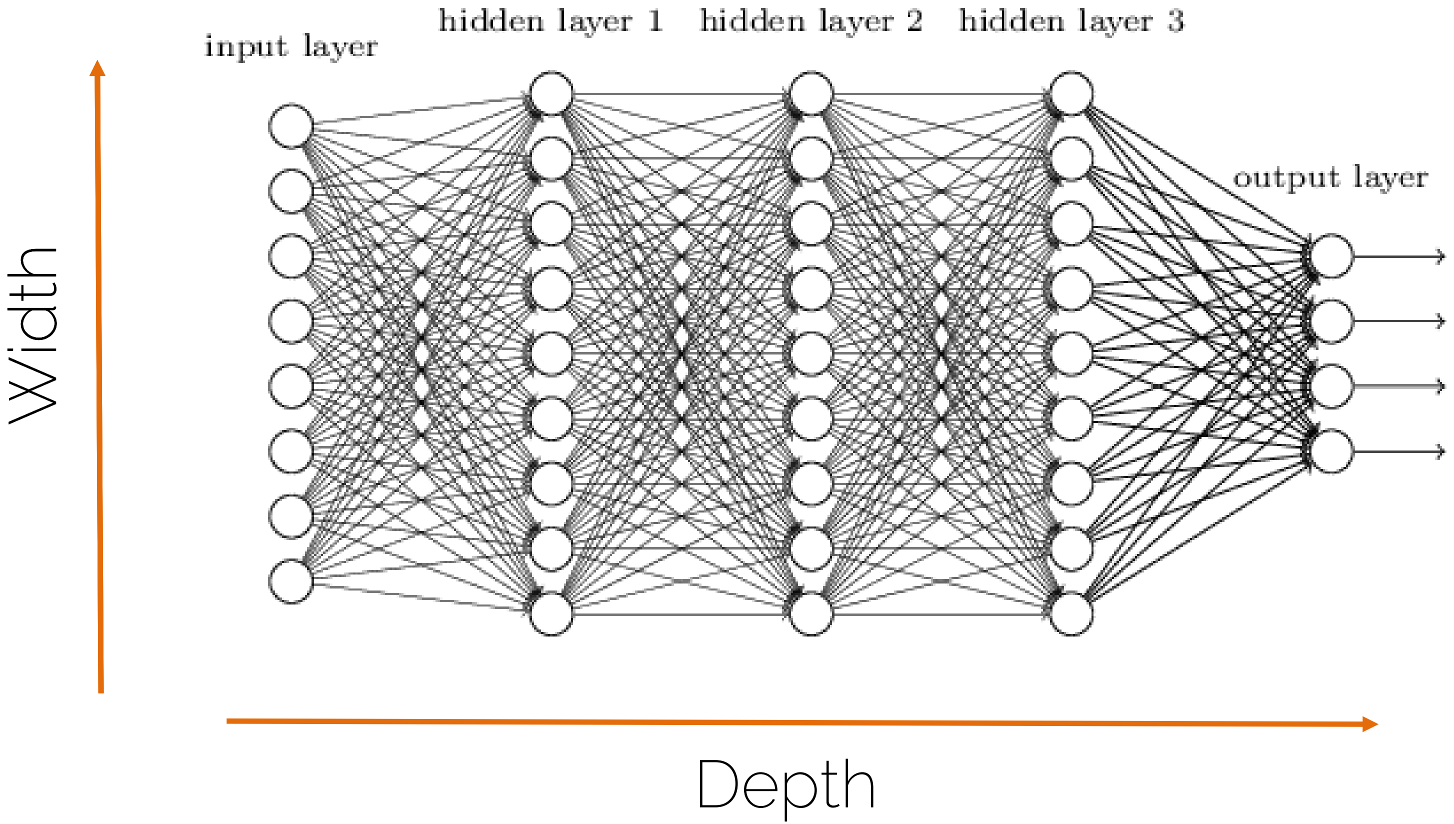
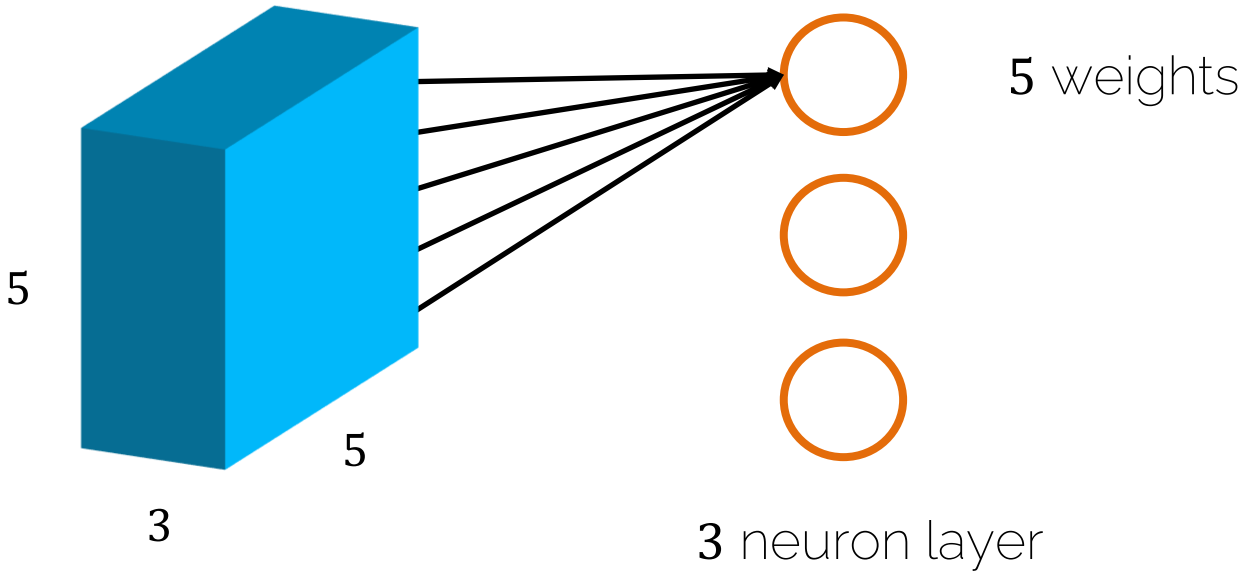
Lecture 9 - Convolutional Neural Networks

Fully Connected Neural Network



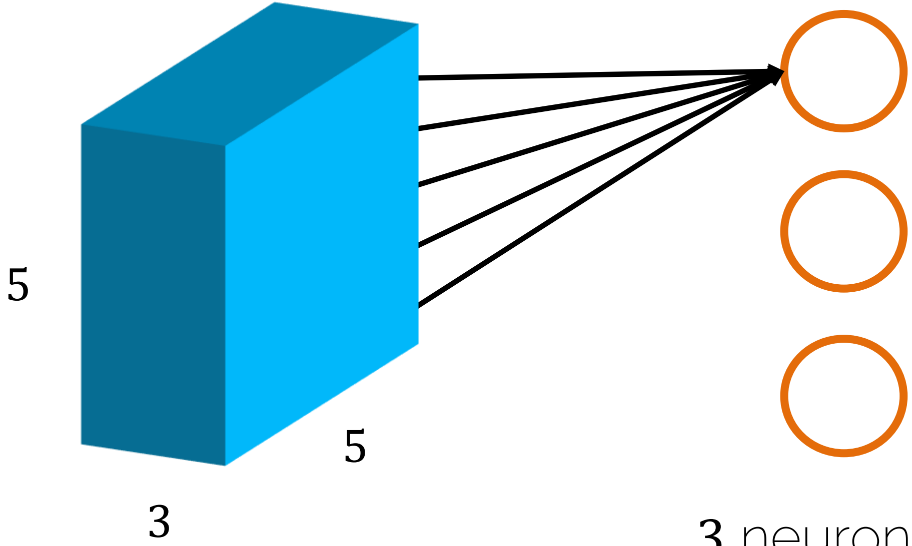
Problems using FC Layers on Images

- How to process a tiny image with FC layers



Problems using FC Layers on Images

- How to process a tiny image with FC layers



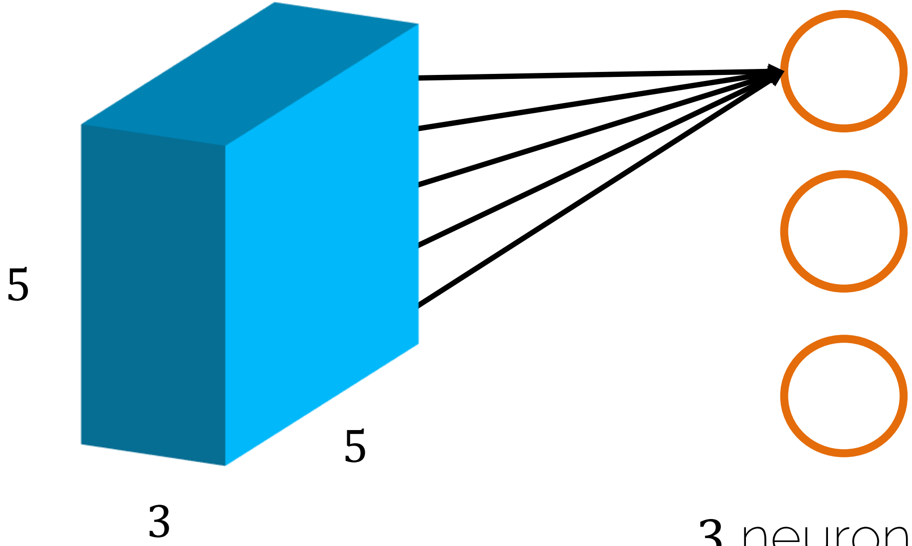
25 weights

For the whole 5×5 image on 1 channel

3 neuron layer

Problems using FC Layers on Images

- How to process a tiny image with FC layers



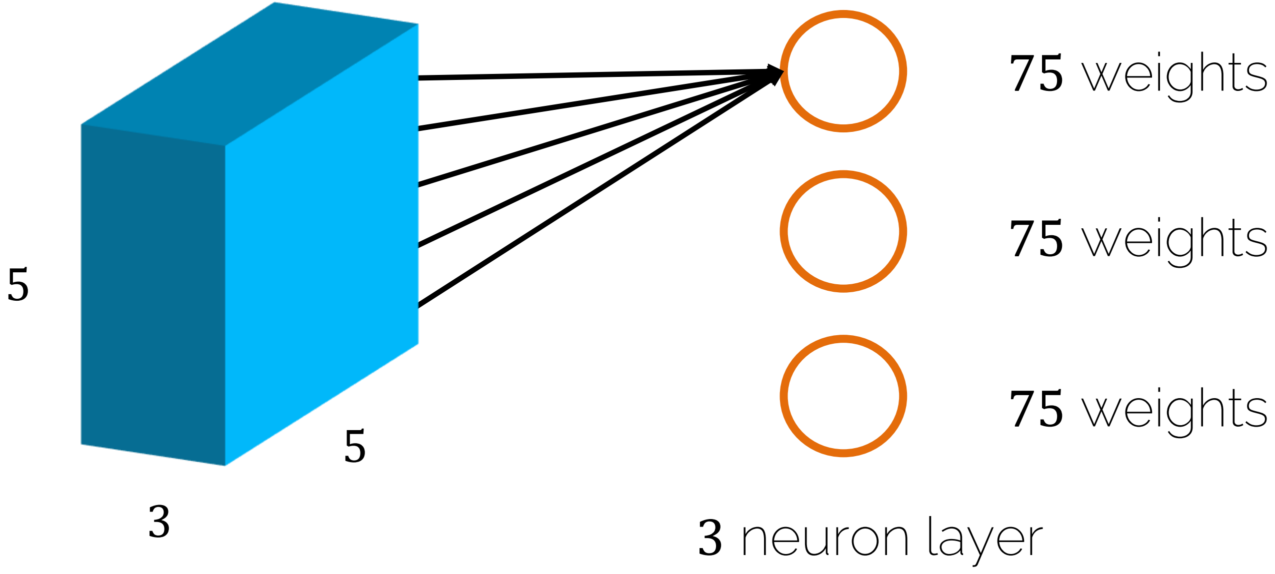
75 weights

For the whole 5×5 image on the 3 channel

3 neuron layer

Problems using FC Layers on Images

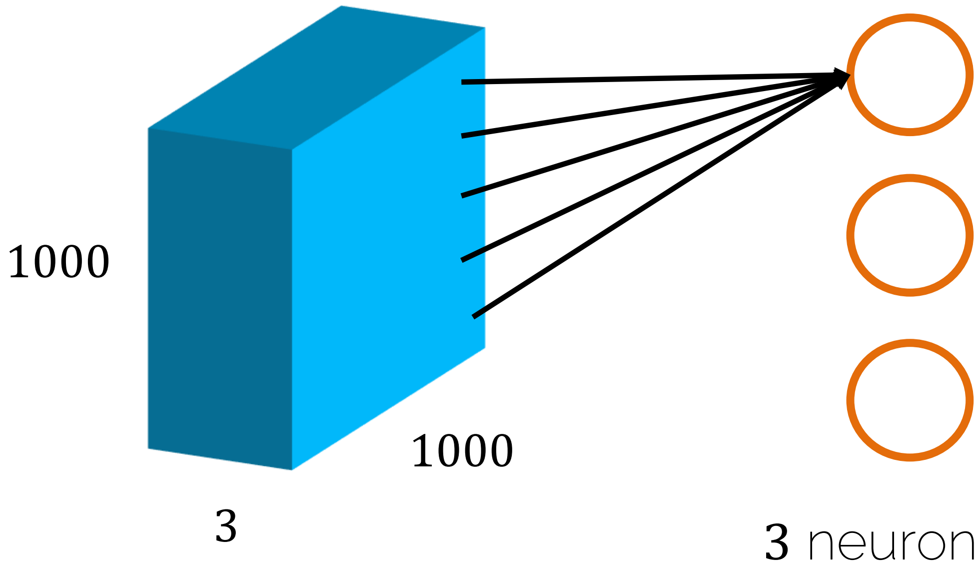
- How to process a tiny image with FC layers



