PHY 110

Newton's Laws for Mechanics

The First Law:

Newton's first law states that:

- If a body is in motion it will continue to move with constant velocity if it is left undisturbed.

Comments:

1. Newton's first law is about isolated systems. But in the real world it is very hard to produce conditions in which the conditions talked about in Newton's first law. Hard but not impossible: Kleppner talks about air tracks and two dimensional smooth plane with layer of evaporating dry ice (cold, heavier than air layer of CO_2). Under such conditions we observe motion in 1 & 2D that confirms Newton's first law.
2. Suppose you, the observer, starts accelerating with respect to the isolated system that is experiencing constant velocity motion. If you were not aware that you yourself were experiencing accelerlation then you would erroneously conclude that the isolated body was accelerating. Thus we see Newton's 1st law also states (implicitly):

   There exists reference frames in which the motion of an isolated body is that of constant velocity.

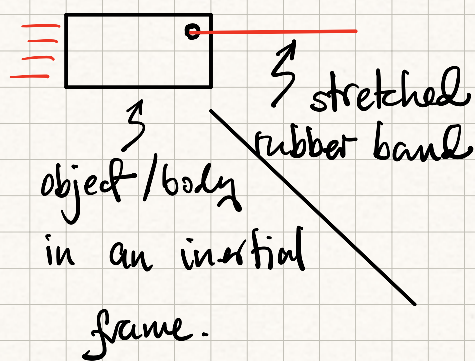
   Such reference frames are called inertial frames.

   Thus we see part of Newton's first law is the <u>definition</u> of inertial frames.
3. Part of Newton's first law is the observation (an observation about the physical universe) that in inertial frames isolated bodies perpetuate in

their motion with constant velocities.

The Second Law

Newton's first law defines what an inertial frame is. Suppose we have a body in an inertial frame. Let us apply a constant push or pull on such a body. We can ensure this by, say, pulling it with a rubber band which is stretched by a specific amount. (Or similarly using a spring...).



<u>We observe:</u> The body accelerates at a constant value as long as the rubber band is stretched by the same amount.

Now suppose we repeat the experiment with another body which is made of the same material as the first one but is larger in size. For this 2nd body we see that for the same stretching of an identical rubber band we get a lower but but still constant acceleration.

Thus we see that the acceleration of the bodies have to do with something intrinsic of the body. We call this quantity mass. For the 1st body we assign to it an arbitrary mass m_1 in some units. For the second body we then assign or define its mass to be

$$m_2 \equiv \frac{a_1}{a_2} m_1$$

In this way we can define masses for any object by measuring its acceleration for the same stretching of an identical rubber band.

$$m_i \equiv \frac{a_1}{a_i} m_1$$

Force:

Now what if we change the amount of pull/push we applied to the 1st body. Suppose we now apply <u>two</u> rubber bands and pull them so that they stretch the <u>same</u> amount as before. It is observed that the body's acceleration is $2a_1$. If we defined the original pull/push as force, $F_1 \equiv K m_1 a_1$, where K is a constant we choose to be 1 by defining m_1 appropriately then we conclude that $2F_1$ produces twice the acceleration. Taking into account the direction of the push/pull we can write

$$\vec{F} = m_1 \vec{a}.$$

By apply force via different kind of devices (rubber bands, magnets, springs etc.) that produces the same acceleration we see that the same force on different bodies produce acceleration that is independent of what kind of force it is. And we see that when we apply the same force on different bodies the product F/a remains constant and it is given by $F = ma$ for a body of mass m. Thus we conclude

$$\vec{F} = m\vec{a}$$

as a general rule.

The Third Law

Newton's 1st & 2nd law does not guarantee that whenever we see acceleration of an object it is necessarily in an inertial frame experiencing force. Why? Because for all that we know the 'force' could be an artefact of the body being in an accelerated and therefore non-inertial reference frame.

Thus for each force acting on a body there must be another body or system of bodies that is the origin of the force. Objects in Newtonian universe interact with each other by applying forces on each other. The precise way this is done is the subject of Newton's third law:

If a body A applies a force on the body B and we denote that force as $\vec{F}_{AB}$, then Newton's third law says that body B also applies a force on body A, which if we denote it by $\vec{F}_{BA}$, satisfies:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Comments:

1. $\vec{F}_{AB}$ and $\vec{F}_{BA}$ have the same magnitude but their directions are opposite.
2. $\vec{F}_{AB}$ and $\vec{F}_{BA}$ are being applied on different bodies.
3. $\vec{F}_{AB}$ & $\vec{F}_{BA}$ are called action-reaction pair.

<u>Units & Dimensions</u>

We have just defined two new concepts: mass & force.
The unit of mass in SI units is Kilogram or kg for short. Remember that the definition of mass referred to an arbitrary object. Historically 1 kg was defined to be the mass of 1 litre of water at the surface of the earth at room temp. But obviously this is not a very precise definition. For sometime the kilogram was the mass of a cylinder of platinum-iridium kept in an archive in France. More recently, scientists have figured out a way to define fundamental units using the fundamental constants in nature.
The modern definition of kilogram involves setting the Planck constant

$$h = 6.626\,070\,15 \times 10^{-34}\ kg \cdot m^2 \cdot s^{-1}$$

meter, m, is defined in terms of how far light travels in one second.

And second, s, is defined as in terms the hyperfine energy level structure of cesium-133 atoms. 1 second is defined to be the time needed for exactly 9 192 631 770 cycles of the hyperfine structure transition of cesium-133.

