Lecture 2

God said,

$$\oint\!\!\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint\!\!\oint_S \vec{B}\cdot d\vec{A} = 0$$

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\iint_S \vec{B}\cdot d\vec{A}$$

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\iint_S \vec{E}\cdot d\vec{A}$$

or

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}$$

and then there was light.

These equations in differential and integral forms are called Maxwell's equations, which describes all of electricity and magnetism, electric and magnetic field and consequently the electromagnetic wave. Our goal of this course is to introduce you to these Maxwell's equations and extend this further beyond - to explain electromagnetic waves.

Introduction to electrostatics

We all are familiar with gravitational force. The gravitational force varies inversely as the square of the distance. There is another force in nature that also varies inversly as the square of distance, but is much much stronger. Also, there ~~are~~ is another major difference. The gravitational force is only attractive. So, there is only one kind of matter. However, in the ~~fo~~ mysterious force that we are talking about has two kinds of "matter", which are called positive and negative. The positive "matters" repels each other, negative "matters" repels each other. However, the negative and positive "matters" attracts each other. ~~Thatt~~ That's how we know that there are two different types of "matter".

The force is called the electric force and the "positives" and 'negatives' are called charges. Electric force is much much stronger than gravitational force. Its so strong that a tiny pen rubbed against your hair can hold a small piece of paper against the whole attraction of entire earth!

The atoms are made with positive protons with a charge of 1.6×10^{-19} C and electrons outside with a charge of -1.6×10^{-19} C. Now, the natural question might arise — if the electric force is so much strong, why an electron doesn't just fell to the nucleus. The answer has to do with quantum effects with uncertainty principles. Another question arises — what holds the nucleus together? The protons are all positive charges. They should enormously repel each other. It turns out that apart from electric force, there is another force — the nuclear force. The nuclear force is much stronger

than the electric force. However, it is sho[rt]
ranged. So, it falls very rapidly than $\frac{1}{r^2}$.
That's why for large nuclei, nuclear force can
no longer hold the nucleus in place, since the
nuclear force act mainly between each proton &
or neutron) and its nearest neighbour. The atom
then becomes unstable and undergoes fission by
just being tapped by ~~an~~ a slow neutron.

Now, we talked about charges — positive and negative. They are fundamental to the particles like proton and ~~neutron~~ electrons. There are two properties of electric charge —

(i) Conservation of charge: The total electric charge present in an isolated system, that is the sum of the positive and negative charge in an isolated system present at any time never changes. By isolated, we mean no matter can enter the system. Light can enter, since ~~p~~ photons do not carry any charge. However, a high energy photon may ~~brea~~ undergo pair creation— creating an electron and a positron—

the antiparticle of an electron. It will still follow the conservation of electric charge, since electron and positron has exactly ~~opp~~ equal and opposite charges.

There is an important catch. Electric charge is conserved locally. By locally we mean, an electric charge can't just vanish from one part of the universe and turn up somewhere else. It can ~~sty~~ only leave one point in space ~~and~~ by moving to a neighbouring point.

Quantization of charge

The electric charges that we find in nature come in units of one magnitude only — equal to the amount of charge carried by a single electron. We denote the magnitude of that charge by e with, ~~e~~

$$e = 1.602176634 \times 10^{-19}\ C$$

Electrons carry a charge of $-e$, whereas each proton carries a charge of e. Evidently, any charged particle that we observe in nature has a charge given by,

$$q = ne \quad \text{with} \quad n \in \mathbb{Z}$$

You can say, well, the charge of quarks is actually $q = -\frac{e}{3}$ and $q = 2\frac{e}{3}$ that makes up the protons and ~~net~~ neutrons. ~~However~~, or in general hadrons. Protons are formed with two up quark $(q = 2\frac{e}{3})$ and one down quark $(q = -\frac{e}{3})$. Neutron is made up of two down and one up quark. However, quarks are confined in the protons and neutrons and so we really don't "see" them. So, quantization of electric charges holds here.

