Notes

Fractions:

Addition/Subtraction:

$$\frac{a}{b}+\frac{c}{d}\Rightarrow\frac{ad+bc}{bd}, \quad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\frac{a}{b}-c\Rightarrow\frac{a}{b}-\frac{c}{1}\Rightarrow\frac{a}{b}-\frac{bc}{b}\Rightarrow\frac{a-bc}{b}$$

$$\frac{a}{b}*c=\frac{ac}{b} \quad \frac{a}{b}/c=\frac{a}{bc}$$

Powers:

Negative:

$$-x^y=-(x^y) \quad -2^2=-(2^2)=-4$$

$$(-x)^y=-x*-x \quad (-2)^2=-2*-2=4$$

Multiple powers:

$$a^0=1, \quad a^1=a, \quad a^{-1}=\frac{1}{a^1}$$

$$a^b*a^c=a^{b+c}, \quad \frac{a^b}{a^c}=a^{b-c}$$

example:

$$x^a*x^b=x^{a+b} \quad x^2*x^4=x^6$$

$$(x^a)^b=x^{a*b} \quad (2^3)^2=(2*2*2)^2=(2*2*2)(2*2*2)=2^6$$

Calculus:

Derivatives:

$$\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Differentiation		Integration	
Function $f(x)$	Derivative $f'(x)$	$f(x)$	$\int f(x)\,dx$
c	0	a	$ax+c$
x^n	nx^{n-1}	x^n	$\frac{1}{n+1}x^{n+1}+c$
$\sin x$	$\cos x$	$\frac{1}{x}$	$\ln(x)+c$
$\cos x$	$-\sin x$	e^{ax}	$\frac{1}{a}e^{ax}+c$
e^x	e^x	$\cos(ax)$	$\frac{1}{a}\sin(ax)+c$
$\ln x\,(x>0)$	$\frac{1}{x}(x>0)$	$\sin(ax)$	$\frac{-1}{a}\cos(ax)+c$

Notations:

Leibniz: $f'(x)=\frac{dy}{dx} \quad f''(x)=\frac{d^{2y}}{dx^2}$

Newton : $f'(x)=\dot{s} \quad f''(x)=\ddot{s}$

Sum rule: $k(x)=f(x)+g(x) \Rightarrow k'(x)=f'(x)+g'(x)$
Constant multiple rule: $k(x)=cf(x) \Rightarrow k'(x)=cf'(x)$
Product rule: $k(x)=f(x)g(x) \Rightarrow k'(x)=f'(x)g(x)+f(x)g'(x)$
Quotient rule: $k(x)=f(x)/g(x) \Rightarrow k'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$
Composite rule: $k(x)=g(f(x)) \Rightarrow k'(x)=g'(f(x))f'(x)$
Double angle formulas:

$$\sin(2\theta)=2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)=2\cos^2(\theta)-1=1-2\sin^2(\theta)$$

<u>Logarithms:</u>

$$x=a^p \quad \Rightarrow \quad \log_a x=p$$

$$1=a^0 \quad \Rightarrow \quad \log_a 1=0$$

$$a^1=a \quad \Rightarrow \quad \log_a a=1$$

$$\frac{1}{a}=a^{-1} \quad \Rightarrow \quad \log_a\left(\frac{1}{a}\right)=-1$$

$$\log_a(xy)=\log_a x+\log_a y$$

$$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$$

$$\log_a\left(\frac{1}{y}\right)=-\log_a y$$

$$\log_a(x^p)=p*\log_a x$$

<u>Statistics</u>

Sample mean: $\bar{x}$
population mean: μ

Sample standard deviation: s
population standrad deviation: σ

Standard error of the mean: $SE=\frac{s}{\sqrt{(n)}}$ where n is the number of samples