The unit of force is defined to be newtons. 1 newton of force on an object of 1 kg inertial mass will creat 1 m/s^2 acceleration.

$$[F] = [m][\text{acceleration}] = [m][\text{length}][\text{time}]^{-2}$$
$$= [M][L][T]^{-2}$$

$$[\text{acceleration}] = [L][T]^{-2}$$
$$[\text{velocity}] = [L][T]^{-1}.$$

Some special cases of Newton's law:

1. $\vec{F} = 0$

Then Newton's second law becomes

$$\frac{d^2\vec{x}}{dt^2} = 0$$

This is a second order differential equation so it needs two sets boundary conditions to solve:

$$\vec{x}(t) = \vec{x}_0 + \vec{v}\,t$$

wher $\vec{x}_0$ is the position of the body at $t=0$ & $\vec{v}$ is the constant velocity in accordance with the first law.

2. Constant force: $\vec{F}$ = constant

$$\int_{t_0}^{t_f} \vec{F}\, dt = m \int_{t_0}^{t_f} \frac{d^2\vec{x}}{dt^2}\, dt$$

$$\Rightarrow \quad \vec{F}(t_f - t_0) = m \left.\frac{d\vec{x}}{dt}\right|_{t_0}^{t_f} = m(\vec{v}_f - \vec{v}_0)$$

$$\Rightarrow \quad \vec{F}(t - t_0) = m\left(\frac{d\vec{x}}{dt} - \vec{v}_0\right)$$

$$\Rightarrow \quad \int_{t_0}^{t_f} \vec{F}(t - t_0)\, dt = m \int_{t_0}^{t_f} \frac{d\vec{x}}{dt}\, dt - m\vec{v}_0 \int_{t_0}^{t_f} dt$$

$$\Rightarrow \quad \vec{F}\frac{t^2}{2} = m\vec{x} - m\vec{x}_0 - m\vec{v}_0 t$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\frac{\vec{F}}{m}t^2$$

For gravitational force $\vec{F} = m\vec{g}$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}\,t^2$$

Let us look at it in a particular coordinate system:

Let us choose $\vec{g}$ to be in the negative $\hat{k}$ direction $\quad \vec{g} = -g\hat{k}$

If we choose $\vec{x}_0$ to be coplanar with $\vec{v}$ in the x-z plane then our equation becomes:

$$\left.\begin{aligned} x &= x_0 + v_0 \cos\theta\, t \\ z &= z_0 + v_0 \sin\theta\, t - \frac{1}{2}gt^2 \\ y &= y_0 \end{aligned}\right\} \text{ parabolic motion of projectiles}$$

These are parametric equations for a parabola with t as the parameter.

Eliminate t:

$$t = \left(\frac{x - x_0}{v_0 \cos\theta}\right)$$

$$z - z_0 = \tan\theta\,(x - x_0) - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2\cos^2\theta}$$

$$RHS = \tan\theta\,(X - X_0) - \frac{g}{2V_0^2\cos^2\theta}\,(X - X_0)^2$$

$$= -\frac{g}{2V_0^2\cos^2\theta}\left\{\left[(x-x_0)^2 - \frac{2V_0^2\cos^{\cancel{2}}\theta}{g}\cdot\frac{\sin\theta}{\cancel{\cos\theta}}(x-x_0) + \frac{V_0^4\sin^2\theta\cos^2\theta}{g^2}\right]\right.$$

$$\left. - \frac{V_0^4\sin^2\theta\cos^2\theta}{g^2}\right\}$$

$$= -\frac{g}{2V_0^2\cos^2\theta}\left(x - x_0 - \frac{V_0^2\sin\theta\cos\theta}{g}\right)^2 + \frac{\cancel{g}}{2\cancel{V_0^2}\,\cancel{\cos^2\theta}}\cdot\frac{V_0^{\cancel{4}\,2}\sin^2\theta\,\cancel{\cos^2\theta}}{g^{\cancel{2}}}$$

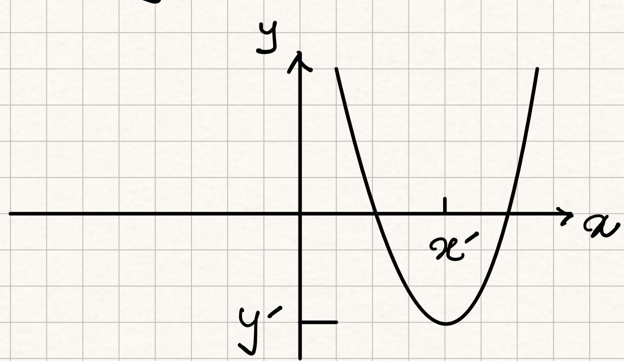
$$= -\frac{g}{2V_0^2\cos^2\theta}\left(x - \left(x_0 + \frac{V_0^2\sin\theta\cos\theta}{g}\right)\right) + \frac{V_0^2\sin^2\theta}{2g}$$

Thus our equation becomes:

$$\boxed{z - \left(z_0 + \frac{V_0^2\sin^2\theta}{2g}\right) = -\frac{g}{2V_0^2\cos^2\theta}\left[x - \left(x_0 + \frac{V_0^2\sin\theta\cos\theta}{g}\right)\right]^2} \qquad (1)$$

Compare this to the equation of a parabola:

$$y - y' = a\,(x - x')^2$$



Equation (1) is an upside down parabola with its vertex at:

$$x \text{ coordinate of vertex} = x_0 + \frac{V_0^2\sin\theta\cos\theta}{g}$$

$$z \text{ coordinate of vertex} = z_0 + \frac{V_0^2\sin^2\theta}{2g}$$