PHY 110: Mechanics and Properties of Matter

Tutorial 1: Vectors

Tibra Ali
MNS Brac University
tibra.ali@bracu.ac.bd

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Notation

Suppose

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \tag{1}$$

is the representation of the vector $\mathbf{A}$ in a given right-handed Cartesian coordinate system. We shall denote this as a 3-tuple

$$\mathbf{A} = (A_x, A_y, A_z). \tag{2}$$

Note that this notation is unambiguous only if we specify the coordinate system.

1 Dot Product

1. Consider a vector $\mathbf{A} = (3, 5, 0)$. Which of the following vectors are orthogonal/perpendicular to $\mathbf{A}$?

   i) $\mathbf{B} = (3, 5, 4)$
   ii) $\mathbf{C} = (3, -5, 4)$
   iii) $\mathbf{D} = (-5, 3, 0)$
   iv) $\mathbf{E} = (0, 0, 4)$

2. What is the general form of the vector $\mathbf{A}^{\perp}$ that is perpendicular to $\mathbf{A}$? Consider solving the equation

$$\mathbf{A} \cdot \mathbf{A}^{\perp} = 0 \tag{3}$$

   for $\mathbf{A}^{\perp}$ to answer this question.

3. Discuss the following statement with two or more of your fellow classmates:

> *"When we know the dot product with a given vector* **A** *with an unknown vector* **X***, it is, in general, impossible to uniquely specify* **X***."*

2 Addition and Subtraction of Vectors

Consider the vectors $\mathbf{A} = (2, -5, 2)$ and $\mathbf{B} = (5, 6, 1)$. Compute:

1. $\mathbf{A} + \mathbf{B}$
2. $\mathbf{A} - \mathbf{B}$
3. Find the cosine of the angle between **A** and **B**. Also find the cosine of the angle between $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$.

For the next three questions assume that **A** and **B** are general vectors (and not necessarily the vectors above specified above).

4. If **A** and **B** are the sides of a parallelogram, which aspects of the parallelogram do $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$ represent?
5. Show that the diagonals of an equilateral parallelogram are perpendicular. (Prove geometrically) trigonometrically
6. Show that if $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$, then **A** and **B** are perpendicular.

3 The Cross Product

Given two vectors $\mathbf{A} = (1, 2, 3)$ and $\mathbf{B} = (2, 3, 4)$ find the cross product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ using

$$\mathbf{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}. \tag{4}$$

We prove

4 Problem

This is a take-home problem if there is no time during the tutorial.

Given a vector $\mathbf{A} = (3, 4, -4)$

1. find a *unit* vector $\hat{\mathbf{B}}$ that lies in the x-y plane and is perpendicular to **A**.
2. find a unit vector $\hat{\mathbf{C}}$ that is perpendicular to both **A** and $\hat{\mathbf{B}}$.
3. Show that **A** is perpendicular to the plane defined by $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$.

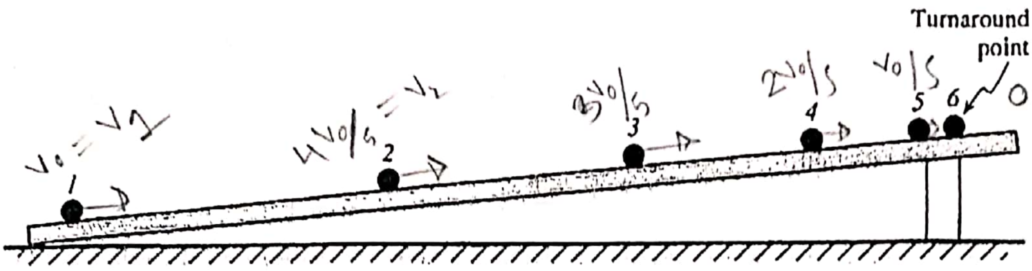
TUTORIAL - 2 (25/05/2022)

YSN

ACCELERATION IN ONE DIMENSION

I. Motion with decreasing speed

The diagram below represents a strobe photograph of a ball as it rolls *up* a track. (In a strobe photograph, the position of an object is shown at instants separated by *equal time intervals.*)



A. Draw vectors on your diagram that represent the instantaneous velocity of the ball at each of the labeled locations. If the velocity is zero at any point, indicate that explicitly. Explain why you drew the vectors as you did.

We assume At 1, V_0 is max. As the ball rolls up the track, at point 6, the ball's velocity is zero

We will call diagrams like the one you drew above *velocity diagrams*. Unless otherwise specified, a velocity diagram shows both the location and the velocity of an object at instants in time that are separated by equal time intervals.

B. In the space at right, compare the velocities at points *1* and 2 by sketching the vectors that represent those velocities. Draw the vectors side-by-side and label them $\vec{v}_1$ and $\vec{v}_2$, respectively.

$\vec{v}_1$, $\vec{v}_2$, and $\Delta\vec{v}$
$\rightarrow \vec{V_1}$
$\rightarrow \vec{V_2}$
$\leftarrow \vec{\Delta V}$

Draw the vector that must be *added* to the velocity at the earlier time to equal the velocity at the later time. Label this vector $\Delta\vec{v}$.

$\vec{\Delta V} + \vec{V_1} = \vec{V_2}$ | $\vec{\Delta V} = \frac{4}{5}\vec{V_0} - \vec{V_0}$

$\Rightarrow \vec{\Delta V} = \vec{V_2} - \vec{V_1}$ | $= \vec{V_0}\left(\frac{-1}{5}\right) = -\vec{V_0}/5$

Why is the name *change in velocity* appropriate for this vector?

We are doing vector subtraction to determine the change.

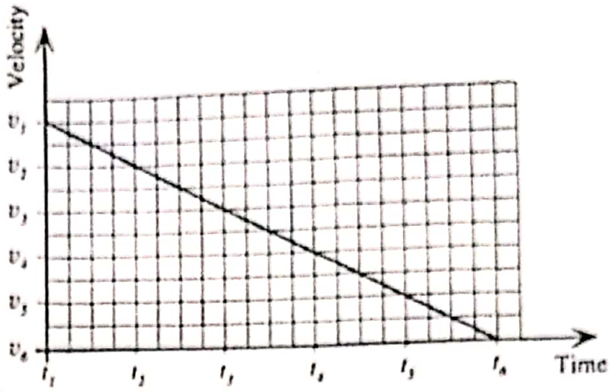
How does the direction of the change in velocity vector compare to the direction of the velocity vectors?

they are opposite to each other

Would your answer change if you were to select two *different* consecutive points (*e.g.*, points *3* and *4*) while the ball was slowing down? Explain.

At every point the change in velocity vector & the direction of velocity vector would be opposite

How would the magnitude of the change in velocity vector between points *1* and *2* compare to the magnitude of the change in velocity vector between two *different* consecutive points (*e.g.*, points *3* and *4*)? Explain. (You may find it useful to refer to the graph of velocity versus time for the motion.)



Magnitude of change of velocity vector between two points (consecutive) will always be the same.

which $|\Delta \vec{v}| = \frac{v_0}{5}$

Note: The positive direction has been chosen to be *up* the track.

[there is not jerk, also, change in acceleration it's accelarting constantly]

C. Consider the change in velocity vector between two points on the velocity diagram that are not consecutive, *e.g.*, points *1* and *4*.

Is the direction of the change in velocity vector different than it was for consecutive points? Explain.

Since it is a linear motion the direction of the change in velocity vector will be same for points which are not consecutive

Is the length of the change in velocity vector different than it was for consecutive points? If so, how many times larger or smaller is it than the corresponding vector for consecutive points? Explain.

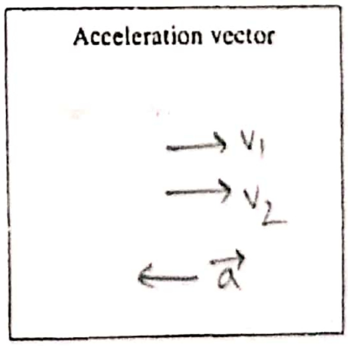
length of $\Delta\vec{v}$ is different for none consecutive points. It is always larger.

$|\Delta\vec{v}| = |\vec{v}_2 - \vec{v}_1|$ $\vec{v}_2 < \vec{v}_1$ → For all possible case

D. Use the definition of acceleration to draw a vector in the space at right that represents the acceleration of the ball between points *1* and 2.

How is the direction of the acceleration vector related to the direction of the change in velocity vector? Explain.

$\boxed{\text{Same}}$



E. Does the acceleration change as the ball rolls up the track? No change
Would the acceleration vector you obtain differ if you were to choose (1) two different successive points on your diagram or (2) two points that are not consecutive? Explain.

1. Successive points: No different

2. ~~For not consecutive points, acceleration vector is same as the velocity vector bcz~~

2. Non-conseleutive Point: No different

F. Generalize your results thus far to answer the following question:

What is the relationship between the direction of the acceleration and the direction of the velocity for an object that is moving in a straight line and slowing down? Explain.

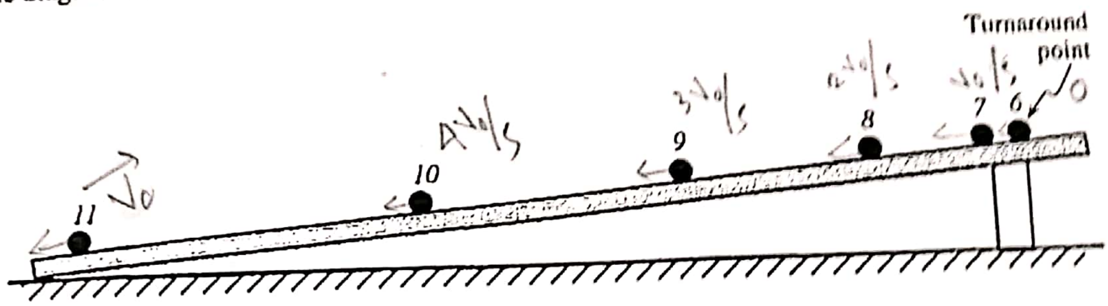
opposite

Describe the direction of the acceleration of a ball that is rolling up a straight incline.

$\vec{a}$ is in the opposite direction of a ball's $\vec{v}$ as

II. Motion with increasing speed

The diagram below represents a strobe photograph of a ball as it rolls *down* the track.



A. Choose two successive points. In the space at right, sketch the velocity vectors corresponding to those points. Draw the vectors side-by-side and label them $\vec{v}_i$ and $\vec{v}_f$, respectively.

$\vec{v}_i$, $\vec{v}_f$, and $\Delta\vec{v}$

$\vec{v}_1 \leftarrow$

$\vec{v}_2 \leftarrow$

$\Delta\vec{v} \leftarrow$

Determine the vector that must be added to the velocity at the earlier time to equal the velocity at the later time. Is the name *change in velocity* appropriate for this vector?

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 \Rightarrow \Delta\vec{v} = \vec{v}_0 - \frac{4}{5}\vec{v}_0 = (1-\frac{4}{5})\vec{v}_0 = \frac{\vec{v}_0}{5}$

How does the direction of the change in velocity vector compare to the direction of the velocity vectors in this case?

same direction

Would your answer change if you were to select two *different* points during the time that the ball was speeding up? Explain.

direction of $\Delta\vec{v}$ will remain same However, magnitude will differ from two non-consecutive point

B. In the space at right, draw a vector to represent the acceleration of the ball between the points chosen above.

Acceleration vector

$\leftarrow \vec{a}$

acceleration

How is the direction of the change in velocity vector related to the direction of the acceleration vector? Explain.

$\Delta\vec{v}$ & $\vec{a}$ have the same direction However, $\Delta\vec{v}$ is merely change of velocity vector. while $\vec{a}$ is the change of velocity vector w.r.t. of time.

Generalize your results thus far to answer the following question:

What is the relationship between the direction of the acceleration and the direction of the velocity for an object that is moving in a straight line and speeding up? Explain.

same direction

Describe the direction of the acceleration of a ball that is rolling down a straight incline.

same as the direction of $\vec{v}$

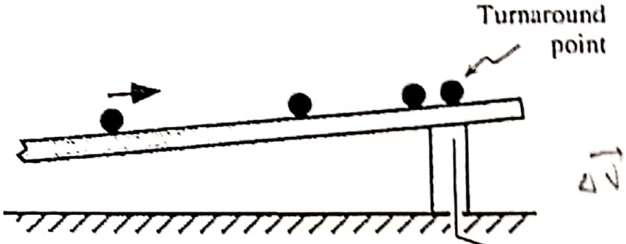
III. Motion that includes a change in direction

Complete the velocity diagram below for the portion of the motion that includes the turnaround.

A. Choose a point *before* the turnaround and another *after*.

In the space below, draw the velocity vectors and label them $\vec{v}_i$ and $\vec{v}_f$.

Draw the vector that must be added to the velocity at the earlier time to obtain the velocity at the later time.

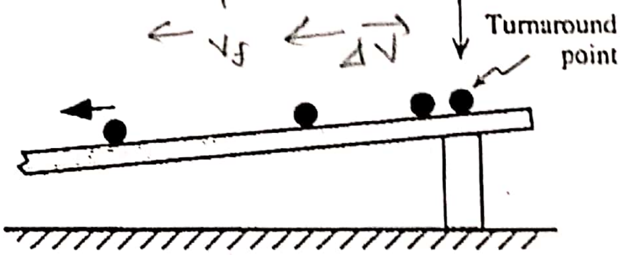


$\vec{\Delta V} = \vec{V_f} - \vec{V_i}$
$= (-) c V_0$

Is the name *change in velocity* that you used in sections I and II also appropriate for this vector?

$\rightarrow V_i$

$\leftarrow V_f$ $\leftarrow \vec{\Delta V}$



B. Suppose that you had chosen the turnaround as one of your points.

What is the velocity at the turnaround point? zero

Would this choice affect the direction of the change in velocity vector? Explain why or why not.

$\vec{v}_i$, $\vec{v}_f$, and $\Delta\vec{v}$

C. In the space at right, draw a vector that represents the acceleration of the ball between the points you chose in part B above.

Acceleration vector

Compare the direction of the acceleration of the ball at the turnaround point to that of the ball as it rolls: (1) up the track and (2) down the track.

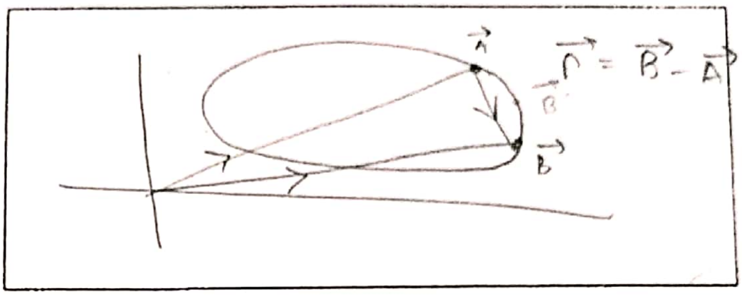
I. Velocity

An object is moving around an oval track. Sketch the trajectory of the object on a large sheet of paper. (Make your diagram *large*.)

