**Influence of Multi-directional Shear Stress**

**on Liquefaction Resistance**

Yusong Hana, Ryosuke Uzuokab, Kyohe Uedac, and Natsuki Yamazakid

Manuscript No.:

a Doctoral Student, Department of Civil and Earth Resources Engineering, Kyoto University, Gokasho, Uji City, Kyoto 611-0011, Japan (E-mail: [han.yusong.53f@st.kyoto-u.ac.jp](mailto:han.yusong.53f@st.kyoto-u.ac.jp))

b Professor, Kyoto University, Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji City, Kyoto 611-0011, Japan (E-mail: uzuoka.ryosuke.6z@kyoto-u.ac.jp)

c Associate Professor, Kyoto University, Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji City, Kyoto 611-0011, Japan (E-mail: ueda.kyohei.2v@kyoto-u.ac.jp)

d Master Student, Department of Civil and Earth Resources Engineering, Kyoto University, Gokasho, Uji City, Kyoto 611-0011, Japan (E-mail: yamazaki.natsuki.88r@st.kyoto-u.ac.jp)

**ABSTRACT**

A 3D discrete element method (DEM) was used to simulate the liquefaction process under cyclic loading with multi-directional shear stress paths. Compared to previous approaches that primarily compared unidirectional and multidirectional loading patterns, the newly proposed method, which contrasts the 8-like and double 8-like stress paths, offers the following advantages: First, both the 8-like and double 8-like paths maintain equal stress magnitudes throughout the simulation. Second, the shear stress variation rates in both paths remain consistent and smooth, eliminating the influence of stress variation rate on liquefaction resistance. Moreover, in the double 8-like loading mode, the major axis periodically rotates, allowing for an investigation of the influence of shear stress directionality on liquefaction resistance.

Keywords: liquefaction, simple shear test, undrained cyclic shear, discrete element method

**1. Introduction**

Soil is subjected to a complex three-dimensional stress state during seismic loadings. To simulate the liquefaction process under seismic conditions, horizontal loading is often simplified to unidirectional cyclic loading due to the limitations of laboratory testing equipment. However, such simplification raised concerns about underestimating the impact of multi-directional cyclic shear on liquefaction resistance. Findings by Ishihara and Yamazaki (1980) indicated that multi-directional shear with equal amplitude in different directions requires only about 70% of the cyclic stress ratio of unidirectional shear to achieve the same shear strain. To better understand the role of multidirectional shear conditions on liquefaction, Kammerer et al. (2005) conducted simple shear tests on Monterey 0/30 Sand with varying relative densities and initial shear stresses, using linear, oval/circular, and figure-8 shaped loading paths. The results demonstrated that, compared to equivalent unidirectional shear with the same relative density and maximum cyclic stress ratio, multidirectional shear induced faster liquefaction. The rotation of stresses in multidirectional shear likely accounts for the more pronounced build-up of pore pressure.

In addition to laboratory experiments, model tests have also provided evidence of the susceptibility of liquefaction behavior to multidirectional shaking. Pyke et al. (1975) found that multidirectional shaking on dry sand caused greater settlement and required less stress to induce liquefaction compared to unidirectional shaking. Results from Su and Li's (2008) centrifuge experiments indicate that, compared to unidirectional shaking, multi-directional shaking accelerates excess pore water pressure build-up as depth increases. A series of centrifuge tests conducted by El Shafee et al. (2017) compared uniaxial and biaxial excitations, showing that an increase of 40% in the uniaxial shaking amplitude was required to produce a similar excess pore water pressure response to that of biaxial shaking. This suggests that the common practice of increasing uniaxial shaking amplitude by 10% to approximate 2D shaking effects underestimates the true response of multidirectional shaking on soil liquefaction behavior.

Wei et al. (2020) investigated the effects of different shear stress paths, including unidirectional, oval, circular, and figure-8, on the number of cycles to liquefaction using DEM from the perspective of micro-scale fabric evolution. They found that the stress ratio in different directions and the shear path significantly influenced the number of cycles required to reach initial liquefaction, and fabric anisotropy evolved progressively throughout the liquefaction process. Yang et al. (2022) also explored the factors influencing the number of cycles to liquefaction in DEM simulations using similar unidirectional and multidirectional shear stress paths. They found that the figure-8 stress path required fewer cycles to reach liquefaction compared to the circular stress path. The coordination number and particle connectivity revealed that the system becomes temporarily under-constrained during 1-D linear and figure-8 shear paths at the moment when mean stress vanishes, whereas it remains over-constrained under 2-D linear and circular shear paths.

