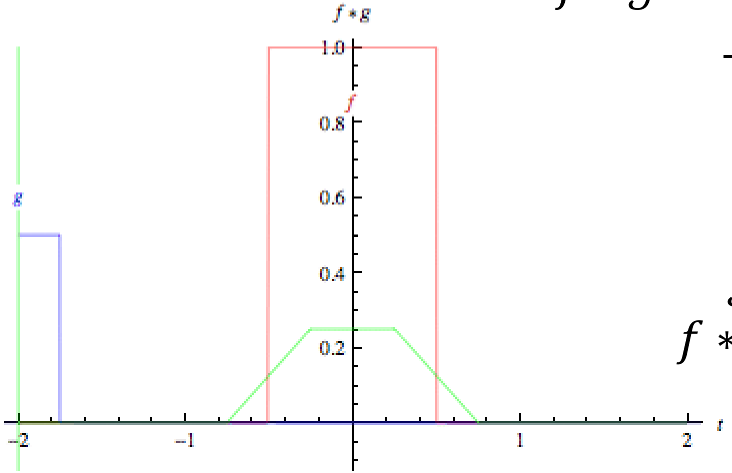
Lecture 9 Recap

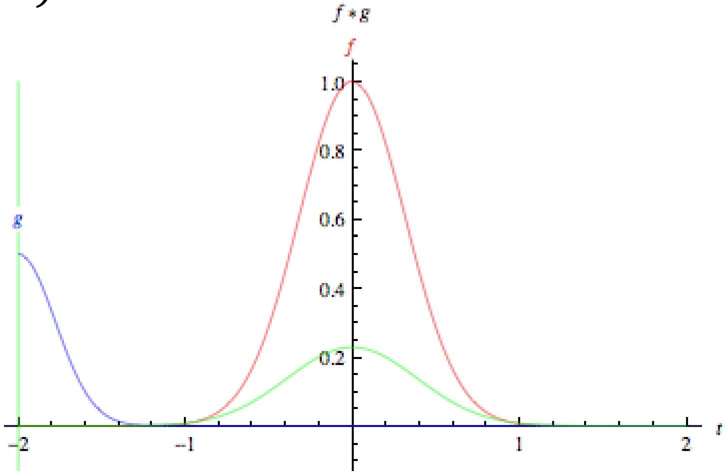
What are Convolutions?

$$f * g = \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$$



Convolution of two box functions

f = red
g = blue
$f * g$ = green



Convolution of two Gaussians

application of a filter to a function
the 'smaller' one is typically called the filter kernel

What are Convolutions?

Discrete case: box filter

4	3	2	-5	3	5	2	5	5	6

1/3	1/3	1/3

??	3	0	0	1	10/3	4	4	16/3	??

What to do at boundaries?

1) Shrink

3	0	0	1	10/3	4	4	16/3

2) Pad
often '0'

7/3	3	0	0	1	10/3	4	4	16/3	11/3

Convolutions on Images

Image 5x5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3x3

0	-1	0
-1	5	-1
0	-1	0

Output 3x3

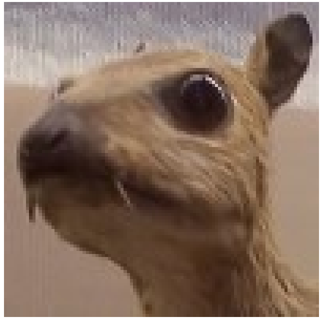
6	1	8
-7	9	2
-5	-9	3

$$5 \cdot 4 + (-1) \cdot 3 + (-1) \cdot 4 + (-1) \cdot 9 + (-1) \cdot 1 =$$
$$20 - 17 = 3$$

Image Filters

- Each kernel gives us a different image filter

Input



Edge detection

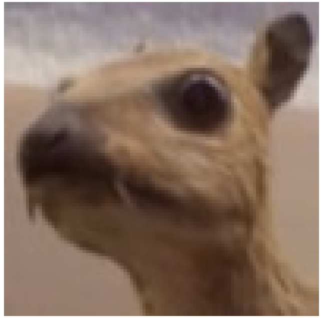
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Box mean

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpen

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

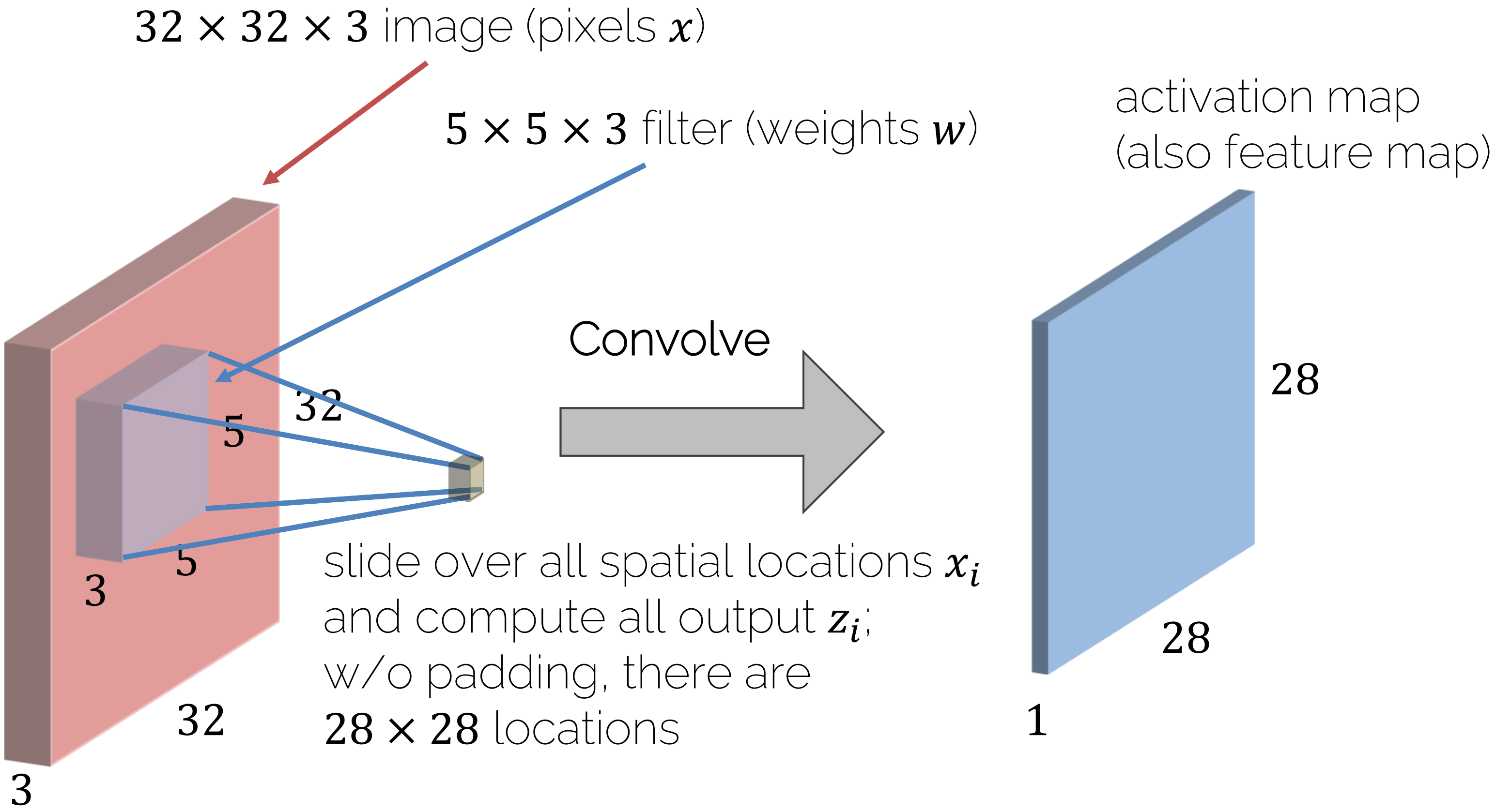


