.0 obtained using the first IC-AC protocol. The axial strain rate was set as ±1%/second, and the lateral cylinders were controlled to maintain a constant- condition along the π-plane in stress space (IC-AC-TS protocol). For clarity, the term "triaxial shear" here refers to the shearing process applied to HCA specimens, which simulates the same stress state as in triaxial shear tests. Fig. 3(a) and Fig. 3(b) respectively show the relationship between deviator stress and axial strain , as well as the variation in void ratio up to ±1.6% .

Compared to the gradually increasing at a constant for the IC-AC-TS approach, the IC-AC method initiates at a lower , with increasing as rises. The impact of these differences on liquefaction resistance is worth exploring. To simplify, the results with the first IC-AC protocol is labeled as "AC", whereas the results with IC-AC-TS specimens refer to “TS” in the accompanying figures. As indicated by Fig. 2(b) and Fig. 3(b), the differences in between different states after IC-AC or IC-AC-TS were minimal, with ranging from 0.8080 to 0.8083. This variation is small compared to the changes during the IC-AC or IC-AC-TS processes, making it reasonable to emphasize the effects of microscopic quantities, instead of , on liquefaction resistance.

**2.2 Implementation of undrained condition**

In DEM simulations of undrained tests, the interaction between water and particles is disregarded, employing a constant volume approach to replicate the undrain condition. The effectiveness of this constant volume approach has been validated in numerous DEM simulations. (Sitharam et al., 2002; Yimsiri and Soga, 2010). With the specimen height held constant, following laboratory testing procedures (Vargas et al., 2020), an unchanged cross-sectional area ensures the overall volume remains consistent. As described by Eq. (1), the variation of the outer and inner radii was controlled proportionally to maintain a constant cross-sectional area of the specimen, where and denote the inner and outer radii, respectively, and represents time. As shown in Eq. (2), the difference in effective stresses between the inner and outer cylinders is regulated by controlling diameter variation, aiming to achieve equivalent stresses acting on inner and outer cylinders, with respect to laboratory tests. Here, represents the height of the specimen, denotes the difference in effective stresses between the outer and inner cylinders, and is an averaged equivalent modulus. is influenced by the dimensions of the inner and outer cylinders, along with the contact stiffness between the cylinders and particles, and , as defined in Eq. (3).

The essence of the method lies in a combined servo mechanism. This mechanism achieves two objectives simultaneously: maintaining constant volume to replicate undrained conditions and minimizing differences in effective stress between the inner and outer cylinders to replicate stress boundary conditions observed in laboratory tests. This method is equivalent to the mixed stress and strain-controlled loading with being zero described in algorithm Ⅱ by Ma et al. (2024), trickily achieving the simulation of undrained condition and stress condition in cyclic torsional tests.

(1)

(2)

(3)

The effective stresses were evaluated by measuring the contact stresses between the boundary and particle skeleton. The assumptions of undrained condition and full saturation result in variations in effective stress on lateral cylinders and EPWP that are equal in magnitude but opposite in sign, quantifying this relationship in Eq. (4). Here, represents the excess pore water pressure, denotes the radial effective stress, which is derived from the inner and outer effective stress and , and is its initial value (Yimsiri and Soga, 2010).

(4)

**2.3 Application of shear force**

Shear forces are applied to the specimens in the form of a sine wave. Similar to the servo mechanism in consolidation process, the torque application method also considers the difference between the target and current values, along with the total contact stiffness between the blades and particles. The main distinction from the servo mechanism for the lateral cylinder is in how stiffness is calculated, specifically considering the distance from the center of rotation. Fig. 4 illustrates the contact between a particle and a blade, where the distance from the center of rotation to the contact point is denoted as , and the angle between the contact normal and the horizontal plane is . Eq. (5) describes the angular velocity of the torsional blade, where represents the difference between the target torque and the current value, and denotes the moment of inertia of the contact stiffness. As indicated by Eq. (6), is determined by the contact stiffness and square of the distance . is adjusted by because a larger reduces its contribution to the shear stiffness.

(5)

(6)

AC and IC specimens obtained from both the IC-AC and IC-AC-TS protocols, with ranging from 0.33 to 3.33, underwent shear forces with 4 different cyclic stress ratios CSR ranging from 0.25 to 0.40 until liquefaction occurred. A discussion concerning the factors influencing liquefaction strength is provided from both macroscopic and microscopic perspectives.

**3. Results and discussion**

**3.1 Macroscopic response**

**3.1.1 Stress strain relationship**

Fig. 5, 6, and 7 respectively depict the relationships between mean principal stress and shear stress , shear strain and shear stress , as well as the variation in EPWP ratio during the cyclic shear, for the IC state with =1.0 and AC state with =0.40. The AC state with =0.40 state was obtained with the IC-AC protocol and liquefaction here is defined as occurring when reaches 95% of .

In Fig. 5(a) and (b), as the shear stress cyclically acts on the hollow cylindrical specimen, the mean principal stress exhibited an overall decrease for both IC and AC specimens. Once falls below approximately 60 kPa, an increase in the magnitude of drove an upward trend in , displaying a butterfly-shaped relationship in - space. After liquefaction onset, and delineate the critical state line and then converge at the origin, periodically. To better understand the evolution of stress state for the =1.0 and =0.40, the relationship between and deviatoric von-Mises stress is depicted in Fig. 5(c) and Fig. 5(d) for CSR=0.250 and CSR=0.350, respectively. The =1.0 specimen started with at 0, while the =0.40 specimen initially exhibited higher level of initial . As cyclic shear progressed, the =1.0 specimen exhibited larger fluctuation in . In contrast, the AC specimen initially showed smaller amplitude in , which gradually increased in amplitude but decreased in magnitude over time.

