Lecture 8

Conductors

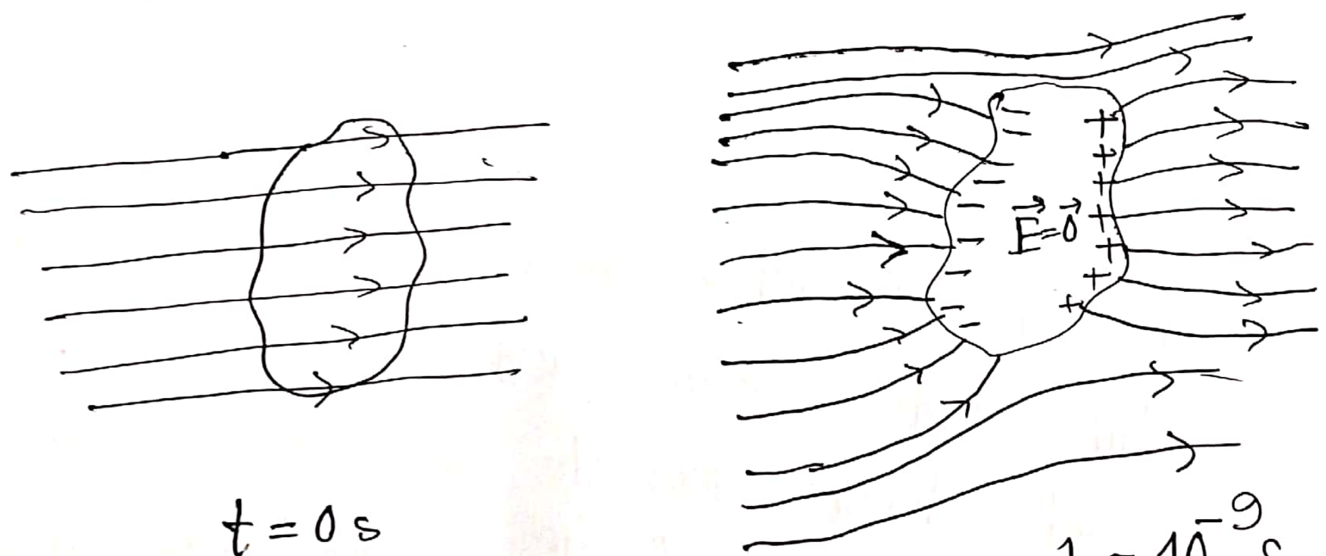
In insulators, each electron is attatched to a particular atom and can't move. An electrical conductor is a solid that contains many free electrons, perhaps one or two per atom. A perfect conductor is a material with infinite amount of free electrons. Although there are no perfect conductors in reality, some material are very close to being perfect. For example, iron and copper carries close to 8.5×10^{28} number of electrons per unit volume, which is huge. So, for all practical purpoeses, there are kind of an infinite amount of free moving electrons.

Now, conductors have some peculiar (!) properties. Let's see.

(i) The electric field inside a conductor is zero. It might seem a strong statement, and to be honest, its not totally true. Electric field inside a conductor is zero in electrostatic equilibrium. Electrostatic equilibrium is achieved when ~~non~~ free charges are moving, every charges are at its position. So, inside conductor, the charges assemble themselves in such a manner, such that no electric field is present to further move the charges. If there are forces other than electrostatic force, ~~this force just ...~~ then there will be some field present inside the conductor, such that it can counterbalance that other forces and electrostatic equilibrium is achieved.

Even in the idealized situation, there are electric fields present inside the conductors in very very small scales. ~~However~~ For example, there is huge electric field very close to a nucleus. However, the overall effect of the nuclei and electrons are such that there is negligibly small electric fields in a larger scale. As long as there is some electric field present in the locations of mobile electrons, they will move. Finally they will orient themselves in such a manner that the electric field becomes zero. in larger scales.

(ii) <u>Conductor in external electric field</u>: Let's consider an ideal conductor placed in an external electric field, say a uniform one. There are mobile charges (electrons) inside the conductor. They electrons will tend to move in opposite direction of the external electric field, creating a ~~negal~~ net positive charge in the other side. As long as the external electric field prevails, thes charges will start piling up in two opposite sides, creating stronger and stronger electric fields as time goes on. After some finite time (characteristically few nano-seconds), the external electric field is completely nullified by the ~~ele~~ opposing electric field created (or oriented) by the charges inside the conductor. The net effect is that, the electric field inside a conductor becomes zero.



$t = 0\,s$ $\qquad$ $t = 10^{-9}\,s$

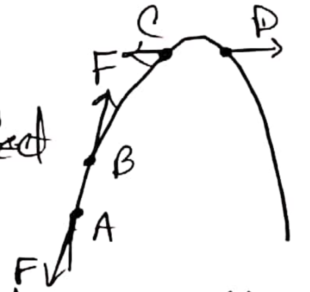
(iii) Charge density inside a conductor under electrostatic equilibrium:

Under the electrostatic equilibrium, there are no electric field inside the conductor. Now, from Gauss's law it follows that,

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Now, since $\vec{E}$ is dentically zero everywhere inside the conductor, $\rho = 0$ inside the conductor. Of course when we say $\rho = 0$, we mean macroscopically 0. Obviously ~~close to a proton~~ in the position of a proton, the charge density is not zero.

