Notes

Fractions:

Addition/Subtraction:

$$\frac{a}{b}+\frac{c}{d}\Rightarrow\frac{ad+bc}{bd}, \qquad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

$$\frac{a}{b}-c\Rightarrow\frac{a}{b}-\frac{c}{1}\Rightarrow\frac{a}{b}-\frac{bc}{b}\Rightarrow\frac{a-bc}{b}$$

$$\frac{a}{b}*c=\frac{ac}{b} \qquad \frac{a}{b}/c=\frac{a}{bc}$$

Powers:

Negative:

$$-x^{y}=-(x^{y}) \qquad -2^{2}=-(2^{2})=-4$$

$$(-x)^{y}=-x*-x \qquad (-2)^{2}=-2*-2=4$$

fraction:

$$x^{\frac{a}{b}}=\sqrt[b]{x^{a}} \qquad x^{-\frac{a}{b}}=\frac{1}{\sqrt[b]{x^{a}}}$$

example:

$$x^{\frac{1}{2}}=\sqrt{x} \qquad x^{-\frac{3}{4}}=\frac{1}{\sqrt[4]{x^{3}}}$$

Multiple powers:

$$a^{0}=1, \qquad a^{1}=a, \qquad a^{-1}=\frac{1}{a^{1}}$$

$$a^{b}*a^{c}=a^{b+c}, \qquad \frac{a^{b}}{a^{c}}=a^{b-c}$$

example:

$$x^{a}*x^{b}=x^{a+b} \qquad x^{2}*x^{4}=x^{6}$$

$$(x^{a})^{b}=x^{a*b} \qquad (2^{3})^{2}=(2*2*2)^{2}=(2*2*2)(2*2*2)=2^{6}$$

Surds:

$$\sqrt{a}*\sqrt{b}=\sqrt{ab} \qquad \sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$$

rationalising:

$$\sqrt{\frac{x}{y}}=\frac{\sqrt{x}}{\sqrt{y}}=\frac{\sqrt{x}}{\sqrt{y}}*\frac{\sqrt{y}}{\sqrt{y}}=\frac{\sqrt{xy}}{y}$$

$$\frac{x}{\sqrt{y}+z}=\frac{x}{\sqrt{y}+z}*\frac{\sqrt{y}-z}{\sqrt{y}-z}=\frac{x\sqrt{y}-xz}{y-z^{2}}$$

Calculus:

Derivatives:

$$\lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Differentiation	Integration
Function $f(x)$ — Derivative $f^{'}(x)$ c — 0 x^n — nx^{n-1} $\sin x$ — $\cos x$ $\cos x$ — $-\sin x$ e^x — e^x $\ln x (x>0)$ — $\frac{1}{x}(x>0)$	$f(x)$ — $\int f(x)dx$ a — $ax+c$ x^n — $\frac{1}{n+1}x^{n+1}+c$ $\frac{1}{x}$ — $\ln(x)+c$ e^{ax} — $\frac{1}{a}e^{ax}+c$ $\cos(ax)$ — $\frac{1}{a}\sin(ax)+c$ $\sin(ax)$ — $\frac{-1}{a}\cos(ax)+c$

Notations:

Leibniz: $f^{'}(x)=\frac{dy}{dx}$ $\quad f^{''}(x)=\frac{d^{2y}}{dx^2}$

Newton : $f^{'}(x)=\dot{s}$ $\quad f^{''}(x)=\ddot{s}$

Sum rule: $k(x)=f(x)+g(x) \Rightarrow k'(x)=f'(x)+g'(x)$

Constant multiple rule: $k(x)=cf(x) \Rightarrow k'(x)=cf'(x)$

Product rule: $k(x)=f(x)g(x) \Rightarrow k'(x)=f'(x)g(x)+f(x)g'(x)$

Quotient rule: $k(x)=f(x)/g(x) \Rightarrow k'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$

Composite rule: $k(x)=g(f(x)) \Rightarrow k'(x)=g'(f(x))f'(x)$

Double angle formulas:

$$\sin(2\theta)=2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)=2\cos^2(\theta)-1=1-2\sin^2(\theta)$$