The fact that a proton has the same charge as the electron, was proved by many experiments. At least, we can say for sure that the charges of electron and proton do not differ by one part in 10^{20}.

Charge density and distribution

So, far, we talked about only point charges. In reality, we mostly deal with a distribution of charges. Therefore, rather than talking about a point charge, we talk about the volume charge density, ρ. Volume charge density ρ is a function of position and bears the unit of charge/volume. So, ρ times a volume element will give you the charge contained in the volume element. If the volume charge density is $\rho(x,y,z)$ at some coordinate (x,y,z), then charge enclosed in a little box of volume $d\tau = dx\,dy\,dz$ is given by,

$$dq = \rho(x,y,z)\,dx\,dy\,dz$$

The total charge in a region of volume V is then just the volume integral,

$$q = \int_V \rho(x,y,z)\,d\tau$$

Now, charge density in general can be a function of time also. So, we might write,

$$\rho = \rho(\vec{r},t)$$

For a point charge, we can still describe the charge density by introducing the Dirac-delta function, which will be given by,

$$\rho = q\,\delta(\vec{r}-\vec{r}\,')$$

So, the particle is at $\vec{r} = \vec{r}\,'$

$$q = \int \rho \, d\tau = \int q(\vec{r})\delta(\vec{r}-\vec{r}')\, d^3x$$
$$= q(\vec{r}')$$

Generally, when we will need to describe the movement of charge from one place to another, we define some quantity called the current density $\vec{J}(\vec{r},t)$, which is defined as –

$$I = \iint_S \vec{J} \cdot d\vec{A}$$

I is the current, which counts the charge per unit time passing through the surface S. In this sense current density is current per area.

$$\text{Current, } I = \frac{dQ}{dt}$$

Now, the conservation of charge means ρ can change in time only is there is a compensating current that flows into or out of that region. We can express this in the continuity equation,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

This equation always comes in when some

quantity is locally conserved.

$$\frac{dQ}{dt} = \iiint_V d\tau \, \frac{\partial \rho}{\partial t} = -\iiint_V d\tau \, \vec{\nabla}\cdot\vec{J}$$

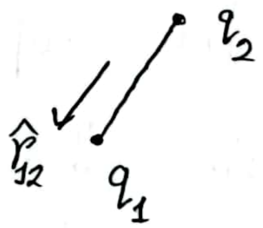
$$= -\iint_S \vec{J}\cdot d\vec{A}$$

The right hand side is basically the current. The minus sign is ensuring that if the net flow of current is outwards, then the charge in that region decreases.

Coulomb's law

We already talked about the fact that charges attracts and repel each other. The force between two charges at rest was first described by Coulomb's law. In vector form we can write the law as —

$$\vec{F}_1 = K \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$



where $\vec{F}_1$ is the force on the charge q_1, $\hat{r}_{12}$ is the unit vector directed from q_2 to the charge q_1, r_{12} is the distance between q_1 and

q_2. q_1 and q_2 are number that gives the magnitude and sign of the respective charges.

$$\vec{F}_{21} = K \frac{q_2 q_1}{r_{21}^2} \hat{r}_{21}$$

And so, $\vec{F}_{12} = -\vec{F}_{21}$. So the force obeys Newton's third law.

In SI unit, the value of the constant K is given by,

$$K = 8.988 \times 10^9 \ Nm^2C^{-2}$$

In stead of K, we can write another constant ϵ_0, which is related to K by,

$$K = \frac{1}{4\pi\epsilon_0} \quad \text{with} \quad \epsilon_0 = 8.854 \times 10^{-12} \ C^2N^{-1}m^{-2}.$$