These studies have made significant contributions to understanding liquefaction by comparing unidirectional and multidirectional shear stress paths and their influence on the number of cycles required for liquefaction. However, the methodologies used remain questionable. For instance, despite maintaining the same maximum shear force, the magnitude of unidirectional and multidirectional shear stress differs throughout the shear process. On the other hand, 2D shear paths like oval or circular maintain shear forces, preventing the effective stress from reaching zero. In contrast, unidirectional and figure-8 shear paths allow the effective stress to repeatedly cycle to zero, making it difficult to evaluate the impact of shear stress direction on liquefaction based on stress criterion. These factors introduced additional factors influencing the liquefaction, complicating the assessment of the specific influence of stress direction on the liquefaction process. Therefore, this paper aims to present an improved approach that minimizes these limitations to explore whether changes in direction of shear stress affect the liquefaction behavior under different cyclic shear.

**2. DEM simulation setup**

**2.1. Specimen preparation**

The discrete element method (DEM) simulates the mechanical response of granular materials, such as sand, by calculating the inter-particle contacts and the motion of individual particles. This approach effectively overcomes the limitations of laboratory experiments, where replicating the same initial state for comparative analyses can be challenging. Additionally, DEM enables the application of multi-directional shear and the use of periodic boundaries, which mimic the continuous conditions in natural soil. In this study, the software Itasca PFC3D was used to simulate simple shear tests on granular sand. The rolling resistance contact model was employed to replicate the inter-particle interactions of non-spherical sand grains.

As shown in Fig. 1, spherical particles with diameters ranging from 1.0 to 3.0 mm, approximating the distribution of Toyoura sand, were generated within a cubic space enclosed by periodic boundaries. Sample compaction was then achieved by adjusting the positions of ribbed shear walls oriented in the vertical direction and periodic boundaries in the horizontal directions, using a servo mechanism for the compaction process. A sample in an isotropic stress state, with all three principal stresses set to 100 kPa, was prepared for subsequent undrained cyclic shear tests as shown in Fig.2.

**2.2. Application of unidirectional and multidirectional shear force**

In studying the effects of multidirectional shear loading on the liquefaction process, other factors, such as changes in the magnitude of shear force, were often introduced. The primary aim of this paper is to minimize the influence of these additional factors when modifying the direction of shear stress. Inspired by previous studies, this research employs a figure-8 pattern for the first type of multidirectional shear. As shown in Eqs (1) and (2), the figure-8 multidirectional shear loading consists of a sinusoidal component in the x-direction and a sinusoidal component in the y-direction, with the amplitude being half of that in the x-direction and the period being half of the x-direction’s cycle.

(1)

(2)

To more quantitatively evaluate the impact of multidirectional shear loading compared to unidirectional loading on the liquefaction process, the unidirectional shear stress is defined to equal the total magnitude of the multidirectional shear stress, and its sign is unified with that of , as shown in Eq (3). By ensuring the unidirectional and multidirectional figure-8 shear stresses have the same magnitude throughout the cyclic shear, the only difference between them is their direction.

(3)

Although the magnitudes of unidirectional and multidirectional figure-8 shear stresses are controlled to be equal, the multidirectional shear not only experiences variations in the shear force magnitude but also undergoes continuous changes in direction. This results in completely different rates of change between the two types of shear forces. This study introduces a new double figure-8 shear loading method to eliminate the impact of differing shear stress variation rates during undrained shear processes. In odd-numbered cycles, the double figure-8 shear stress has the same x and y components as the standard figure-8 pattern. However, in even-numbered cycles, the x and y components are swapped, with the x component equaling the y component of the figure-8, and vice versa. As the cycles progress, the "figure-8" axes of the shear stress periodically alternate between odd and even cycles, as described by Eqs (4) and (5).

(4)

(5)

The introduction of a comparison between the double figure-8 shear loading and the figure-8 shear loading provides several benefits. First, it ensures that the magnitude of the shear force always remains constant. Second, it maintains similarity in the change rate of shear stress: during odd-numbered cycles, the shear stress of the double figure-8 loading matches that of the figure-8 loading, while during even-numbered cycles, the shear force components are opposite but share the equal magnitudes. Finally, the phase change between odd and even cycles results in shear stress components with continuity of the rate of change, ensuring a smooth transition in shear stress. Through this comparative analysis, other factors that may influence liquefaction can be effectively excluded, allowing for a more rational evaluation of the impact of shear direction on the liquefaction process.