In Fig. 6, stiff shear modulus with slight reduction was observed in initial stage. As cyclic loading progressed towards the liquefaction state, shear stiffness markedly decreased, plastic deformation occurred, and strong nonlinearity became evident. As shown in Fig. 7, pore water pressure responses differ between =1.0 and =0.40 states. For the =1.0 state, increased more rapidly in the initial stages. However, after approximately 20 cycles, for the =0.40 state surpassed that of the =1.0 state. Liquefaction occurred after about 41 cycles for =0.40, whereas =1.0 case reached liquefaction after approximately 84 cycles, nearly twice as many as the =0.40 state, suggesting a higher liquefaction strength for initially IC state with =1.0. Fig. 5, 6, and 7 demonstrate that typical stress and strain evolution in laboratory tests has been replicated in DEM simulations, providing preliminary evidence for the effectiveness of the method proposed. On the other hand, it is crucial to investigate how this resistance changes with varying CSR and . Fig. 8(a) and Fig. 8(b) compare the number of cycles required to reach liquefaction under varying CSR and initial values. Fig. 8(a) demonstrates liquefaction resistance for specimens prepared with IC-AC protocol and found that under different CSR conditions, the number of cycles to liquefaction decreases with decreasing when , and decreases with increasing when . Hence, greater stress anisotropy results in fewer cycles needed to trigger liquefaction, indicating lower resistance.

The influence of stress paths on liquefaction resistance is also noteworthy. Fig. 8(b) analyses the variation in the number of cycles required for triggering liquefaction under CSR=0.25, including specimens prepared with both protocols. Observations reveal that the protocols for preparing specimens, whether following the IC-AC or IC-AC-TS protocol, does not significantly affect the liquefaction resistance for the same . Yet, other forms of protocols for preparation may have an impact on liquefaction resistance and remains to be investigated, but this falls outside the scope of this study and should be noted.

**3.1.2. Cumulative shear work**

Cumulative shear work refers to the energy input during undrained cyclic shear. It is valuable to examine the correlation between the liquefaction resistance of soils and their susceptibility to the input energy. Towhata and Ishihara (1985) conducted a series of experiments where specimens were subjected to undrained cyclic shear under various loading conditions. They revealed a unique relationship between excess pore water pressure and shear work, despite differences in stress anisotropy. Figueroa et al. (1994) similarly confirmed that the shear work required for triggering liquefaction is independent of the amplitude of strain through strain-controlled tests. Georgiannou and Konstadinou (2014) concluded from the comparison between IC and AC specimens that the energy associated with terminal water pressure is positively correlated with relative density. For equivalent relative densities, AC specimens require greater energy than IC specimens to induce liquefaction.

The cumulative unit volume shear work is defined as shown in Eq. (7), expressed as an integral of shear strain rate, shear stress, and incremental time. This represents the accumulated input energy per unit volume in the specimen. As indicated by Fig. 9(a), in the liquefaction process normalized by , it is difficult to distinguish the differences between the cases with different values. Fig. 9(b) explores the impact of different on the liquefaction by relating and . For the initial stages, particularly when is less than 0.2, a larger contributes to a higher for the same amount of work ​. decreases as shear force performs positive work and increases as shear force performs negative work. The shear work required to achieve liquefaction for different values range from 0.07 to 0.13, with a trend of increasing ​ as stress anisotropy increases. This implies that although the IC state with =1.0 specimen requires more cyclic numbers to reach liquefaction, its higher initial shear stiffness results in lower strain velocity, hence requiring the least shear work.

(7)

**3.1.3. Initial shear wave velocity**

The response in cumulative shear work inspired an exploration in relationship between initial stiffness and liquefaction resistance (Xu et al., 2015). The shear modulus is the derivative of shear stress with respect to shear strain and Eq. (8) describes the initial shear wave velocity in terms of the initial shear modulus and the saturated density (Yang and Taiebat, 2024).

(8)

In the =1.0 state, reaches its highest value, nearly 450m/s. Regardless of whether is less than 1.0 or greater than 1.0, decreases with increasing stress anisotropy, corresponding to the trend of revolution in liquefaction resistance. At =0.33 and =3.33, drops to its lowest level, approximately 340m/s. This behavior provides a macroscopic guideline for assessing liquefaction strength: a higher indicates greater liquefaction resistance. Additionally, the cases for <1.0 and >1.0 diverge: for the same , <1.0 exhibits higher liquefaction resistance compared to >1.0. For example, =0.40 with at 384m/s achieves close to 42, while =2.50 with around 400m/s has only at 37. Yet, these macroscopic findings remain difficult to elucidate the difference in liquefaction resistance under different initial stress conditions. This emphasizes the necessity for a more detailed discussion at the microscale to comprehend the factors influencing liquefaction resistance.

**3.2 Microscopic interpretation**

**3.2.1 Evolution of coordination number**

The coordination number indicates the number of contacts each individual particle has. Its average value for all particles isotropically evaluates the compactness of fabric in granular materials (Oda, 1977; Shire and O’Sullivan, 2012; Fei and Narsilio, 2020). Particles with fewer than two contacts are generally unable to stably transmit interparticle forces and do not effectively contribute to the soil skeletons. These particles are classified as floaters and are excluded from the calculation of the coordination number (Thornton, 2000; Hu et al., 2023). As shown in Eq. (8), represents the mechanical coordination number. is the number of interparticle contacts, and is the total number of particles. and indicate the number of particles with one and zero contacts, respectively, to eliminate the influence of floaters. Fig. 10 shows the evolution of the coordination number under different initial states during the cyclic shear. The results indicate that the influence of initial stress states on the coordination number is significant, with greater stress anisotropy corresponding to lower coordination numbers both at the initial state and during the cyclic shear process. As cyclic loading was applied to the undrained specimen, exhibited oscillatory behavior while progressively decreasing. The decline in indicates a reduction in interparticle contacts and a loss of fabric integrity and robustness. Liquefaction was observed to occur when fell to approximately 3.5, signifying a critical threshold where the specimen became susceptible to cyclic shear stresses. Regardless of the initial , the post-liquefaction repeatedly cycles between approximately 3.5 and lower values. This implies that the initial stress anisotropies have minimal impact on the post-liquefaction fabric.

