

■ Worksheet: Heap Problem Set

Problem 1 [15 Points] — Water Buckets

You are given several buckets with water levels represented by a min-heap. In each operation, choose the smallest non-zero water level, denoted by x , and remove x units of water from all non-empty buckets. Determine the minimum number of operations required to empty all the buckets. Implement `extract_min()`, `sink()`, and `minimum_operation_find()`. Sample Input: heap = [1, 5, 9, 10, 12] Sample Output: 5

—■ Answer Section:

Problem 2 [15 Points] — Adding Arrays

You are given two arrays A and B (size n). Starting from index 0, for each element in B, add it to the current smallest integer in A. After processing all elements in B, return array A sorted in non-decreasing order. Allowed methods: `insert()`, `extract()` from a min-heap. Sample Input: $n = 3$ A = [1, 3, 5] B = [2, 4, 6] Sample Output: [5, 7, 9]

—■ Answer Section:

Problem 3 [15 Points] — K Largest Elements

You are given an array `arr` of size n and a number k . Return the k largest elements of the array in decreasing order. Use a min-heap of size k . Sample Input: `arr` = [7, 10, 4, 3, 20, 15], $k = 3$ Sample Output: [20, 15, 10]

—■ Answer Section:

Problem 4 [20 Points] — Running Median

You are given a stream of integers. After each insertion, print the median of all elements seen so far. Use two heaps: - A max-heap for the left half - A min-heap for the right half Sample Input: stream = [5, 15, 1, 3] Sample Output: 5, 10, 5, 4

—■ Answer Section:

Problem 5 [20 Points] — Merge K Sorted Lists

You are given k sorted arrays. Merge them into a single sorted array. Use a min-heap to repeatedly extract the smallest element among the heads of each list. Sample Input: A1 = [1, 4, 7] A2 = [2, 5, 8]

A3 = [3, 6, 9] Sample Output: [1, 2, 3, 4, 5, 6, 7, 8, 9]

—■ Answer Section:

Problem 6 [15 Points] — Heap Sort

You are given an unsorted array. Sort it in ascending order using a heap. Implement `build_heap()`, `heapify()`, and use repeated `extract_max()` or `extract_min()`. Sample Input: `arr = [4, 10, 3, 5, 1]`
Sample Output: [1, 3, 4, 5, 10]

—■ Answer Section:

Problem 7 [15 Points] — Rope Connection

You are given n ropes of different lengths. You need to connect them into one rope. The cost of connecting two ropes is equal to the sum of their lengths. Find the minimum cost of connecting all ropes. Use a min-heap. Sample Input: `ropes = [4, 3, 2, 6]` Sample Output: 29

—■ Answer Section:

Problem 8 [20 Points] — Task Scheduler

You are given n tasks with their execution times. At every step, always pick the shortest task (min-heap). After finishing it, print the remaining tasks in heap order. Implement `insert()`, `extract_min()`, `sink()`. Sample Input: `tasks = [7, 3, 5, 1]` Sample Output: Order of execution: 1, 3, 5, 7
Heap states after each extraction: [3, 7, 5] [5, 7] [7] []

—■ Answer Section: