



## Assignment # 8

**Due date: May 14th 2015 (slip underneath my office door please)**

The dynamics equations of a 2-axis planar manipulator in joint space are as following:

$$\boldsymbol{\tau} = \begin{bmatrix} m_2 l_2^2 (\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 c_2 (2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_1 l_2 c_2 \ddot{\theta}_1 + m_2 l_2^2 (\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 + m_2 l_2 g c_{12} \end{bmatrix}$$

Assume that  $m_1 = 1.5 \text{ kg}$ ,  $m_2 = 0.5 \text{ kg}$ ,  $l_1 = 0.3 \text{ m}$ ,  $l_2 = 0.1 \text{ m}$ ,  $g = 9.81 \text{ m/s}^2$

(1) Given the following joint trajectories

$$\boldsymbol{\theta}(t) = \begin{bmatrix} \frac{\pi}{2} \sin\left(\frac{\pi}{8} t\right) \\ \frac{\pi}{4} (t - 2) \end{bmatrix}$$

Compute the required joint control torques for time  $t$  from 0 to 4 seconds. Plot the torques versus time.

(2) Use the torques computed from (1) as input, simulate the motion of the end point of the manipulator for 4 seconds. Use the modified Euler integration method and set the time step to 0.01 seconds. Please plot the simulation results:

$\theta_1(t)$ ,  $\theta_2(t)$ ,  $\dot{\theta}_1(t)$ , and  $\dot{\theta}_2(t)$  versus time.

$x(t)$ ,  $y(t)$ ,  $\dot{x}(t)$ , and  $\dot{y}(t)$  versus time.

(3) Compute the tip trajectory  $x(t)$  and  $y(t)$  using the forward kinematics and then compare the result to the simulated tip trajectory. Plot the errors between the kinematically computed  $x(t)$  and  $y(t)$  and dynamically simulated  $x(t)$  and  $y(t)$ .

**\*\* Important note: please clearly index and label your axes and units in your plots, otherwise the plots are meaningless**