A. Choose a point to serve as an origin for your coordinate system. Label that point *O* (for origin). Select two locations of the object that are about one-eighth of the oval apart and label them *A* and *B*.

1. Draw the position vectors for each of the two locations *A* and *B* and draw the vector that represents the displacement from *A* to *B*.

Copy your group's drawing in this space after discussion.



2. Describe how to use the displacement vector to determine the direction of the average velocity of the object between *A* and *B*. Draw a vector to represent the average velocity.

direction of the average velocity is same/parallel to the displacement vector

3. Choose a point on the oval between points *A* and *B*, and label that point *B'*.

As point *B'* is chosen to lie closer and closer to point *A*, does the direction of the average velocity over the interval *AB'* change? If so, how?

A→B' the direction of the average velocity over AB' becomes tangential. (direction of the instantaneous velocity)

4. Describe the direction of the instantaneous velocity of the object at point *A*.

tangenti. tangentale to the point A

How would you characterize the direction of the instantaneous velocity at *any* point on the trajectory?

tangential to the curve.

Does your answer depend on whether the object is speeding up, slowing down, or moving with constant speed? Explain.

No, it will remain same

B. If you were to choose a different origin for the coordinate system, which of the vectors that you have drawn in part A would change and which would not change?

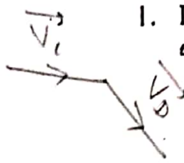
position vector of $\vec{A}, \vec{B}$ will change but displacement vector will remain the same

II. Acceleration for motion with constant speed

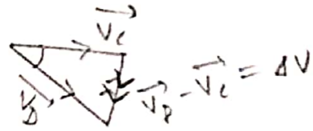
Suppose that the object in section I is moving around the track at *constant speed.* Draw vectors to represent the velocity at two points on the track that are relatively close together. (Draw your vectors *large.*) Label the two points *C* and *D*.

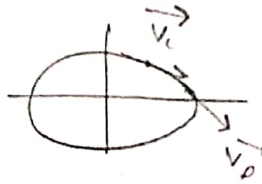
A. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_C$ and $\vec{v}_D$. From these vectors, determine the *change in velocity vector,* $\Delta\vec{v}$.

1. Is the angle formed by the "head" of $\vec{v}_C$ and the "tail" of $\Delta\vec{v}$ *greater than, less than,* or *equal to* 90°?



(parallel transport)





As point *D* is chosen to lie closer and closer to point *C*, does the above angle *increase, decrease,* or *remain the same?* Explain how you can tell.

decreases and approaches 0.

Does the above angle approach a *limiting* value? If so, what is its limiting value?

$\lim_{D}$ ~~points D→C as θ→0~~ θ → 0 as D → C

2. Describe how to use the change in velocity vector to determine the average acceleration of the object between *C* and *D*. Draw a vector to represent the average acceleration between points *C* and *D*.

direction of $\Delta\vec{v}$ is parallel to $\vec{a}$ | $a_{avg} = \frac{\Delta\vec{v}}{\Delta t}$ (average velocity is being scaled by $1/\Delta t$)

What happens to the magnitude of $\Delta\vec{v}$ as point *D* is chosen to lie closer and closer to point *C?* Does the acceleration change in the same way? Explain.

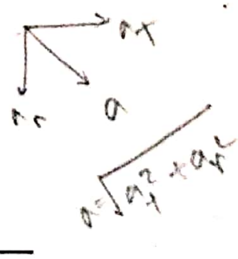
$\lim_{D\to C} \vec{\Delta t} = \Delta\vec{v} \to 0$ as $D \to C$ but $\Delta\vec{v}/\Delta t$ doesn't ~~go~~ become zero (acceleration Doesn't change the same way as $\vec{v}$)

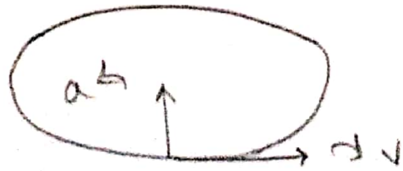
Consider the direction of the acceleration at point *C*. Is the angle between the acceleration vector and the velocity vector *greater than, less than,* or *equal to* 90°? (*Note:* Conventionally, the angle between two vectors is defined as the angle formed when they are placed "tail-to-tail.")

~~less than 90° (when speeding up)~~
~~greater than 90° (when slowing down)~~

equal to 90°

at constant speed, acceleration & velocity vectors are orthogonal





B. Suppose you were to choose a new point on the trajectory where the *curvature is different* from that at point *C*.

Is the magnitude of the acceleration at the new point *greater than, less than,* or *equal to* the magnitude of the acceleration at point *C*? Explain.

magnitude of the acceleration is same everywhere on the curve (e.g. ellipse)

Describe the direction of the acceleration at the new point.

➪ Check your reasoning for section II with a tutorial instructor before proceeding.

III. Acceleration for motion with changing speed

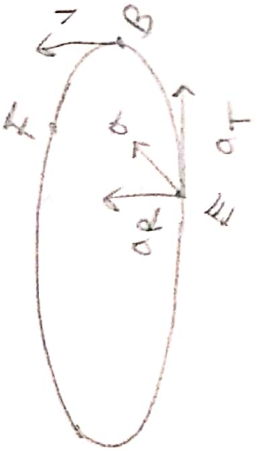
Suppose that the object is *speeding up* as it moves around the oval track. Draw vectors to represent the velocity at two points on the track that are relatively close together. (Draw your vectors *large*.) Label the two points *E* and *F*.

A. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_E$ and $\vec{v}_F$. From these vectors, determine the change in velocity vector, $\Delta\vec{v}$.

$\Delta\vec{V} = \vec{V}_E - \vec{V}_F$

1. Is the angle θ, formed by the head of $\vec{v}_E$ and the tail of $\Delta\vec{v}$, *greater than, less than,* or *equal to* 90°?

less than 90°



Consider how θ changes as point *F* is chosen to lie closer and closer to point *E*.

What value or range of values is possible for this angle for an object that is speeding up? Explain.

(0, 2)

What happens to the magnitude of $\Delta\vec{v}$ as point *F* is chosen to lie closer and closer to point *E?*

$\Delta\vec{V} \to 0$

2. Describe how you would determine the acceleration of the object at point *E*.

Find $\Delta\vec{V}$ and scale it by a factor of $1/\Delta t$ $\left(\vec{a}_{avg} = \frac{\Delta\vec{V}}{\Delta t}\right)$

Consider the direction of the acceleration at point *E*. Is the angle between the acceleration vector and the velocity vector (placed "tail-to-tail") *greater than, less than,* or *equal to* 90°?

less than 90°

B. Suppose the object started *from rest* at point E and moved towards point F with increasing speed. How would you find the acceleration at point E?

parallel to the velocity vector

Describe the direction of the acceleration of the object at point E.

~~para~~ parallel to the velocity.

