Notes

Fractions:

Addition/Subtraction:

$$\frac{a}{b}+\frac{c}{d}\Rightarrow\frac{ad+bc}{bd}, \quad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\frac{a}{b}-c\Rightarrow\frac{a}{b}-\frac{c}{1}\Rightarrow\frac{a}{b}-\frac{bc}{b}\Rightarrow\frac{a-bc}{b}$$

$$\frac{a}{b}*c=\frac{ac}{b} \quad \frac{a}{b}/c=\frac{a}{bc}$$

Powers:

Negative:

$$-x^{y}=-(x^{y}) \quad -2^{2}=-(2^{2})=-4$$

$$(-x)^{y}=-x*-x \quad (-2)^{2}=-2*-2=4$$

Multiple powers:

$$a^{0}=1, \quad a^{1}=a, \quad a^{-1}=\frac{1}{a^{1}}$$

$$a^{b}*a^{c}=a^{b+c}, \quad \frac{a^{b}}{a^{c}}=a^{b-c}$$

example:

$$x^{a}*x^{b}=x^{a+b} \quad x^{2}*x^{4}=x^{6}$$

$$(x^{a})^{b}=x^{a*b} \quad (2^{3})^{2}=(2*2*2)^{2}=(2*2*2)(2*2*2)=2^{6}$$

Calculus:

Derivatives:

$$\lim_{h\to0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Differentiation		Integration	
Function $f(x)$	Derivative $f^{'}(x)$	$f(x)$	$\int f(x)\,dx$
c	0	a	$ax+c$
x^{n}	nx^{n-1}	x^{n}	$\frac{1}{n+1}x^{n+1}+c$
$\sin x$	$\cos x$	$\frac{1}{x}$	$\ln(x)+c$
$\cos x$	$-\sin x$	e^{ax}	$\frac{1}{a}e^{ax}+c$
e^{x}	e^{x}	$\cos(ax)$	$\frac{1}{a}\sin(ax)+c$
$\ln x\,(x>0)$	$\frac{1}{x}(x>0)$	$\sin(ax)$	$\frac{-1}{a}\cos(ax)+c$

Notations:

Leibniz: $f^{'}(x)=\frac{dy}{dx} \quad f^{''}(x)=\frac{d^{2y}}{dx^{2}}$

Newton : $f^{'}(x)=\dot{s} \quad f^{''}(x)=\ddot{s}$

Sum rule: $k(x)=f(x)+g(x) \Rightarrow k'(x)=f'(x)+g'(x)$
Constant multiple rule: $k(x)=cf(x) \Rightarrow k'(x)=cf'(x)$
Product rule: $k(x)=f(x)g(x) \Rightarrow k'(x)=f'(x)g(x)+f(x)g'(x)$
Quotient rule: $k(x)=f(x)/g(x) \Rightarrow k'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$
Composite rule: $k(x)=g(f(x)) \Rightarrow k'(x)=g'(f(x))f'(x)$
Double angle formulas:

$$\sin(2\theta)=2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)=2\cos^2(\theta)-1=1-2\sin^2(\theta)$$

Logarithms:

$x=a^p \quad \Rightarrow \quad \log_a x=p$

$1=a^0 \quad \Rightarrow \quad \log_a 1=0$

$a^1=a \quad \Rightarrow \quad \log_a a=1$

$\frac{1}{a}=a^{-1} \quad \Rightarrow \quad \log_a\left(\frac{1}{a}\right)=-1$

$\log_a(xy)=\log_a x+\log_a y$

$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$

$\log_a\left(\frac{1}{y}\right)=-\log_a y$

$\log_a(x^p)=p*\log_a x$

Statistics
Sample mean: $\bar{x}$
population mean: μ

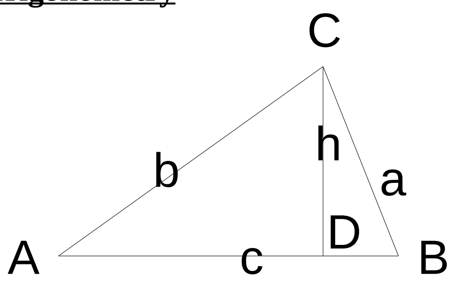
Sample standard deviation: s
population standrad deviation: σ

Standard error of the mean: $SE=\frac{s}{\sqrt{(n)}}$ where n is the number of samples
95% of $\bar{x}$ (sample mean) is in $\mu \pm 1.96$SE
Central limit theorem $(n \leq 25)$
The sampling distribution of the mean for
samples of size n from a population with mean μ and standard deviation σ
may be approximated by a normal distribution with mean μ and standard deviation SE

$$(\bar{x}=\mu, s=SE)$$

Two sample z test

Trigonometry



Pythagoras theorem (for right angle triangles) hypothenous^2=adjacent^2 + opposite^2 ($b^2=c^2+h^2$)

$$\sin(\theta)=\frac{Opposite}{Hypothenous} \quad \left(\sin(A)=\frac{h}{b}\right)$$

$$\cos(\theta)=\frac{adjacent}{hypotenuse} \quad \left(\cos(A)=\frac{c}{b}\right)$$

$$\tan(\theta)=\frac{Opposite}{Adjacent} \quad \left(\cos(A)=\frac{h}{c}\right)$$

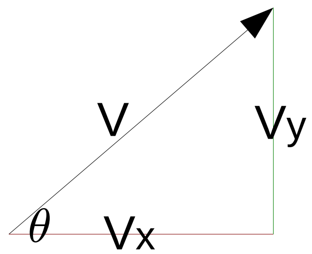
Sin Rule:

$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

and:

$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$$

Vectors:



To Component form: $V_x=Vcos(\theta)$ $V_y=Vsin(\theta)$

The Magnitude: $|V|=\sqrt{V_x^2+V_y^2}$

The angle: $\theta=\tan^{-1}(\frac{V_y}{V_x})$

To add vectors $V+U=(V_x+U_x)i+(V_y+U_y)j$

Polynomials

solving

To solve a Polynomial means to find all values for which the polynomial equals 0.