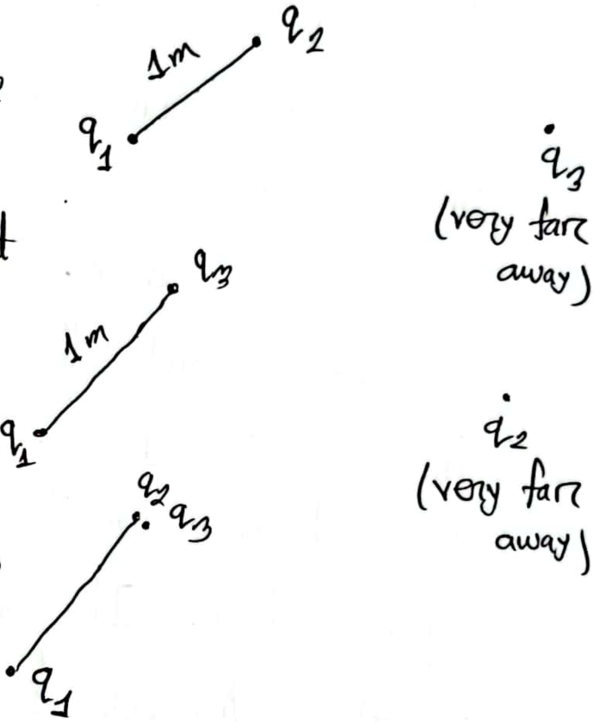
So, in terms of ϵ_0, Coulomb's law can be written as,

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

The superposition principle

When there are more than ~~one~~ two charges, things could become complicated, but it doesn't. It is another property of nature that, the forces on any charge is the vector sum of the Coulomb forces on the individual charges. This is called the principle of superposition. So, if we want to calculate the force on q_1 due to two charges q_2 and q_3, its basically,

$$\vec{F} = \vec{F}_2 + \vec{F}_3$$



We can first measure the force on q_1 due to q_2, when q_3 is very far away, so its effect is negligible. Then, we can measure the force on q_1 due to q_3 when q_2 is very far away. If we now measure the force by bringing both q_2 and q_3 very close to q_1, then it will be found that the force felt by q_1 now is equal to the ~~two~~ sum of two previous forces. So, the force with ~~with~~ which two charges interact is not changed by the presence of a third charge. So, the force on q_1 is,

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13}$$

The verification of the inverse square law was however first found by Coulomb, who used a torsion pendulum to find the result experimentally in ~1876. Later Maxwell repeated an experiment done by Cavendish in 1772, which would immediately confirm the inverse square law. In that experiment, there was a spherical conducting shell, inside of which there was another smaller sphere connected to the large shell. The large spherical shell was charged and then carefully removed after separating it into two parts. If the inner sphere didn't have any charge on it, it would confirm the inverse square law. If the exponent was $2+\delta$ instead of 2, then Maxwell found δ to be $<10^{-6}$, and present experiments ~~also~~ approximated $\delta < 10^{-16}$. So, the inverse square law is something that is pretty much accurate. Also, Coulomb's law as tested at distances starting from 10^{-16} m to 10^{8} m, and it still holds. But the breaks down in the limit of $<10^{-16}$ m and like those distances.

~~Charge distribution and finding electric field~~

The electric field

While talking about Coulomb's law, ~~we often~~ it is convenient to introduce the idea of electric field. Suppose, we have some arrangement of charges, $q_1, q_2, \dots, q_N$, fixed in space and we are not interested in the force that they exert on each other, rather only the effect of those charges at some point when some other charge q_0 is placed at that point. We already know how to calculate the resultant force, which will be given by,

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{N} \frac{q_0 q_j}{r_{0j}^2} \hat{r}_{0j}$$

where $\hat{r}_{0j}$ is the vector from j^{th} charge to the q_0 charge at let's say (x,y,z). The force is proportional to q_0, and we might divide $\vec{F}$ by q_0 to obtain a vector quantity that depends only on the structure of the original system. We call this vector quantity the electric field, which will be given by,

$$\vec{E}(x,y,z) = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{N} \frac{q_j}{r_{0j}^2} \hat{r}_{0j}$$

The force on some other charge q at (x, y, z) is then given by,

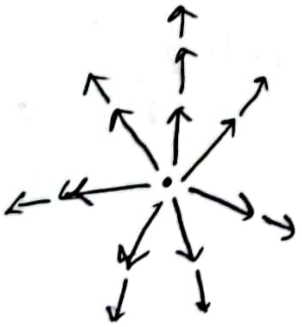
$$\vec{F} = q\vec{E}$$

The unit of electric field can obviously be written as N/C, but we will see another unit later.