95% of $\bar{x}$ (sample mean) is in $\mu \pm 1.96\text{SE}$

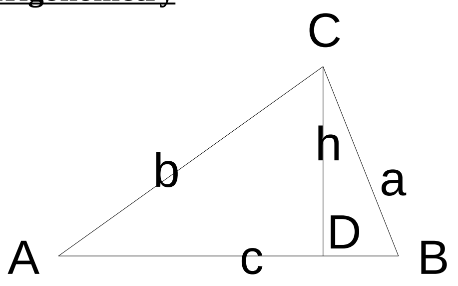
<u>Central limit theorem</u> $(n \leq 25)$

The sampling distribution of the mean for
samples of size n from a population with mean μ and standard deviation σ
may be approximated by a normal distribution with mean μ and standard deviation SE

$$(\bar{x}=\mu, s=SE)$$

<u>Two sample z test</u>

Trigonometry



Pythagoras theorem (for right angle triangles) hypothenous^2=adjacent^2 + opposite^2 ($b^2=c^2+h^2$)

$$\sin(\theta)=\frac{Opposite}{Hypothenous} \quad \left(\sin(A)=\frac{h}{b}\right)$$

$$\cos(\theta)=\frac{adjacent}{hypotenuse} \quad \left(\cos(A)=\frac{c}{b}\right)$$

$$\tan(\theta)=\frac{Opposite}{Adjacent} \quad \left(\cos(A)=\frac{h}{c}\right)$$

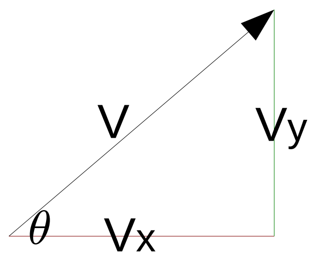
Sin Rule:

$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

and:

$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$$

Vectors:



To Component form: $V_x=Vcos(\theta)$ $V_y=Vsin(\theta)$

The Magnitude: $|V|=\sqrt{V_x^2+V_y^2}$

The angle: $\theta=\tan^{-1}(\frac{V_y}{V_x})$

To add vectors $V+U=(V_x+U_x)i+(V_y+U_y)j$

Polynomials

solving

To solve a Polynomial means to find all values for which the polynomial equals 0.

Quadratic polynomial formula:

$$ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{(b^2-4\mathrm{ac})}}{2\mathrm{a}}$$

Steps for completing the square and proof:

$$ax^2+bx+c$$

$$a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] \quad \boxed{\left(\frac{1}{2}\right)\left(\frac{b}{a}\right)=\left(\frac{b}{2\mathrm{a}}\right)} \quad \boxed{\left(\frac{b}{2\mathrm{a}}\right)^2=\frac{b^2}{4\mathrm{a}^2}}$$

$$a\left[x^2+\frac{b}{a}x+\frac{b^2}{4\mathrm{a}^2}+\frac{c}{a}-\frac{b^2}{4\mathrm{a}^2}\right]$$

$$a\left[\left(x+\frac{b}{2\mathrm{a}}\right)^2+\frac{c}{a}-\frac{b^2}{4\mathrm{a}^2}\right]$$

$$a\left[\left(x+\frac{b}{2\mathrm{a}}\right)^2+\frac{4\mathrm{a}^2c-b^2a}{4\mathrm{a}^3}\right] \quad a\left[\left(x+\frac{b}{2\mathrm{a}}\right)^2+\frac{4\mathrm{ac}-b^2}{4\mathrm{a}^2}\right]$$

$$a\left(x+\frac{b}{2\mathrm{a}}\right)^2+\frac{4\mathrm{ac}-b^2}{4\mathrm{a}} \quad \text{Vertex: } \boxed{x=-\frac{b}{2\mathrm{a}}} \text{ and } \boxed{y=\frac{4\mathrm{ac}-b^2}{4\mathrm{a}}}$$

$$a\left(x+\frac{b}{2\mathrm{a}}\right)^2=-\frac{4\mathrm{ac}-b^2}{4\mathrm{a}}$$

$$\left(x+\frac{b}{2\mathrm{a}}\right)^2=-\frac{4\mathrm{ac}-b^2}{4\mathrm{a}^2} \quad x+\frac{b}{2\mathrm{a}}=\pm\sqrt{\frac{4\mathrm{ac}-b^2}{4\mathrm{a}^2}} \quad x+\frac{b}{2\mathrm{a}}=\frac{\pm\sqrt{4\mathrm{ac}-b^2}}{2\mathrm{a}}$$