For uniform circular motion (Horizontal)

$a_T = 0$

$a = \sqrt{a_R^2 + a_T^2} = a_R$

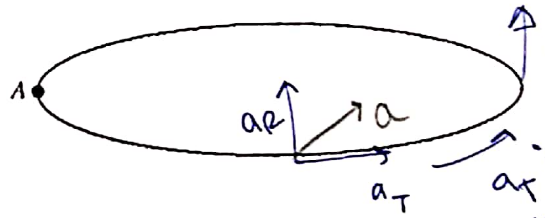
if reference frame is earth

C. At several points on each of the diagrams below, draw a vector that represents the acceleration of the object.

Acceleration vectors for constant speed

Top view diagram

Acceleration vectors for speeding up from rest at point A



Top view diagram

a_T is the component that speeds up the particle

Characterize the direction of the acceleration at each point on the trajectory for each case.

Is the acceleration directed toward the "center" of the oval at every point on the trajectory for either of these cases?

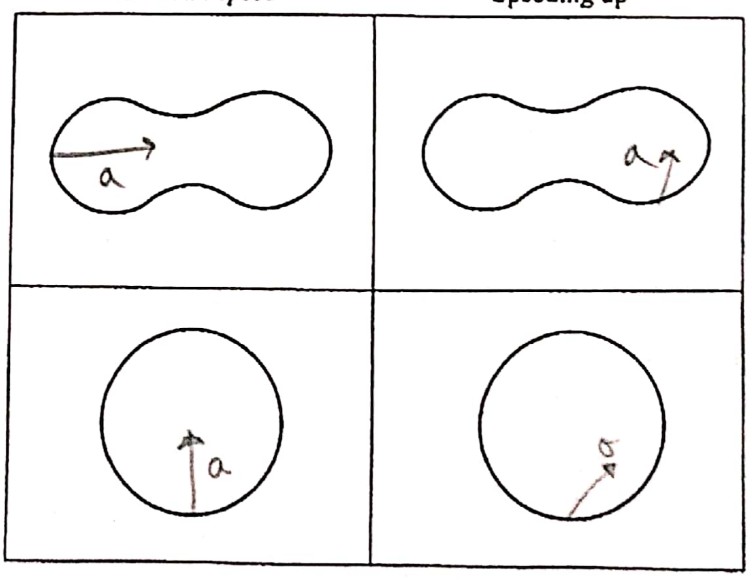
No, for constant speed acceleration is directed to the center But for constant speeding up. a will have two components, one across the tangent of the curve. another is directed to the center

Sketch arrows to show the direction of the acceleration for the following trajectories:

Constant speed | Speeding up

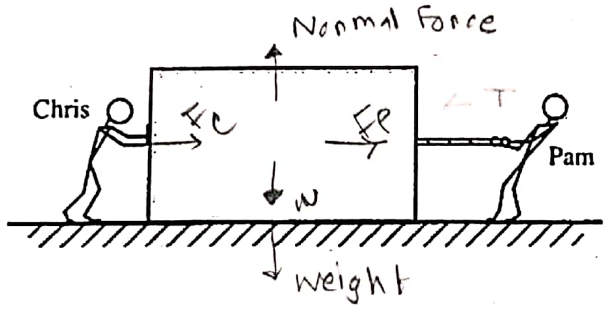
• peanut-shaped

• circular

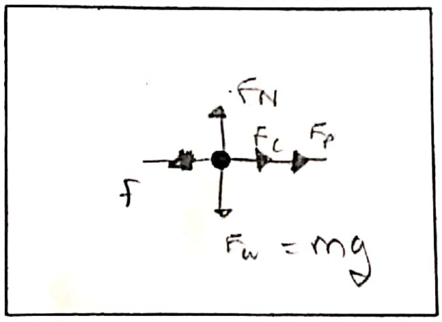


I. Identifying forces

Two people are attempting to move a large block. The block, however, does not move. Chris is pushing on the block. Pam is pulling on a rope attached to the block.



Copy your group's sketch here after discussion.



A. Draw a large dot on your large sheet of paper to represent the block. Draw vectors with their "tails" on the dot to show the forces exerted *on* the block. Label each vector and write a brief description of that force next to the vector.

In Newtonian physics, all forces are considered as arising from an interaction between *two* objects. Forces are specified by identifying the object *on which* the force is exerted and the object *that is exerting* the force. For example, in the situation above, a gravitational force is exerted *on* the block *by* the Earth.

B. Describe the remaining forces you have indicated above in a similar fashion.

$\vec{N}$ → ~~Due~~ The block is exert force on earth, w, and earth is exerting on the block

The diagram you have drawn is called a *free-body diagram*. A free-body diagram should show only the forces exerted *on* the object or system of interest (in this case, forces exerted *on the block*). Check your free-body diagram and, if necessary, modify it accordingly.

A proper free-body diagram should *not* have anything on it except a representation of the object and the (labeled) forces exerted on that object. A free-body diagram *never* includes (1) forces exerted by the object of interest on other objects or (2) sketches of other objects that exert forces on the object of interest.

C. All forces arise from interactions between objects, but the interactions can take different forms.

Which of the forces exerted on the block require *direct contact* between the block and the object exerting the force?

friction force, Normal force

Which of the forces exerted on the block *do not* arise from direct contact between the block and the object exerting the force?

~~the~~ Gravitational force, ~~Tension~~ Force
= weigh in the same in this case

We will call forces that depend on contact between two objects *contact forces*. We will call forces that do not arise from contact between two objects *non-contact forces*.

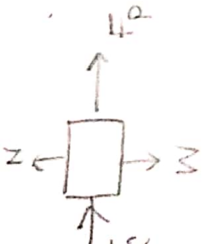
D. There are many different types of forces, including: friction ($\vec{f}$), tension ($\vec{T}$), magnetic forces ($\vec{F}^{mag}$), normal forces ($\vec{N}$), and the gravitational force ($\vec{W}$, for weight). Categorize these forces according to whether they are contact or non-contact forces.

Contact forces	Non-contact forces
Normal force, tension, friction	magnetic force, gravitational Force

E. Consider the following discussion between two students.

Student 1: *"I think the free-body diagram for the block should have a force by Chris, a force by the rope, and a force by Pam."*

Student 2: *"I don't think the diagram should show a force by Pam. People can't exert forces on blocks without touching them."*



With which student, if either, do you agree? Explain your reasoning. Both are wrong

⇒ A force by rope won't be included

⇒ non-contact force exists!

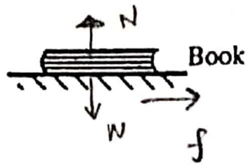
It is often useful to label forces in a way that makes clear (1) the type of force, (2) the object on which the force is exerted, and (3) the object exerting the force. For example, the gravitational force exerted *on* the block *by* the Earth might be labeled $\vec{W}_{BE}$. Your instructor will indicate the notation that you are to use.

F. Label each of the forces on your free-body diagram in part A in the manner described above.

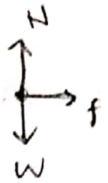
⇨ Do not proceed until a tutorial instructor has checked your free-body diagram.

II. Drawing free-body diagrams

A. Sketch a free-body diagram for a book at rest on a level table. (*Remember:* A proper free-body diagram should not have anything on it except a representation of the book and the forces exerted *on* the book.)



Make sure the label for each force indicates:



- the type of force (gravitational, frictional, *etc.*),
- the object on which the force is exerted, and
- the object exerting the force.

1. What evidence do you have for the existence of each of the forces on your diagram?

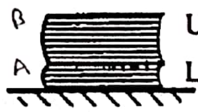
W → Gravitation force   N → gravitational force
f → arises when a body tries move along a surface of second body

2. What observation can you make that allows you to determine the relative magnitudes of the forces acting on the book?

$\Sigma F_y = 0 \Rightarrow 0 = F_g - N \Rightarrow N = F_g$ as the book is at rest. $[F_g = W]$

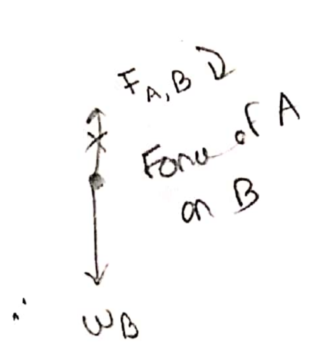
How did you show the relative magnitudes of the forces on your diagram?

B. A second book of greater mass is placed on top of the first.

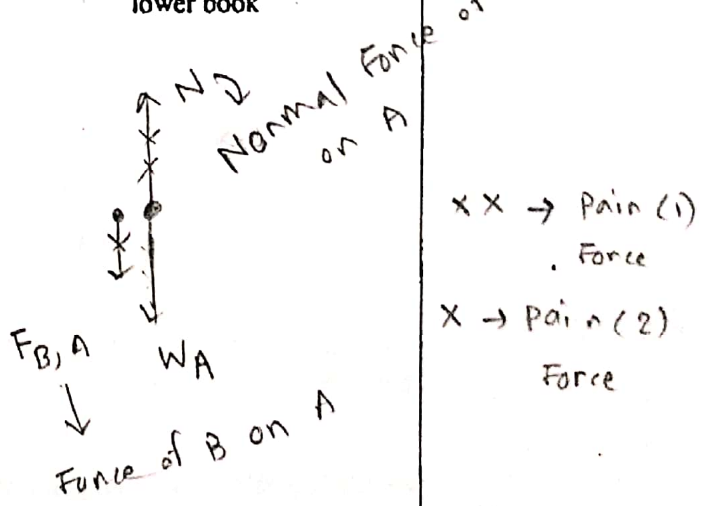


Sketch a free-body diagram for each of the books in the space below. Label all the forces as in part A.

Free-body diagram for upper book



Free-body diagram for lower book



×× → Pair (1) Force

× → Pair (2) Force

Specify which of the forces are contact forces and which are non-contact.

W → Non contact

$F_{A,B}$, N, F_{BA} → contact Forces

Normal Force arises from electromagnetic Force (molecular scale)

1. Examine all the forces on the two free-body diagrams you just drew. Explain why a force that appears on one diagram *should not* appear on the other diagram.

   Because a diagram only includes the force exerted 'on' the object

2. What *type* of force does the upper book exert on the lower book (*e.g.*, frictional, gravitational)?

   Gravitational Force

   Why would it be *incorrect* to say that the weight of the upper book acts on the lower book?

   weight acts on earth

3. What observation can you make that allows you to determine the relative magnitudes of the forces on the *upper* book?

   Upper book is at rest. So, $\sum F_y = 0 \Rightarrow F_{A,B} = W_B$

4. Are there any forces acting on the *lower* book that have the same magnitude as a force acting on the *upper* book? Explain.

   $F_{A/B}$ & $F_{B,A}$ are partner force that ~~th~~ have same ~~mg~~ magnitude

C. Compare the free-body diagram for the lower book to the free-body diagram for the same book in part A (*i.e.*, before the upper book was added).

   Which of the forces changed when the upper book was added and which remained the same?

   Force of B on A was added

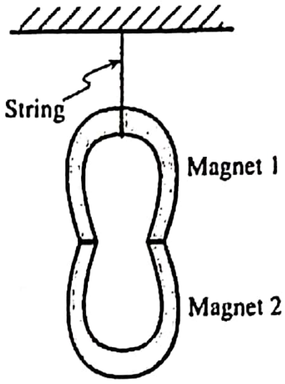
As discussed earlier, we think of each force acting on an object as being exerted by another object. The first object exerts a force of equal magnitude and opposite direction on the second object. The two forces together are called an *action-reaction* or *Newton's third law* force pair.

D. Which, if any, Newton's third law force pairs are shown in the diagrams you have drawn? On which object does each of the forces in the pair act?

   Identify any third law force pairs on your diagrams by placing one or more small "X" symbols through each member of the pair. For example, if you have two sets of third law force pairs shown on your diagrams, mark *each* member of the first pair as ⟶×⟶, and each member of the second pair as ⟶××⟶.

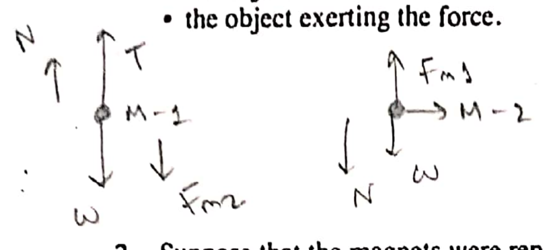
III. Supplement: Contact and non-contact forces

A. A magnet is supported by another magnet as shown at right.



1. Draw a free-body diagram for magnet 2. The label for each of the forces on your diagram should indicate:

   - the type of force (*e.g.*, gravitational, normal),
   - the object on which the force is exerted, and
   - the object exerting the force.



2. Suppose that the magnets were replaced by stronger magnets of the same mass.

   If this changes the free-body diagram for magnet 2, sketch the new free-body diagram and describe how the diagram changes. (Label the forces as you did in part 1 above.) If the free-body diagram for magnet 2 does not change, explain why it does not.

A stronger magnet would just apply a greater force. the direction of the force would remain the same. So diagram will remain the same

3. Can a magnet exert a non-contact force on another object? Yes (Magnetic Force)

   Can a magnet exert a contact force on another object? Yes (Normal Force)

   Describe how you can use a magnet to exert *both* a contact force and a non-contact force on another magnet. Non-contact force. magnetic Force

   contact force in here is the normal force ~~that~~ so when a magnet touches the surface of another magnate, the surface of contact gets 'dumped', so the first magnate experiences a force exerted by the later one.

4. To ensure that you have accounted for all the forces acting on magnet 2 in parts 1 and 2:

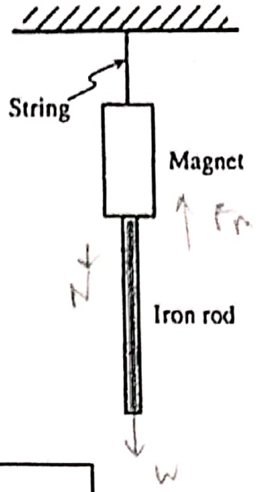
   List all the non-contact forces acting on magnet 2.

   W (gravitational Force) aka weight

   List all the contact forces acting on magnet 2. (*Hint:* Which objects are in *contact* with magnet 2?)

   Magnetic Force, Normal Force

B. An iron rod is held up by a magnet as shown. The magnet is held up by a string.

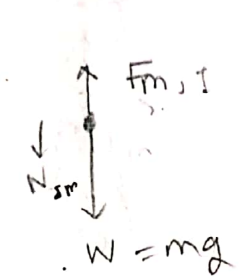
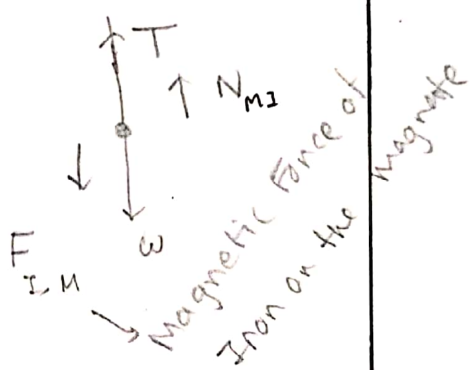


1. In the spaces below, sketch a free-body diagram for the iron rod and a separate free-body diagram for the magnet.

   The label for each of the forces on your diagrams should indicate:

   - the type of force (*e.g.*, gravitational, normal),
   - the object on which the force is exerted, and
   - the object exerting the force.