**3. Results and discussion**

**3.1 Macroscopic response**

**3.1.1 Stress strain relationship**

With the application of unidirectional and multidirectional cyclic shear loads to undrained samples, the mean effective stress gradually decreases as shown in Fig. 4. When drops below approximately 50% of its initial value, an increase in shear load causes a slight rise in , while a decrease in shear load leads to a significant reduction in , eventually resulting in liquefaction as reaches zero. Even as continuously varies, the shear patterns of unidirectional, single-8, and double-8 modes project onto and as anticipated, effectively maintaining their respective linear, single-8, and double-8 stress path shapes.

Fig. 5 presents the liquefaction resistance under different forms of shear loading. The occurrence of liquefaction is defined as when the pore water pressure reaches 95% of the initial confining stress. It is evident that the unidirectional shear loading requires the highest cyclic number to reach liquefaction at the same CSR, compared to the multidirectional loadings. Following this, there is a difference between the required cyclic numbers for single-8 and double-8 shear stresses, with single-8 needing slightly more cycles than double-8. For instance, at CSR = 0.25, single-8 requires 30 cycles to reach liquefaction, whereas double-8 only requires about 22 cycles.

From a macroscopic perspective, the rotation of shear force direction has a significant impact on the liquefaction process. Although the magnitudes of unidirectional and multidirectional shear forces are equal at any given moment, the shear force direction in unidirectional loading remains fixed, unlike in multidirectional loading. This lack of directional rotation in unidirectional loading results in a higher number of cycles needed to reach liquefaction. Additionally, besides the influence of shear stress direction on the liquefaction process, the shear loading history also plays a role. For instance, although single-8 and double-8 paths share the same shear stress magnitude and rate of change at any given moment, the single-8 path maintains a constant figure-8 orientation, while the double-8 path alternates its orientation with each odd and even cycle. This alternating orientation in the double-8 path results in a higher number of cycles needed to reach liquefaction. In addition to differences in the number of cycles required for liquefaction, the strain development behaviors of single-8 and double-8 also differ. As cyclic loading continues, both single-8 and double-8 shear paths exhibit figure-8 patterns in their strain development. However, distinct differences emerged in the two shear paths. The strain path under single-8 shear stress maintains a consistent orientation throughout the cycles, with each cycle expanding outward along both and directions. This results in a steady accumulation of strain that gradually shifts in a direction perpendicular to the primary figure-8 axis. The strain path alternates orientation with each cycle, causing the "8-shape" axis to shift periodically. As a result, the double-8 pattern shows more complex and irregular strain growth compared to the single-8 path, with less directional offset over time.

**3.1.2. Cumulative shear work**

Shear work, defined as the work performed by the shearing rib on the specimen during cyclic loading, provides a scalar measure to evaluate liquefaction differences across various shear stress paths from a macroscopic perspective. In Fig.7, before the EPWP reaches approximately 60 kPa, the double-8 path shows the highest EPWP at equivalent shear work levels, followed by single-8 path, with unidirectional path exhibiting the lowest EPWP. This order of EPWP increase is inversely related to the number of cycles required to reach liquefaction, with double-8 reaching liquefaction faster than single-8, and unidirectional taking the longest. Despite these differences, the shear work required to reach liquefaction is roughly similar across all three loading types. This indicates that while directional variations in shear stress influence the rate at which EPWP builds up, the total energy input needed to induce liquefaction remains consistent.

**3.2 Microscopic interpretation**

**3.2.1 Evolution of coordination number**

The coordination number is a key indicator of the microstructural characteristics, representing the average number of contacts per particle. Particles with fewer than two contact are considered unable to effectively transmit contact forces and therefore do not contribute to the skeletal microstructure of the granular material. These particles are identified as "floaters" and are excluded from the coordination number calculation, as shown in Eq. (6). Here, denotes the mechanical coordination number. is​ the total number of contacts, and ​, , and represent the total number of particles, particles with one contact, and particles with zero contacts, respectively.

(6)

The evolution of ​ under different shear stress paths shows distinct patterns. Initially, the three shear stress types exhibit high coordination numbers around 5.00, indicating a well-structured granular skeleton with significant interparticle contacts. As cyclic shear progresses and the cyclic number normalized by the total cycles ​ approaches 1.0, the coordination number decreases in all cases, reflecting structural degradation and the approach toward liquefaction. However, the rate and extent of reduction differ among the three shear stress types.

The unidirectional shear stress maintains a relatively higher coordination number compared to single-8 and double-8 stresses throughout the process, suggesting greater structural integrity under unidirectional cyclic loading. Single-8 shear exhibits a moderate decrease, compared to double-8, which shows the most rapid reduction in , indicating that the double-8 stress path leads to more frequent structural rearrangements and a weaker granular skeleton.