For the whole 5×5 image on the three channels **per neuron**

Problems using FC Layers on Images

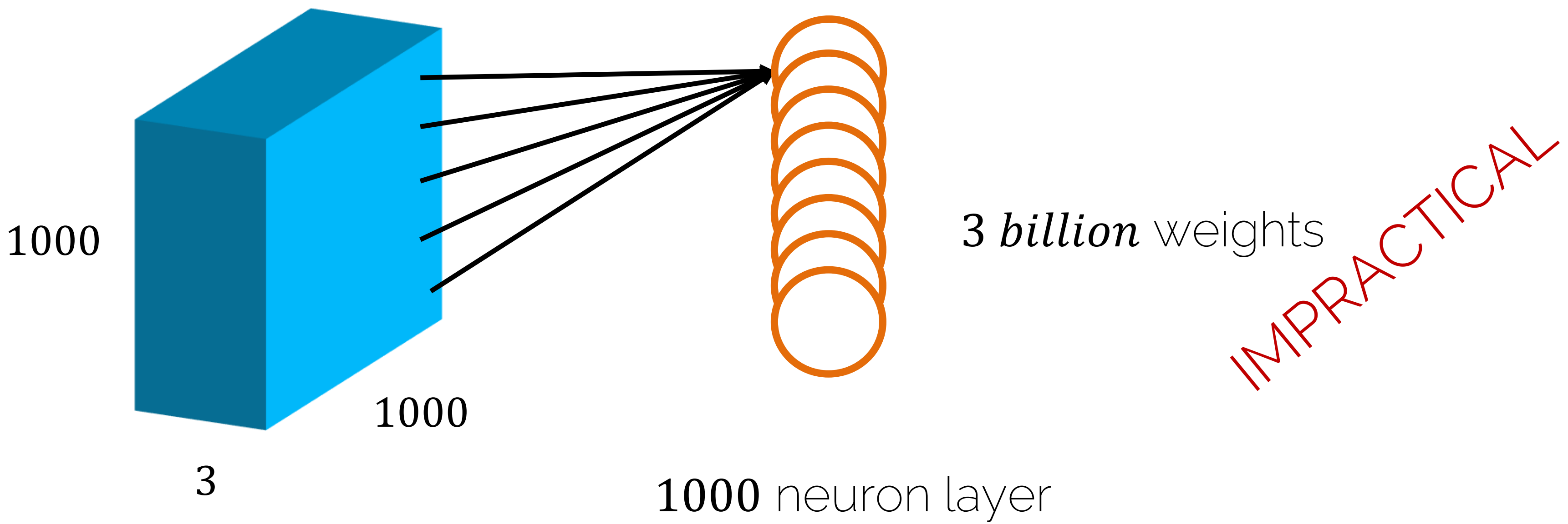
- How to process a **normal** image with FC layers



3 neuron layer

Problems using FC Layers on Images

- How to process a **normal** image with FC layers



Why not simply more FC Layers?

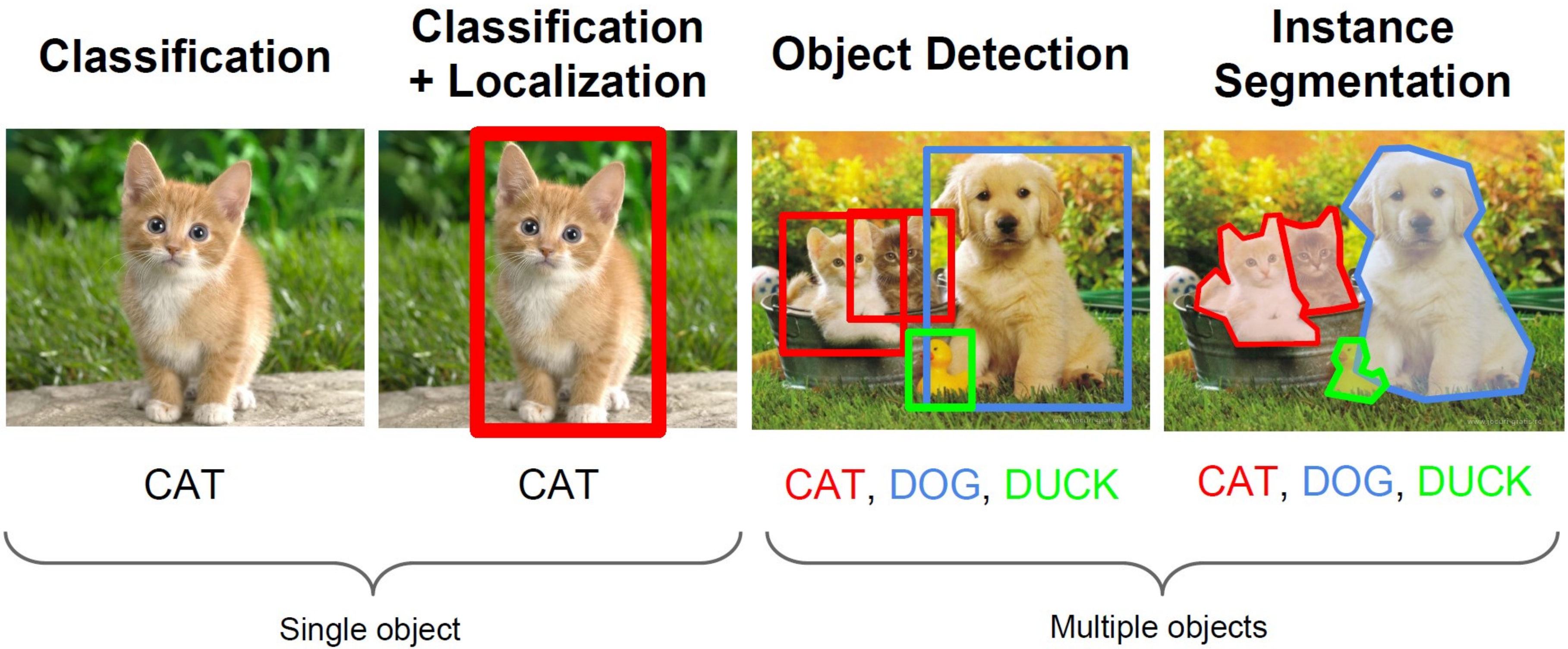
We cannot make networks arbitrarily complex

- Why not just go deeper and get better?
  - No structure!!
  - It is just brute force!
  - Optimization becomes hard
  - Performance plateaus / drops!

Better Way than FC ?

- We want to restrict the degrees of freedom
  - We want a layer with structure
  - Weight sharing → using the same weights for different parts of the image

Using CNNs in Computer Vision

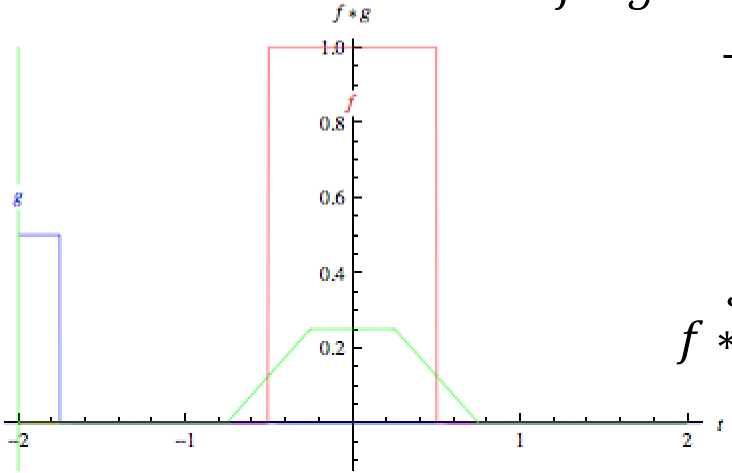


[Li et al., CS231n Course Slides] Lecture 12: Detection and Segmentation

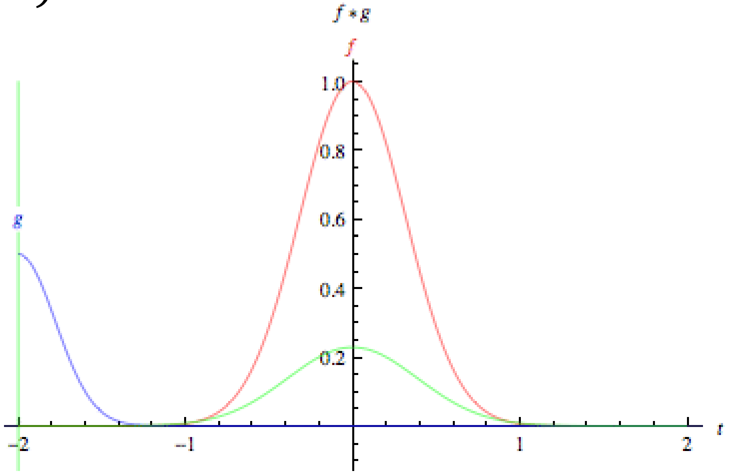
Convolutions

What are Convolutions?

$$f * g = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$



f = red
g = blue
$f * g$ = green



Convolution of two box functions

Convolution of two Gaussians

Application of a filter to a function

— The 'smaller' one is typically called the filter kernel

What are Convolutions?

Discrete case: box filter

f	**4**	**3**	**2**	**-5**	**3**	**5**	**2**	**5**	**5**	**6**
g	**1/3**	**1/3**	**1/3**							

'Slide' **filter kernel** from left to right; at each position, compute a single value in the output data

What are Convolutions?

Discrete case: box filter

f	**4**	**3**	**2**	**-5**	**3**	**5**	**2**	**5**	**5**	**6**
g	**1/3**	**1/3**	**1/3**							
$f * g$		**3**								

$$4 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = 3$$

What are Convolutions?

Discrete case: box filter

f	4	3	2	-5	3	5	2	5	5	6
g		1/3	1/3	1/3						
$f * g$		3	0							

$$3 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + (-5) \cdot \frac{1}{3} = 0$$

What are Convolutions?

Discrete case: box filter

f	**4**	**3**	**2**	**-5**	**3**	**5**	**2**	**5**	**5**	**6**
g			**1/3**	**1/3**	**1/3**					
$f * g$		**3**	**0**	**0**						

$$2 \cdot \frac{1}{3} + (-5) \cdot \frac{1}{3} + 3 \cdot \frac{1}{3} = 0$$

What are Convolutions?

Discrete case: box filter

f	**4**	**3**	**2**	**-5**	**3**	**5**	**2**	**5**	**5**	**6**
g				**1/3**	**1/3**	**1/3**				
$f * g$		**3**	**0**	**0**	**1**					

$$(-5) \cdot \frac{1}{3} + 3 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} = 1$$

What are Convolutions?

Discrete case: box filter

f	4	3	2	-5	3	5	2	5	5	6
g					1/3	1/3	1/3			
$f * g$		3	0	0	1	10/3				

$$3 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = \frac{10}{3}$$

What are Convolutions?

Discrete case: box filter

f	4	3	2	-5	3	5	2	5	5	6
g						1/3	1/3	1/3		
$f * g$		3	0	0	1	10/3	4			

$$5 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} = 4$$

What are Convolutions?

Discrete case: box filter

f	4	3	2	-5	3	5	2	5	5	6
g							1/3	1/3	1/3	
$f * g$		3	0	0	1	10/3	4	4		

$$2 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} = 4$$

What are Convolutions?

Discrete case: box filter

f	4	3	2	-5	3	5	2	5	5	6
g								1/3	1/3	1/3
$f * g$		3	0	0	1	10/3	4	4	16/3	

$$5 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} + 6 \cdot \frac{1}{3} = \frac{16}{3}$$

What are Convolutions?

Discrete case: box filter

4	3	2	-5	3	5	2	5	5	6

1/3	1/3	1/3

??	3	0	0	1	10/3	4	4	16/3	??

What to do at boundaries?

What are Convolutions?

Discrete case: box filter

4	3	2	-5	3	5	2	5	5	6

1/3	1/3	1/3

??	3	0	0	1	10/3	4	4	16/3	??

What to do at boundaries?

Option 1: Shrink   1. -> ignore

3	0	0	1	10/3	4	4	16/3

What are Convolutions?

Discrete case: box filter

0	4	3	2	-5	3	5	2	5	5	6	0

1/3	1/3	1/3

??	3	0	0	1	10/3	4	4	16/3	??

$$0 \cdot \frac{1}{3} + 4 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3} = \frac{7}{3}$$

What to do at boundaries?

→ Pad with 0

Option 2: **Pad** (often 0's)

7/3	3	0	0	1	10/3	4	4	16/3	11/3

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6		

$$5 \cdot 3 + (-1) \cdot 3 + (-1) \cdot 2 + (-1) \cdot 0 + (-1) \cdot 4$$
$$= 15 - 9 = 6$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	

$$5 \cdot 2 + (-1) \cdot 2 + (-1) \cdot 1 + (-1) \cdot 3 + (-1) \cdot 3$$
$$= 10 - 9 = 1$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	8

$$5 \cdot 1 + (-1) \cdot (-5) + (-1) \cdot (-3) + (-1) \cdot 3$$
$$+(-1) \cdot 2$$
$$= 5 + 3 = 8$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	8
-7		

$$5 \cdot 0 + (-1) \cdot 3 + (-1) \cdot 0 + (-1) \cdot 1 + (-1) \cdot 3$$
$$= 0 - 7 = -7$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	8
-7	9	

$$5 \cdot 3 + (-1) \cdot 2 + (-1) \cdot 3 + (-1) \cdot 1 + (-1) \cdot 0$$
$$= 15 - 6 = 9$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	8
-7	9	2

$$5 \cdot 3 + (-1) \cdot 1 + (-1) \cdot 5 + (-1) \cdot 4 + (-1) \cdot 3$$
$$= 15 - 13 = 2$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	8
-7	9	2
-5		

$$5 \cdot 0 + (-1) \cdot 0 + (-1) \cdot 1 + (-1) \cdot 6 + (-1) \cdot (-2) = -5$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	8
-7	9	2
-5	-9	

$$5 \cdot 1 + (-1) \cdot 3 + (-1) \cdot 4 + (-1) \cdot 7 + (-1) \cdot 0$$
$$= 5 - 14 = -9$$

Convolutions on Images

Image 5×5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3×3

0	-1	0
-1	5	-1
0	-1	0

Output 3×3

6	1	8
-7	9	2
-5	-9	3

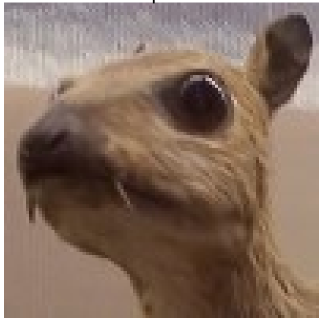
$$5 \cdot 4 + (-1) \cdot 3 + (-1) \cdot 4 + (-1) \cdot 9 + (-1) \cdot 1$$
$$= 20 - 17 = 3$$

Image Filters

- Each kernel gives us a different image filter

LET'S LEARN THESE FILTERS!