Quadratic polynomial formula:

$$ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$$

Steps for completing the square and proof:

$$ax^2+bx+c$$

$$a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] \qquad \boxed{\left(\frac{1}{2}\right)\left(\frac{b}{a}\right)=\left(\frac{b}{2a}\right)} \qquad \boxed{\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}}$$

$$a\left[x^2+\frac{b}{a}x+\frac{b^2}{4a^2}+\frac{c}{a}-\frac{b^2}{4a^2}\right]$$

$$a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}\right]$$

$$a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4a^2c-b^2a}{4a^3}\right] \qquad a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right]$$

$$a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a} \qquad a\left(x+\frac{b}{2a}\right)^2=-\frac{4ac-b^2}{4a}$$ Vertex: $\boxed{x=-\frac{b}{2a}}$ when $\boxed{y=-\frac{4ac-b^2}{4a}}$

$$\left(x+\frac{b}{2a}\right)^2=-\frac{4ac-b^2}{4a^2} \qquad x+\frac{b}{2a}=\pm\sqrt{\frac{4ac-b^2}{4a^2}} \qquad x+\frac{b}{2a}=\frac{\pm\sqrt{4ac-b^2}}{2a}$$

Solution: $\boxed{x=\frac{-b\pm\sqrt{4ac-b^2}}{2a}}$ (then y=0)

Discriminant: $\Delta=b^2-4ac$

if $\Delta>0$ then there are 2 real roots (solutions).

if $\Delta=0$ then there is 1 root.

if $\Delta<0$ then there are no roots.

example:

$$3x^2+7x-6 \Rightarrow 3\left[x^2+\frac{7}{3}x-\frac{6}{3}\right] \qquad \left(\frac{1}{2}\right)\left(\frac{7}{3}\right)=\frac{7}{6} \qquad \left(\frac{7}{6}\right)^2=\left(\frac{49}{36}\right)$$

$$3\left[x^2+\frac{7}{3}x+\frac{49}{36}-\frac{6}{3}-\frac{49}{36}\right] \qquad 3\left[\left(x+\frac{7}{6}\right)^2-\frac{121}{36}\right] \qquad 3\left(x+\frac{7}{6}\right)^2-\frac{121}{12}$$

Vertex: $\boxed{3\left(x+\frac{7}{6}\right)^2=\frac{121}{12}}$ (x=-7/6 y=121/12)

$$\left(x+\frac{7}{6}\right)^2=\frac{121}{36} \qquad x+\frac{7}{6}=\pm\sqrt{\frac{121}{36}} \qquad \boxed{x=-\frac{7\pm 11}{6}} \qquad (\Delta=121>0)$$

factorising

for $ax^2+bx+c=0$, if a=1 then find two numbers that multiplied together = c and added together =b. if $a\neq 1$ then multiply a by c and find the two factors that add up to b.

example:

$8x^2+18x+9=0 \quad 8*9=72$ (that's ac=8*9=72)

the factors of 72 which are added together to give 18 are 6 and 12

rewrite formula splitting bx into those factors

$8x^2+12x+6x+9$
$4x(2x+3) \quad + \quad 3(2x+3)$ (NOTE if the brackets are different then something is wrong)
$(4x+3)(2x+3)$

so solutions are $-\frac{3}{4}$ (-0.75) and $-\frac{3}{2}$ (-1.5)

<u>expanding</u>

$(a+b)^2=a^2+2ab+b^2$
$(a-b)^2=a^2-2ab+b^2$
$(a+b)(a-b)=a^2-b^2$

$(a+b)^3=a^3+a^2b+ab^2+b^3$ $\quad$ $(a-b)^3=(a-b)(a-b)^2=(a-b)(a^2-2ab+b^2)=a^3-3a^2b+3ab^2-b^3$

<u>Pascal's Triangle</u>

Start with two 1s and then add the sum of the two cells above to get the coefficient.

Power							
1	**1**	**1**					$(x+y)^1=1x^1+1y^1$
2	**1**	**2**	**1**				$(x+y)^2=1x^2+2x^1y^1+1y^2$
3	**1**	**3**	**3**	**1**			$(x+y)^3=1x^3+3x^2y^1+3x^1y^2+1y^3$
4	**1**	**4**	**6**	**4**	**1**		$(x+y)^4=1x^4+4x^3y^1+6x^2y^2+4x^1y^3+1y^4$
5	**1**	**5**	**10**	**10**	**5**	**1**	$(x+y)^5=1x^5+5x^4y^1+10x^3y^2+10x^2y^3+5x^1y^4+1y^5$

power of **x** decrease (from **<u>power</u>** to 0) as power of **y** increase(from 0 to **<u>power</u>**).
coefficient taken from pascal's triangle.
i.e.: $(x+y)^2=1x^2y^0+2x^1y^1+1x^0y^2$