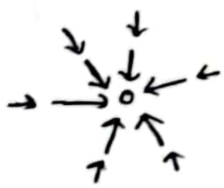
Now, the question arises, why do we even need electric field. For one, we have a tremendous advantage. Once we know $\vec{E}$ in some small neighbourhood, we immediately know what will happen to any charge in that neighbourhood. So, electric field attaches to every point in a system, a local property. Often we say, the force between two particles doesn't act directly. ~~Rather not~~ Rather, nature chooses intermediaries, which are called fields. So, field is a dynamical quantity which takes a value (and possibly direction) at each and every point in space and time. When we talk ~~abat to~~ about force in modern Physics, we really talk about the interactions of the particles with the field. First, ~~p~~ charged particles create electric fields, and then the field tells other charged particles how to move. This motion, in turn changes the fields that the particle create and ~~were~~ we are left with a beautiful dance with the particles and fields, each dictating the moves of the other.

The idea of fields ~~prod~~ provides an advantage that all interactions are local. Any object - particles and fields only affect things around their neighbourhood. This influence then propagate through field with time to reach another point in space to influence a particle there. ~~The~~ This lack of ~~inta~~ instantaneousness interactions actually allow ~~&~~ forces to be compatible with special theory of relativity.

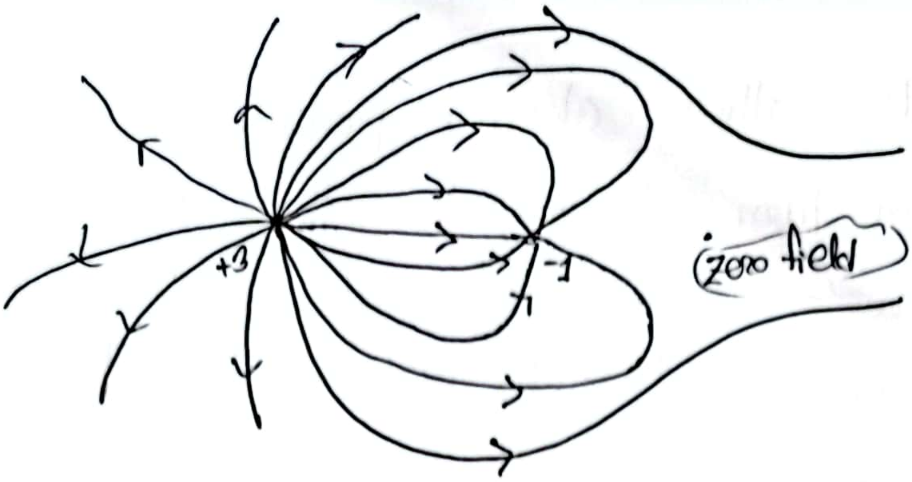
Now, we can draw the electric field lines. We indicate $\vec{E}$ at various points by drawing little arrows near that points and making the arrows longer where $|\vec{E}|$ is larger.



$q_1 = +3C$



$q_2 = -1C$



In this figure, we have drawn the fild lines, where the ~~fild~~ field at each point can be found by drawing a tangent at that point to the curve. These curves are smooth and continuous except at the locations of the charges and at a point where the electric field is zero. The drawing is in two dimension, which can only depict the field lines in a particular plane. However we can always rotate this picture around the symmetry axis to find a glimpse of how the field will look like in 3D.

Field for a continuous charge distribution

$$\vec{E}(x,y,z) = \frac{1}{4\pi\epsilon}\int_V \frac{\rho(x',y',z')\,dx'dy'dz'}{r^2}\,\hat{r}$$