Input



Edge detection

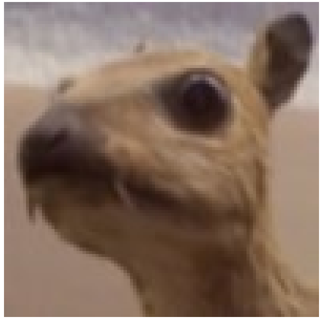
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Box mean

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpen

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$



Gaussian blur

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

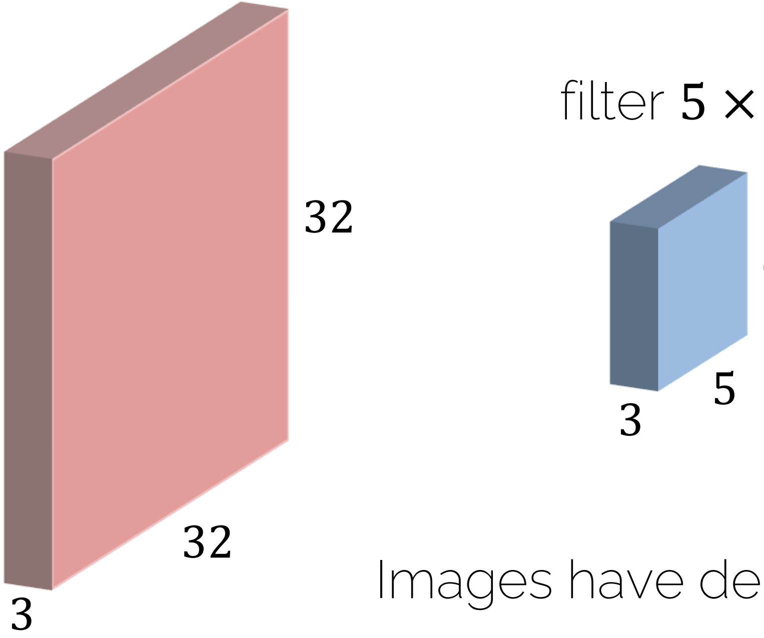
Convolutions on RGB Images

width height depth

image $32 \times 32 \times \boxed{3}$

Depth dimension *must* match;
i.e., filter extends the full depth of the input

filter $5 \times 5 \times \boxed{3}$



Convolve filter with image
i.e., 'slide' over it and:

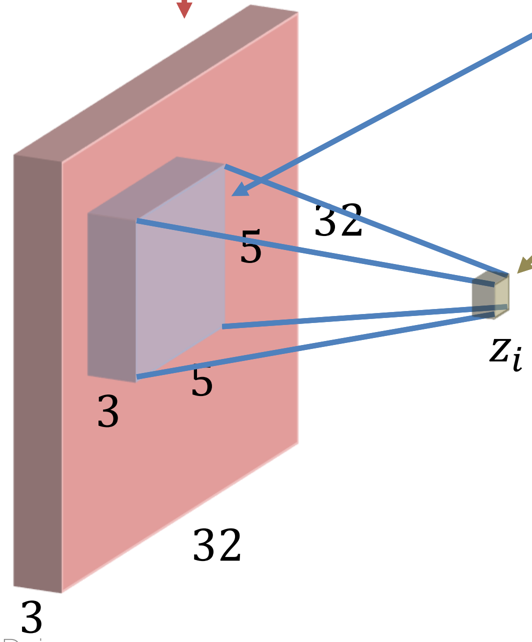
- apply filter at each location
- dot products

Images have depth: e.g. RGB -> 3 channels

Convolutions on RGB Images

$32 \times 32 \times 3$ image (pixels $\boldsymbol{X}$)

$5 \times 5 \times 3$ filter (weights vector $\boldsymbol{w}$)



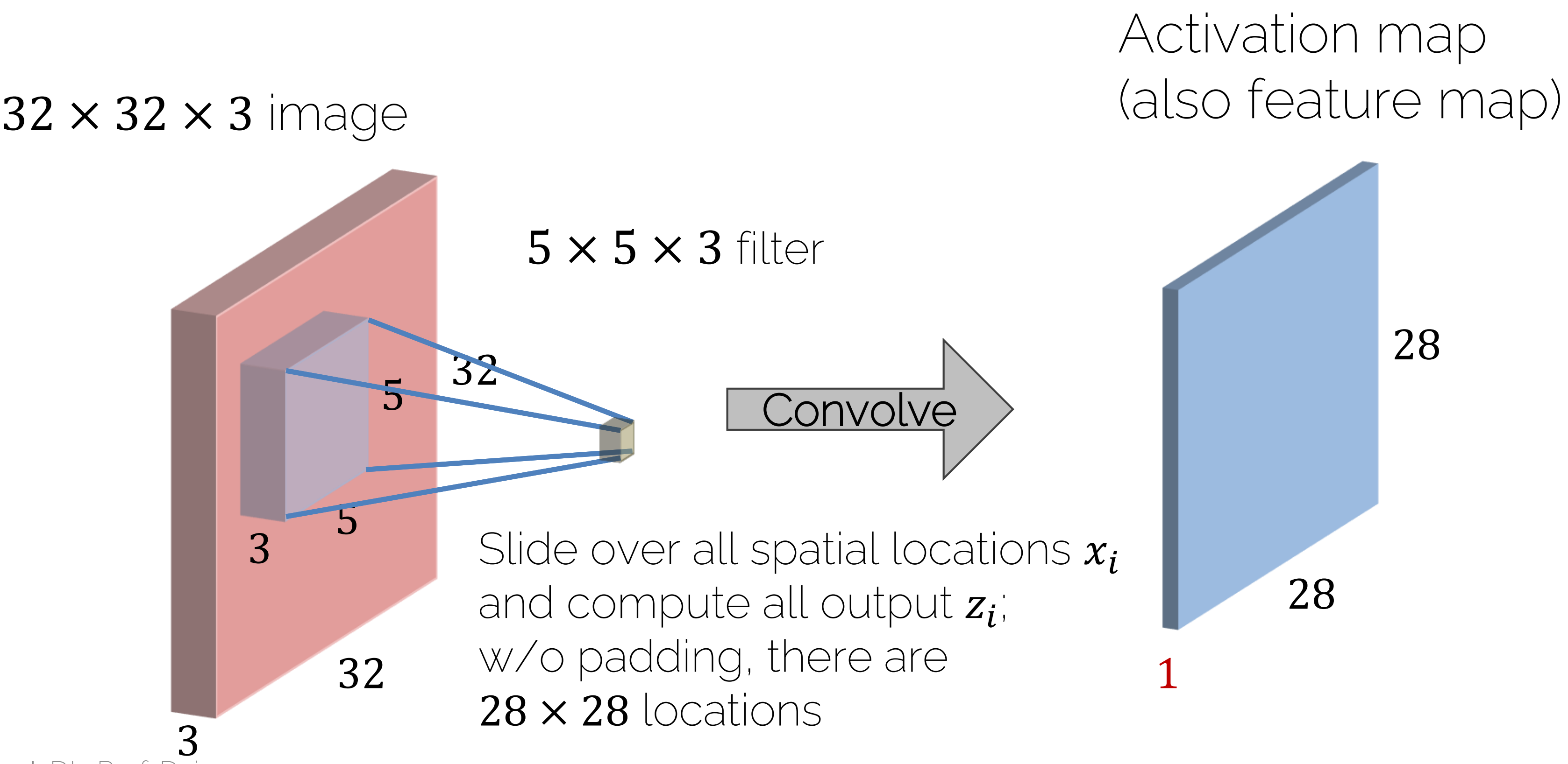
$\color{red}{1}$ number at a time:

equal to dot product between filter weights $\boldsymbol{w}$ and $\boldsymbol{x_i} - th$ chunk of the image. Here: $5 \cdot 5 \cdot 3 = 75$-dim dot product + bias

$$\boxed{z_i = \boldsymbol{w}^T \boldsymbol{x}_i + b}$$

$(5 \times 5 \times 3) \times 1$ $(5 \times 5 \times 3) \times 1$ 1

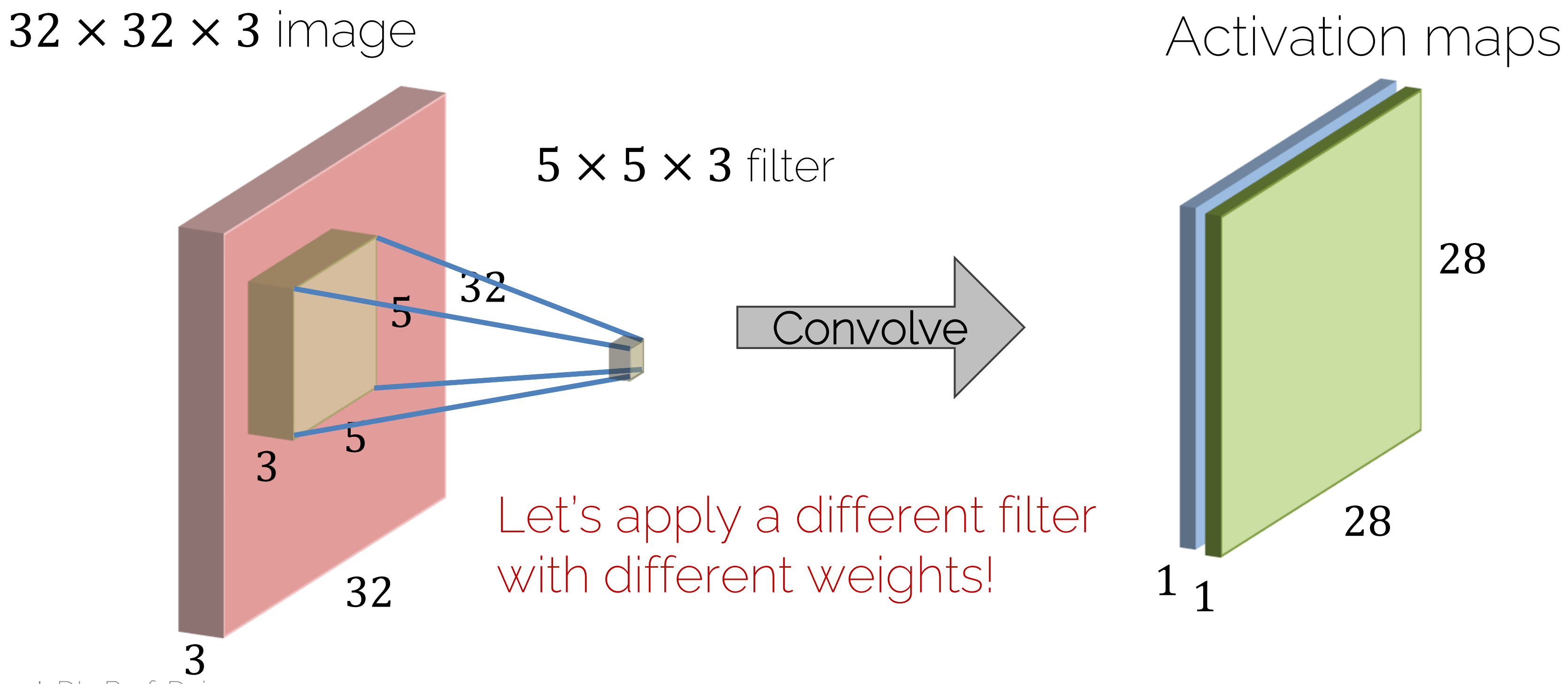
Convolutions on RGB Images



Slide over all spatial locations x_i and compute all output z_i; w/o padding, there are 28×28 locations

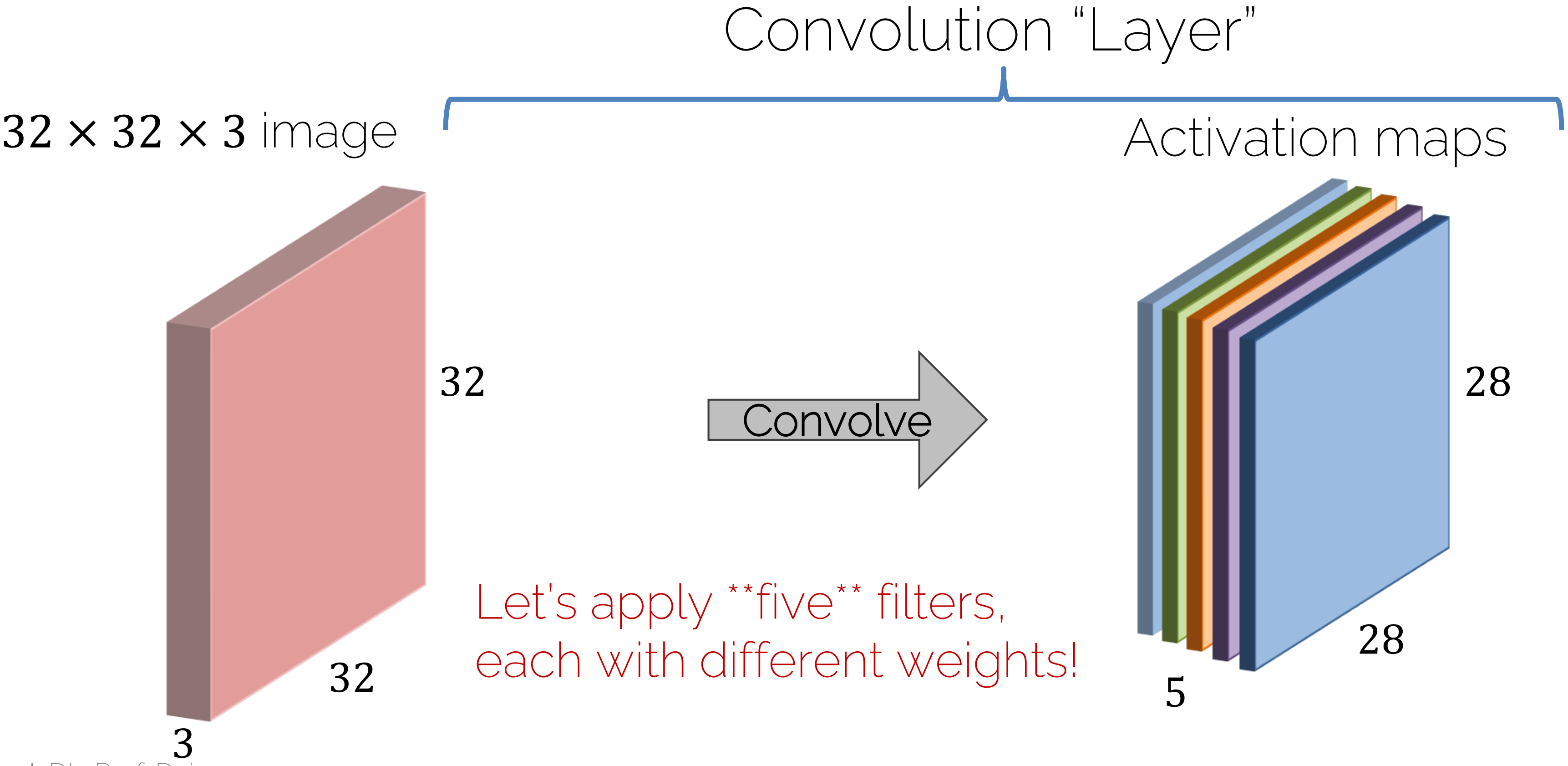
Convolution Layer

Convolution Layer



Let's apply a different filter with different weights!

