1- a) $\log_2 n^2+1 \in O(n)$

$$\lim_{n\to\infty} \frac{\log_2 n^2+1}{n} = \frac{\infty}{\infty} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{2n}{(n^2+1)\ln 2} = 0$$

So that n increases faster than $\log_2 n^2+1$

So the statement is True.

b) $\sqrt{n(n+1)} \in \Omega(n)$

$$\lim_{n\to\infty} \frac{\sqrt{n(n+1)}}{n} = \lim_{n\to\infty} \sqrt{\frac{n^2+n}{n^2}} = \lim_{n\to\infty} \sqrt{\frac{1+\cancel{\frac{1}{n}}^{0}}{1}} = 1$$

Since result bigger than 0, the statement is True.

c) $n^{n-1} \in \Theta(n^n)$

$$\lim_{n\to\infty} \frac{n^{n-1}}{n^n} = \lim_{n\to\infty} \frac{n^n}{n.n^n} = \lim_{n\to\infty} \frac{1}{n} = 0$$

Since the result of limit is zero then the statement should be $O(n^n)$, not $\Theta(n^n)$ So, False.

d) $O(2^n+n^3) \subset O(4^n)$

$$\lim_{n\to\infty} \frac{2^n+n^3}{4^n} = \lim_{n\to\infty} \frac{2^n}{4^n} + \lim_{n\to\infty} \frac{n^3}{4^n} = 0+0=0$$

Since the result of limit is 0 then the $O(2^n+n^3) \subset O(4^n)$ is True.

e) $O(2\log_3^9\sqrt{n}) \subset O(3\log_2 n^2)$

$$\lim_{n\to\infty} \frac{2\log_3^9\sqrt{n}}{3\log_2 n^2} = \frac{\log_3 n}{9\log_2 n} = \frac{\ln n.\ln 2}{9\ln n.\ln 3} = \lim_{n\to\infty} \frac{\ln 2}{9\ln 3}$$

Since the result constant the statement is False.

f) $\log_2\sqrt{n}$ and $(\log_2^n)^2$

$$\lim_{n\to\infty} \frac{\log_2\sqrt{n}}{(\log_2^n)^2} = \lim_{n\to\infty} \frac{\frac{1}{2}.\log_2 n}{\log_2 n.\log_2 n} = \lim_{n\to\infty} \frac{1}{2\log_2 n} = 0$$

Since the result of limit 0 then $\log_2\sqrt{n}$ and $(\log_2^n)^2$ are not same asymtotic order. False.

2- Order the following functions by growth rate and explain your reasoning for each of them

n^2, n^3, $n^2 \log_2 n$, $\sqrt{n}$, $\log_2 n$, 10^n, 2^n, $8^{\log_2 n}$

Comparing 10^n, 2^n $\rightarrow \lim\limits_{n \to \infty} \frac{10^n}{2^n} = \lim\limits_{n \to \infty} 5^n = \infty$ so that $\boxed{10^n > 2^n}$

Comparing 2^n, n^3 $\rightarrow$ Exponential functions grows rate bigger than polymomial functions so $\boxed{2^n > n^3}$

comparing n^3, $8^{\log_2 n}$ $\rightarrow 8^{\log_2 n} = n^{\log_2 8} = n^3$ so $\boxed{n^3 = 8^{\log_2 n}}$

Comparing $8^{\log_2 n}$, $n^2 \log_2 n$ $\rightarrow \lim\limits_{n \to \infty} \frac{8^{\log_2 n}}{n^2 \log_2 n} = \lim\limits_{n \to \infty} \frac{n^3}{n^2 \log_2 n} = \infty$ so that $\boxed{8^{\log_2 n} > n^2 \log_2 n}$

Comparing $n^2 \log_2 n$, n^2 $\rightarrow \lim\limits_{n \to \infty} \frac{n^2 \log_2 n}{n^2} = \infty$ so $\boxed{n^2 \log_2 n > n^2}$