Free-body diagram doesn't include (partner forces)?

Free-body diagram for iron rod	Free-body diagram for magnet
	
(*Hint:* There should be three forces.)	(*Hint:* There should be four forces.)

2. For each of the forces shown in your diagram for the iron rod, identify the corresponding force that completes the Newton's third law (or action-reaction) force pair.

① F_{IM} and F_{MI}

② F_{IE} and F_{EI} (Force on iron by earth & Force on earth by iron)

③ N_{Im} and N_{MI}

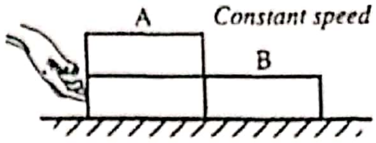
3. How would your diagram for the iron rod change if the magnet were replaced with a stronger magnet? Which forces would change (in type or in magnitude)? Which forces would remain the same?

The diagram would remain the same

I. Applying Newton's laws to interacting objects: constant speed

Three identical bricks are pushed across a table at *constant speed* as shown. The hand pushes horizontally. (*Note:* There is friction between the bricks and the table.)

Call the stack of two bricks system A and the single brick system B.



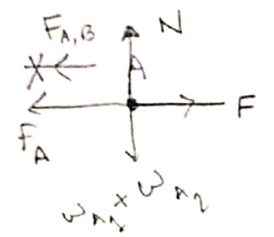
→ Net force of system A and B is zero

A. Compare the *net force* (magnitude and direction) on system A to that on system B. Explain how you arrived at your comparison.

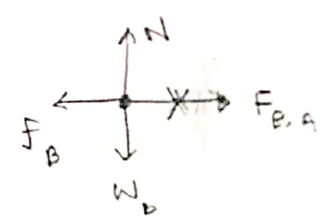
System A and system B both are moving at constant speed so their acceleration is 0

B. Draw separate free-body diagrams for system A and system B. Label each of the forces in your diagrams by identifying: the type of force, the object on which the force is exerted, and the object exerting the force.

Free-body diagram for system A



Free-body diagram for system B



C. Is the magnitude of the force exerted on system A by system B *greater than, less than*, or *equal to* the magnitude of the force exerted on system B by system A? Explain.

Equal, but opposite direction, Newton's third law (Action-Reaction Pair Forces)

Would your answer change if the hand were pushing system B to the left instead of pushing system A to the right? If so, how? If not, why not?

No, third law holds the same for Action-Reaction Pair

$\vec{F}_{AB} = -\vec{F}_{BA}$ & $|F_{AB}| = |F_{BA}|$

D. Identify all the *Newton's third law (action-reaction)* force pairs in your diagrams by placing one or more small "X" symbols through each member of the pair (*i.e.*, mark each member of the first pair as —x→, each member of the second pair as —xx→, *etc.*).

What criteria did you use to identify the force pair(s)?

Two forces of same type is being exerted on two different objects

Is your answer to part C consistent with your identification of Newton's third law (or action-reaction) force pairs? If so, explain how it is consistent. If not, resolve the inconsistency.

① Two forces are $\boxed{\text{not}}$ in the same body diagram e.g. on the same object

② Both forces have same magnitude with opposite direction

E. Rank the magnitudes of all the *horizontal* forces that you identified on your free-body diagrams in part B. (*Hint:* Recall that the bricks are pushed so that they move at constant speed.)

System A: $F - F_{A,B} - f = 0 \Rightarrow F = F_{A,B} + f_A$

System B: $-f + F_{B,A} = 0 \Rightarrow F_{B,A} = f_B$

Force by Hand (F)
f_A
$\left.\begin{matrix} F_{B,A} \\ F_{A,B} \\ f_B \end{matrix}\right\}$ same

Did you apply Newton's second law in comparing the magnitudes of the horizontal forces? If so, how?

Determied the net force $\Sigma F_x = 0$ For both system A, B

Did you apply Newton's third law in comparing the magnitudes of the horizontal forces? If so, how?

$F_{B,A} = F_{A,B}$ [Each force in Action-Reaction pair has equal ~~Mgt~~ ~~Mgnt~~ Magnitude]

What information besides Newton's laws did you need to apply in comparing the magnitudes of the horizontal forces? also net vertical acceleration

frictional force, $f = \mu N$, where N depends on individual masses of system A & B. $M_A \neq M_B$ ($M_A = 2M_B$)

F. Suppose the mass of each brick is 2.5 kg, the coefficient of kinetic friction between the bricks and the table is 0.2, and the bricks are moving at a constant speed of 0.50 m/s.

Determine the magnitude of each of the forces that you drew on your free-body diagrams in part B. (Use the approximation $g = 10\ \text{m/s}^2$.)

$f_B = \mu N = 0.2(2.5 \times 10) = \mu mg = 5N$

$N = mg = 25\ N$

$W = mg = 25N$

and $F_{B,A} = f_B = 5N$

$\therefore F_{B,A} = F_{A,B} = 5\ N$

$f_A = 2 \times 2.5 \times 10 \times 0.2$
$= 10\ N$

$N = W = 50\ N$

$\therefore F = F_{A,B} + f_A$
$= 5 + 10 = 15\ N$

Would your answers change if the bricks were moving half as fast? If so, how? If not, why not?

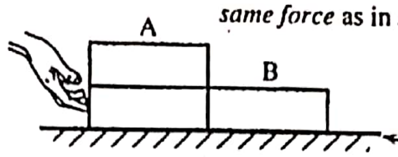
No, because net force is zero, and there is no accelration

➪ Discuss your answers to section I with a tutorial instructor before continuing.

II. Applying Newton's laws to interacting objects: varying speed

Suppose the bricks were pushed by the hand with the same force as in section I; however, the coefficient of kinetic friction between the bricks and the table is *less than* that in section I.

$f_A' < f_A$



Hand pushes with *same force* as in section I

Coefficient of friction less than in section I

A. Describe the motions of systems A and B. How does the motion compare to that in part I?

Net force is no longer zero. as f_A' and f_B' are less than f_A & f_B. Acceleration will lead to a change in speed.

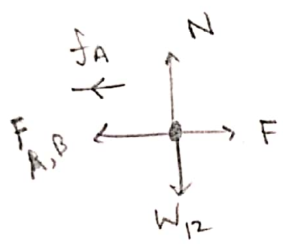
B. Compare the *net force* (magnitude and direction) on system A to that on system B. Explain.