This difference in evolution helps explain the varying cyclic numbers required to reach liquefaction under different shear paths. The unidirectional path, with its relatively stable , requires the highest cycles to induce liquefaction. In contrast, the double-8 path, with its rapid loss, reaches liquefaction more quickly, requiring fewer cycles. The single-8 path falls in between, requiring a moderate number of cycles to liquefy. This trend aligns with the increased instability and structural degradation induced by complex shear paths like double-8.

**3.2.2 Evolution of fabric anisotropy**

Table 1. Parameters in DEM simulation

|  |  |
| --- | --- |
| Description | Value |
| Number of particles | 29273 |
| Density, (kg/m3) | 2650 |
| Young’s modulus, E (GPa) | 1.2 |
| Normal-to-shear stiffness, | 2.0 |
| Tangential frictional coefficient between particles, | 0.10\*/0.50\*\* |
| Tangential frictional coefficient between particle and wall, | 0.0 |
| Normal critical damping ratio, | 0.7 |
| Shear critical damping ratio, | 0.5 |
| Rolling friction coefficient, | 0.5 |

\*IC

\*\* Generation, AC, TS, and Cyclic shear

图表, 折线图

描述已自动生成

Figure 1. Grain size distribution of curves of Toyoura sand and DEM simulation

图片包含 图形用户界面

描述已自动生成

Figure 2. Particle and contact force distribution after initial compaction, along with boundary and shearing rib configuration.

图表, 折线图, 直方图

描述已自动生成图表

描述已自动生成

1. Unidirectional shear stress evolution
2. Unidirectional shear stress path

图表, 折线图

描述已自动生成图表

描述已自动生成

(c) Single-8 shear stress path

1. Double-8 shear stress path

Figure 3. Shear stress path in unidirectional and multidirectional loading

图表, 雷达图

描述已自动生成

1. Stress evolution in unidirectional loading

图表, 表面图

描述已自动生成图表, 图示

描述已自动生成

Figure 4. Shear and mean effective stress evolution in unidirectional and multidirectional shear (CSR=0.250)

1. Stress evolution in single-8 loading
2. Stress evolution in double-8 loading

图表, 散点图

描述已自动生成

Figure 5. Liquefaction resistance under unidirectional and multidirectional shear stress

图表

描述已自动生成图表

描述已自动生成

1. Strain evolution in single-8 loading

(b) Strain evolution in single-8 loading

Figure 6. Comparison of strain evolution between single-8 and double-8 loading

图表

描述已自动生成

Figure 7. Cumulative shear work evolution in liquefaction under unidirectional, single-8, and double-8 cyclic shear stress

图表, 散点图

描述已自动生成

Figure 8. Coordination number evolution in liquefaction under unidirectional, single-8, and double-8 cyclic shear stress

**REFERENCES**

El Shafee, O., Abdoun, T., & Zeghal, M. (2017). Centrifuge modelling and analysis of site liquefaction subjected to biaxial dynamic excitations. Géotechnique, 67(3), 260–271. <https://doi.org/10.1680/jgeot.16.p.049>

Ishihara, K., & Yamazaki, F. (1980). Cyclic simple shear tests on saturated sand in multi-directional loading. Soils and Foundations, 20(1), 45–59. <https://doi.org/10.3208/sandf1972.20.45>

Kammerer, A. M., Pestana, J. M., & Seed, R. B. (2005). Behavior of monterey 0/30 sand under multidirectional loading conditions. Geomechanics, 154–173. <https://doi.org/10.1061/40797(172)8>

Pyke, R. M., Chan, C. K., & Seed, H. B. (1975). Settlement of sands under multidirectional shaking. Journal of the Geotechnical Engineering Division, 101(4), 379–398. <https://doi.org/10.1061/ajgeb6.0000162>

Su, D., & Li, X. S. (2008). Impact of multidirectional shaking on liquefaction potential of level sand deposits. Géotechnique, 58(4), 259–267. <https://doi.org/10.1680/geot.2008.58.4.259>

Wei, J., Huang, D., & Wang, G. (2020). Fabric evolution of granular soils under multidirectional cyclic loading. Acta Geotechnica, 15(9), 2529–2543. <https://doi.org/10.1007/s11440-020-00942-8>

Yang, M., Taiebat, M., Mutabaruka, P., & Radjaï, F. (2021). Evolution of granular media under constant-volume multidirectional cyclic shearing. Acta Geotechnica, 17(3), 779–802. <https://doi.org/10.1007/s11440-021-01239-0>