Gaussian blur

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

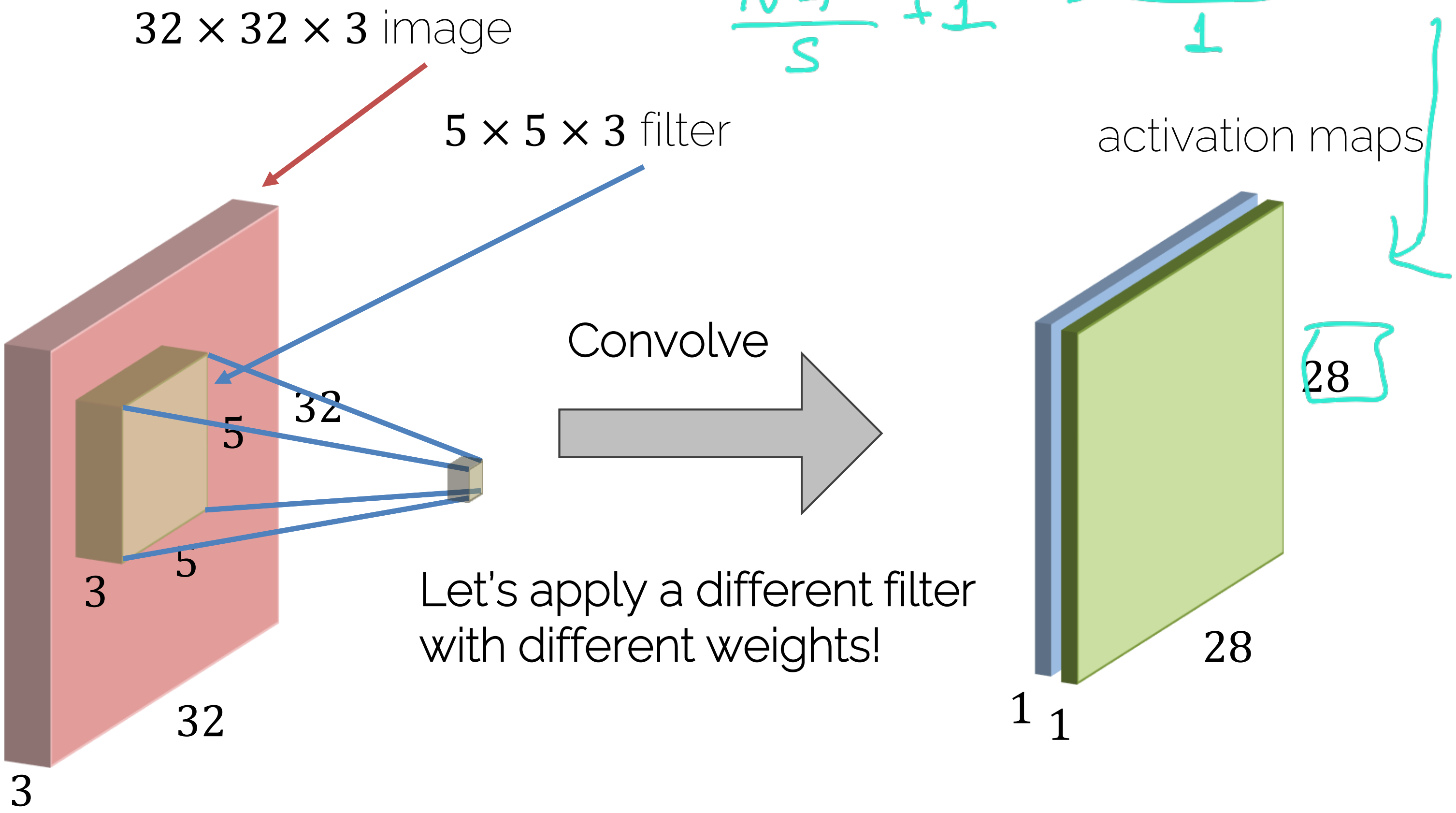
LET'S LEARN THESE FILTERS!

