The seven crystal system and fourteen Bravais lattice

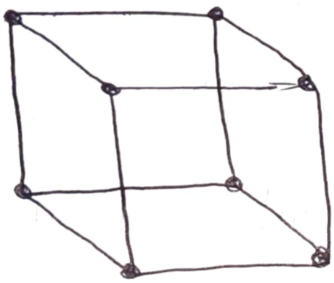
We have seen that, in 2D there are five Bravais lattices that we can speak about. In 3D, we can also characterize the lattices according to the underlying point symmetry groups. All possible 3D crystallographic point groups can be divided into a total of 7 crystal systems, based on the presence of a specific symmetry element or combination of them present in the point group. We will talk about the specific distribution of point groups later. Lets first introduce the seven crystal system. For each of the crystal system, the lattice translation symmetry allows one of the following lattice point centerings -

(i) Primitive (P) : All the lattice points are at the vertex of the crystal system structure.

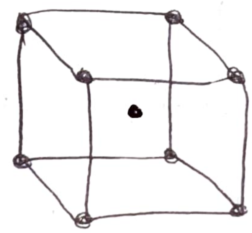
(ii) Body centered (I): It has one additional lattice point at the center of the cell.

(iii) <u>Face centered (F)</u>: It has additional lattice points at each of the faces of the cell.

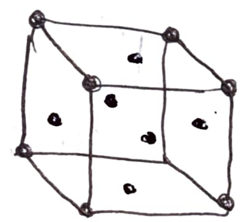
(iv) <u>Base centered (A, B or C)</u>: It has additional lattice points at the center of each pair of opposite cell faces.



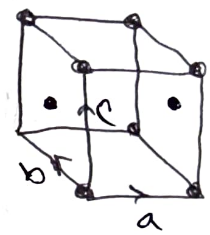
P



I



F

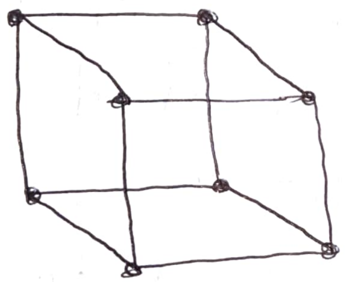


A

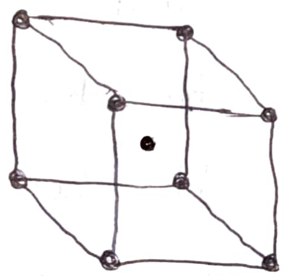
Introduction to different centerings of then seven crystal systems makes the total number of distinct Bravais lattice to 14. It seems that the total number should be much higher. However, it turns out that, while some of the combinations are not unique, some are not possible due to the point symmetry requirement of the crystal system. This reduces the total number to 14.

We list below the seven crystal system and the associated Bravais lattices belonging to them.

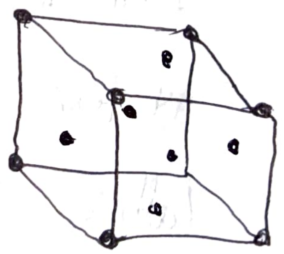
(i) Cubic (3) : The cubic crystal system contains those Bravais lattices whose point group is just the point groups of a cube. There are three cubic Bravais lattices with non-equivalent space group, all having the cubic point group. They are shown below.



Simple Cubic (P)



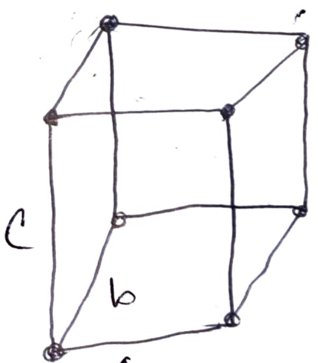
Body centered cubic (I)



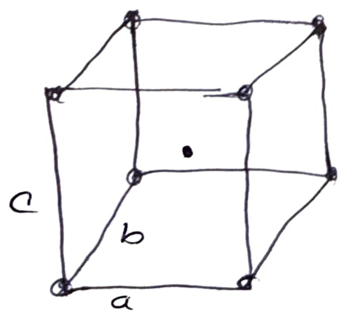
Face centered cubic (F)

The characteristic symmetry of the cubic system is the four 3-fold rotation axis (along the body diagonals).