Logarithms:

$x=a^p \quad \Rightarrow \quad \log_a x=p$

$1=a^0 \quad \Rightarrow \quad \log_a 1=0$

$a^1=a \quad \Rightarrow \quad \log_a a=1$

$\frac{1}{a}=a^{-1} \quad \Rightarrow \quad \log_a\left(\frac{1}{a}\right)=-1$

$\log_a(xy)=\log_a x+\log_a y$

$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$

$\log_a\left(\frac{1}{y}\right)=-\log_a y$

$\log_a(x^p)=p*\log_a x$

Statistics

Sample mean: $\overline{x}$

population mean: μ

Sample standard deviation: s

population standrad deviation: σ

Standard error of the mean: $SE=\frac{s}{\sqrt{(n)}}$ where n is the number of samples

95% of $\overline{x}$ (sample mean) is in $\mu\pm1.96$SE

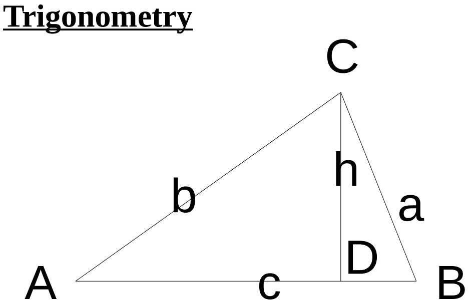
Central limit theorem $(n\leq25)$

The sampling distribution of the mean for samples of size n from a population with mean μ and standard deviation σ may be approximated by a normal distribution with mean μ and standard deviation SE

$$(\overline{x}=\mu\,,s=SE)$$

Two sample z test

Trigonometry



Pythagoras theorem (for right angle triangles) hypothenous^2=adjacent^2 + opposite^2 ($b^2=c^2+h^2$)

$\sin(\theta)=\frac{Opposite}{Hypothenous}$ $\left(\sin(A)=\frac{h}{b}\right)$

$\cos(\theta)=\frac{adjacent}{hypotenuse}$ $\left(\cos(A)=\frac{c}{b}\right)$

$\tan(\theta)=\frac{Opposite}{Adjacent}$ $\left(\cos(A)=\frac{h}{c}\right)$

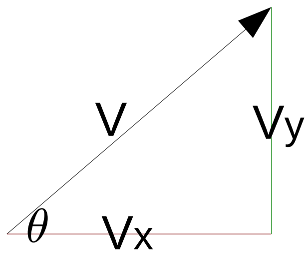
Sin Rule:

$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

and:

$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$$

Vectors:

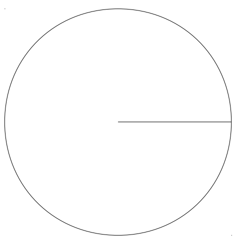
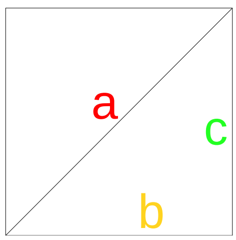
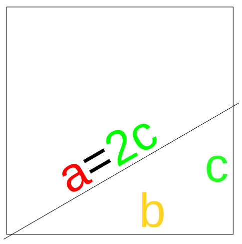


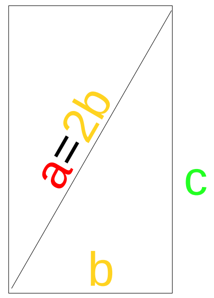
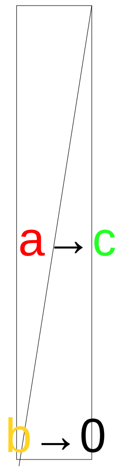
To Component form: $V_x = V\cos(\theta)$ $V_y = V\sin(\theta)$
The Magnitude: $|V| = \sqrt{V_x^2 + V_y^2}$

The angle: $\theta = \tan^{-1}(\frac{V_y}{V_x})$

To add vectors $V + U = (V_x + U_x)i + (V_y + U_y)j$

Trig ratios:

	θ	proof
	0	Adjacent(b)=Hypotenuse(a), opposite(c)=0 $\sin(0) = \frac{c}{a} = \frac{0}{a} = 0$ $\cos(0) = \frac{b}{a} = 1$ $\tan(0) = \frac{0}{b} = 0$
	45	$c = b \Rightarrow \tan(45) = \frac{c}{b} = 1$ $a^2 = b^2 + c^2$ given: $b = c = 1$ $a^2 = 1^2 + 1^2$ $a = \sqrt{2}$ $\sin(45) = \frac{c}{a} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}$ $\cos(45) = \frac{b}{a} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}$
	30	$c = \left(\frac{1}{2}\right)a$ or $a = 2c$ $\sin(30) = \frac{c}{a} = \frac{c}{2c} = \frac{1}{2}$ given: $a = 1, c = \frac{1}{2}$ $a^2 = b^2 + c^2 \Rightarrow 1^2 = b^2 + \left(\frac{1}{2}\right)^2$ $b^2 = 1 - \left(\frac{1}{2}\right)^2$ $b = \sqrt{1 - \left(\frac{1}{4}\right)} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ $\cos(30) = \frac{b}{a} = \frac{\sqrt{3}/2}{1} = \frac{\sqrt{3}}{2}$ $\tan(30) = \frac{c}{b} = \frac{(1/2)}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \Rightarrow \frac{\sqrt{3}}{3}$

<table>
<tr><td></td><td>60</td><td colspan="6">$$\cos(60)=\frac{b}{a}=\frac{b}{2b}=\frac{1}{2}$$
given $a=1, b=\frac{1}{2} \Rightarrow 1^2=\left(\frac{1}{2}\right)^2+c^2$
$$c=\sqrt{1-\frac{1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$
$$\sin(60)=\frac{c}{a}=\frac{\sqrt{3}/2}{1}=\frac{\sqrt{3}}{2} \qquad \tan(60)=\frac{c}{b}=\frac{\sqrt{3}/2}{1/2}=\sqrt{3}$$</td></tr>
<tr><td rowspan="2"></td><td colspan="2">$\theta \to 90^\circ$
$a \to c$
$b \to 0$</td><td colspan="5">$$\sin(\theta)=\frac{c}{a}\to 1 \qquad \cos(\theta)=\frac{b}{a}\to\frac{0}{a}\to 0$$
$$\tan(\theta)=\frac{c}{b}=\frac{c}{b\to 0}\to\infty$$ i.e. $$\frac{c}{\text{small number}}=\text{large number}$$</td></tr>
<tr><td colspan="2">$\theta > 90^\circ$
$\theta \to 90^\circ$
$a \to c$
$b \to 0$</td><td colspan="5">$$\tan(\theta)=\frac{c}{b}=\frac{c}{-b\to 0}\to -\infty$$
i.e. $$\frac{c}{\text{small negative number}}=\text{large negative number}$$</td></tr>
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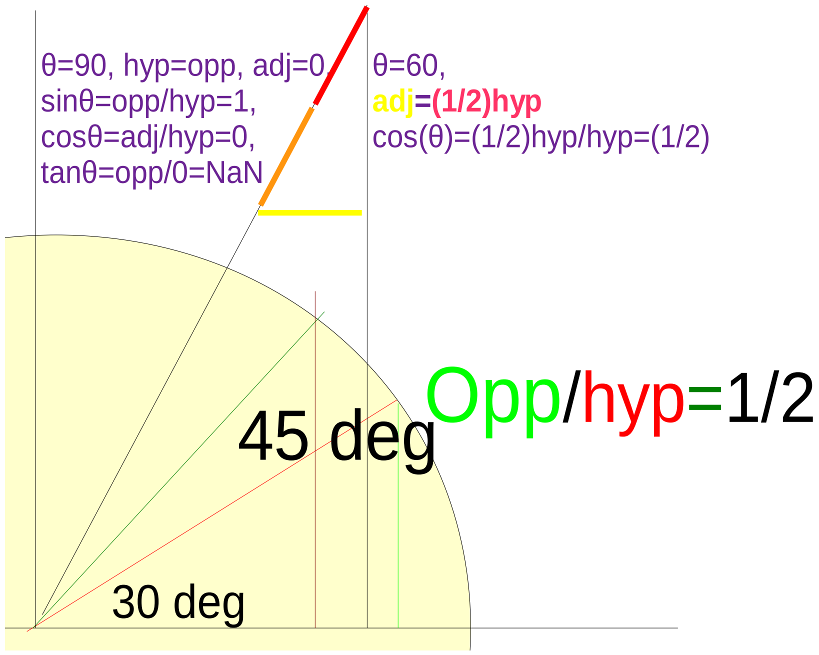
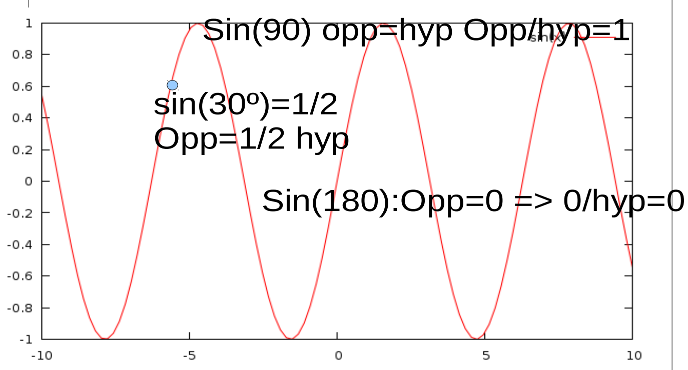
Trig. functions