Convolutions on RGB Images



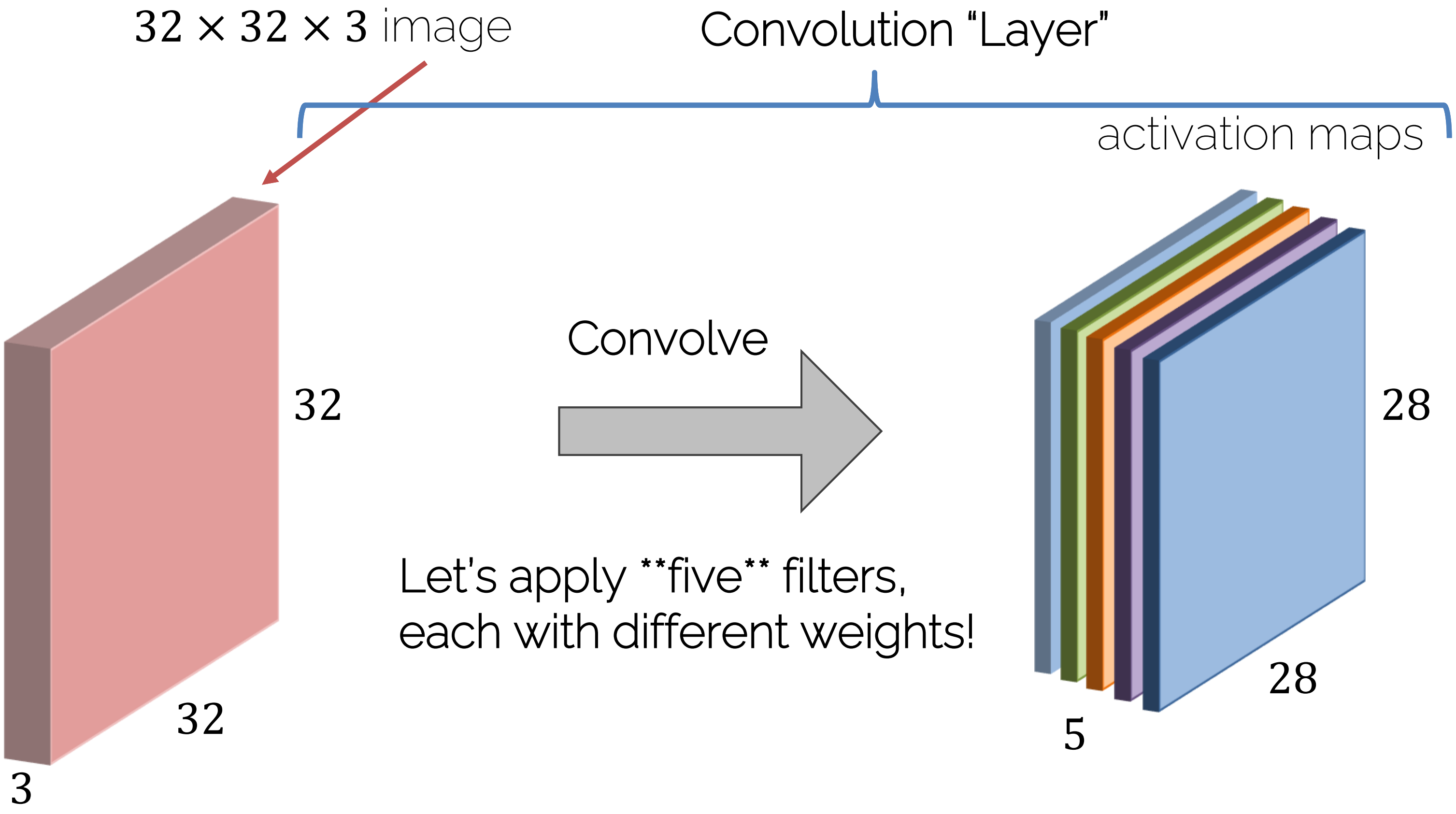
Convolution Layer

$$\frac{N-F}{S}+1 \rightarrow \frac{32-5}{1}+1=\boxed{28}$$



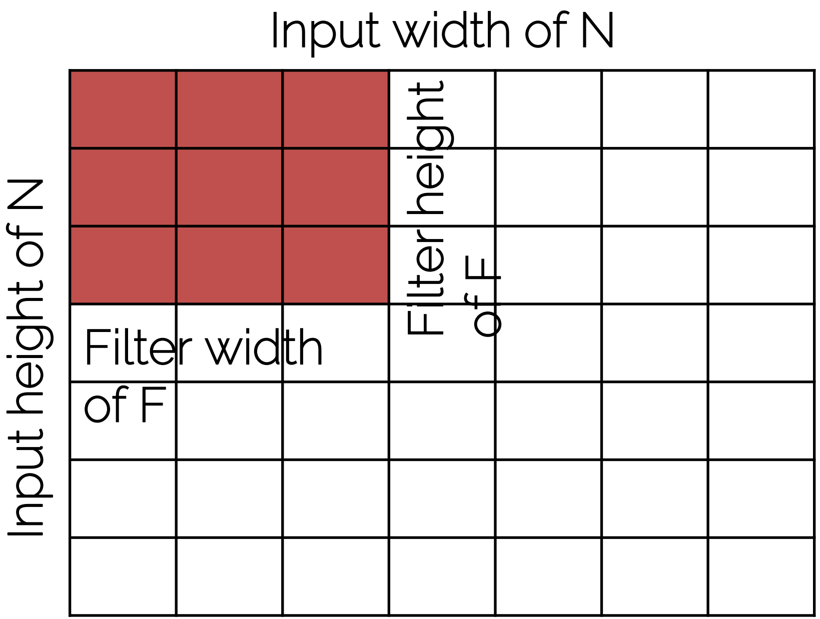
Let's apply a different filter with different weights!

Convolution Layer



Let's apply **five** filters,
each with different weights!

Convolution Layers: Dimensions



Input: $N \times N$
Filter: $F \times F$
Stride: S
Output: $\left(\frac{N-F}{S} + 1\right) \times \left(\frac{N-F}{S} + 1\right)$