(ii) Tetragonal (2): One can reduce the symmetry of a cube by pulling on two opposite faces to stretch it into a rectangular prism with a square base, where the height of the prism is not equal to the length of the bases. By such manner, we can construct simple tetragonal lattice, which is characterized by three mutually perpendicular primitive vectors, only two of which are equal in length. The third axis is called the c-axis (the unequal length). We can similarly create body centered tetragonal lattice. We could also create face centered tetragonal lattice. However this is equivalent to the body centered tetragonal lattice.



Simple



Body centered

The characteristic symmetry element is one four fold rotation axis (about an axis passing through the midpoint of the bases)

How body centered tetragonal is the same as face centered tetragonal?

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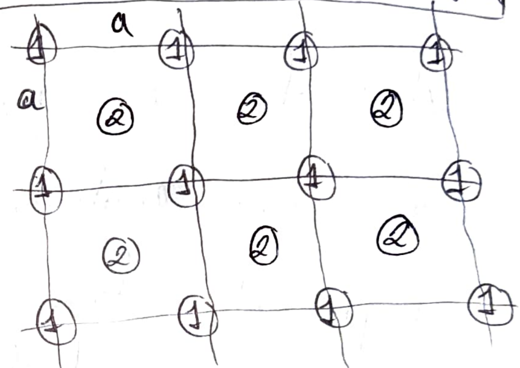
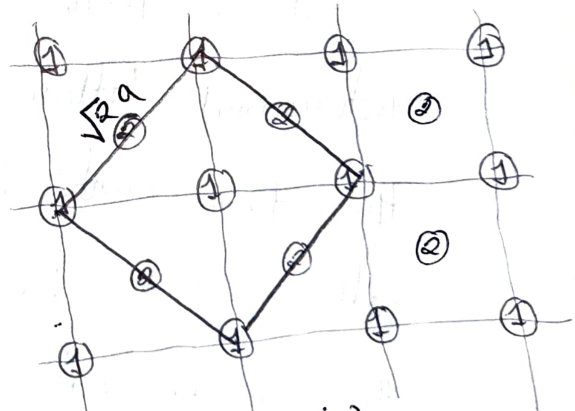
Base centered tetragonal – Simple tetragonal



ii (a)

Consider the lattice, viewed from the c-axis of a body centered tetragonal lattice. Points ① lie at the corners of the lattice, ~~where~~ base, where point ② is at a height of $c/2$ from the base of the plane made by ① points.

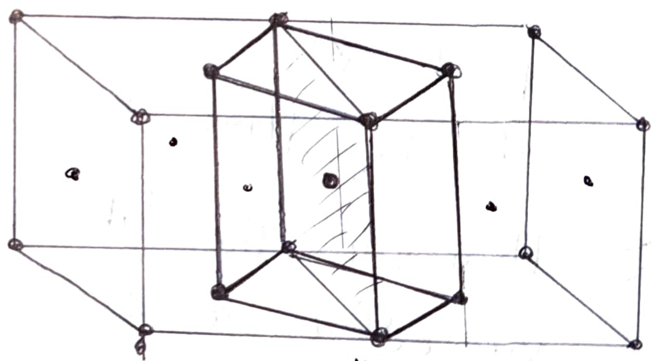


ii (b)

However, the lattice can be viewed as a square array of length $a' = \sqrt{2}a$, as shown in the figure. Since points ① are at the base and ② are at the height of $c/2$, this gives rise to the face centered tetragonal lattice.

Note: If you started with a body centered cubic lattice (in fig. 1) such that $c = a$, then in the second figure $a' = b' = \sqrt{2}a$ and already we had $c = a$. So, the resulting figure would be a face centered tetragonal lattice. Not a fcc

And, for a general c value, the first figure shows a body centered tetragonal while the second figure represents face centered tetragonal, however both being equivalent. Here is a figure that clearly shows that you can find a body centered tetragonal lattice from the face centered tetragonal lattice (both has the same symmetry).