(8)

**3.2.2 Evolution of fabric anisotropy**

The fabric tensor quantifies the directionality and distribution of the fabric in granular materials (Oda, 1972; Oda et al., 1982). It can be represented in various forms (Oda, 1982; Kanatani, 1984), with the second-order tensor product being widely used. As shown in Eq. (9), the fabric tensor is the sum of tensor products of contact normal vectors divided by the number of contacts, where denotes the contact normal vector. The diagonal elements of are positive and sum to one. The fraction of the diagonal elements describes the concentration of contact normals, with higher value indicating a greater concentration in that direction. The off-diagonal elements represent the asymmetry of the distribution of contact normals. The fabric tensor is a crucial indicator for characterizing the state of granular materials, microscopically refining the critical state theory (Li and Dafalias, 2012).

It is important to note that when calculating the fabric tensor, the local circumferential, radial, and axial coordinates of the elements within the HCA are position-dependent and are converted from the global coordinate system as shown in Fig 12. Here, ​ refers to the angle of rotation from the negative y-direction to the radial vector that connects the cylinder's axis to the contact point. , , and denote the components of the contact normal in the circumferential, radial, and axial directions, respectively. The global -axis aligns with the local -axial direction, enabling the determination of , , and components through the rotation transformation presented in Eq. (10), where , , and represent the components in global coordinates.

Eq. (11) defines the anisotropic fabric tensor, which subtracts 1/3 from the diagonal elements of the fabric tensor and then multiplied by 15/2, where represents the Kronecker delta. As the second invariant of , measures the development of fabric anisotropy (Zhao and Guo, 2013), as shown in Eq. (12), where the Einstein summation convention is applied, and indicates the magnitude of fabric anisotropy. Compared to 's sensitivity to , exhibits diminished responsiveness to changes in between 0.5 and 2.5. However, as stress anisotropy increases beyond these thresholds, greater fabric anisotropy was observed, as indicated in Fig. 12. Additionally, with higher stress anisotropy, underwent more pronounced fluctuations. For the same level of initial fabric anisotropy , states with >1.0 fluctuate more significantly compared to states with <1.0. This difference in fabric anisotropy potentially influences the macroscopic behavior of liquefaction resistance.

(9)

(10)

(11)

(12)

**3.2.3 Effect of fabric on liquefaction resistance**

For differentiating the states with <1.0 or >1.0, Otsubo et al. (2022) recommended to adopt a signed indicator (Yimsiri and Soga, 2010) to evaluate fabric anisotropy as shown in Eq.13, where denotes the initial vertical principal component in fabric tensor . A larger means a concentration of contact normal converging in vertical direction after specimen preparation. The impact of fabric on liquefaction resistance is investigated through space of , , and for specimens prepared with various , as shown in Fig. 13 with a similar method utilized by Yang and Taiebat (2024). Specimens prepared using the IC-AC and IC-AC-TS protocols exhibit similar and , indicating that the two stress paths do not significantly affect the microscopic fabric, thereby explaining the similar which indicates liquefaction resistance. For =1.0, and liquefaction strength exhibit the highest levels. Comparing the changes between states with =0.67 and =1.50 reveals that remains nearly constant while slightly decreases and liquefaction resistance significantly decreases. This indicates that little changes in microscopic fabric contribute to significant macroscopic difference. As stress anisotropy increases, AC states with <1.0 and >1.0 diverge along different routes.

(13)

To investigate the combined influence of and on liquefaction resistance, an exponential function that linearly relates and (Yang and Huang, 2023; Yang and Taiebat, 2024), was employed to fit their relationship, as shown in Eq. (13). The fitted surface equation results in a positive , suggesting that an increasing , which indicates a fabric of contact normal converging in axial direction enhances liquefaction resistance compared to < 0.0. The initial conditions of the specimens in this study vary considerably compared to the literatures, which could explain the distinct conclusions. Firstly, in this study, the specimen preparation method was standardized with the method by reducing the friction coefficient for achieving specimen densification. Unlike the comparison between two stress ratios of =1.0 and =0.5 with different relative densities, this study evaluates liquefaction resistance under various between and beyond the two thresholds under uniform relative density. The intermediate principal stress , which is defined in Eq. (14), also differs from that in the literature. In contrast to the anisotropic state with =0.5 (=0.0), this study further expands the discussions to include =1.0, when the specimens are subjected to >1.0.

(13)

(14)

From the projection on the - plane, it can be inferred that has a significant impact on liquefaction resistance, with higher being associated with greater liquefaction resistance.

The effect of on the mechanical behaviour of granular materials is of interest (Huang et al., 2014), and is thereby worth noting when evaluating liquefaction response. HCA laboratory experiments (Tastan and Carraro, 2022) were conducted to compare liquefaction strength under different initial . Nevertheless, it remains crucial to provide a microscale explanation. The initial stress states in this study are categorized into two types: one where the initial is less than 1.0 with =0.0, and another where the initial is greater than 1.0 with =1.0. Although, overall, liquefaction resistance decreases with an increasing initial stress anisotropy, there are notable differences in liquefaction behavior for the two AC conditions. For instance, by comparing the two conditions of =0.4 and =2.5, it is observed that both share a similar value around 4.43. However, the cyclic number required for liquefaction is more than 10% higher for = 0.4, compared to = 2.5. Additionally, for the two conditions of =0.33 and =3.0, although their values are similar, for =0.33 is at least 0.05 lower than that for =3.0. This indicates that with =0.33, a smaller is sufficient to achieve the same liquefaction resistance as with =3.0. This characteristic can also be observed through fitting for the two conditions: the case represented by the blue solid line for <1.0 with a higher vertical intercept and the green dotted line for >1.0 with a lower vertical intercept. For the cases of =0.67 and ​=1.5, although they exhibit difference in , the limited degree of stress and fabric anisotropy between them results in minimal variation in liquefaction resistance. The discussion here is limited to exploring the influence of only at the extremes of 0.0 and 1.0. Investigating how the liquefaction resistance changes with in-between values would also attract interest and warrants further study, but it lies beyond the scope of this research.