rad	deg	$\sin(\theta)$	$\tan(\theta)$
	0	0	10
	15	$-\frac{1+\sqrt{3}}{2\sqrt{2}}$	
	30	$\frac{1}{2}$	
	45		
	60		
	75		
	90		

proof

$\sin(\theta)=opp/hyp$ and $\theta=30^{degrees}$

$A^2=B^2+C^2$

given Adj=1 for the ratio of B to C would be

$\frac{1}{2}$

, ie the opposite is half the hyp.

So

$Hyp^2=\frac{1}{2}^2+1^2 \quad hyp=\sqrt{\frac{1}{4}+1}$ and

of hyp is this relation to adj

at 0 degrees:
opp=0, sin(0)=0/hyp =0
tan=0/adj=0
cos(0):adj=hyp => adj/hyp=1

tan(h)=opp/h h tendsto 0,

at 30 degrees Opposite is half hyp, or $\frac{hyp}{2hyp}=\frac{1}{2}$

at 45 degrees the adjacent and the opposide are equal, it is a ...forgot the name...
so given A^2=B^2+C^2 , and given b=c=1, hyp=sqrt{1+1} or sqrt{2} of lengths C and B

and $\tan\theta=\frac{1}{1}=1$

hence $\sin 45=\frac{opp}{hyp}=\frac{1}{\sqrt{2}}$

at 60 degrees
at 90 degrees

adj=0

$$\tan(90)=\frac{Opp}{0}=NaN$$

$$|_{\theta\to 90}\, adj\to 0 \Rightarrow \tan(\theta)=\frac{opp}{adj\to 0}\to\infty$$

at 180 degrees
sin: opp=0 =>0/hyp=0
cos: adj=hyp =>adj/hyp=1
tan:

hyp=opp so sin=opp/hyp=1/1=1 (given hyp and opp are 1)

Polynomials
<u>solving</u>
To solve a Polynomial means to find all values for which the polynomial equals 0.

Quadratic polynomial formula:

$$ax^2+bx+c=0\Rightarrow x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$$