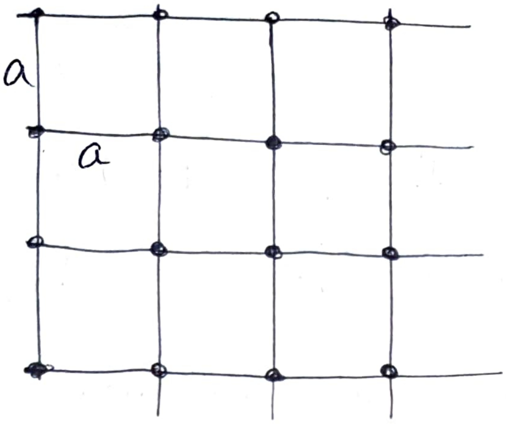
ii (c)

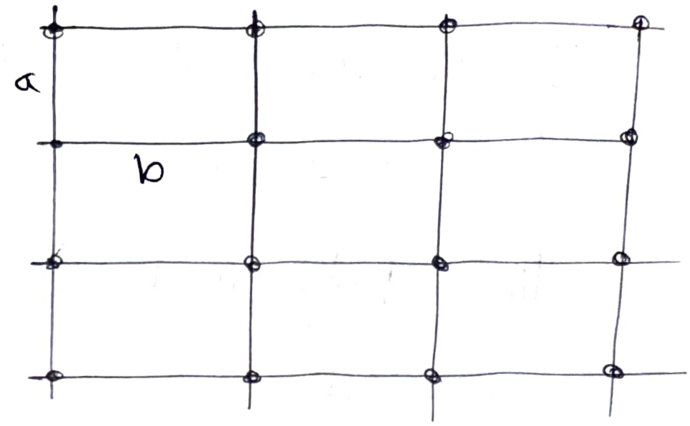
You can also find the body centered tetragonal lattice from fcc lattice in the similar way. However, the fcc lattice has higher symmetries different from the bc tetragonal one and hence we consider fcc a separate lattice, and bc and fc tetragonal to be the same.

(iii)

Orthorhombic (4): Continuing to less symmetric deformation of the cubic lattice one can reduce the symmetry by deforming the square base of the tetragonal lattice into rectangle. This produces an ~~latti~~ object with three mutually perpendicular sides with unequal lengths. The orthorhombic group is symmetry group of such objects. Here, $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90°$. By stretching the tetragonal lattice along an "a" axis conveniently one can produce the orthorhombic lattice. The characteristic symmetry is three mutually orthogonal two-fold rotation axis. Since the symmetry is two-fold, base-centering is possible here, along with body centering and face centering.

The following figure shows a way to reduce the symmetry of a simple tetragonal lattice by stretching it in the b direction. The view is along the c axis, where for the simple





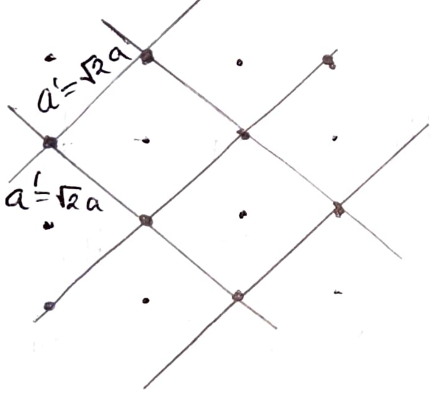
Simple tetragonal

iii (a)

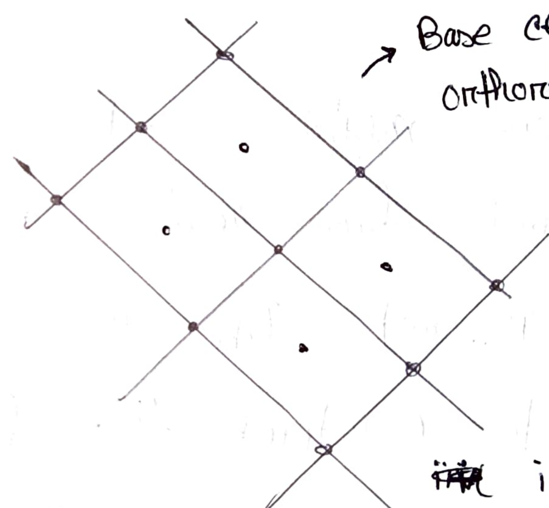
Simple orthorhombic

iii (b)

The same net of simple tetragonal can be considered as a centered square array, as shown in the following figure. Then, stretching along ~~the~~ one side



iii (c)



iii (d)

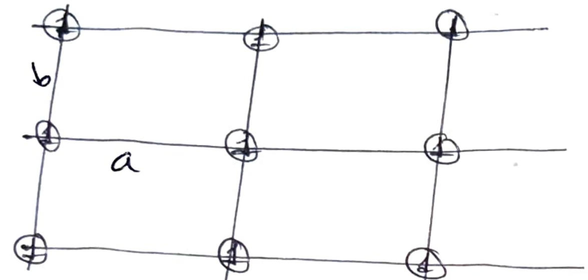
of the ~~sq~~ centered square array gives you the centered rectangular array. Stacked directly above one another gives the base-centered orthorhombic Bravais lattice. You can think of the operation as, stretching the simple tetragonal lattice along it's square diagonal produces base centered orthorhombic lattice.