Comparing n^2, $\sqrt{n}$ $\rightarrow \lim\limits_{n \to \infty} \frac{n^2}{\sqrt{n}} = \infty$ so $\boxed{n^2 > \sqrt{n}}$

comparing $\sqrt{n}$, $\log_2 n$ $\rightarrow \lim\limits_{n \to \infty} \frac{\sqrt{n}}{\log_2 n} = \frac{\infty}{\infty}$ then L'Hospital

$\lim\limits_{n \to \infty} \frac{(\sqrt{n})'}{(\log_2 n)'} = \lim\limits_{n \to \infty} \frac{\ln 2 \; n}{2\sqrt{n}} = \infty$ so that $\boxed{\sqrt{n} > \log n}$

Result = $\boxed{10^n > 2^n > n^3 = 8^{\log_2 n} > n^2 \log_2 n > n^2 > \sqrt{n} > \log_2 n}$

3- a) Let size of Array is n. Then time complexity of program $O(n)$. Because, all operations inside for loop, take $O(1)$ constant time.

Since there is no action affecting the loop variable "i", inside the loop, the loop works arraySize time.

So time complexity of algorithm $O(n)$.

b) When we analyzed the for loop;

when; $i=2$	count++	
$i=3$	$i=2.3=6$	
$i=6$	count ++	
$i=7$	$i=6.7=42$	
$i=42$	count ++	

If we just look at the values of i where count is increased;

$i=2,6,42 \dots$

We obtain the formula i^2+i

$2^2+2=6$, $6^2+6=42 \dots$

Then, we can modify the for loop;

```
for(int i=2; i<=n; i^2+i)
    count ++;
```

Then, the loop variables "i" increased exponentially because i increases i^2+i on each step.

So, time complexity of the function f is

$O(\text{LogLog}\, n)$

4 - a) $\sum_{i=1}^{n} i^2 \log_2 i$ lets say $f(n)=\sum_{i=1}^{n} i^2\log_2 i$ $g(i)=i^2\log_2 i$

$g(i)=i^2\log_2 i \rightarrow$ Non-decreasing ✓

$\rightarrow \int_0^n g(i)\,di \leq f(n) \leq \int_1^{n+1} g(i)\,di$

$\rightarrow \int_0^n i^2\log_2 i\,di \leq f(n) \leq \int_1^{n+1} i^2\log_2 i\,di$

Note= We use the integration by part method to get integration

$\int i^2\log_2 i\,di = \frac{1}{\ln 2}\int i^2 \ln i\,di \rightarrow$ Then using method $\boxed{uv-\int v\,du}$

$u=\ln i$ $dv=i^2\,di$ $\Rightarrow \frac{1}{\ln 2}\left(\overset{u}{\ln i}\cdot\overset{v}{\frac{i^3}{3}}-\int \overset{v}{\frac{i^3}{3}}\cdot\overset{du}{\frac{1}{i}\,di}\right)$

$du=\frac{1}{i}\,di$ $v=\frac{i^3}{3}$

$\rightarrow \frac{i^3\ln i}{3\ln 2}-\frac{1}{3\ln 2}\int i^2\,di = \frac{i^3\ln i}{3\ln 2}-\frac{i^3}{9\ln 2}=\frac{3i^3\ln i-i^3}{9\ln 2}$ //

So; $\left.\frac{3i^3\ln i-i^3}{9\ln 2}\right|_0^n \leq f(n) \leq \left.\frac{3i^3\ln i-i^3}{9\ln 2}\right|_1^{n+1}$

undefined $0.-\infty$ $\leq f(n) \leq \frac{3(n+1)^3\ln n+1-(n+1)^3+1}{9\ln 2}$

upper bound

$\boxed{f(n)\in O(n^3\log n)}$

For lower bond;

$f(n)=\sum_{i=1}^{n} i^2\log_2 i = 0+\sum_{i=2}^{n} i^2\log_2 i$ then; $0+\int_1^n i^2\log_2 i\,di \leq f(n)$

