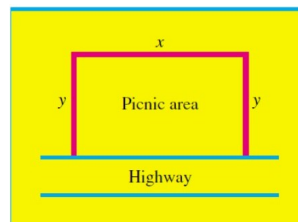


COMP6229 (2017/18): Machine Learning
Optional worksheet on background mathematics (Not for Assessment)

Here is a set of random problems forming an entirely optional exercise and meant ONLY for students who think their mathematical skills are not strong. Please upload your work (scan of **neat** handwritten attempts) to any **two** of the problems, if you wish to join the class on Friday 13/10.

Issue	9 Oct 2017
Deadline	12 Oct 2017 (16:00)
Discussion on	13 Oct. 2017

1. The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of 5,000 square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?



Hint: Write out a constrained minimization problem, a Lagrangian, differentiate, equate to zero and try to solve.

2. Find the maximum and minimum values of the function $f(x, y) = xy$, subject to the constraint $x^2 + y^2 = 8$. Use MATLAB to plot the function $f(x, y)$ (3D or contours on it) over some convenient range and illustrate your answer.
3. An editor has been given 60,000 to spend on the development and promotion of a new book. An empirical study has found that if x thousand dollars is spent on development and y thousand on promotion, approximately $f(x, y) = 20x^{3/2}y$ copies of the book will be sold. How much money should the editor allocate to development and how much to promotion in order to maximize sales?
4. Set up a system of linear equations for the following problem and then solve it: The three-digit number N is equal to 15 times the sum of its digits. If you reverse the digits of N , the resulting number is larger by 396. Also, the units (ones) digit of N is one more than the sum of the other two digits. Find N .
5. Let \mathbf{u} and \mathbf{v} be vectors in \mathcal{R}^3 . Let $k = \|\mathbf{u}\|$ and $l = \|\mathbf{v}\|$. Show that the vector

$$\mathbf{w} = l\mathbf{u} + k\mathbf{v}$$

bisects the angle between \mathbf{u} and \mathbf{v} .

6. The matrix A_θ is defined as

$$\begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- For what values of θ is A_θ invertible? (Hint: Consider $A_\theta \mathbf{x} = \mathbf{0}$)
- Show that $A_\theta^{-1} = A_\theta^t$.

7. A symmetric matrix A can be diagonalized as

$$A = S D S^{-1}$$

Can the repeated multiplication of A^n be done efficiently using this relationship?

8. Let x be a random variable with probability density given by

$$f(x) = \begin{cases} c(1 - x^2) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is the value of c ?
 - (b) Derive the cumulative distribution of x .
9. You arrive at a bus stop at 10 : 00, knowing that the bus will arrive at some time uniformly distributed between 10 : 00 and 10 : 30.
- (a) What is the probability that you will have to wait longer than 10 minutes?
 - (b) If at 10 : 15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?