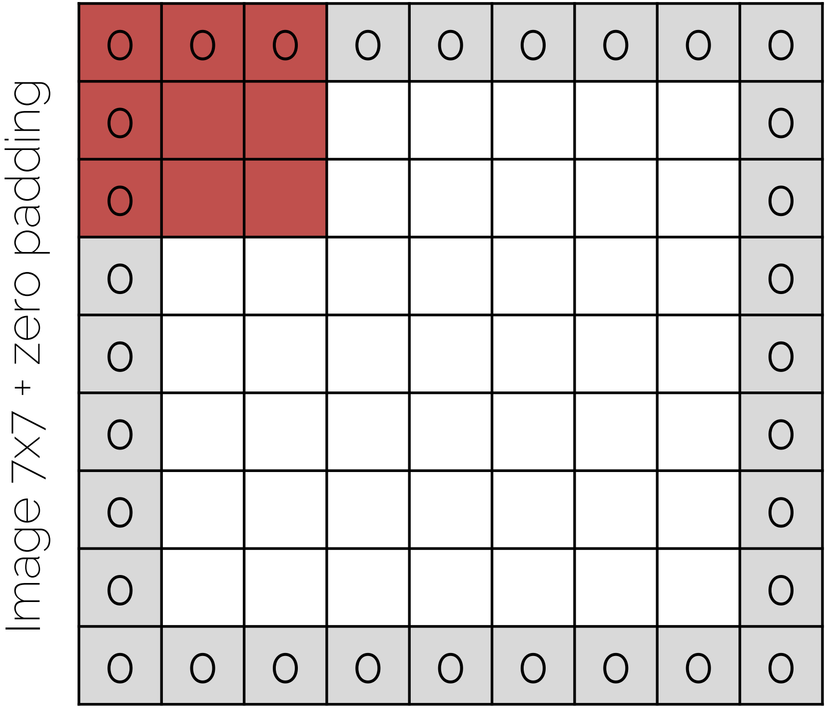
$N = 7, F = 3, S = 1$: $\frac{7-3}{1} + 1 = 5$

$N = 7, F = 3, S = 2$: $\frac{7-3}{2} + 1 = 3$

$N = 7, F = 3, S = 3$: $\frac{7-3}{3} + 1 = 2.3333$

Fractions are illegal

Convolution Layers: Padding



Types of convolutions:

- **Valid** convolution: using no padding
- **Same** convolution: output=input size

Set padding to $P = \frac{F-1}{2}$

Convolution Layers: Dimensions

Remember: **Output** $= \left(\frac{N+2\cdot P-F}{S}+1\right) \times \left(\frac{N+2\cdot P-F}{S}+1\right)$

REMARK: in practice, typically integer division is used (i.e., apply the floor–operator!)

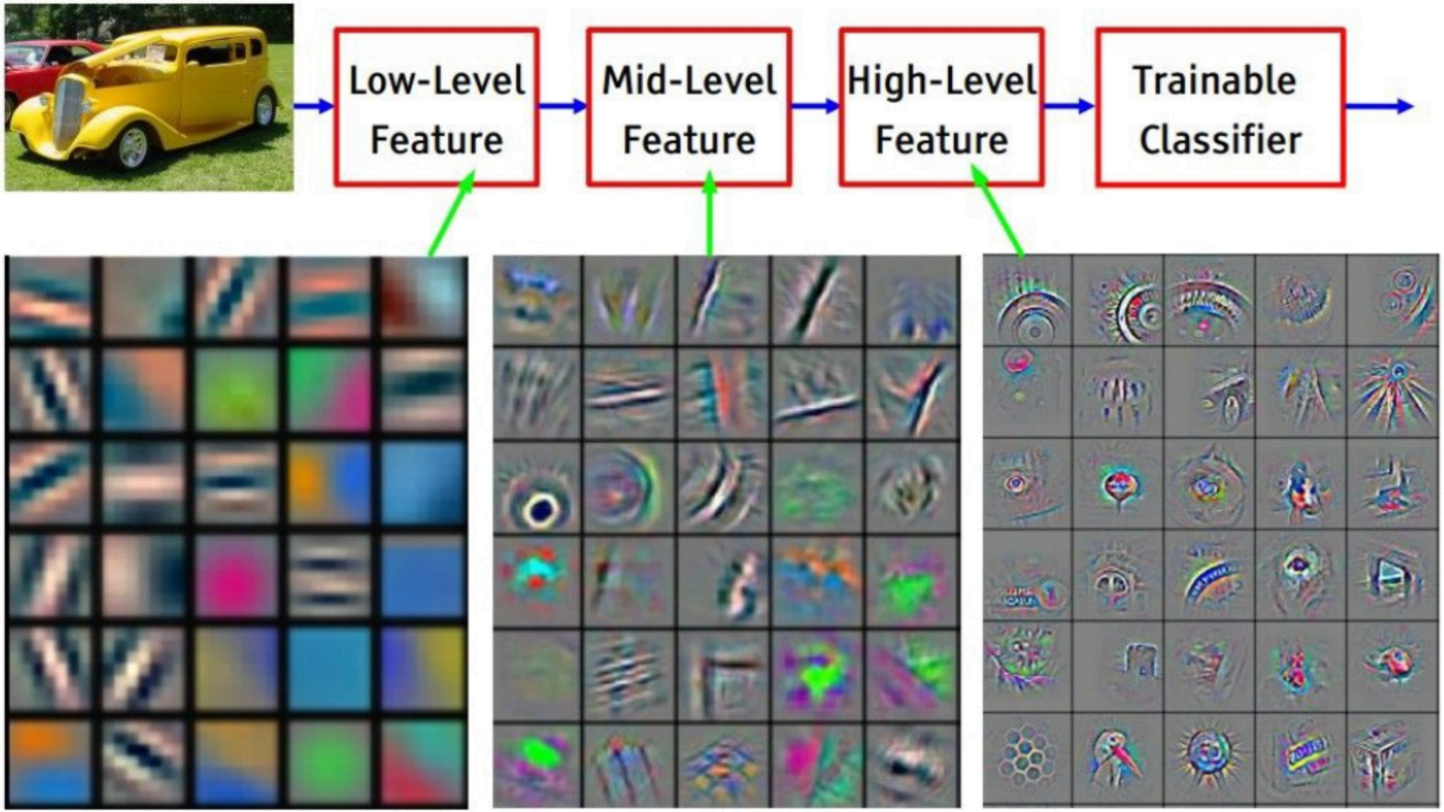
Example: 3x3 conv with same padding and strides of 2 on an 64x64 RGB image -> N = 64, F = 3, P = 1, S = 2