Additionally, under higher stress anisotropy, is greater for <1.0 compared to >1.0 at the same level. For instance, =0.4 corresponds to =0.0315, whereas =2.5 corresponds to =0.0204. Comparing the IC state and an AC state with a fixed obtains a descending trend in liquefaction resistance with an increasing fabric anisotropy but incorporating multiple AC conditions with <1.0 and >1.0 gives a positive contribution from fabric anisotropy. This explains why fitting a surface as shown in Fig. 13 yields a positive value. Therefore, when evaluating the fabric anisotropy, not limited to the magnitude of anisotropy, considering fabric change induced by other factors, such as , matters as well.

**3.2.4 Interparticle contact force**

To interpret the influence by fabric and particle interactions intuitively, contact forces and individual particle movement in the initial state with =0.33, post-liquefaction state with =0.33, initial state with =3.00, and post-liquefaction state with =3.00 are depicted in Fig. 14(a), Fig.14(b), Fig.14(c), and Fig.14(d). The specimens with =0.33 and =3.00 exhibit significant differences in the distribution of contact forces during the initial stages of cyclic shear. For =0.33, the contact forces converge in the axial direction, whereas for =3.00, the contact forces tend to be distributed horizontally, aligning with the direction of the maximum principal stress in both cases. As cyclic loading progresses, the number of interparticle force gradually decreases, and the magnitude of these forces diminishes, until liquefaction and large deformation occurs. From a macroscopic perspective, the liquefaction process involves a decline in stiffness and an increase in nonlinearity. From a microscopic viewpoint, liquefaction occurs because the particle-constituting skeleton becomes increasingly difficult to sustain through relative particle displacement, hindering the transfer of external forces. In contrast to the influence of on initial interparticle contact forces, the effect of different initial values on these forces becomes negligible in post-liquefaction stage. Whether =0.33 or =3.00, the contact forces no longer align with the initial direction of the maximum principal stress but tend to orient at a 45 angle to the horizontal plane. This microscopic observation explains why post-liquefaction and oscillate within a similar range, regardless of the initial .

**3.2.5 Contact normal orientation**

The fabric tensor is orientation-dependent, meaning its elements vary based on the specified coordinate directions (Kanatani, 1984). This sparked interest in using statistical methods, such as probability density function (PDF) to analyze the distribution of contact normal (Rothenburg and Bathurst, 1989). To simultaneously capture changes in both the direction and quantity of contact normals during cyclic shear, it is recommended to use contact density for visualization. Contact density (Han et al., 2023; Ma et al., 2024) describes the average number of contacts per unit surface area for a particle with normalized radius, as shown in Eq. (15). Here, represents the contact density, and indicate the polar and azimuthal angles in the spherical polar coordinate system, respectively. indicates the number of contacts with normals within the range of . and the subsequent integral denote the total number of particles and the corresponding surface area on the unit sphere, respectively. This method effectively evaluates interparticle contacts in granular systems during an undrained cyclic shear test and accommodates various particle numbers.

(15)

The evolutions of contact density during the liquefaction process for K0=0.33 and K0=3.00, are shown in Fig. 15 and Fig. 16. Fig. 15(a) represents the state with initial K0=0.33, exhibiting an elongated columnar shape extending along the axial direction, whereas Fig. 16(a) depicts the contact density with initial K0=3.00, characterized by a dimpled ellipsoid oriented toward the axial direction. As cyclic shear progresses, the direction of maximum contact density varies following the rotating principal stress axis, and the overall contact density gradually decreases as shown in Fig. 15 (b) and Fig. 16 (b). Fig. 15(c), Fig. 15(d), Fig. 16(c), and Fig.16 (d) indicate that the post-liquefaction distribution of contact density is largely independent of the initial state, shifting between two inclined elongated columnar distributions along different directions.

Although has the most significant impact on liquefaction resistance, the variations in liquefaction resistance and microscopic quantities caused by morphological differences in fabric are also worth investigating. This morphological difference in fabric explains why, in the -- space, as shown in Fig. 13, paths with <1.0 and >1.0 diverge as stress anisotropy increases, and why exhibits more pronounced fluctuation with >1.0 than that with <1.0 during the cyclic shear in Fig. 12. For comparable stress anisotropy or stress ratio of and , represented by =0.4 with =2.50 or =0.33 with =3.00, the elongated columnar distribution of fabric for <1.0 produces a greater than the dimpled ellipsoidal distribution of fabric observed for >1.0. When is similar, the state with <1.0 and an elongated columnar morphology of fabric results in more contact normals along the axial direction, which is perpendicular to the shear force, potentially enhancing liquefaction resistance (Zhang et. al., 2023), compared to the dimpled ellipsoidal morphology observed for >1.0. Thus, a positive was obtained with the fitted surface in Fig. 13. Additionally, cyclic shear induces larger amplitude of fluctuations in during cyclic shear for >1.0 compared to <1.0 due to the underlying morphological differences in contact normal fabric.

This discussion is based on the analyses of 10 different initial shear states consolidated from the same condition at =10.0 kPa through either IC-AC or IC-TS processes, ranging from =0.33 to =3.33. These states involve different stress anisotropies, including both =0.0 (<1.0) and =1.0 (>1.0). While these states show consistency in the macroscopic void ratio, they exhibit significant differences in microscopic parameters, which notably affect liquefaction resistance. The initial settings contrast Yang and Taiebat's (2024) discussion, produced a different conclusion. It becomes evident that introducing a broader range of values, can alter the findings about the impact of microscopic factors, such as fabric anisotropy, on liquefaction resistance, underscoring the need to validate a generalized conclusion.

**4. Conclusions**

This study utilized the DEM to investigate the impact of initial stress anisotropy on the liquefaction resistance of sand soils under undrained cyclic shear conditions. By employing a combined servo mechanism, the evolution of stress resembling that in laboratory HCA tests were observed, indicating that the undrained condition, stress variation, as well as principal stress axes rotation have been successfully reproduced with the method. The analysis covered a broad range of values from 0.33 to 3.33 and incorporated shear loading with CSR from 0.250 to 0.400. It explored the effects of different initial stress conditions and stress paths on undrained cyclic behavior. The findings are summarized as follows:

Initial stress anisotropy, represented by , significantly affects liquefaction resistance. Liquefaction resistance decreases with decreasing when is less than 1.0, and with increasing when is greater than 1.0. Thus, greater stress anisotropy results in reduced liquefaction resistance. The protocols for specimen preparation, whether through initial isotropic consolidation followed by linear anisotropic consolidation (IC-AC) or initial isotropic consolidation followed by constant- triaxial shear (IC-TS), does not significantly impact liquefaction resistance.

