Pre-Algebra Formulas

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MATH

Numbers

Set notation

Number sets

$\mathbb{N}$	natural (counting) numbers	$1,2,3,4,5,\ldots$
$\mathbb{W}$	whole numbers (natural and 0)	$0,1,2,3,4,5,\ldots$
$\mathbb{Z}$	integers (whole and negatives)	$-3,-2,-1,0,1,2,3$
$\mathbb{Q}$	rational numbers (reals that can be fractions)	$1/2,3/4,117/121$
$\mathbb{R}-\mathbb{Q}$	irrational numbers (reals that can't be fractions)	$\pi,\sqrt{2},\sqrt{3}$
$\mathbb{R}$	real numbers (all of the above, just not imaginary)	not $\sqrt{-1},\pm\infty$

Hierarchy of number sets

$\mathbb{R}$ - **Real** numbers includes

$\mathbb{R}-\mathbb{Q}$ - **Irrational** numbers and

$\mathbb{Q}$ - **Rational** numbers, which includes

$\mathbb{Z}$ - **Integers**, which includes

$\mathbb{W}$ - **Whole** numbers, which includes

$\mathbb{N}$ - **Natural** numbers

Set notation

$\in$	element of, “belongs to,” or “in”
$\cup$	union
$\cap$	intersection

Examples

$A = \{1,2\}$	A is the set that includes 1 and 2
$A = \{x \in \mathbb{Z}\}$	A is the set where x is an element of integers

Identity numbers

0	is the identity number for addition and subtraction, because adding 0 to something or subtracting 0 from something doesn’t change its identity
1	is the identity number for multiplication and division, because multiplying or dividing something by 1 doesn’t change its identity

Negative numbers

Algebraic subtraction

If a and b are real numbers, then

$$a - b = a + (-b)$$

where $-b$ is the opposite of b.

Rules of signed numbers

For addition and subtraction, the sign of the result is the same sign as the original numbers if the original numbers have the same sign. The sign of the result is the sign of the larger number if the signs of the original numbers have different signs.

Addition

Positive + Positive = Positive

Negative + Negative = Negative

Positive + Negative =

Positive	if the positive number is larger than the negative number
Negative	if the negative number is larger than the positive number

For multiplication and division, the sign of the result is positive if the original numbers have the same sign. The sign of the result is negative if the original numbers have different signs.

Multiplication and Division

Positive × Positive = Positive Positive ÷ Positive = Positive

Negative × Negative = Positive Negative ÷ Negative = Positive

Positive × Negative = Negative Positive ÷ Negative = Negative

Negative × Positive = Negative Negative ÷ Positive = Negative

Therefore...

The product of an even number of negative factors is positive

The product of an odd number of negative factors is negative

Factors and multiples

Prime numbers

A prime number is a whole number greater than 1 whose only whole number factors are 1 and the number itself.

Composite numbers

Unlike a prime number, which has exactly two factors (1 and itself), composite numbers have three or more factors.

Least common multiple (LCM)

A common multiple of two numbers a and b is evenly divisible by both a and b. The least common multiple is the smallest of a and b's common multiples.

Divisibility rules

2	last digit is $0, 2, 4, 6, 8$
3	sum of the digits is divisible by 3
4	last two digits are divisible by 4
5	last digit is $0, 5$
6	divisible by 2 and 3
7	$5 \times$ last digit + rest of the number is divisible by 7
8	last three digits are divisible by 8
9	sum of the digits is divisible by 9
10	last digit is 0

Decimals

Place value

100,000,000,000,000	Hundred trillions
10,000,000,000,000	Ten trillions
1,000,000,000,000	Trillions
100,000,000,000	Hundred billions
10,000,000,000	Ten billions
1,000,000,000	Billions
100,000,000	Hundred millions
10,000,000	Ten millions
1,000,000	Millions
100,000	Hundred thousands
10,000	Ten thousands
1,000	Thousands
100	Hundreds
10	Tens
1	Ones (units)

0.1	1/10	Tenths
0.01	1/100	Hundredths
0.001	1/1,000	Thousandths
0.0001	1/10,000	Ten-thousandths
0.00001	1/100,000	Hundred-thousandths
0.000001	1/1,000,000	Millionths
0.0000001	1/10,000,000	Ten-millionths
0.00000001	1/100,000,000	Hundred-millionths
0.000000001	1/1,000,000,000	Billionths
0.0000000001	1/10,000,000,000	Ten-billionths
0.00000000001	1/100,000,000,000	Hundred-billionths
0.000000000001	1/1,000,000,000,000	Trillionths
0.0000000000001	1/10,000,000,000,000	Ten-trillionths
0.00000000000001	1/100,000,000,000,000	Hundred-trillionths

Fractions

Adding fractions with equal denominators

Fractions with equal denominators are added by adding the numerators algebraically and recording the sum over a single denominator.