Convolution Layer



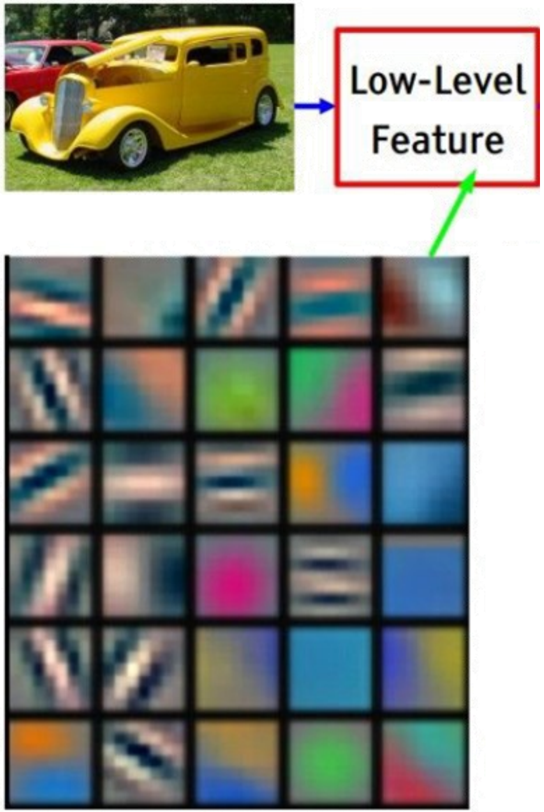
Let's apply **five** filters,
each with different weights!

Convolution Layer

- A basic layer is defined by
  - Filter width and height (depth is implicitly given)
  - Number of different filter banks (#weight sets)

- Each filter captures a different image characteristic

Different Filters

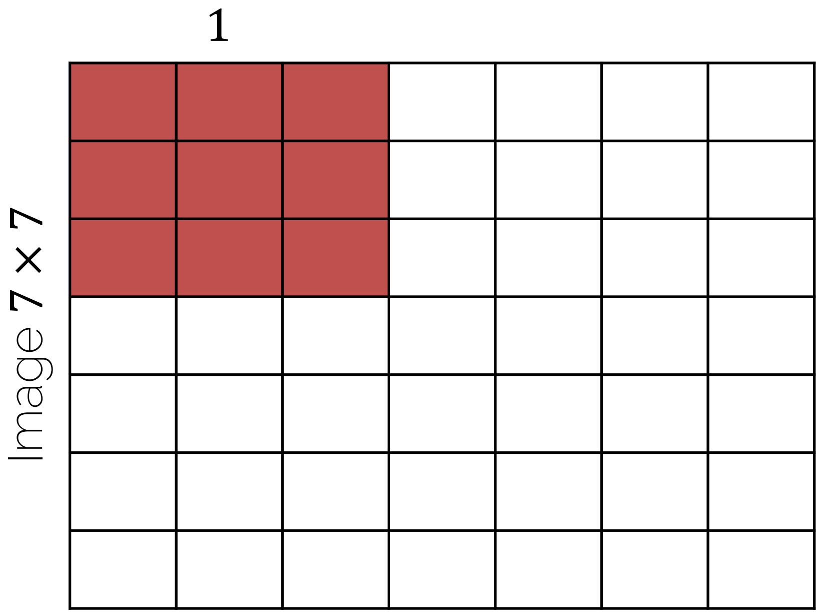


- Each filter captures different image characteristics:
  - Horizontal edges
  - Vertical edges
  - Circles
  - Squares
  - …

[Zeiler & Fergus, ECCV'14] Visualizing and Understanding Convolutional Networks

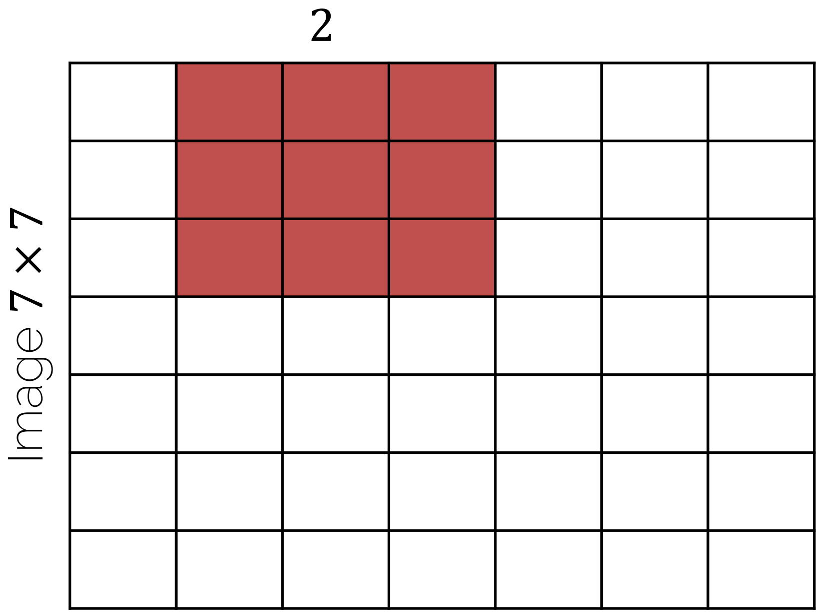
Dimensions of a Convolution Layer

Convolution Layers: Dimensions



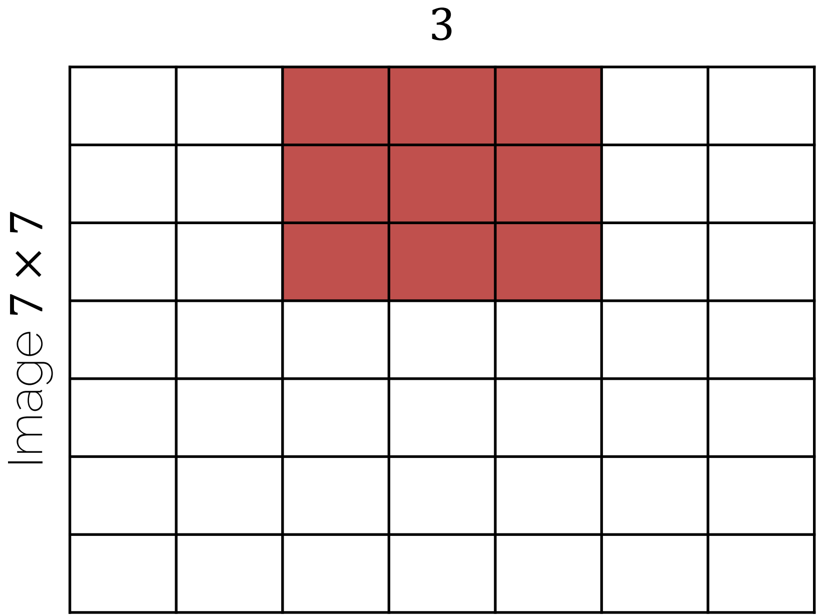
Input: 7×7
Filter: 3×3
Output: 5×5

Convolution Layers: Dimensions



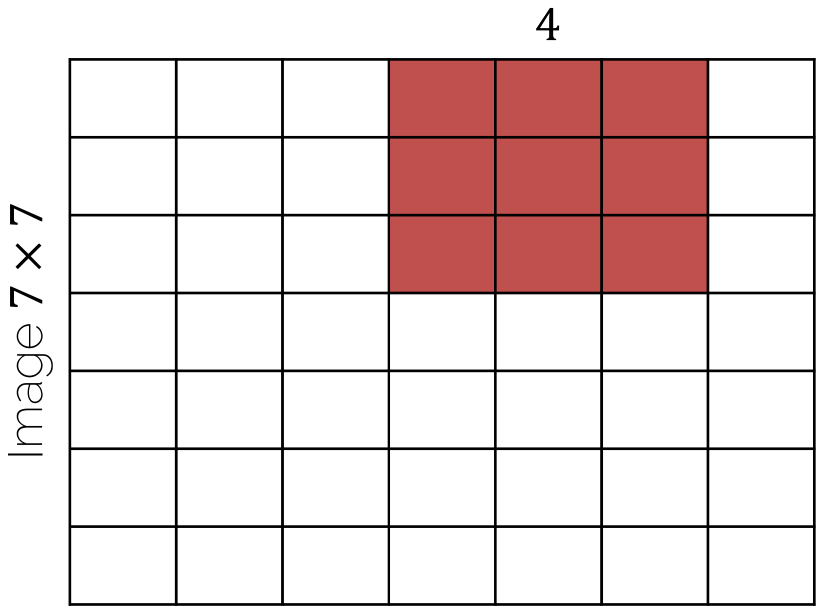
Input: 7×7
Filter: 3×3
Output: 5×5

Convolution Layers: Dimensions



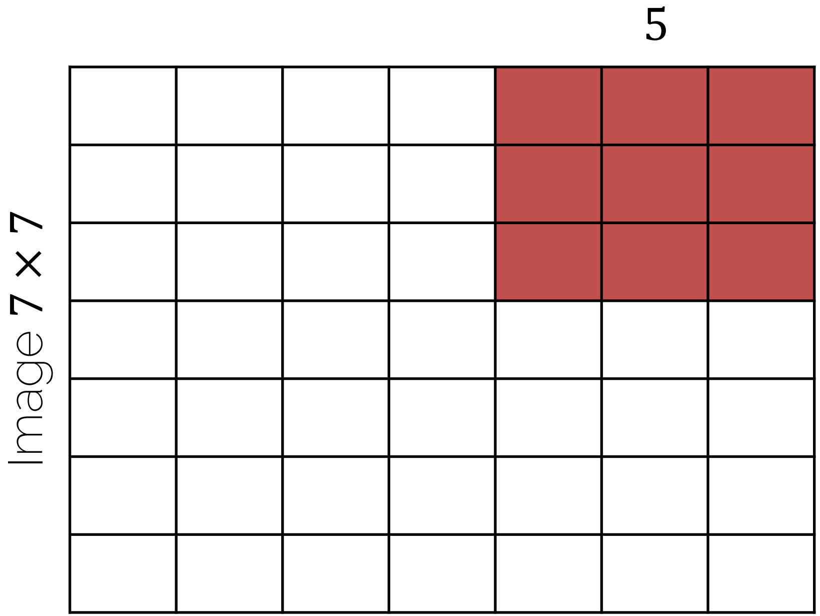
Input: 7×7
Filter: 3×3
Output: 5×5

Convolution Layers: Dimensions



Input: 7×7
Filter: 3×3
Output: 5×5

Convolution Layers: Dimensions

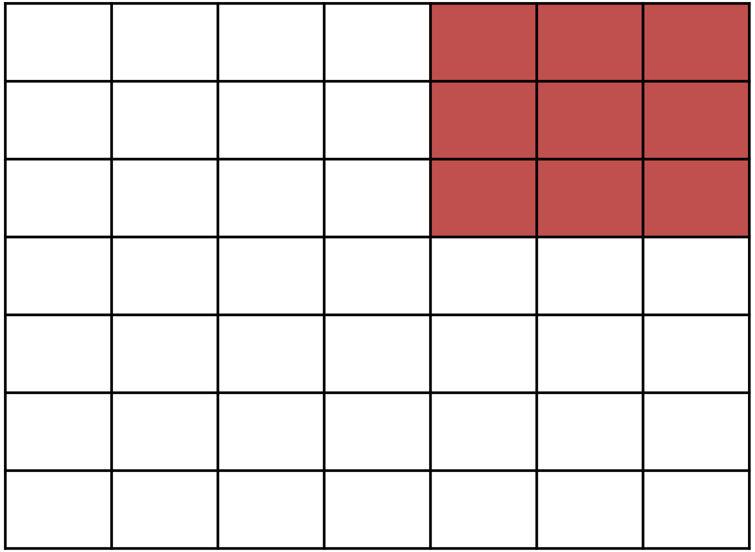


Input: 7×7
Filter: 3×3
Output: 5×5

Convolution Layers: Stride

With a **stride** of 1

Image 7×7



Input: 7×7

Filter: 3×3

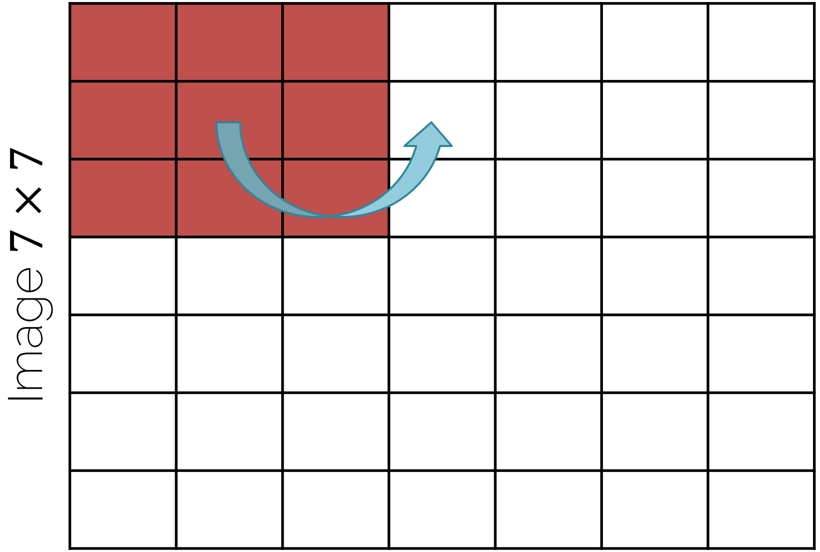
Stride: 1 → How much filter go after applied