Greater initial stress anisotropy is associated with a lower initial mechanical coordination number . A correlation is observed between and the cyclic number required for liquefaction , which indicates liquefaction resistance, providing a microstructural explanation for the influence on liquefaction strength. However, the - relationship varies under different stress conditions: for <1.0 corresponding to an intermediate principal stress ratio =0.0, the fitted curve is higher, indicating slightly greater liquefaction resistance at the same compared to >1.0 corresponding to =1.0.

Anisotropic consolidation states with <1.0 or >1.0 at the same level not only differ in but also produce different levels of fabric anisotropy . In the -- space, as stress anisotropy increases, the predicted relationships for <1.0 and >1.0 further diverge. This divergence impacts the evaluation of the influence of on liquefaction, highlighting the significance of comparing in an extensive range.

Contact density not only reflects the distribution characteristics of contact normals in different orientations, like a probability density function (PDF), but also captures the overall changes in contact number during undrained cyclic shear, with the advantage of being independent of the total number of particles. With this approach, it was found that anisotropic consolidation states with <1.0 or >1.0 produce different morphologies of contact density. When <1.0, an elongated columnar distribution tends to result in larger compared to the dimpled ellipsoidal distribution for >1.0. Additionally, when <1.0, more contact normals cluster in the direction perpendicular to the shear force and its adjacent surroundings, potentially enhancing liquefaction resistance.

Table 1. Parameters in DEM simulation

|  |  |
| --- | --- |
| Description | Value |
| Number of particles | 40,249 |
| Density, (kg/m3) | 2600 |
| Young’s modulus, E (GPa) | 1.2 |
| Normal-to-shear stiffness, | 2.0 |
| Tangential frictional coefficient between particles, | 0.10\*/0.50\*\* |
| Tangential frictional coefficient between particle and wall, | 0.0 |
| Normal critical damping ratio, | 0.7 |
| Shear critical damping ratio, | 0.5 |
| Rolling friction coefficient, | 0.5 |

\*IC

\*\* Generation, AC, TS, and Cyclic shear

Table 2. Equations of stress and strain in the HCA

|  |  |  |
| --- | --- | --- |
| Description | Stress | Strain |
| Vertical |  |  |
| Inner |  |  |
| Outer |  |  |
| Circumferential |  |  |
| Radial |  |  |
| Shear |  |  |
| Major principal |  |  |
| Intermediate principal |  |  |
| Minor principal |  |  |

Table 3. Specification of dense specimens in initial cyclic undrained shear stage

表格

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Table 4. Specification of loose specimens in initial cyclic undrained shear stage

表格

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图片包含 应用程序

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1. Pouring method for generating particles

图片包含 图形用户界面

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1. Insertion of torsional blades

Fig. 1. Specimen generation process in the initial stage using the pouring method and insertion of torsional blades

图表

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1. Mean effective stress vs. deviator stress

图表

描述已自动生成

1. Mean effective stress vs. void ratio

Fig.2. Stress and void ratio evolution in anisotropic consolidation for specimens with different stress anisotropies from isotropic consolidation state with =10.0kPa and different target

图形用户界面, 应用程序, 表格, Excel

描述已自动生成

1. Mean effective stress vs. deviator stress (dense)

图表, 折线图

描述已自动生成

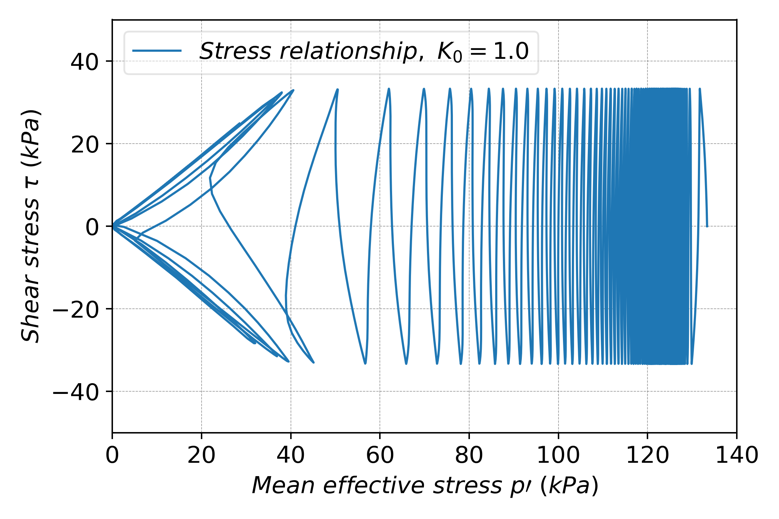
1. Relationship between e and (dense)

Fig. 3. Stress and void ratio evolution in constant- triaxial shear for specimens with different stress anisotropies after anisotropic consolidation with =133.33kPa and target =1.0

图示

中度可信度描述已自动生成

Fig. 4. Determination of moment of inertia of shear stiffness in servo mechanism for torque application



1. Shear stress vs. mean effective stress (CSR = 0.250, K0=1.00, dense)

图表

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1. Shear stress vs. mean effective stress (CSR = 0.250, K0=0.40, dense)

图表

描述已自动生成

1. Deviatoric stress vs. mean effective stress (CSR=0.250, dense)

图表

中度可信度描述已自动生成

1. Deviatoric stress vs. mean effective stress , CSR=0.350

Fig. 5. Stress evolution of IC with =1.0 and AC specimen with =0.40 in undrained cyclic shear

图表, 折线图

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1. Shear stress vs. shear strain , CSR = 0.250, K0=1.00