Output: $\left(\frac{64+2\cdot 1-3}{2}+1\right) \times \left(\frac{64+2\cdot 1-3}{2}+1\right)$

$= floor(32.5) \times floor(32.5)$

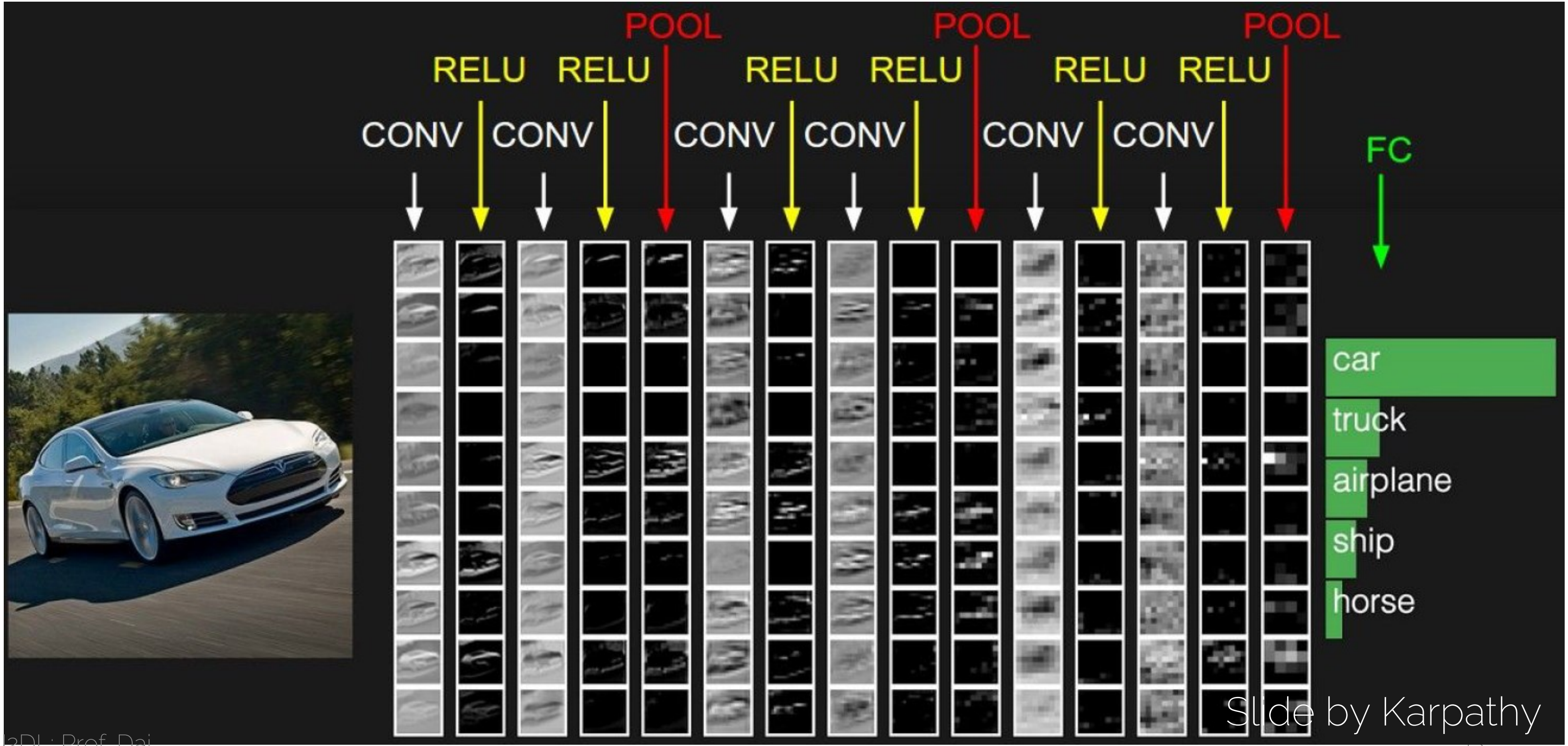
$= 32 \times 32$

CNN Learned Filters



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

CNN Prototype



Pooling Layer: Max Pooling