$\Sigma F_{x,A} = F - f_A' - F_{A,B}$ (on right)
(System A)

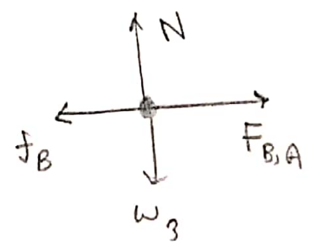
$\Sigma F_{x,B} = F_{B,A} - f_B'$ [on right because Net force in both system is no longer zero]

C. Draw and label separate free-body diagrams for systems A and B.

Free-body diagram for system A



Free-body diagram for system B



D. Consider the following discussion between two students.

Student 1: *"System A and system B are pushed by the same force as before, so they will have the same motion as in section I."*

Student 2: *"I disagree. I think that they are speeding up since friction is less. So now system A is pushing on system B with a greater force than system B is pushing on system A."*

With which student, if either, do you agree? Explain your reasoning.

student 2 is right. Because with less frictional force,

E. Rank the magnitudes of all the *horizontal* forces that appear on your free-body diagrams in part C. Explain your reasoning. (Describe explicitly how you used Newton's second and third laws to compare the magnitudes of the forces.)

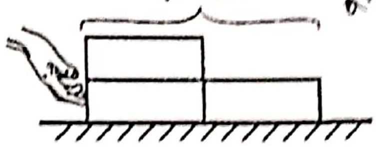
Is it possible to *completely* rank the horizontal forces in this case?

No, not possible.

C = System of more than two particles, Net internal force is zero, but internal forces are NOT zero.

III. Applying Newton's laws to a system of interacting objects

Let C represent the system consisting of all three bricks. The motion of the bricks is the same as in section II.



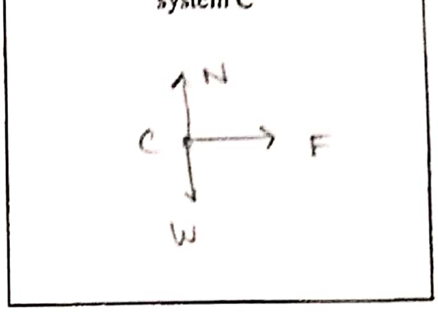
A. Compare the magnitude of the *net force* on system C to the magnitudes of the *net forces* on systems A and B. Explain.

$\Sigma F^{int} = 0 \quad \Sigma F^{ext} = F - f_c$

B. Draw and label a free-body diagram for system C.

Compare the forces that appear on your free-body diagram for system C to those that appear on your diagrams for systems A and B in section II.

For each of the forces that appear on your diagram for system C, list the corresponding force (or forces) on your diagrams for systems A and B.



Free-body diagram for system C

Are there any forces on your diagrams for systems A and B that you did not list? If so, what characteristic do these forces have in common that none of the others share?

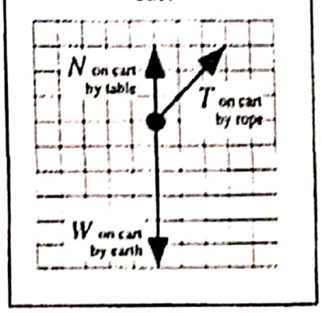
Why is it not necessary to consider these forces in determining the motion of system C?

According to newton's third law, they have equal but opposite (magnitude and direction) So, they cancels out, giving net zero internal force

Note that such forces are sometimes called *internal forces*, to be distinguished from *external forces*.

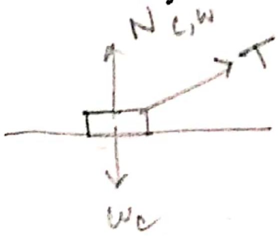
IV. Interpreting free-body diagrams

At right is a free-body diagram for a cart. All forces have been drawn to scale.



Free-body diagram for cart

In the space below, sketch the cart, rope, *etc.*, as they would appear in the laboratory.



What can you say about the motion of the cart based on the free-body diagram? For example, could the cart be: moving to the left? moving to the right? stationary? Explain whether each case is possible and, if so, describe the motion of the cart.

The cart moves to the right. Because T on cart by the rope has a horizontal force component along the right

$\Sigma F_x = T_x$

$\Sigma F_y = W - N_{c,T}$

I. Relating work and changes in kinetic energy

A. A block is moving to the left on a frictionless, horizontal table. A hand exerts a constant horizontal force on the block.

1. Suppose that the work done on the block by the hand is positive. Draw arrows at right to show the direction of the displacement of the block and the direction of the force by the hand.

Displacement of block	Force on block by hand
← d ← F	← F

Explain how you chose the direction of the force on the block by the hand.

$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$ $\theta = 0°$ is restricted as the block moving $\boxed{\text{along}}$ the table

Is the block *speeding up, slowing down*, or *moving with constant speed?* Explain.

Due to the constant force, the block speeds up and move along the plane

2. Suppose that the block again moves to the left but now the work done by the hand is negative. In the space at right, draw arrows to represent the direction of the displacement of the block and the direction of the force by the hand.

Displacement of block	Force on block by hand
← d	→ F

Explain how you chose the direction of the force on the block by the hand.

$W = \vec{F} \cdot \vec{d}$; the displacement of the block is the ~~posi~~ opposite to the direction of Force; $\theta = 180°$

Is the block *speeding up, slowing down*, or *moving with constant speed?* Explain.

slowing down bc of negative acceleration (which is retarding the motion)

B. In a separate experiment, two hands push horizontally on the block. Hand 1 does positive work and hand 2 does negative work ($W_{H_1} > 0$; $W_{H_2} < 0$).

For each of the following cases, draw a free-body diagram for the block that shows all the horizontal and vertical forces exerted on the block. Tell whether the sum, $W_{H_1} + W_{H_2}$, is *positive, negative*, or *zero*.

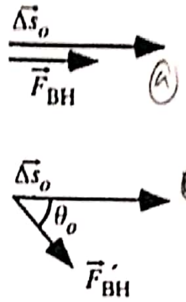
$W_{H_1} > 0$
$W_{H_2} < 0$

W_{H2} → neg
W_{H1} → positive

(1) The block moves to the right and speeds up. $F_1 > F_2$ → d N $F_{net} > 0$ $W_1 + W_2 > 0$ F_2 W F_1	(2) The block moves to the left and speeds up. N ← d ← $F_{net} > 0$ F_1 W F_2 $W_1 + W_2 > 0$
(3) The block moves to the right and slows down. $F_2 > F_1$ N → d ← F $F_{net} < 0$ $W_1 + W_2 < 0$ F_2 W F_1	The block moves to the left with constant speed. N ← $F_{net} = 0$ $W_1 + W_2 = 0$ F_1 W F_2

C. Shown at right is a side-view diagram of the displacement, $\Delta\vec{s}_o$, that a block undergoes on a table when pushed by a hand. The horizontal force on the block by the hand, $\vec{F}_{BH}$, is also shown.

Side view



W = Fs

W Fs cos θ

1. Suppose instead that a hand pushes with a force of the same magnitude, F_{BH}, as before but now at an angle below the horizontal, as shown in the side-view diagram at right. Is the work done by the new force *greater than, less than,* or *equal to* the work done by the original force?

Less than previous one $0 < \theta_0 < 90$

$1 < \cos\theta_0 < 0$

Explain how you used the definition of work to obtain your answer.

Work is displacement scaled by the projection of force along the displacement at (a) s and F are parallel so projection is max. at (b) projection $F\cos\theta$ is smaller than F

2. Suppose instead that a hand pushes with a force of the same magnitude, F_{BH}, as before but instead does zero work. In the space at right, draw an arrow to represent the direction of the force by the hand in this case.

Displacement of block	Force on block by hand
→	↑ FBH

D. Recall the motion of the block in part B. For each force that you identified, state whether that force did *positive work, negative work,* or *zero work* on the block. Explain.