图表, 折线图

描述已自动生成

1. Shear stress vs. shear strain , CSR = 0.250, K0=0.40

Fig. 6. Evolution of shear stress and shear strain in undrained cyclic shear loading

图表, 折线图

描述已自动生成

Fig.7. Comparison of evolution of excess pore water pressure ratio between IC state with and AC state with =0.40

**图表, 散点图

描述已自动生成**

1. Cyclic liquefaction resistance with various CSR

图表, 散点图

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1. Number of cyclic loadings vs. under cyclic shear loading with cyclic shear stress ratio of 0.250

Fig. 8. Cyclic liquefaction resistance of specimens subjected to different initial stress anisotropies and cyclic shear stress ratios

图表

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1. vs. (dense)

图表

描述已自动生成

1. vs. (dense)

Fig. 9. Evolution of cumulative unit volume shear work and shear work required to trigger liquefaction for different

图表, 折线图

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Fig. 10. Relationship between cyclic number and initial shear wave velocity (CSR=0.250)

图表, 散点图

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Fig. 11. Mechanical coordination number evolution in cyclic shear and relationship between cyclic number required for liquefaction and initial mechanical coordination number (CSR=0.250)

图形用户界面

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Fig. 12. Conversion of contact normal from global x-y-z orthogonal coordinate to circumferential-radial-axial local cylindrical coordinates

图表

描述已自动生成

Fig. 13. Evolution of second invariant of mechanical anisotropic fabric tensor for specimens subjected to different initial stress anisotropies

图表

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1. Effect of and on liquefaction resistance

图表

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1. Decoupled effect of on liquefaction resistance

Fig. 14. Effect of initial coordination number and initial fabric anisotropy on liquefaction resistance for specimens with different initial stress anisotropies (CSR=0.250)

图片包含 图形用户界面

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1. AC state with initial =0.33

图形用户界面

中度可信度描述已自动生成

1. Post-liquefaction state with initial =0.33

电脑萤幕画面

中度可信度描述已自动生成

1. AC state with initial =3.00

图片包含 图形用户界面

描述已自动生成

1. Post-liquefaction state with initial =3.00

Fig. 15. Contact force chain and displacement of particles under initial and post-liquefaction states subjected to different initial

图表, 雷达图

描述已自动生成 图表, 图示

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1. /=0.00 (b) /=0.75

图表

中度可信度描述已自动生成 图表

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(c) /=1.01 (d) /=1.04

Fig. 16. Contact density evolution in liquefaction process with initial =0.33

图表

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1. /=0.00 (b) /=0.73

图表

描述已自动生成 图表

描述已自动生成

1. /=0.97 (d) /=1.03

Fig. 17. Contact density evolution in liquefaction process with initial =3.00

**REFERENCES**

Arthur, J. R. F., Bekenstein, S., Germaine, J. T., & Ladd, C. C. (2009). Stress Path Tests with Controlled Rotation of Principal Stress Directions. In *ASTM International eBooks* (pp. 516–525). <https://doi.org/10.1520/stp28768s>

Arthur, J. R. F., Del C, J. I. R., Dunstan, T., & Chua, K. S. (1980). Principal Stress Rotation: A Missing Parameter. *Journal of the Geotechnical Engineering Division*, *106*(4), 419–433. <https://doi.org/10.1061/ajgeb6.0000946>

Banerjee, S. K., Yang, M., & Taiebat, M. (2023). Effect of coefficient of uniformity on cyclic liquefaction resistance of granular materials. Computers and Geotechnics, 155, 105232. <https://doi.org/10.1016/j.compgeo.2022.105232>

Cundall, P. A., & Strack, O. D. (1979). A discrete numerical model for granular assemblies. Géotechnique, 29(1), 47–65. https://doi.org/10.1680/geot.1979.29.1.47

Fei, W., & Narsilio, G. A. (2020). Impact of three-dimensional sphericity and roundness on coordination number. Journal of Geotechnical and Geoenvironmental Engineering, 146(12). https://doi.org/10.1061/(asce)gt.1943-5606.0002389

Figueroa, J. L., Saada, Adel. S., Liang, L., & Dahisaria, N. M. (1994). Evaluation of soil liquefaction by energy principles. Journal of Geotechnical Engineering, 120(9), 1554–1569. <https://doi.org/10.1061/(asce)0733-9410(1994)120:9(1554)>

Georgiannou, V. N., & Konstadinou, M. (2014). Effects of density on cyclic behaviour of anisotropically consolidated Ottawa sand under undrained torsional loading. Géotechnique, 64(4), 287–302. <https://doi.org/10.1680/geot.13.p.090>

Han, Y., Kato, S., & Kim, B.-S. (2023). DEM analysis of fabric evolution and behavior of granular geomaterials in true triaxial test with flexible boundary. KSCE Journal of Civil Engineering, 27(8), 3341–3354. https://doi.org/10.1007/s12205-023-0336-1

Hu, J., Wu, H., Gu, X., & Zhou, Q. (2023). Particle shape effects on dynamic properties of granular soils: A DEM study. Computers and Geotechnics, 161, 105578. https://doi.org/10.1016/j.compgeo.2023.105578

Huang, X., Hanley, K. J., O’Sullivan, C., Kwok, C. Y., & Wadee, M. A. (2014). DEM analysis of the influence of the intermediate stress ratio on the critical-state behaviour of granular materials. Granular Matter, 16(5), 641–655. https://doi.org/10.1007/s10035-014-0520-6

Huang, Xin, Kwok, C., Hanley, K. J., & Zhang, Z. (2018). DEM analysis of the onset of flow deformation of sands: Linking monotonic and cyclic undrained behaviours. Acta Geotechnica, 13(5), 1061–1074. https://doi.org/10.1007/s11440-018-0664-3

Hyodo, M., Murata, H., Yasufuku, N., & Fujii, T. (1991). Undrained Cyclic Shear Strength and Residual Shear Strain of Saturated Sand by Cyclic Triaxial Tests. Soils and Foundations, 31(3), 60–76. <https://doi.org/10.3208/sandf1972.31.3_60>