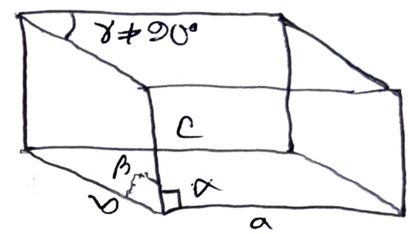
meaning, you just increased the distance of the points along the diagonal of the simple tetragonal lattice.

It is easy to find the body centered and face centered orthorhombic lattice. Just stretch the body centered tetragonal lattice in fig. ii(a) along either of the parallel lines direction, which gives you body centered ~~tetragonal~~ orthorhombic. Similarly, by stretching along either of the parallel lines in fig. ii(b) gives you ~~b~~ face centered ~~te~~ orthorhombic Bravais lattice.

(iv) Monoclinic (2):

One can reduce the orthorhombic symmetry by distorting the rectangular faces perpendicular to the c-axis into general parallelogram. The symmetry group of such object is the monoclinic group. Distorting the simple orthorhombic lattice in such a way one gets the simple monoclinic Bravais lattice. The lattice has $a \neq b \neq c$ and ~~$\alpha = \beta = 90°$~~ $\alpha = \beta = 90° \neq \gamma$. The characteristic symmetry is one two fold rotation axis (along the c-axis).



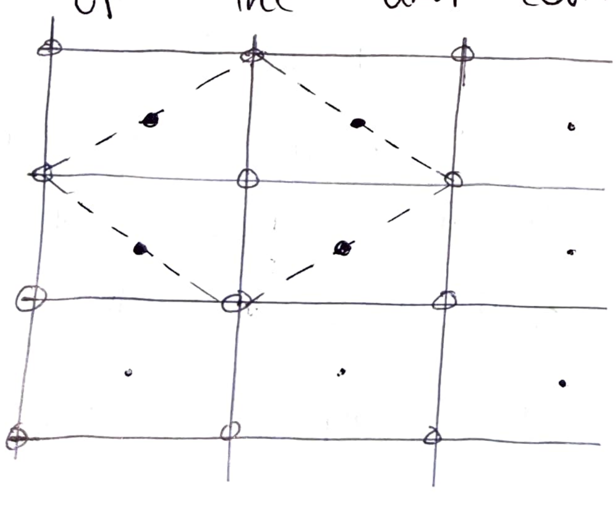


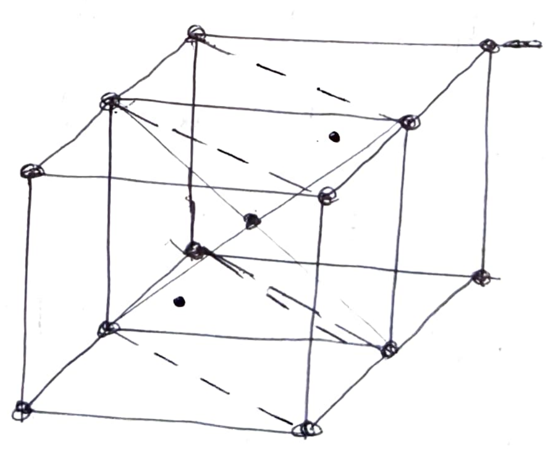
iv(a)

iv(b)

$a \neq b \neq c$; c is perpendicular to the ab plane.

We can have body centered monoclinic lattice. However, the face centered and base centered monoclinic lattices can be thought of as body centered monoclinic lattice with appropriate choice of the unit cell.





iv(c)

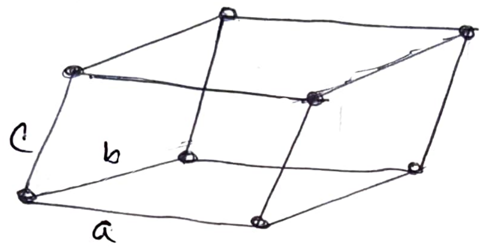
iv(d)

In fig. iv(c) we show that the body centered monoclinic ~~cell~~ lattice can also be considered as the face centered monoclinic lattice. ~~Here both o and • lie in the~~ Here o points lie in the plane of the paper and • points are at a height of 1/2. Stacking this up for solid lines give you a body centered monoclinic lattice, while staking up for the dotted line gives you face centered monoclinic lattice.