Steps for completing the square and proof:

$$ax^2+bx+c$$

$$a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] \quad \boxed{\left(\frac{1}{2}\right)\left(\frac{b}{a}\right)=\left(\frac{b}{2a}\right)} \quad \boxed{\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}}$$

$$a\left[x^2+\frac{b}{a}x+\frac{b^2}{4a^2}+\frac{c}{a}-\frac{b^2}{4a^2}\right]$$

$$a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}\right]$$

$$a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4a^2c-b^2a}{4a^3}\right] \qquad a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right]$$

$$a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$$ Vertex: $\boxed{x=-\frac{b}{2a}}$ and $\boxed{y=\frac{4ac-b^2}{4a}}$

$$a\left(x+\frac{b}{2a}\right)^2=-\frac{4ac-b^2}{4a}$$

$$\left(x+\frac{b}{2a}\right)^2=-\frac{4ac-b^2}{4a^2} \qquad x+\frac{b}{2a}=\pm\sqrt{\frac{4ac-b^2}{4a^2}} \qquad x+\frac{b}{2a}=\frac{\pm\sqrt{4ac-b^2}}{2a}$$

Solution: $\boxed{x=\frac{-b\pm\sqrt{4ac-b^2}}{2a}}$ (then y=0)

Discriminant(Δ): $-1(4ac-b^2)$

$$\Delta=b^2-4ac$$

if $\Delta>0$ then there are 2 real roots (solutions).
if $\Delta=0$ then there is 1 root.
if $\Delta<0$ then there are no roots.
<u>example:</u>

$$3x^2+7x-6 \Rightarrow 3\left[x^2+\frac{7}{3}x-\frac{6}{3}\right] \qquad \left(\frac{1}{2}\right)\left(\frac{7}{3}\right)=\frac{7}{6} \qquad \left(\frac{7}{6}\right)^2=\left(\frac{49}{36}\right)$$

$$3\left[x^2+\frac{7}{3}x+\frac{49}{36}-\frac{6}{3}-\frac{49}{36}\right] \qquad 3\left[\left(x+\frac{7}{6}\right)^2-\frac{121}{36}\right]$$

vertex: $\boxed{3\left(x+\frac{7}{6}\right)^2-\frac{121}{12}}$ (x=-7/6 y=-121/12)

$$\left(x+\frac{7}{6}\right)^2=\frac{121}{36} \qquad x+\frac{7}{6}=\pm\sqrt{\frac{121}{36}} \qquad \boxed{x=-\frac{7\pm 11}{6}} \qquad (\Delta=121>0)$$

<u>factorising</u>

for $ax^2+bx+c=0$, if a=1 then find two numbers that multiplied together = c and added together =b. if $a\neq 1$ then multiply a by c and find the two factors that add up to b.

example:

$8x^2+18x+9=0 \quad 8*9=72$ (that's ac=8*9=72)

the factors of 72 which are added together to give 18 are 6 and 12

rewrite formula splitting bx into those factors

$8x^2+12x+6x+9$

$4x(2x+3) \;+\; 3(2x+3)$ (NOTE if the brackets are different then something is wrong)

$(4x+3)(2x+3)$

so solutions are $-\frac{3}{4}$ (-0.75) and $-\frac{3}{2}$ (-1.5)

<u>expanding</u>

$(a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

$(a+b)(a-b)=a^2-b^2$ (perfect square)

$(a+b)^3=a^3+a^2b+ab^2+b^3 \qquad (a-b)^3=(a-b)(a-b)^2=(a-b)(a^2-2ab+b^2)=a^3-3a^2b+3ab^2-b^3$

<u>Pascal's Triangle</u>

Start with two 1s and then add the sum of the two cells above to get the coefficient.

Power		
1	**1 1**	**$(x+y)^1=1x^1+1y^1$**
2	**1 2 1**	**$(x+y)^2=1x^2+2x^1y^1+1y^2$**
3	**1 3 3 1**	**$(x+y)^3=1x^3+3x^2y^1+3x^1y^2+1y^3$**
4	**1 4 6 4 1**	**$(x+y)^4=1x^4+4x^3y^1+6x^2y^2+4x^1y^3+1y^4$**
5	**1 5 10 10 5 1**	**$(x+y)^5=1x^5+5x^4y^1+10x^3y^2+10x^2y^3+5x^1y^4+1y^5$**

power of **x** decrease (from **<u>power</u>** to 0) as power of **y** increase(from 0 to **<u>power</u>**).

coefficient taken from pascal's triangle.

i.e.: **$(x+y)^2=1x^2y^0+2x^1y^1+1x^0y^2$**