Take highest signal
it allows us down sampling

Single depth slice of input

3	1	3	5
6	0	7	9
3	2	1	4
0	2	4	3

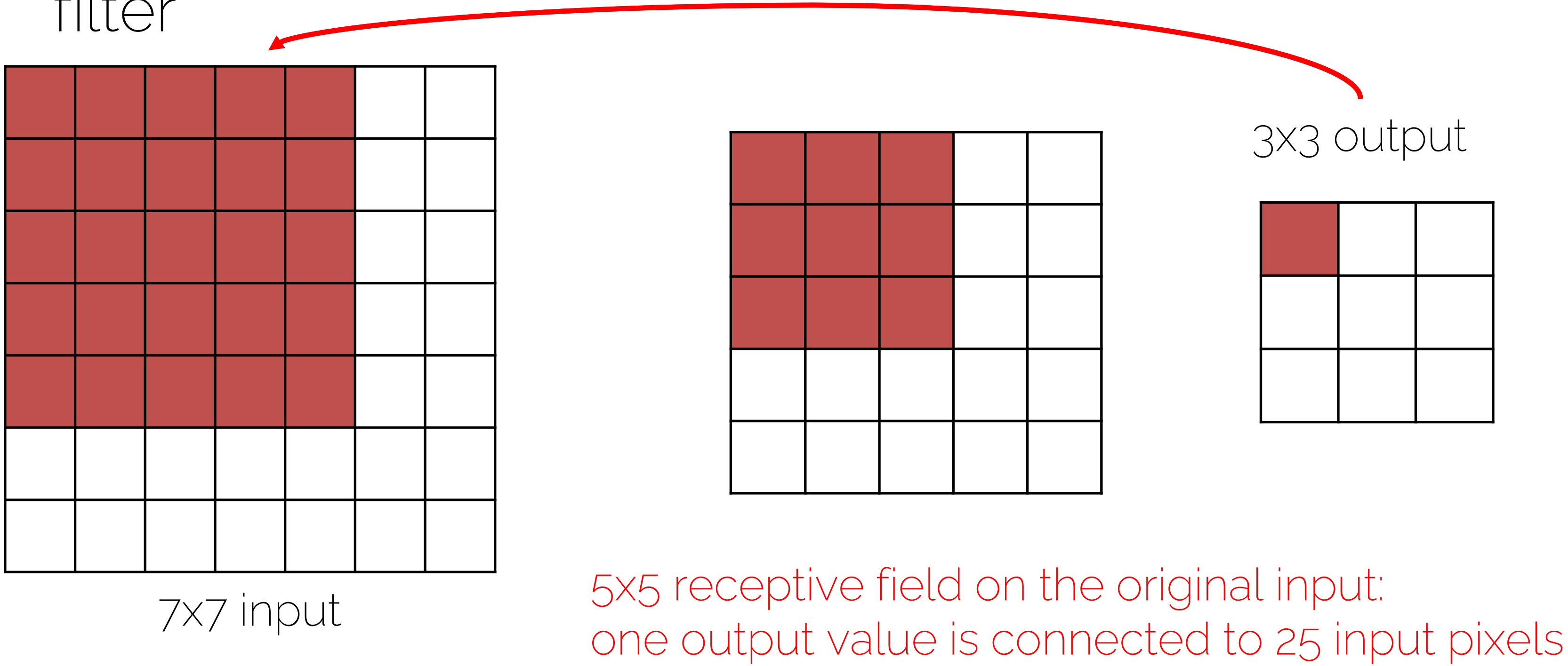
Max pool with
2×2 filters and stride 2

'Pooled' output

6	9
3	4

Receptive Field

- Spatial extent of the connectivity of a convolutional filter



Lecture 10 – CNNs (part 2)