$\rightarrow \left.\frac{3i^3\ln i-i^3}{9\ln 2}\right|_1^n \leq f(n)$

$\rightarrow \frac{3n^3\ln n-n^3+1}{9\ln 2} \leq f(n)$

$\boxed{f(n)\in\Omega(n^3\log n)}$

lower bound

So; $f(n)\in O(n^3\log n)$

$f(n)\in\Omega(n^3\log n)$

$\boxed{f(n)\in\Theta(n^3\log n)}$

b) $\sum_{i=1}^{n} i^3$ Lets say $f(n)=\sum_{i=1}^{n} i^3$ $g(i)=i^3$

$g(i)=i^3 \longrightarrow$ Non-decreasing ✓

$\rightarrow \int_0^n g(i)\,di \le f(n) \le \int_1^{n+1} g(i)\,di$

$\rightarrow \int_0^n i^3\,di \le f(n) \le \int_0^{n+1} i^3\,di$

$\longrightarrow \frac{i^4}{4}\Big|_0^n \le f(n) \le \frac{i^4}{4}\Big|_1^{n+1}$

$\longrightarrow \underbrace{\frac{n^4}{4}}_{f(n)\in\Omega(n^4)} \le f(n) \le \underbrace{\frac{(n+1)^4-1}{4}}_{f(n)\in O(n^4)}$

So $\boxed{f(n)\in\Theta(n^4)}$

c) $\sum_{i=1}^{n} \frac{1}{2\sqrt{i}}$ Lets say $f(n)=\sum_{i=1}^{n}\frac{1}{2\sqrt{i}}$ $g(i)=\frac{1}{2\sqrt{i}}$

$g(i)=\frac{1}{2\sqrt{i}}$ Harmonic Series $\rightarrow$ non-increasing function

$\rightarrow \int_1^{n+1}\frac{1}{2\sqrt{i}}\,di \le f(n) \le \int_0^n \frac{1}{2\sqrt{i}}\,di$

$\rightarrow \sqrt{i}\Big|_1^{n+1} \le f(n) \le \sqrt{i}\Big|_0^n$

$\rightarrow \underbrace{\sqrt{n+1}-1}_{f(n)\in\Omega(\sqrt{n})} \le f(n) \le \underbrace{\sqrt{n}}_{f(n)\in O(\sqrt{n})}$

So $\boxed{f(n)\in\Theta(\sqrt{n})}$

d) $\sum_{i=1}^{n}\frac{1}{i}$ Lets say $f(n)=\sum_{i=1}^{n}\frac{1}{i}$ $g(i)=\frac{1}{i}$

$g(i)=\frac{1}{i}$ Harmonic series $\rightarrow$ non increasing function

$\int_1^{n+1}\frac{1}{i} \le f(n) \le \int_0^n \frac{1}{i}$

$\underbrace{\ln(n+1)-\ln 1} \le f(n) \le \underbrace{\ln i - \ln 0}_{\infty}$

$f(n)\in\Omega(\log n)$

For upper bound;

$\sum_{i=1}^{n}\frac{1}{i} = 1+\sum_{i=2}^{n}\frac{1}{i}$

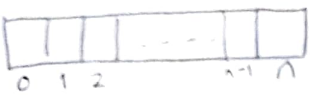
$f(n)\le 1+\int_1^n \frac{1}{i}$

$f(n)\le 1+\ln x\,\Big|_1^n$

$\underbrace{f(n)\le \ln n+1}$

$f(n)\in O(\log n)$

So $\boxed{f(n)\in\Theta(\log n)}$

5 - Arr  (can have repeated elmants)

Solutions 1; The searched element is returned the moment it is found.

Lets say searched element is x. Then;

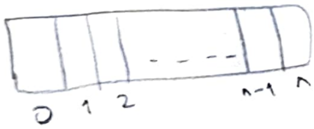
If $x = Arr[0]$ then best case occures.

$\boxed{B(n) = 1 \in \Theta(1)}$

If $x = Arr[n]$ or not on the list then worst case occures.

$\boxed{W(n) = n \in \Theta(n)}$

Solutions 2; When the searching element is found, it continues to search if there is more than searching element. Then returns all locations of searched element.



In this case, even if we find the wanted element in the first index, we have to look through the whole list to see if there is another.

So, Best case $n \in \Theta(n)$
Worst case $n \in \Theta(n)$