Denominator-numerator rule

Multiplying the numerator and denominator of a fraction by the same nonzero quantity won't change the value of the fraction.

$$\frac{a}{b} = \frac{ac}{bc} \qquad \text{because} \qquad \frac{c}{c} = 1 \qquad (b, c \neq 0)$$

Note: Use this rule to rationalize a denominator.

Signs of fractions

Every fraction has three signs, one for the numerator, one for the denominator, and one out in front of the fraction. If the sign is not written, then you know it's a + sign.

$$-\frac{+3}{+4} = -\frac{3}{4}$$

You can change ANY TWO signs of a fraction without changing its value.

$$-\frac{+3}{+4} = +\frac{-3}{+4} = +\frac{+3}{-4} = -\frac{-3}{-4}$$

Reciprocal (multiplicative inverse)

For any nonzero real number a, the reciprocal of the number is $1/a$. In other words, the product of any number and its reciprocal is 1.

Percent markup

Selling price = purchase price + markup

Ratio and proportion

Ratio

A ratio is a way to describe the relationship between two numbers.

Proportion

A proportion is the statement that two ratios are equal.

Exponents

Definition of exponential notation

Given n factors of x on the left side of the equation below, then

$$x \cdot x \cdot x \cdot \ldots \cdot x = x^n$$

Rules of exponents

Product rule	$x^a x^b = x^{a+b}$	$x \neq 0$
	$(xy)^a = x^a y^a$	
Power rule	$\left(x^a\right)^b = x^{ab}$	a and b are real numbers and $x \neq 0$
Quotient rule	$\frac{x^a}{x^b} = x^{a-b}$	a and b are real numbers and $x \neq 0$

Powers of 1

1 raised to any power equals 1.

Negative exponents

If n is any real number and x is any real nonzero number, then

$$x^{-n} = \frac{1}{x^n}$$

Radicals

Definition of a square root

If x is a positive real number, then $\sqrt{x}$ is the unique positive real number such that

$$\left(\sqrt{x}\right)^2 = x$$

Definition of a cube root

If x is a real number, then $\sqrt[3]{x}$ is the unique positive or negative real number such that

$$\left(\sqrt[3]{x}\right)^3 = x$$

Definition of a fourth root

If x is a real number, then $\sqrt[4]{x}$ is the unique positive real number such that

$$\left(\sqrt[4]{x}\right)^4 = x$$

Radical terminology

In the example $\sqrt[4]{16} = 2$ (“the fourth root of sixteen equals two”),

$\sqrt{}$ is the radical sign

16 is the radicand

4 is the index (if the index isn't written, it's understood to be 2)

Product of square roots theorem

If m and/or n are nonnegative real numbers, then

$$\sqrt{m}\sqrt{n} = \sqrt{mn} \qquad \text{and} \qquad \sqrt{mn} = \sqrt{m}\sqrt{n}$$

If m and n are both negative real numbers, then

$$\sqrt{m}\sqrt{n} = -\sqrt{mn}$$

Simplified form

An expression that contains square root radicals is in simplified form when no radicand has a perfect-square factor and no radicals are in the denominator.

Scientific notation

Powers of 10

$$10^1 = 10$$

$10^2 = 100$

$10^3 = 1{,}000$

$10^4 = 10{,}000$

etc.

Multiplication

To multiply in scientific notation, multiply the whole or decimal numbers together, then multiply the powers of 10 together, adding their exponents. Make sure the final answer is written in scientific notation.

Measurement

US customary to metric conversion

Distance

12	inches	in	1	foot
3	feet	in	1	yard
5,280	feet	in	1	mile

Weight

16	ounces	in	1	pound

2,000	pounds	in	1	ton

Metric conversion

	kilo	*hecto*	*deka*	*meter*	*deci*	*centi*	*milli*
kilo	1	1/10	1/100	1/1,000	1/10,000	1/100,000	1/1,000,000
hecto	10	1	1/10	1/100	1/1,000	1/10,000	1/100,000
deka	100	10	1	1/10	1/100	1/1,000	1/10,000
meter	1,000	100	10	1	1/10	1/100	1/1,000
deci	10,000	1,000	100	10	1	1/10	1/100
centi	100,000	10,000	1,000	100	10	1	1/10
milli	1,000,000	100,000	10,000	1,000	100	10	1

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