(iv) Any excess charge reside on the surface: As the electric field inside is zero, any excess charge must move to the outer surface. Because, the object is a conductor, the excess charges will repel each other, and will try to move farthest place from one another. On the other hand, they will redistribute themselves as long as no electric fields remain inside the conductor. So, any net charge goes to the outer surface of the conductor. Now, if the conductor is spherical, the charges will be distributed uniformly in the outer surface. If the conductor have sharp ~~o~~ curvature, any the charges will pile up there more. Why? the Lets consider a curved part of a ~~negatively~~ =0



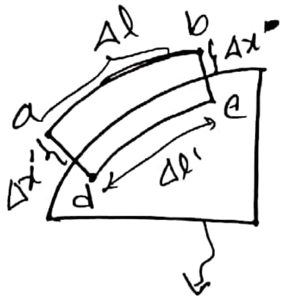
charged conductor. They will try to redistribute themselves due to the effect of their repulsive forces. Consider charge A and B and the force between them. Since the charges can only move along the surface, the outward (normally outward) component of the electric field is irrelevent here. Now you see, there is a very large component of the electric field tangential to the surface, that will try to move the charges further along the surface. Where, charges C and D are near the curved region, where the force is mostly normal to the conductor surface. So, they don't really feel more force to move along the surface. As a result they can pile up in the most curved region. Once the electrostatic equilibrium is reached, then the remain there.

(v) <u>A conductor is an equipotential</u>: Consider any two points a and b inside a conductor. The potential difference is defined as,

$$V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{s} = 0 \quad \text{since } E = 0$$

$\therefore V(\vec{a}) = V(\vec{b})$

So, the potential is a constant inside a conductor. Consequently, the surface of a conductor is an equipotential surface. Because, at electrostatic equilibrium, no charges will be moving along the surface, meaning the surface should be an equipotential.



Part of the conductor

Let's do some mathematics. Consider a loop abcda (closed loop), where some part is inside the conductor material. Let's calculate the line integral of the electric field along this closed path. We choose the loop such that the ab and cd part are parallel to the surface and bc and ad part are perpendicular to the surface at intersecting points. Say, the electric field is in any random direction. So, it will have tangential and normal component.

$$\therefore \vec{E} = \vec{E}_t + \vec{E}_n$$

$$\therefore \oint \vec{E} \cdot d\vec{s} = \int_a^b \vec{E}_t \cdot d\vec{s} + \int_b^c \vec{E}_n \cdot d\vec{s} + \int_c^d \vec{E}_t \cdot d\vec{s} + \int_d^a \vec{E}_n \cdot d\vec{s}$$

$$= \int_a^b E_t \, ds + \int_b^c E_n \, ds + \int_c^d E_t \, ds + \int_d^a E_n \, ds$$

Now, inside the conductor, $\vec{E} = \vec{0}$. So, $E_t = 0$ for going from c to d.

$$\therefore \oint \vec{E} \cdot d\vec{s} = \int_a^b E_t \, dS + \oint E_n \Delta x - E_n \Delta x'$$

Now, if the loop is placed just outside the surface of the conductor, ~~$\Delta x, \Delta x' \approx 0$~~ $\Delta x \cong \Delta x' \approx 0$.

$$\therefore \oint \vec{E} \cdot d\vec{s} = \int_a^b E_t \, dS$$

Now, the line integral along a closed loop of the electrostatic field, being a conservative one, is zero.

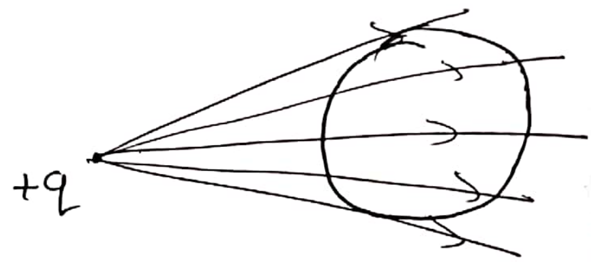
$$\therefore \int_a^b E_t \, dS = 0$$

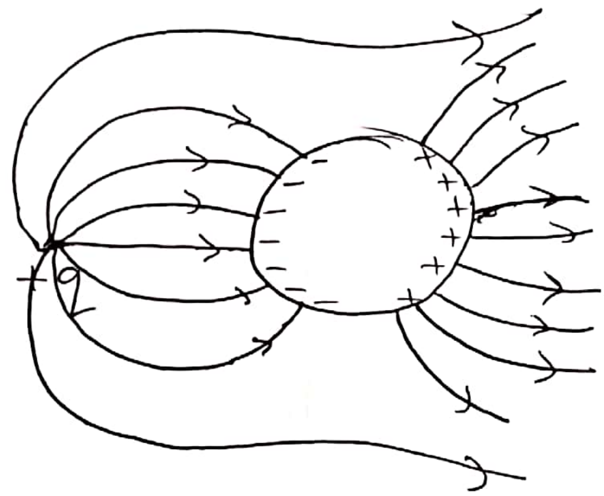
So, $E_t = 0$, since this integral is valid for any interval a and b.

So, the electric field must be normal to the surface of a conductor everywhere. It ~~dos~~ also gurantees that the surface of a conductor must be an equipotential surface, since electric fields are perpendicular to an equipotential surface everywhere.

Induced charges in a conductor

Let's say, we are placing a charge +q
close to a conductor. The electric field from
the ~~conductor~~ charge will penetrate the conductor.
But since there are mobile electrons inside,
they will ~~mo~~ start moving due to the field.
After some small amount of time, they will
redistribute in such a manner that the electric
field due to this induced charges excatly cancel
the field of the point charge, and electrostatic
equilibrium is achieved.





Exact value of induced charge

Consider a random shaped
conductor with a cavity
inside.