In fig. iv(d) we show that base centered monoclinic lattice can again be considered as body centered monoclinic lattice.

Triclinic (1)

The destruction of the cube is totally completed by tilting the c-axis of the monoclinic lattice so that ~~the~~ no angle is 90°. This is the Bravais lattice made out of three primitive vectors with no relationship with each other, and hence has the minimum symmetry. Here, $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma$ and none of them are 90°. The only symmetry element is that, the lattice has a point of inversion (since all Bravais lattice has it's lattice points as inversion centers).



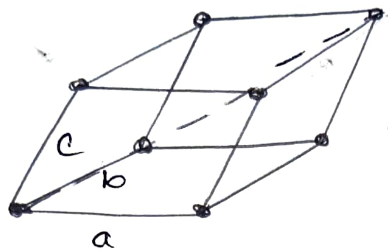
V(a)

To find the left two crystal system and corresponding Bravais lattices, we go back to the original cube and try distorting it another way.

(vi) Trigonal (1)

The trigonal point group describes the symmetry of an object one produces by stretching the cube along a body diagonal. The lattice made by such distortion is called the rhombohedral or trigonal Bravais lattice. It is generated by three primitive vectors of equal length that make equal angle with each other (not 90°).

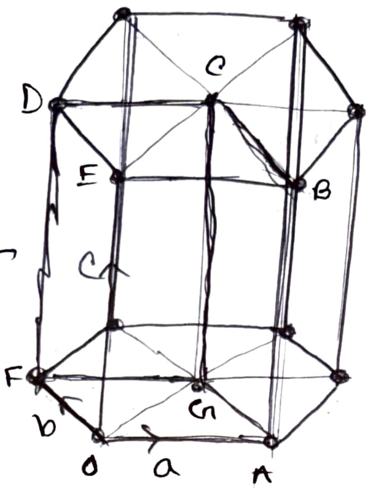
So, here $a = b = c$ and $\alpha = \beta = \gamma \neq 90^\circ$. The characteristic symmetry is one three fold rotation axis (along the body diagonal shown in the figure).

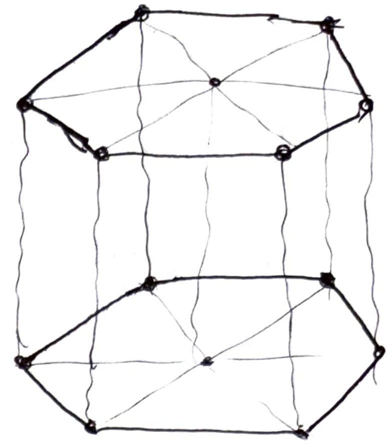


vi(a)

(vii) Hexagonal (1): The last crystal system and corresponding Bravais lattice does not come from cubic system. The hexagonal point group is the symmetry group of a right prism (two identical end faces connected by rectangular side faces) with regular hexagon as the base. The simple hexagonal Bravais lattice is the only Bravais lattice in the hexagonal system. Here, $a = b \neq c$ and $\alpha = \beta = 90^\circ$ and $\gamma = 120^\circ$. The hexagonal crystal system has one six-fold rotation axis (perpendicular to the bases).

Here, the rhombic prism made by OAGF and BCDE base is the unit cell.



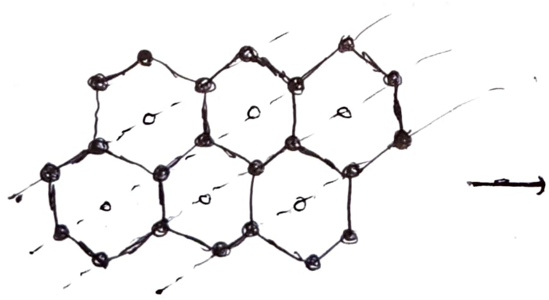


Vii (a)

~~Here, OABCDFO~~

Would it be possible to construct the Hexagonal Bravais lattice without ~~thes~~ those lattice points at the midpoint of the bases? The answer is ~~no now~~ no. This is because, you lose the the traslation symmetry, which should be evident from the two dimensional counterpart, namely Honeycomb lattice. However, once you put

Made by filled circles



a lattice point at the centers (open circles), you immediately get the translation and define the primitive cell that guides the translations.