Output: 5×5

Stride of S: apply filter every S-th spatial location; i.e. subsample the image

Convolution Layers: Stride

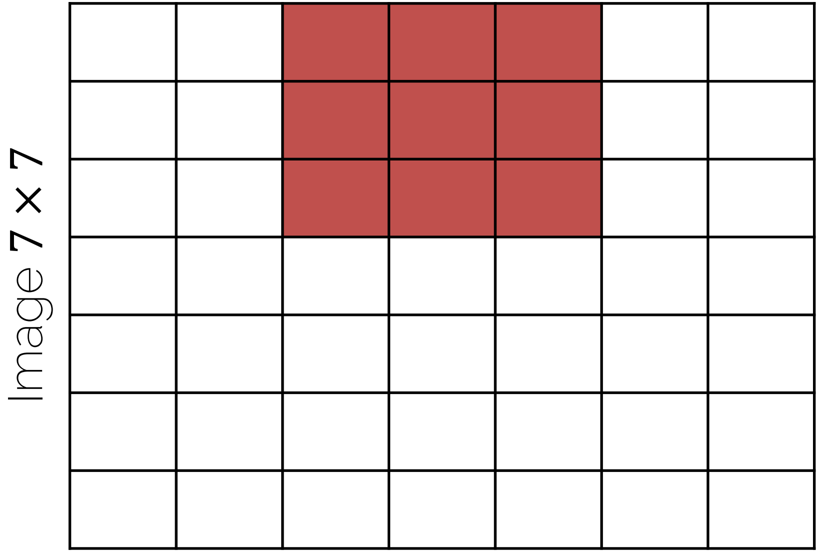
With a **stride** of **2**



Input:	7×7
Filter:	3×3
Stride:	2
Output:	3×3

Convolution Layers: Stride

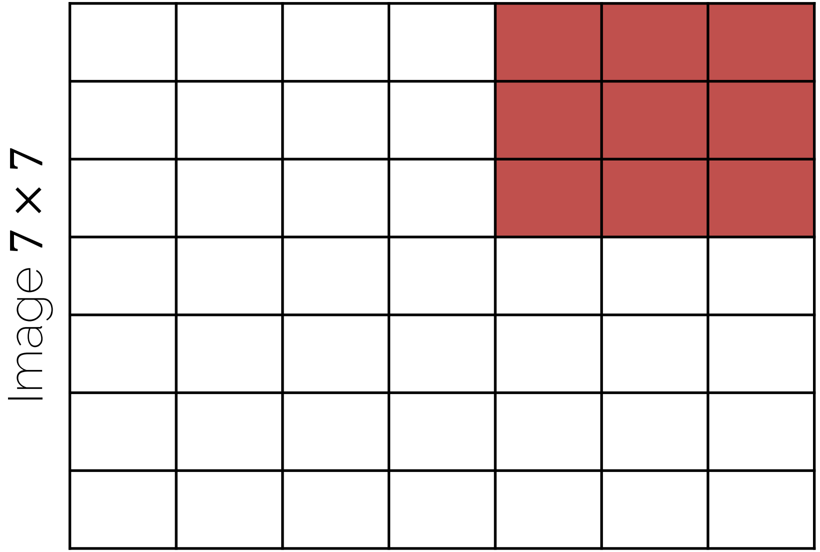
With a **stride** of **2**



Input:	7×7
Filter:	3×3
Stride:	2
Output:	3×3

Convolution Layers: Stride

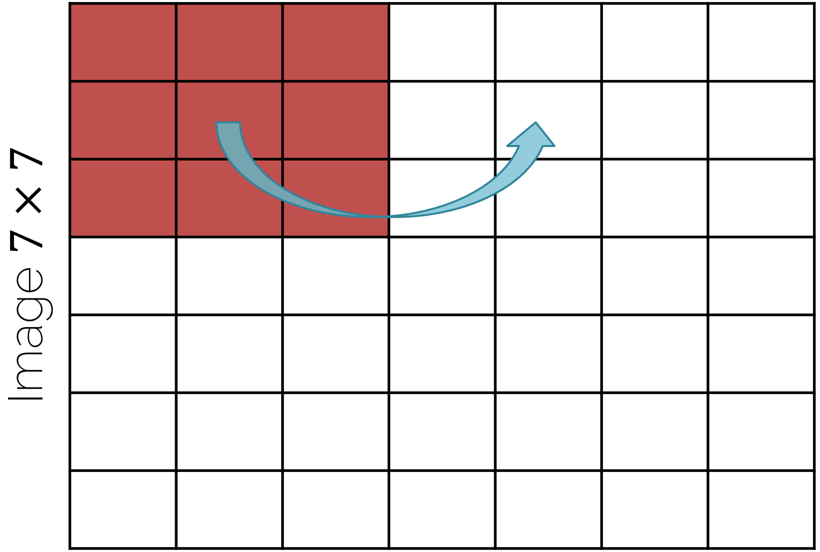
With a **stride** of **2**



Input:	7×7
Filter:	3×3
Stride:	2
Output:	3×3

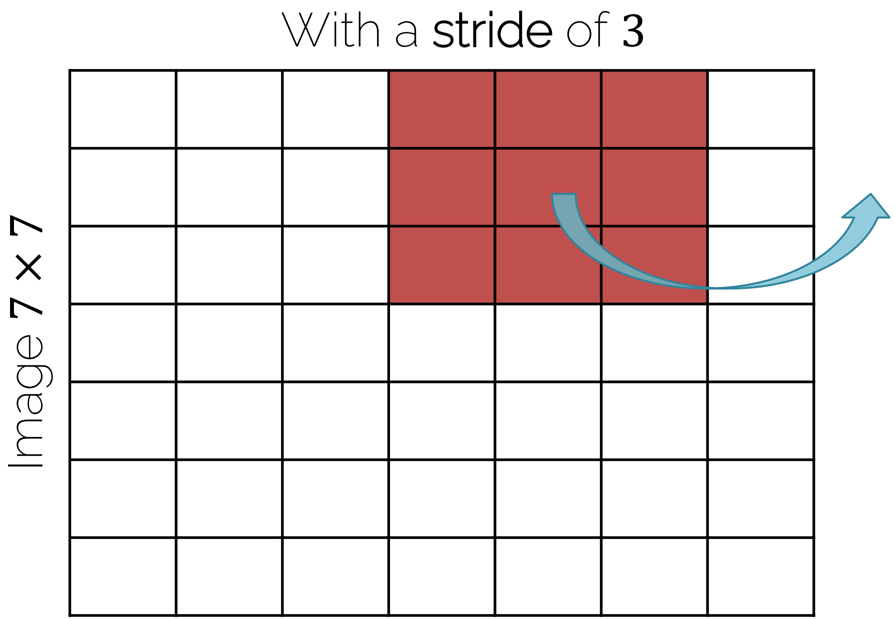
Convolution Layers: Stride

With a **stride** of 3



Input:	7×7
Filter:	3×3
Stride:	3
Output:	$? \times ?$

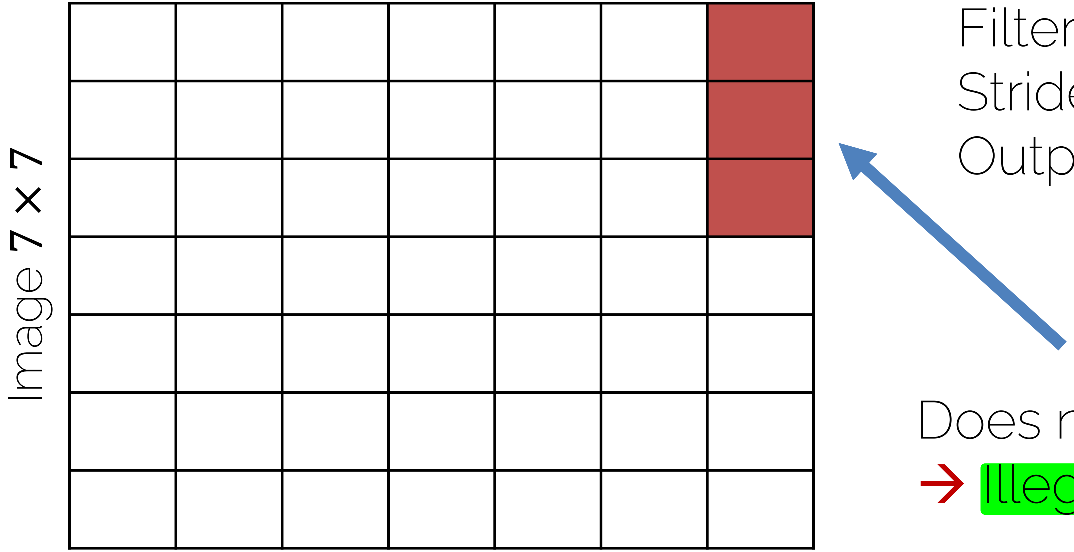
Convolution Layers: Stride



Input: 7×7
Filter: 3×3
Stride: 3
Output: $? \times ?$

Convolution Layers: Stride

With a **stride** of 3

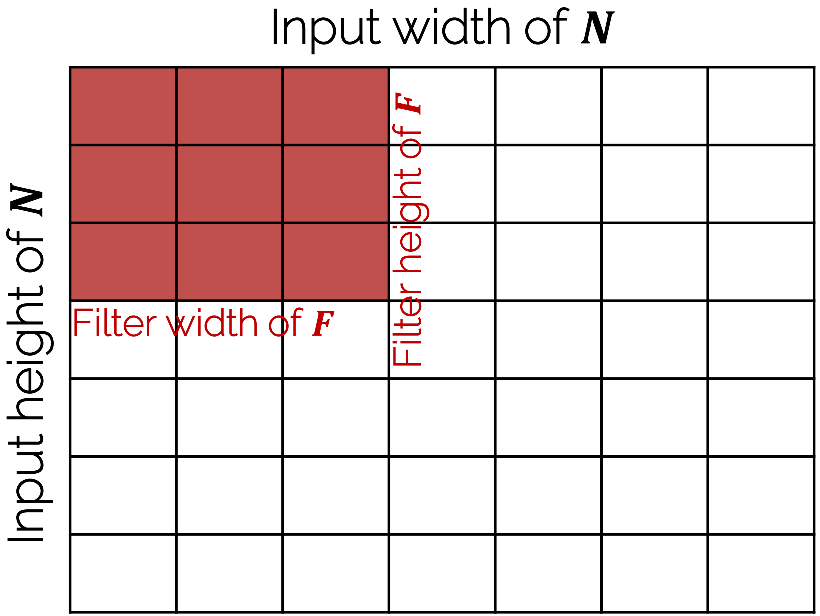


Input:	7×7
Filter:	3×3
Stride:	3
Output:	$? \times ?$

Does not really fit (remainder left)

→ Illegal stride for input & filter size!

Convolution Layers: Dimensions



Input:	$N \times N$
Filter:	$\mathrm{F} \times F$
Stride:	S
Output:	$\left(\frac{N-F}{S}+1\right) \times \left(\frac{N-F}{S}+1\right)$

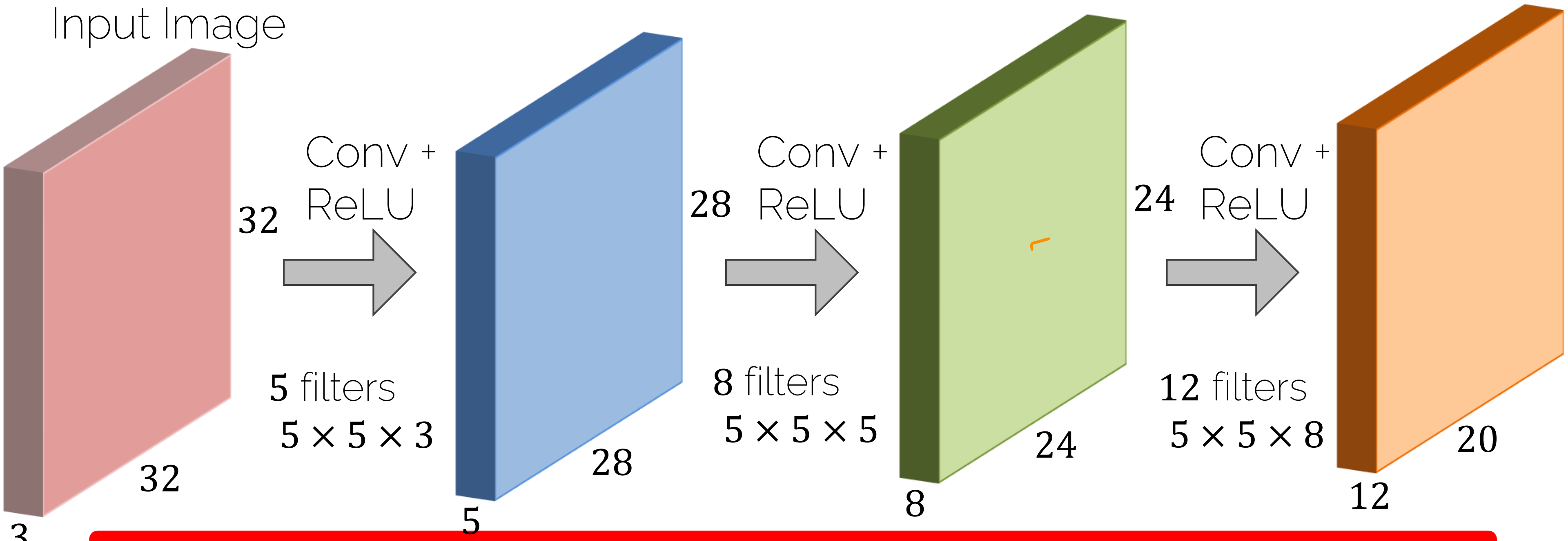
$N = 7, F = 3, S = 1$: $\frac{7-3}{1} + 1 = 5$