Solution: $\boxed{x=\frac{-b\pm\sqrt{4\mathrm{ac}-b^2}}{2\mathrm{a}}}$ (then y=0)

Discriminant: $\Delta=b^2-4\mathrm{ac}$

if $\Delta>0$ then there are 2 real roots (solutions).

if $\Delta=0$ then there is 1 root.

if $\Delta<0$ then there are no roots.

example:

$$3\mathrm{x}^2+7\mathrm{x}-6\Rightarrow 3\left[x^2+\frac{7}{3}x-\frac{6}{3}\right] \quad \left(\frac{1}{2}\right)\left(\frac{7}{3}\right)=\frac{7}{6} \quad \left(\frac{7}{6}\right)^2=\left(\frac{49}{36}\right)$$

$$3\left[x^2+\frac{7}{3}x+\frac{49}{36}-\frac{6}{3}-\frac{49}{36}\right] \quad 3\left[\left(x+\frac{7}{6}\right)^2-\frac{121}{36}\right]$$

vertex: $\boxed{3\left(x+\frac{7}{6}\right)^2-\frac{121}{12}}$ (x=-7/6 y=-121/12)

$$\left(x+\frac{7}{6}\right)^2=\frac{121}{36} \quad x+\frac{7}{6}=\pm\sqrt{\frac{121}{36}} \quad \boxed{x=-\frac{7\pm 11}{6}} \quad (\Delta=121>0)$$

factorising

for $ax^2+bx+c=0$, if a=1 then find two numbers that multiplied together = c and added together =b. if $a\neq 1$ then multiply a by c and find the two factors that add up to b.

example:

$8\mathrm{x}^2+18\mathrm{x}+9=0$ $\quad 8*9=72$ (that's ac=8*9=72)

the factors of 72 which are added together to give 18 are 6 and 12
rewrite formula splitting bx into those factors

$8x^2+12x+6x+9$

$4x(2x+3) \quad + \quad 3(2x+3)$ (NOTE if the brackets are different then something is wrong)

$(4x+3)(2x+3)$

so solutions are $-\frac{3}{4}$ (-0.75) and $-\frac{3}{2}$ (-1.5)

<u>expanding</u>

$(a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

$(a+b)(a-b)=a^2-b^2$

$(a+b)^3=a^3+a^2b+ab^2+b^3$ $\qquad (a-b)^3=(a-b)(a-b)^2=(a-b)(a^2-2ab+b^2)=a^3-3a^2b+3ab^2-b^3$

<u>Pascal's Triangle</u>

Start with two 1s and then add the sum of the two cells above to get the coefficient.

Power		
1	**1 1**	**$(x+y)^1=1x^1+1y^1$**
2	**1 2 1**	**$(x+y)^2=1x^2+2x^1y^1+1y^2$**
3	**1 3 3 1**	**$(x+y)^3=1x^3+3x^2y^1+3x^1y^2+1y^3$**
4	**1 4 6 4 1**	**$(x+y)^4=1x^4+4x^3y^1+6x^2y^2+4x^1y^3+1y^4$**
5	**1 5 10 10 5 1**	**$(x+y)^5=1x^5+5x^4y^1+10x^3y^2+10x^2y^3+5x^1y^4+1y^5$**

power of **x** decrease (from **<u>power</u>** to 0) as power of **y** increase(from 0 to **<u>power</u>**).
coefficient taken from pascal's triangle.
i.e.: **$(x+y)^2=1x^2y^0+2x^1y^1+1x^0y^2$**