Ishihara, K., & Koga, Y. (1981). Case Studies of Liquefaction in the 1964 Niigata Earthquake. *Soils and Foundations*, 21(3), 35-52. <https://doi.org/10.3208/sandf1972.21.3_35>

Ishihara, K., & Takatsu, H. (1979). Effects of overconsolidation and K0, conditions on the liquefaction characteristics of sands. Soils and Foundations, 19(4), 59–68. <https://doi.org/10.3208/sandf1972.19.4_59>

Ishihara, K., & Towhata, I. (1983). Sand Response to Cyclic Rotation of Principal Stress Directions as Induced by Wave Loads. Soils and Foundations, 23(4), 11–26. <https://doi.org/10.3208/sandf1972.23.4_11>

Ishihara, K., Yamazaki, A., & Haga, K. (1985). Liquefaction of K0-Consolidated Sand Under Cyclic Rotation of Principal Stress Direction With Lateral Constraint. *Soils and Foundations*, *25*(4), 63–74. <https://doi.org/10.3208/sandf1972.25.4_63>

Ishihara, K., & Yasuda, S. (1975). Sand Liquefaction In Hollow Cylinder Torsion Under Irregular Excitation. *Soils and Foundations*, *15*(1), 45–59. <https://doi.org/10.3208/sandf1972.15.45>

Itasca Consulting Group, Inc. (2021) PFC — Particle Flow Code, Ver. 7.0. Minneapolis: Itasca.

Jiang, M., Kamura, A., & Kazama, M. (2021). Comparison of liquefaction behavior of granular material under SH- and love-wave strain conditions by 3D DEM. Soils and Foundations, 61(5), 1235–1250. https://doi.org/10.1016/j.sandf.2021.06.013

Kanatani, K. (1984). Distribution of directional data and fabric tensors. International Journal of Engineering Science, 22(2), 149–164. https://doi.org/10.1016/0020-7225(84)90090-9

Li, B., Zhang, F., & Gutierrez, M. (2014). A numerical examination of the hollow cylindrical torsional shear test using dem. Acta Geotechnica, 10(4), 449–467. <https://doi.org/10.1007/s11440-014-0329-9>

Li, X. S., & Dafalias, Y. F. (2012). Anisotropic critical state theory: Role of Fabric. Journal of Engineering Mechanics, 138(3), 263–275. https://doi.org/10.1061/(asce)em.1943-7889.0000324

Liu, Y., Mao, H., Xu, C., &amp; Zhang, Y. (2021). DEM investigation on the mechanical behavior of mudstone in the hollow cylinder torsional shear test. Computers and Geotechnics, 137, 104236. <https://doi.org/10.1016/j.compgeo.2021.104236>

Ma, Q., He, X., Zhou, Y.-G., Chen, Y.-M., & Han, Y.-S. (2024). An approach to DEM simulation of hollow torsional shear tests for achieving general loading paths. Computers and Geotechnics, 172, 106402. https://doi.org/10.1016/j.compgeo.2024.106402

Mijic, Z., Bray, J. D., Riemer, M., Rees, S. D., & Cubrinovski, M. (2021). Cyclic and monotonic simple shear testing of native Christchurch silty soil. *Soil Dynamics and Earthquake Engineering*, *148*, 106834. <https://doi.org/10.1016/j.soildyn.2021.106834>

Morimoto, T., Otsubo, M., & Koseki, J. (2021). Microscopic investigation into liquefaction resistance of pre-sheared sand: Effects of particle shape and initial anisotropy. Soils and Foundations, 61(2), 335–351. <https://doi.org/10.1016/j.sandf.2020.12.008>

Oda, M. (1972). The mechanism of fabric changes during compressional deformation of Sand. Soils and Foundations, 12(2), 1–18. https://doi.org/10.3208/sandf1972.12.1

Oda, M. (1977). Co-ordination number and its relation to shear strength of granular material. Soils and Foundations, 17(2), 29–42. <https://doi.org/10.3208/sandf1972.17.2_29>

Oda, M. (1982). Fabric tensor for discontinuous geological materials. Soils and Foundations, 22(4), 96–108. https://doi.org/10.3208/sandf1972.22.4\_96

Oda, M., Nemat‐Nasser, S., & Mehrabadi, M. M. (1982). A statistical study of fabric in a random assembly of spherical granules. International Journal for Numerical and Analytical Methods in Geomechanics, 6(1), 77–94. https://doi.org/10.1002/nag.1610060106

Oka, F., Yashima, A., Tateishi, A., Taguchi, Y., & Yamashita, A. (1999). A cyclic elasto-plastic constitutive model for sand considering a plastic-strain dependence of the shear modulus. Géotechnique, 49(5), 661–680. https://doi.org/10.1680/geot.1999.49.5.661

Otsubo, M., Chitravel, S., Kuwano, R., Hanley, K. J., Kyokawa, H., & Koseki, J. (2022). Linking inherent anisotropy with liquefaction phenomena of granular materials by means of dem analysis. Soils and Foundations, 62(5), 101202. https://doi.org/10.1016/j.sandf.2022.101202

Pan, K., Cai, Y. Q., Yang, Z. X., & Pan, X. D. (2019). Liquefaction of sand under monotonic and cyclic shear conditions: Impact of drained preloading history. Soil Dynamics and Earthquake Engineering, 126, 105775. <https://doi.org/10.1016/j.soildyn.2019.105775>

Rothenburg, L., & Bathurst, R. J. (1989). Analytical study of induced anisotropy in idealized granular materials. Géotechnique, 39(4), 601–614. https://doi.org/10.1680/geot.1989.39.4.601

Seed, H. B., & Idriss, I. M. (1967). Analysis of Soil Liquefaction: Niigata Earthquake. *Journal of the Soil Mechanics and Foundations Division*, *93*(3), 83–108. <https://doi.org/10.1061/jsfeaq.0000981>

Seed, H. B., & Lee, K. (1966). Liquefaction of Saturated Sands During Cyclic Loading. *Journal of the Soil Mechanics and Foundations Division*, *92*(6), 105–134. <https://doi.org/10.1061/jsfeaq.0000913>

Shire, T., & O’Sullivan, C. (2012). Micromechanical assessment of an internal stability criterion. Acta Geotechnica, 8(1), 81–90. <https://doi.org/10.1007/s11440-012-0176-5>

Silver, M. L., Chan, C. S., Ladd, R. S., Lee, K., Tiedemann, D. A., Townsend, F. C., Valera, J. E., & Wilson, J. F. (1976). Cyclic Triaxial Strength of Standard Test Sand. *Journal of the Geotechnical Engineering Division*, *102*(5), 511–523. <https://doi.org/10.1061/ajgeb6.0000272>

Sitharam, Thallak. G., Dinesh, S. V., & Shimizu, N. (2002). Micromechanical modelling of monotonic drained and undrained shear behaviour of granular media using three‐dimensional dem. International Journal for Numerical and Analytical Methods in Geomechanics, 26(12), 1167–1189. <https://doi.org/10.1002/nag.240>

Tastan, E. O., & Carraro, J. A. (2022). Effect of principal stress rotation and intermediate principal stress changes on the liquefaction resistance and undrained cyclic response of Ottawa sand. Journal of Geotechnical and Geoenvironmental Engineering, 148(5). https://doi.org/10.1061/(asce)gt.1943-5606.0002772

Tatsuoka, F., Ochi, K., Fujii, S., & Okamoto, M. (1986). Cyclic undrained triaxial and torsional shear strength of sands for different sample preparation methods. Soils and Foundations, 26(3), 23–41. https://doi.org/10.3208/sandf1972.26.3\_23

Thornton, C. (2000). Numerical simulations of deviatoric shear deformation of granular media. Géotechnique, 50(1), 43–53. https://doi.org/10.1680/geot.2000.50.1.43

Tobita, Y. (1989). Fabric tensors in constitutive equations for granular materials. Soils and Foundations, 29(4), 91–104. https://doi.org/10.3208/sandf1972.29.4\_91

Toki, S., Tatsuoka, F., Miura, S., Yoshimi, Y., Yasuda, S., & Makihara, Y. (1986). Cyclic Undrained Triaxial Strength of Sand by a Cooperative Test Program. *Soils and Foundations*, *26*(3), 117–128. <https://doi.org/10.3208/sandf1972.26.3_117>

Towhata, I., & Ishihara, K. (1985). Shear work and pore water pressure in undrained shear. Soils and Foundations, 25(3), 73–84. <https://doi.org/10.3208/sandf1972.25.3_73>

Vargas, R. R., Ueda, K., & Uemura, K. (2020). Influence of the relative density and K0 effects in the cyclic response of Ottawa F-65 sand - cyclic Torsional Hollow-Cylinder shear tests for LEAP-ASIA-2019. *Soil Dynamics and Earthquake Engineering*, *133*, 106111. <https://doi.org/10.1016/j.soildyn.2020.106111>

Wei, J., Huang, D., & Wang, G. (2020). Fabric evolution of granular soils under multidirectional cyclic loading. Acta Geotechnica, 15(9), 2529–2543. <https://doi.org/10.1007/s11440-020-00942-8>

Wen, Y., & Zhang, Y. (2023). Evidence of a unique critical fabric surface for granular soils. Géotechnique, *73*(5), 439–454. <https://doi.org/10.1680/jgeot.21.00126>

Xie, X., Ye, B., Zhao, T., Feng, X., & Zhang, F. (2023). DEM investigation into the effects of liquefaction history–induced anisotropy on sand behaviors. International Journal of Geomechanics, 23(3). <https://doi.org/10.1061/ijgnai.gmeng-7492>

Xu, X. M., Ling, D. S., Cheng, Y. P., & Chen, Y. M. (2015). Correlation between liquefaction resistance and shear wave velocity of granular soils: A micromechanical perspective. Géotechnique, 65(5), 337–348. https://doi.org/10.1680/geot.sip.15.p.022

Yamashita, S., & Toki, S. (1993). Effects of fabric anisotropy of sand on cyclic undrained triaxial and torsional strengths. Soils and Foundations, 33(3), 92–104. https://doi.org/10.3208/sandf1972.33.3\_92

Yang, M., Taiebat, M., Mutabaruka, P., & Radjaï, F. (2021). Evolution of granular media under constant-volume multidirectional cyclic shearing. Acta Geotechnica, 17(3), 779–802. <https://doi.org/10.1007/s11440-021-01239-0>

Yang, M., & Taiebat, M. (2024). Effect of anisotropic consolidation on cyclic liquefaction resistance of granular materials via 3D-Dem modeling. Journal of Geotechnical and Geoenvironmental Engineering, 150(5). <https://doi.org/10.1061/jggefk.gteng-11970>

Yang, S., & Huang, D. (2023). Investigating the influence of inherent soil fabrics on reliquefaction resistance of sands using DEM-clump simulation. Computers and Geotechnics, 164, 105817. https://doi.org/10.1016/j.compgeo.2023.105817

Yimsiri, S., & Soga, K. (2010). DEM analysis of soil fabric effects on behaviour of sand. Géotechnique, 60(6), 483–495. <https://doi.org/10.1680/geot.2010.60.6.483>

Yoshimi, Y., Tokimatsu, K., Kaneko, O., & Makihara, Y. (1984). Undrained Cyclic Shear Strength of a Dense Niigata Sand. Soils and Foundations, 24(4), 131–145. <https://doi.org/10.3208/sandf1972.24.4_131>

Zhang, A., Jiang, M., & Wang, D. (2023). Effect of fabric anisotropy on the cyclic liquefaction of sands: Insight from DEM simulations. Computers and Geotechnics, 155, 105188. https://doi.org/10.1016/j.compgeo.2022.105188

Zhao, J., & Guo, N. (2013). Unique critical state characteristics in granular media considering fabric anisotropy. *Geotechnique*, *63*(8), 695–704. <https://doi.org/10.1680/geot.12.p.040>