$N = 7, F = 3, S = 2$: $\frac{7-3}{2} + 1 = 3$

$N = 7, F = 3, S = 3$: $\frac{7-3}{3} + 1 = 2.\overline{3}$

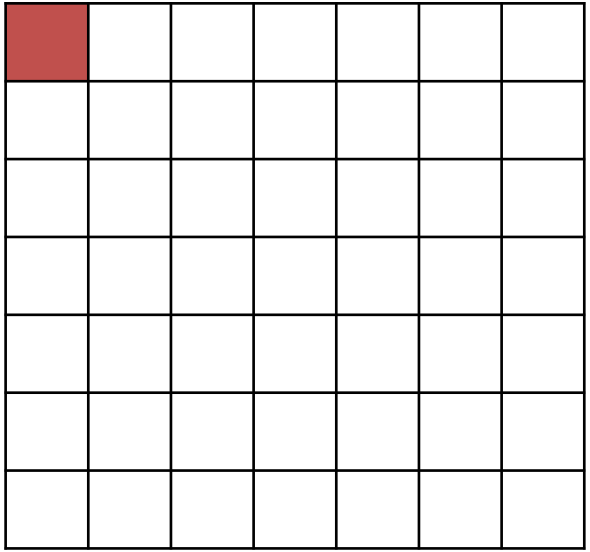
Fractions are illegal

Convolution Layers: Dimensions



Shrinking down so quickly (32→28→24→20) is typically not a good idea…

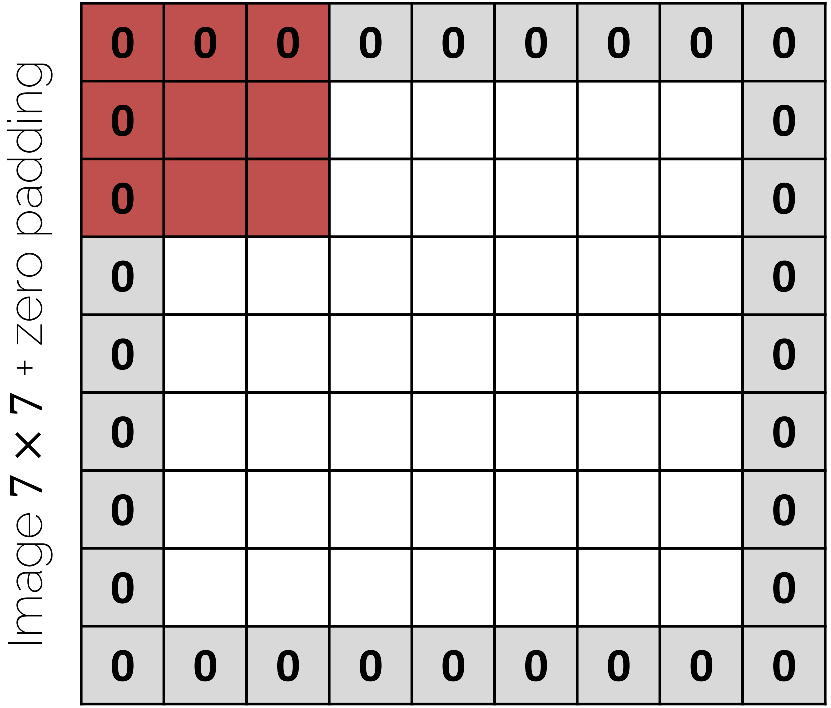
Convolution Layers: Padding



Why padding?

- Sizes get small too quickly
- Corner pixel is only used once

Convolution Layers: Padding

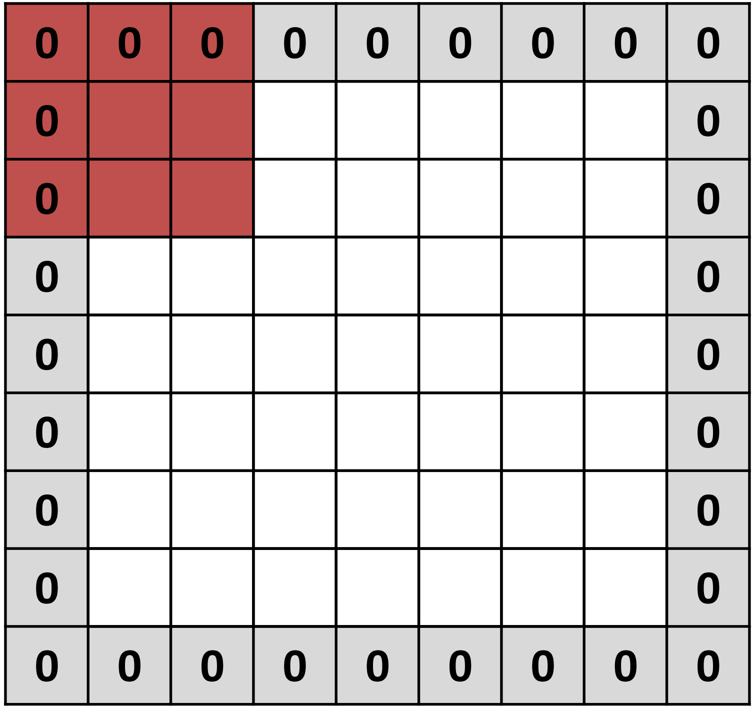


Why padding?

- Sizes get small too quickly
- Corner pixel is only used once

Convolution Layers: Padding

Image 7×7 + zero padding



Input ($N \times N$):	7×7
Filter ($F \times F$):	3×3
Padding (P):	1
Stride (S):	1
Output	7×7

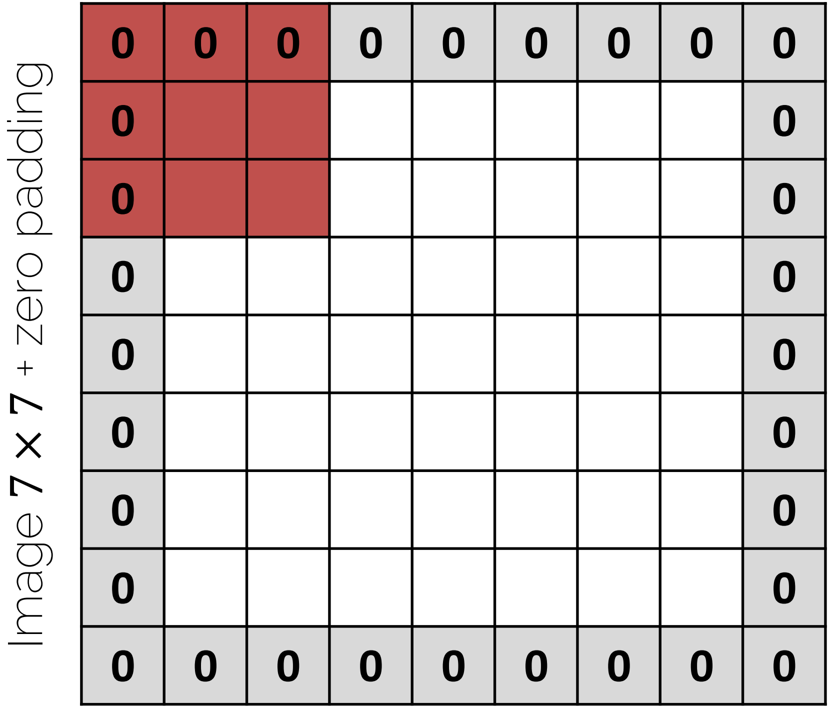
Most common is '*zero*' padding

Output Size:

$$\left(\left\lfloor \frac{N+2\cdot P-F}{S} \right\rfloor + 1\right) \times \left(\left\lfloor \frac{N+2\cdot P-F}{S} \right\rfloor + 1\right)$$

$\lfloor \quad \rfloor$ denotes the floor operator (as in practice an integer division is performed)

Convolution Layers: Padding



Types of convolutions:

- **Valid convolution**: using no padding
- **Same convolution**: output=input size

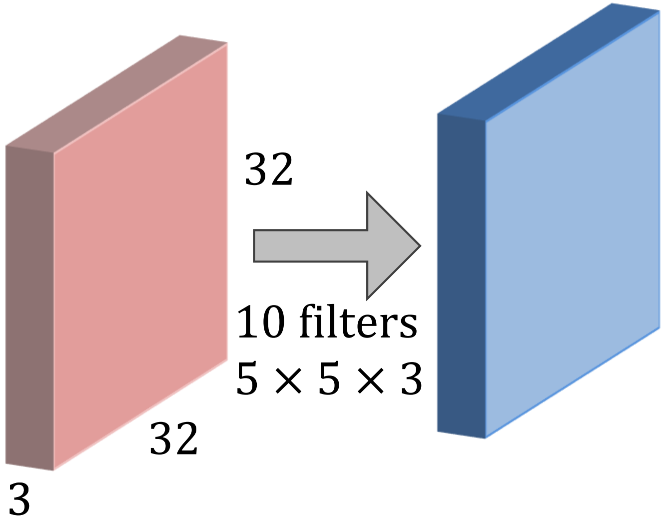
  Set padding to $P = \frac{F-1}{2}$

Convolution Layers: Dimensions

Example

Input image: $32 \times 32 \times 3$
10 filters 5×5
Stride 1
Pad 2

Depth of 3 is implicitly given



Output size is:

$$\frac{32 + 2 \cdot 2 - 5}{1} + 1 = 32$$

i.e. $32 \times 32 \times 10$

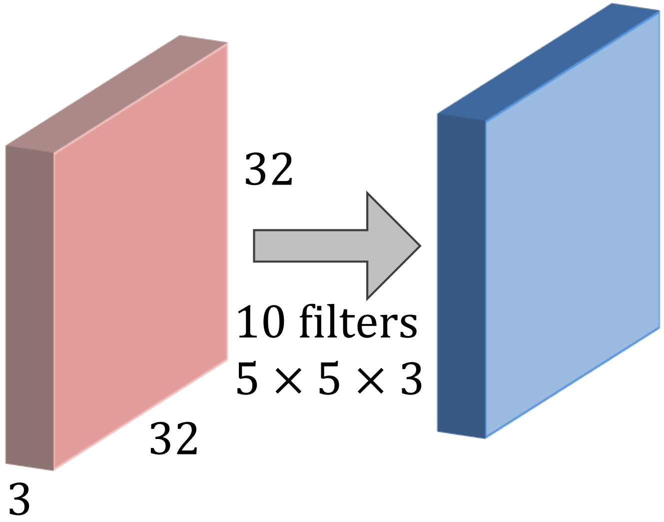
Remember

Output: $\left(\left\lfloor\frac{N+2\cdot P-F}{S}\right\rfloor + 1\right) \times \left(\left\lfloor\frac{N+2\cdot P-F}{S}\right\rfloor + 1\right)$

Convolution Layers: Dimensions

Example

Input image: $32 \times 32 \times 3$
10 filters 5×5
Stride 1
Pad 2



Output size is:

$$\frac{32 + 2 \cdot 2 - 5}{1} + 1 = 32$$

i.e. $32 \times 32 \times 10$

Remember

Output: $\left(\left\lfloor\frac{N+2\cdot P-F}{S}\right\rfloor + 1\right) \times \left(\left\lfloor\frac{N+2\cdot P-F}{S}\right\rfloor + 1\right)$

Convolution Layers: Dimensions

Example

Input image: $32 \times 32 \times 3$
10 filters 5×5
Stride 1
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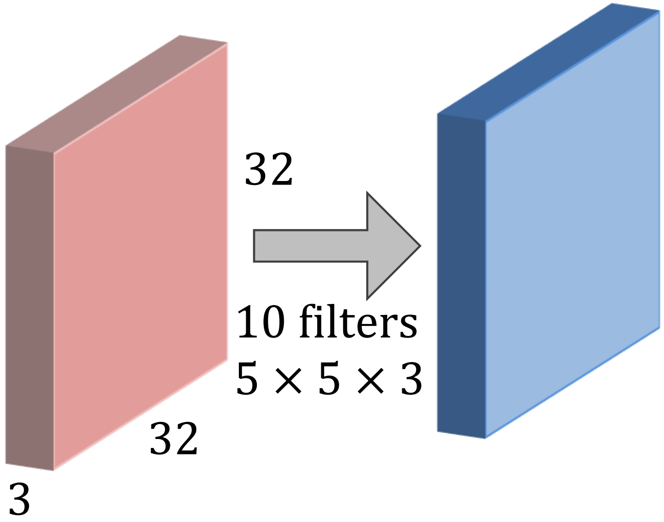


Number of parameters (weights):
Each filter has $5 \times 5 \times 3 + 1 = 76$ params (+1 for bias)
-> $76 \cdot 10 = 760$ parameters in layer

Example

- You are given a convolutional layer with 4 filters, kernel size 5, stride 1, and no padding that operates on an RGB image.

- Q1: What are the dimensions and the shape of its weight tensor?

RGB Filter Kernel. Weights are related with kernels

- ☐ A1: $(3, 4, 5, 5)$
- ☐ A2: $(4, 5, 5)$
- ☐ A3: depends on the width and height of the image

Example

- You are given a convolutional layer with 4 filters, kernel size 5, stride 1, and no padding that operates on an RGB image.

- Q1: What are the dimensions and the shape of its **weight tensor**?

A1: (3, 4, 5, 5)

Input channels (RGB = 3)

