Task 2: Depth alignment via linear least squares

TODO: Find the optimal solution s,t for the following least squares criterion:

$$\mathcal{L}_{ssi}(\mathbf{d}, \mathbf{d}^*) = \min_{s,t} \frac{1}{2M} \sum_{i=1}^{M} (s\mathbf{d}_i + t - \mathbf{d}_i^*)^2$$

$$\mathcal{L}_{ssi}(\mathbf{d}, \mathbf{d}^*) = \min_{h} \frac{1}{2M} \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}}^\top \mathbf{h} - \mathbf{d}_i^*)^2$$

with $\overrightarrow{\mathbf{d_i}} = (\mathbf{d}_i, 1)^\top$ and $\mathbf{h} = (s, t)$ such that $\overrightarrow{\mathbf{d_i}}^\top \mathbf{h} = \mathbf{d}_i * s + 1 * t$

Solution:
At the minimum of the function it's derivative is 0.

$$\frac{\partial \mathcal{L}_{ssi}}{\partial \mathbf{h}} = 0$$

$$\Leftrightarrow$$

Outer derivative $*$ inner derivative:

$$2 * \frac{1}{2M} \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}}^\top \mathbf{h} - \mathbf{d}_i^*) * \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}}) = 0$$

$$\Leftrightarrow \sum_{i=1}^{M} \overrightarrow{\mathbf{d_i}} (\overrightarrow{\mathbf{d_i}}^\top \mathbf{h} - \mathbf{d}_i^*) = 0$$

$$\Leftrightarrow \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \overrightarrow{\mathbf{d_i}}^\top) \mathbf{h} - \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \mathbf{d}_i^*) = 0$$

$$\Leftrightarrow \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \overrightarrow{\mathbf{d_i}}^\top) \mathbf{h} = \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \mathbf{d}_i^*)$$

$$\Leftrightarrow \left(\sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \overrightarrow{\mathbf{d_i}}^\top) \right) \mathbf{h} = \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \mathbf{d}_i^*)$$

$$\Leftrightarrow \mathbf{h} = \left(\sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \overrightarrow{\mathbf{d_i}}^\top) \right)^{-1} \sum_{i=1}^{M} (\overrightarrow{\mathbf{d_i}} \mathbf{d}_i^*)$$

$$\Leftrightarrow \mathbf{h} = \left(\sum_{i=1}^{M} \begin{bmatrix} d_i \\ 1 \end{bmatrix} \begin{bmatrix} d_i & 1 \end{bmatrix} \right)^{-1} \sum_{i=1}^{M} (\begin{bmatrix} d_i \\ 1 \end{bmatrix} \mathbf{d}_i^*)$$

$$\Leftrightarrow \mathbf{h} = \left(\sum_{i=1}^{M} \begin{bmatrix} d_i^2 & d_i \\ d_i & 1 \end{bmatrix} \right)^{-1} \sum_{i=1}^{M} (\begin{bmatrix} d_i \\ 1 \end{bmatrix} \mathbf{d}_i^*)$$