Weight (Force on earth on Block) → zero work

F_2 → negative work | F_1 → positive work

Normal force of table on the block → zero Block

The sum of the works done by *all* forces exerted on an object is called the *net work*, W_{net}. Is the *net work* done on the block in part B *positive, negative,* or *zero?* Base your answer on your free-body diagram and your knowledge of the block's motion.

→ Net force and displacement in same direction (1, 2) → work positive

→ Net force and displacent anti-parallel → work negative

→ Net force zero and displacement in an arbitrary direction → zero work

Is the net work done on the block *greater than, less than,* or *equal to* the work done by the net force on the block? Explain your reasoning.

Net work done on the block is equal to the work done by the net force on the block

E. Generalize from your answers to parts A–D to describe how the speed of an object changes if the net work done on the object is (1) positive, (2) negative, or (3) zero.

(1) positive work → speed increase | (3) zero work - constant speed

(2) negative work → speed decrease

Discuss how your results are consistent with the work-energy theorem discussed in class ($W_{net} = \Delta K = K_{final} - K_{initial}$).

(1) $+W = K_f - K_i$ ∴ $K_f > K_i$ → speed increase

(2) $(-)W = K_f - K_i$ ⇒ $K_i > K_f$ (speed decreasing)

(3) $0 = K_f - K_i$ ⇒ $K_i = K_f$ (constant speed)

⇨ Check your answers with a tutorial instructor before proceeding.

II. Applying the work-energy theorem

Base your answers below on the work-energy theorem and your results from section I.

A. A glider, glider A, is pulled by a string across a level, frictionless table. The string exerts a constant horizontal force.

1. How does the net work done on the glider in moving through a distance $2d$ compare to the net work done on the glider in moving through a distance d?

twice as much as work to move through a distance d

Assume that the glider starts from rest. Find the ratio of the speed after the glider has moved a distance $2d$ to the speed of the glider after moving a distance d. Explain.

$W_d = Fd$; $W_{2d} = 2Fd$

For d, $W_d = K_f - K_i = K_f$ $\Rightarrow mad = \frac{1}{2}mv_d^2$ $\Rightarrow V_d = \sqrt{2ad}$

For 2d, $W_{2d} = K_f - K_i$ $\Rightarrow ma.2d = \frac{1}{2}mv_{2d}^2$ $\Rightarrow V_{2d} = 2\sqrt{ad}$ $\therefore \frac{V_{2d}}{V_d} = \sqrt{2}$

2. A string pulls a second glider, glider B, across a frictionless table. The string exerts the same force on glider B as did the string on glider A. The mass of glider B is greater than that of glider A ($m_B > m_A$). Both gliders start from rest.

After each glider has been pulled a distance d, is the kinetic energy of glider A *greater than, less than,* or *equal to* the kinetic energy of glider B? Explain.

Glider A: ~~$\Rightarrow W_A = K_f$~~

$W_A = \Delta K_A$ ~~$Fd = \frac{1}{2}m_A v_{fA}^2$~~

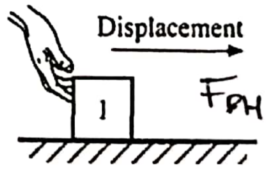
$\therefore W_A = Fd = \Delta K_A$

Glider B: $W_B = \Delta K_B \Rightarrow Fd = \Delta K_B$

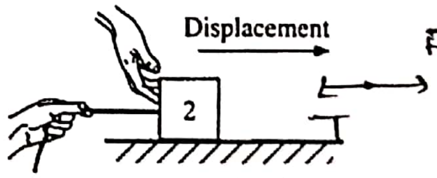
~~$\therefore W_B = Fd = \frac{1}{2}m_B V_B^2$~~

work done on both A & B equal so, $\Delta K_A = \Delta K_B$

B. The diagrams at right show two identical gliders that move to the right *without friction*. The hands exert identical, horizontal forces on the gliders. The second glider experiences an additional, smaller force from a massless string held as shown.



Suppose the gliders move through identical displacements.



Is the work done on glider 1 by the hand *greater than, less than,* or *equal to* the work done on glider 2 by the hand? Explain.

Glider 1:

$F_{Net} = F_{BH}$

$W_1 = F_{BH} \cdot d$

Glider 2:

$F_{net} = F_{BH} - T$

$W_2 = (F_{BH} - T)d = F_{BH}d - Td$

$\Rightarrow W_2 = W_1 - Td$

$\Rightarrow W_1 = W_2 + Td$

$\therefore W_1 > W_2$

So, work done by glider 1 is $\boxed{\text{greater}}$ than glider 2

Is the change in kinetic energy of glider 1 *greater than, less than,* or *equal to* the change in kinetic energy of glider 2? Base your answer on your knowledge of the net work done on each object.

$W_1 = \Delta K_1$

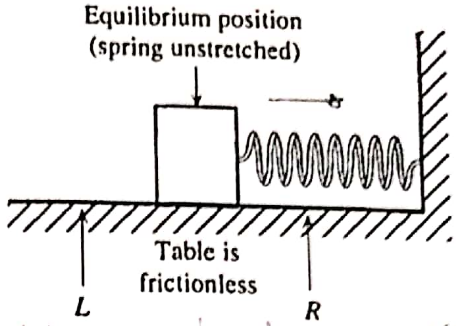
and, $W_2 = \Delta K_2$

as $W_1 > W_2$

$\Delta K_1 > \Delta K_2$

So, kinetice energy of glider 1 is greater than that of glider 2

C. A block on a frictionless table is connected to a spring as shown. The spring is initially unstretched. The block is displaced to the *right* of point *R* and is then released.



1. When the block passes point *R*, is the spring compressed or stretched?

compressed

Does your answer depend on the direction in which the block is moving? Explain.

YES. When the block moves to the right, the spring is compressed. when it moves to the left the spring is stretched

2. In the space provided, draw a free-body diagram for the block at the instant the block passes point *R* moving to the left. Draw arrows to represent the directions of the velocity, the acceleration, and the net force on the block, all at that instant. If any quantity is zero, state so explicitly.

$W = -\frac{1}{2}kx^2 + \frac{1}{2}kx_0^2$

$x < x_0$

$\therefore W = +ve$

x = point R

x_0 = point of release

~~at equilibrium position~~

Free-body diagram at point R

N, F_s

At R, spring is compressed.

$F_s = -k(x - x_0)$

Direction of velocity at point *R*	Direction of net force at point *R*	Direction of acceleration at point *R*
←	←	←

Is the net work done on the block from the point of release to point *R*, *positive*, *negative*, or *zero*? Explain.

x = point R

$\therefore$ so $F_s = -kx$

So, $F_s = -k(x - x_0)$ The force is acting on the left and the block is also moving to the left so, work done is $\boxed{\text{positive}}$

3. At some instant, the block passes point *L* moving to the left. Draw a free-body diagram for the block at that instant. Also, draw arrows to represent the directions of the velocity, the net force, and the acceleration, all at that instant. If any quantity is zero, state so explicitly.

Free-body diagram at point *L*

N

F_s

N

Direction of velocity at point *L*	Direction of net force at point *L*	Direction of acceleration at point *L*

During a small displacement of the block from the right of point *L* to the left of point *L*:

- Is the net work done on the block *positive*, *negative*, or *zero*? Explain.

- Does the speed of the block *increase*, *decrease*, or *remain the same*? Explain how your answer is consistent with the work-energy theorem.