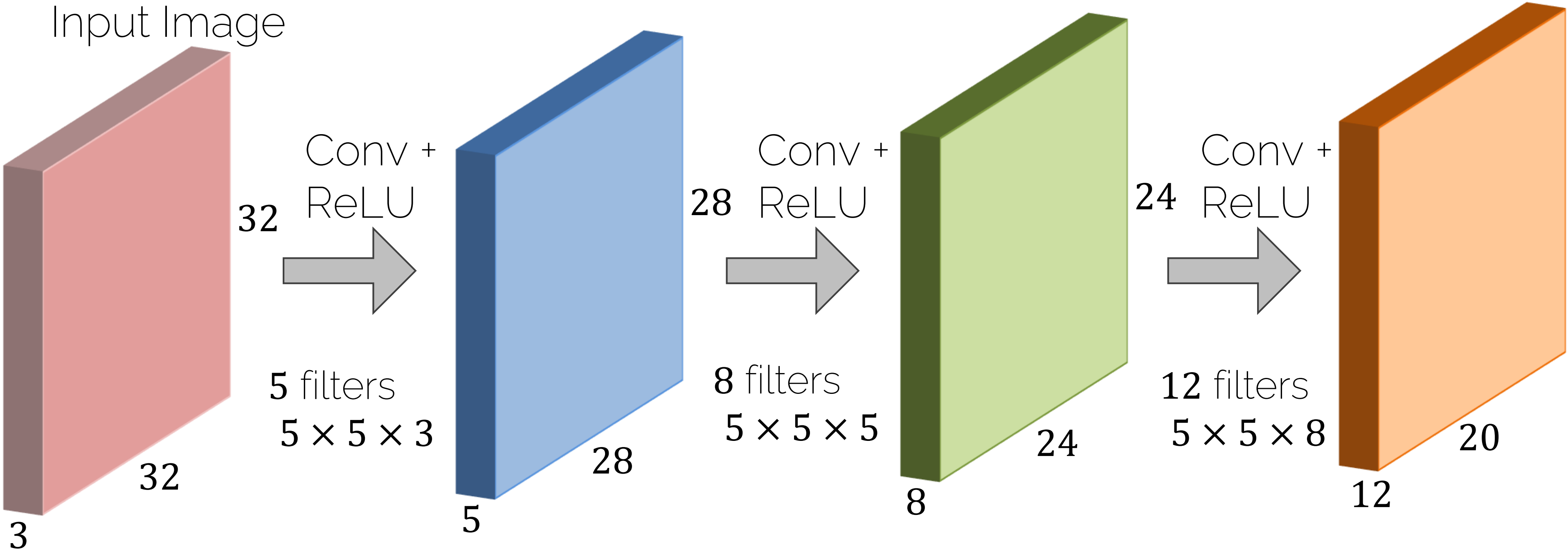
Output size = 4 filters

Filter size = 5×5

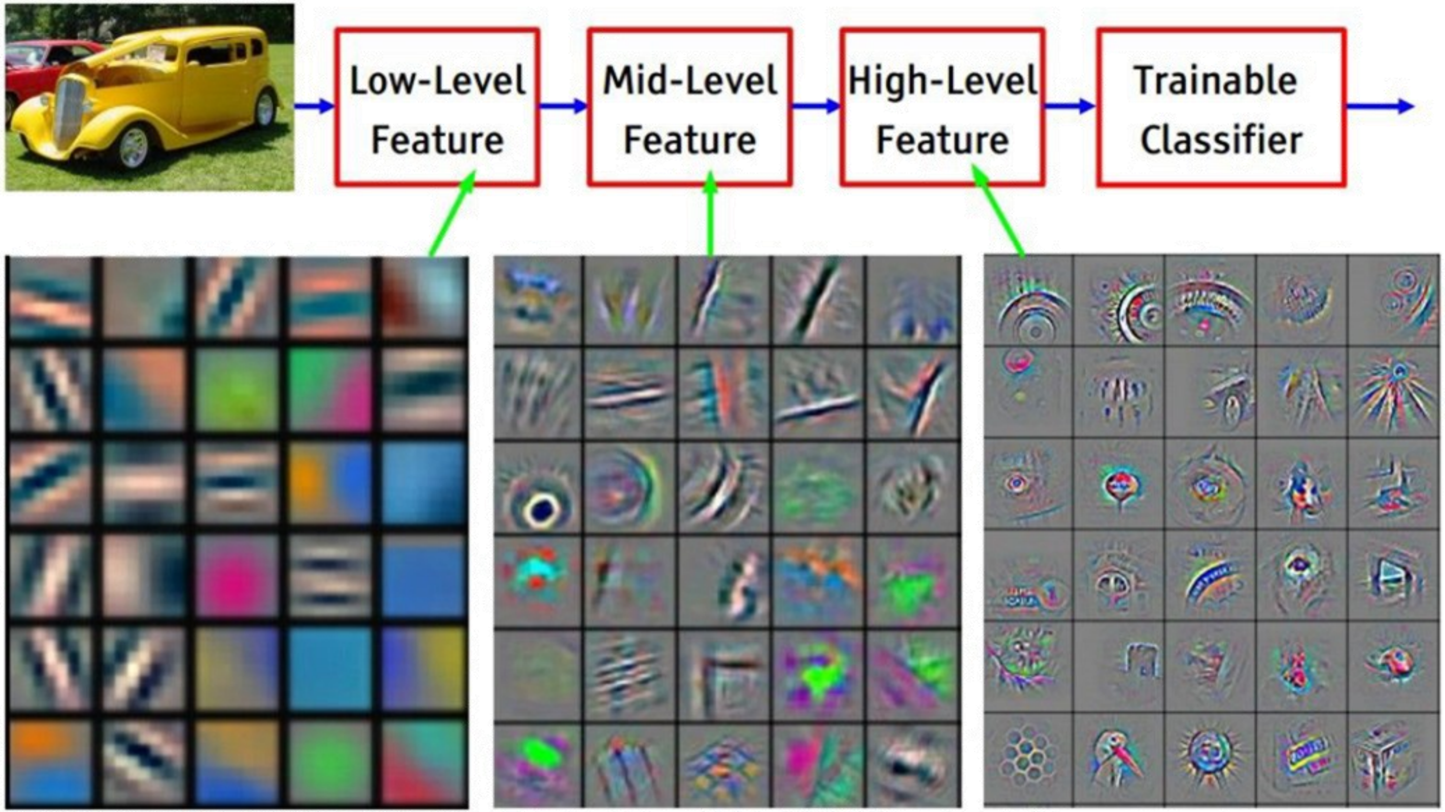
Convolutional Neural Network (CNN)

CNN Prototype

ConvNet is concatenation of Conv Layers and activations



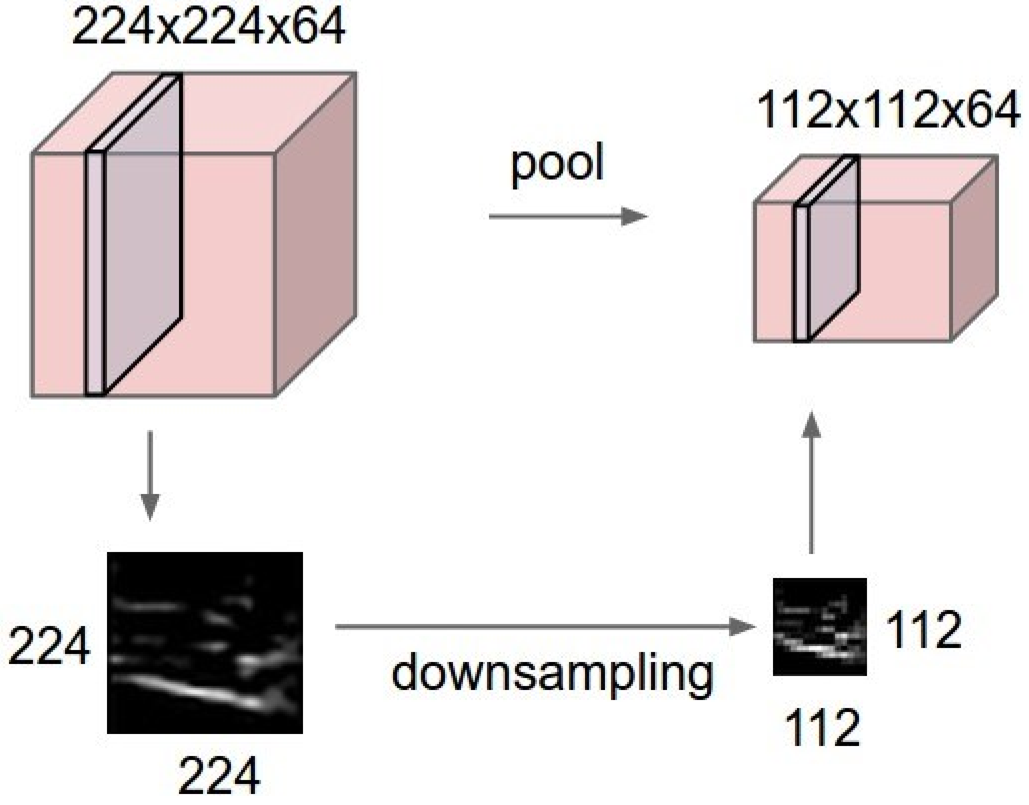
CNN Learned Filters



[Zeiler & Fergus, ECCV'14] Visualizing and Understanding Convolutional Networks

Pooling

Pooling Layer



[Li et al., CS231n Course Slides] Lecture 5: Convolutional Neural Networks

Pooling Layer: Max Pooling

Single depth slice of input

3	1	3	5
6	0	7	9
3	2	1	4
0	2	4	3

Max pool with
2×2 filters and stride 2

'Pooled' output

6	9
3	4

Pooling Layer

- Conv Layer = 'Feature Extraction'
  - Computes a feature in a given region
- Pooling Layer = 'Feature Selection'
  - Picks the strongest activation in a region

Pooling Layer

- Input is a volume of size $W_{in} \times H_{in} \times D_{in}$
- Two hyperparameters
  - Spatial filter extent F
  - Stride S

Filter count K and padding P make no sense here

- Output volume is of size $W_{out} \times H_{out} \times D_{out}$
  - $W_{out} = \frac{W_{in}-F}{S} + 1$
  - $H_{out} = \frac{H_{in}-F}{S} + 1$
  - $D_{out} = D_{in}$
- Does not contain parameters; e.g. it's fixed function

Pooling Layer

- Input is a volume of size $W_{in} \times H_{in} \times D_{in}$
- Two hyperparameters
  - Spatial filter extent F
  - Stride S
- Output volume is of size $W_{out} \times H_{out} \times D_{out}$
  - $W_{out} = \frac{W_{in}-F}{S} + 1$
  - $H_{out} = \frac{H_{in}-F}{S} + 1$
  - $D_{out} = D_{in}$
- Does not contain parameters; e.g. it's fixed function

Common settings:
$F = 2, S = 2$
$F = 3, S = 2$

Pooling Layer: Average Pooling

Single depth slice of input

3	1	3	5
6	0	7	9
3	2	1	4
0	2	4	3

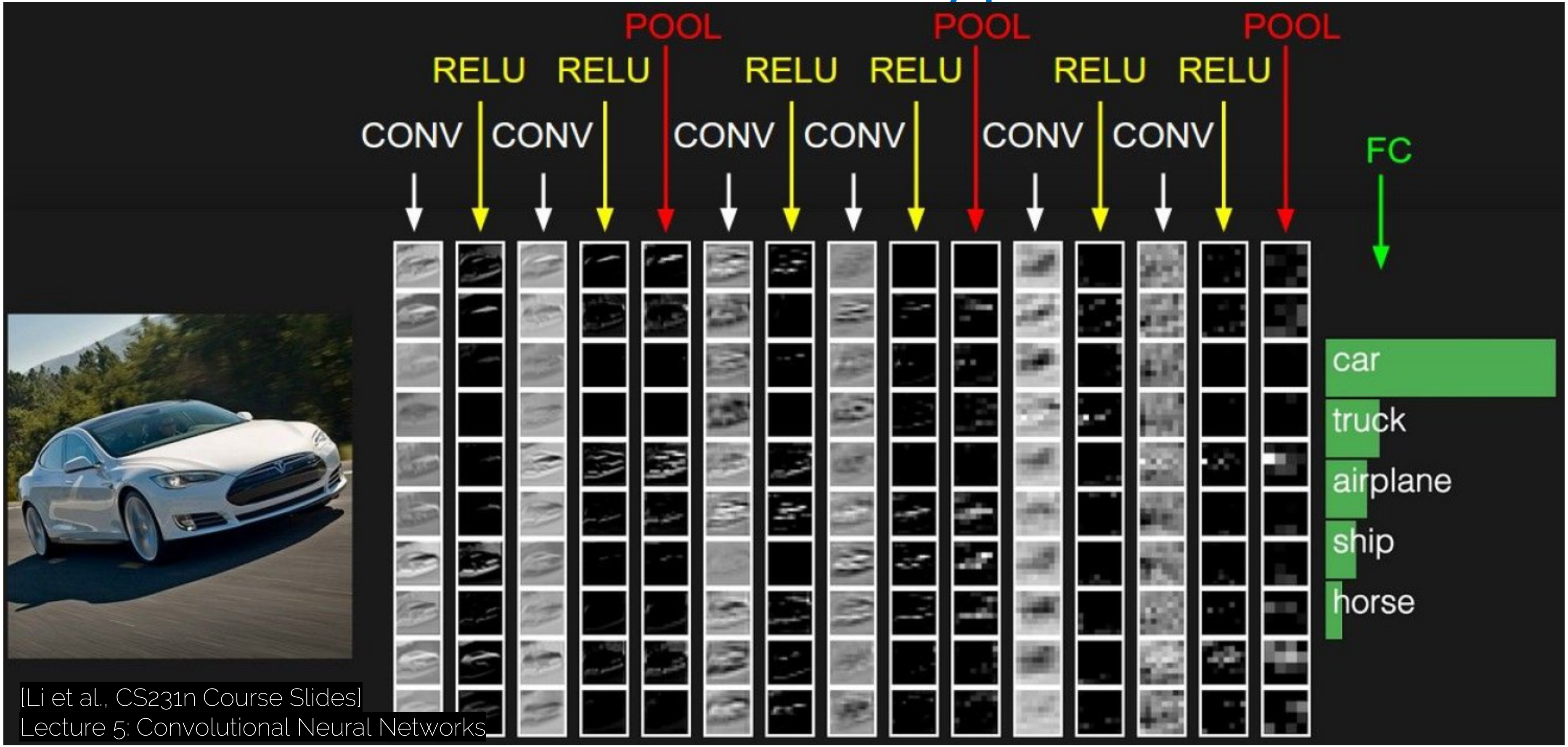
Average pool with
2×2 filters and stride 2

'Pooled' output

2.5	6
1.75	3

- Typically used deeper in the network

CNN Prototype



Final Fully-Connected Layer

- Same as what we had in ‘ordinary’ neural networks
  - Make the final decision with the extracted features from the convolutions
  - One or two FC layers typically

Convolutions vs Fully-Connected

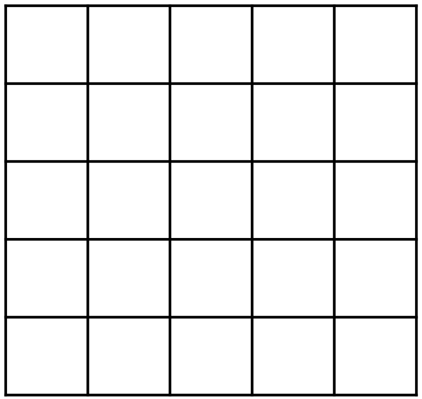
- In contrast to fully-connected layers, we want to restrict the degrees of freedom
  - FC is somewhat brute force
  - Convolutions are **structured**

- Sliding window to with the same filter parameters to extract image features
  - Concept of weight sharing
  - Extract same features independent of location

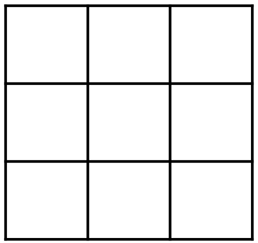
Receptive field

Receptive Field

- Spatial extent of the connectivity of a convolutional filter

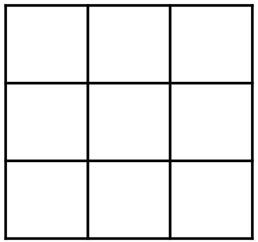


5x5 input



3x3 filter

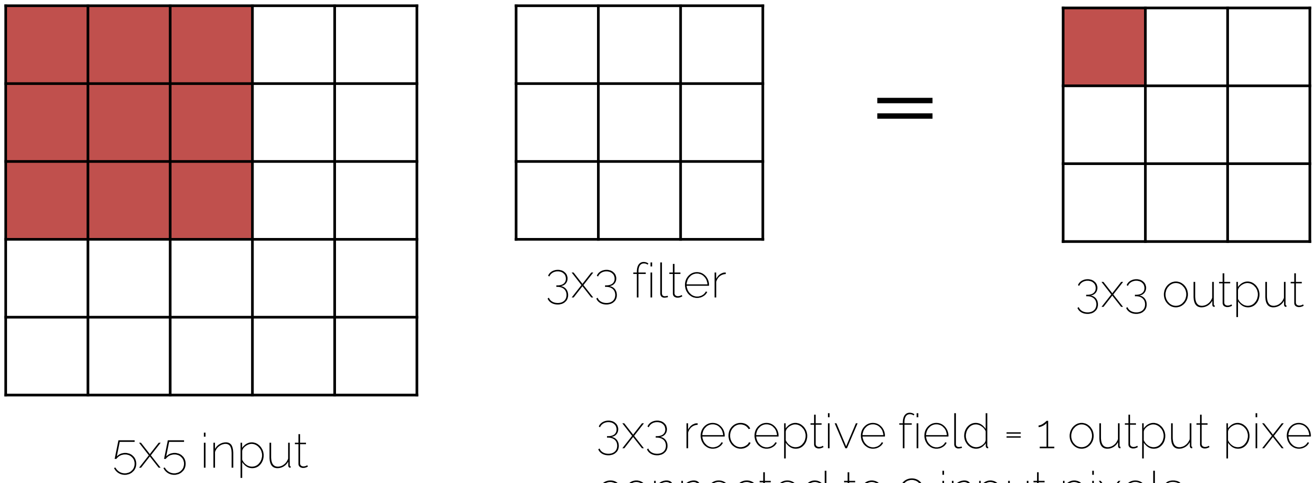
=



3x3 output

Receptive Field

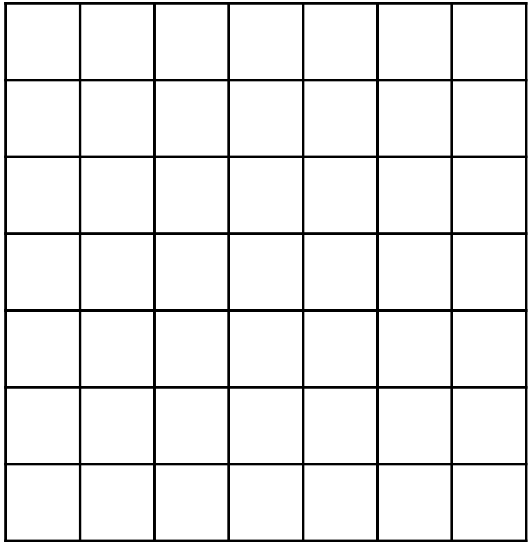
- Spatial extent of the connectivity of a convolutional filter



3x3 receptive field = 1 output pixel is connected to 9 input pixels

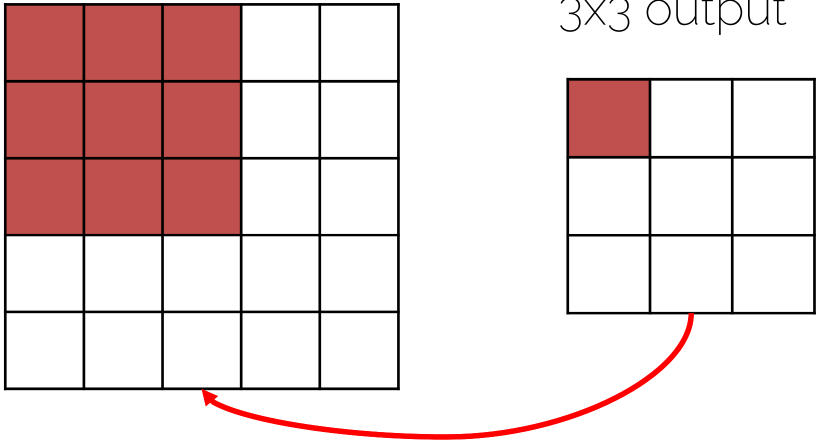
Receptive Field

- Spatial extent of the connectivity of a convolutional filter



7x7 input

3x3 output



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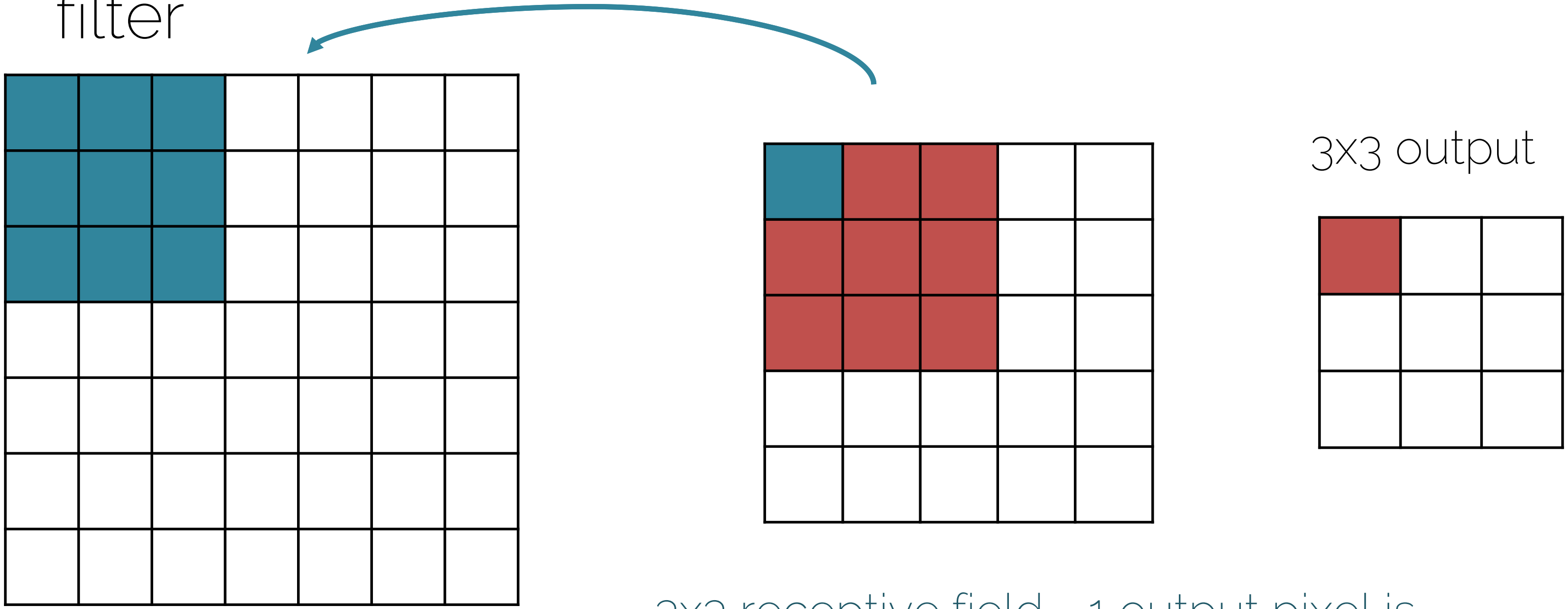
Receptive Field

- Spatial extent of the connectivity of a convolutional filter

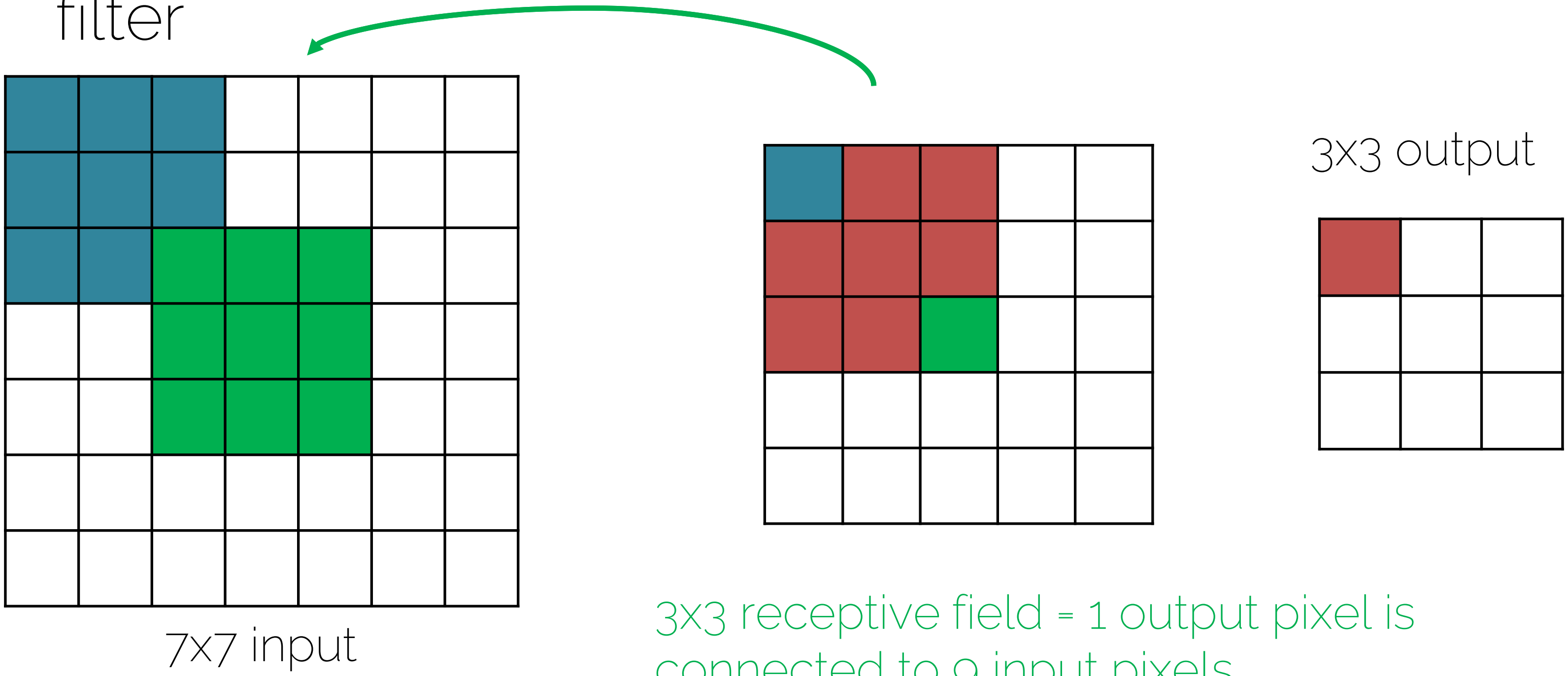


7x7 input

3x3 receptive field = 1 output pixel is connected to 9 input pixels

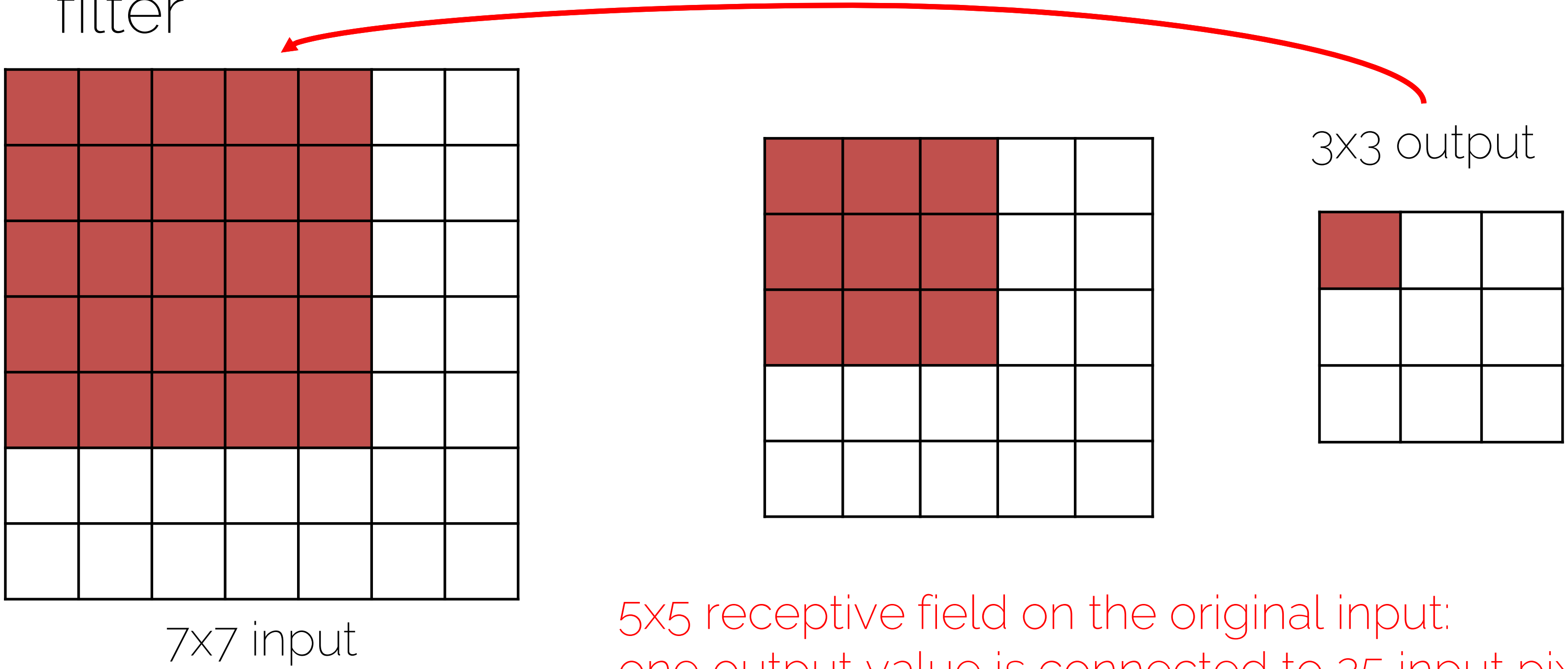
Receptive Field

- Spatial extent of the connectivity of a convolutional filter



Receptive Field

- Spatial extent of the connectivity of a convolutional filter



5x5 receptive field on the original input:
one output value is connected to 25 input pixels

See you next time!

References

- Goodfellow et al. “Deep Learning” (2016),
  - Chapter 9: Convolutional Networks

- http://cs231n.github.io/convolutional-networks/