Classic Architectures

LeNet

- Digit recognition: 10 classes

$32 \times 32 \times 1$

Note RGB

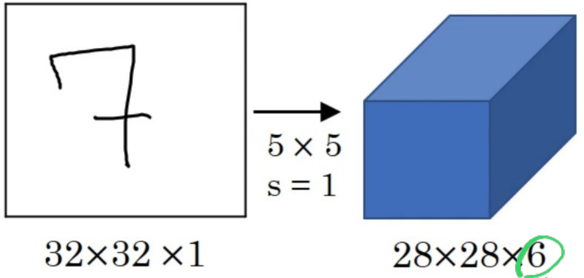
Input: 32×32 grayscale images

This one: Labeled as class "7"

[LeCun et al. '98] LeNet

LeNet

- Digit recognition: 10 classes



$$\frac{N-F}{S}+1$$

$$\frac{32-5}{1}+1=28$$

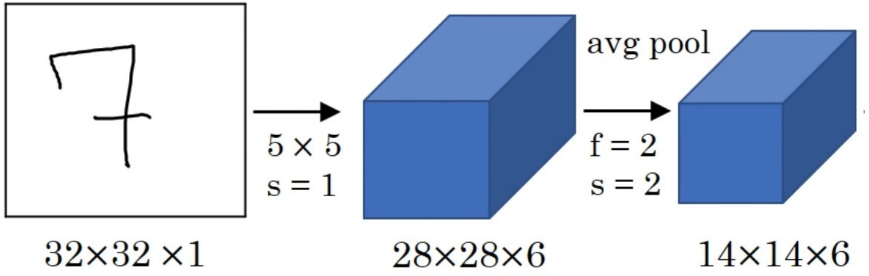
6 filters

- Valid convolution: size shrinks
- How many conv filters are there in the first layer? 6

6

LeNet

- Digit recognition: 10 classes



$\frac{N-F}{S}+1$

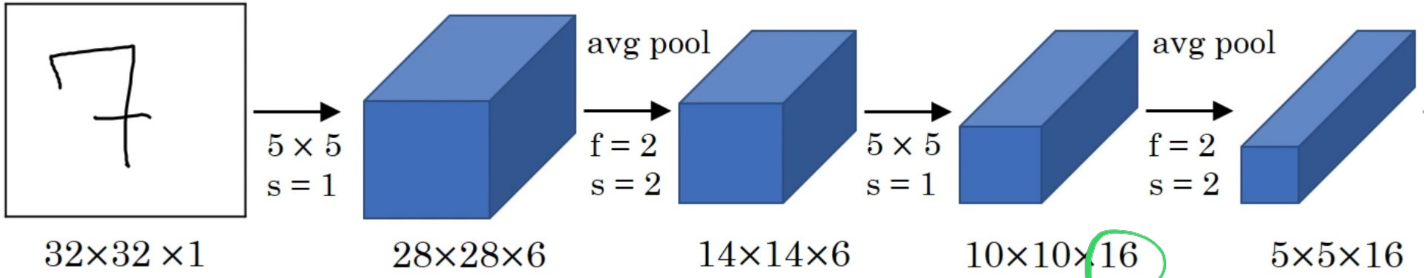
$\frac{28-2}{2}+1=14$

- At that time average pooling was used, now max pooling is much more common

- Now max pooling much more popular since it extracts highest signal

LeNet

- Digit recognition: 10 classes

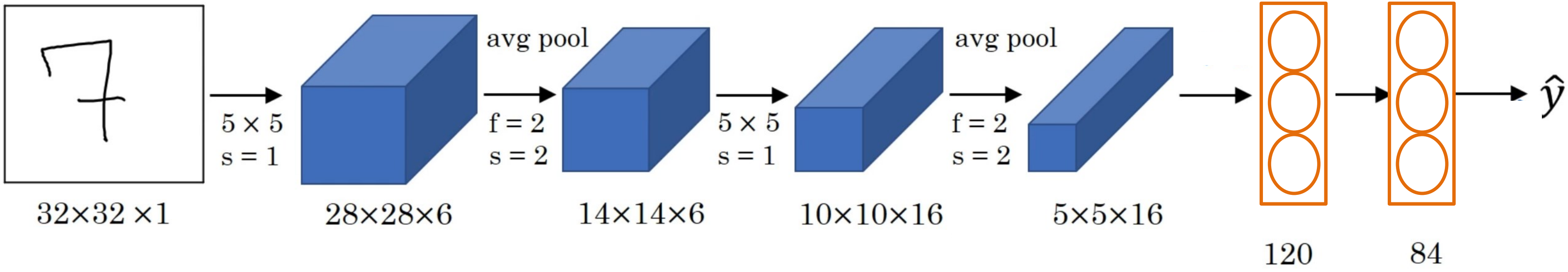


- Again valid convolutions, how many filters?

16

LeNet

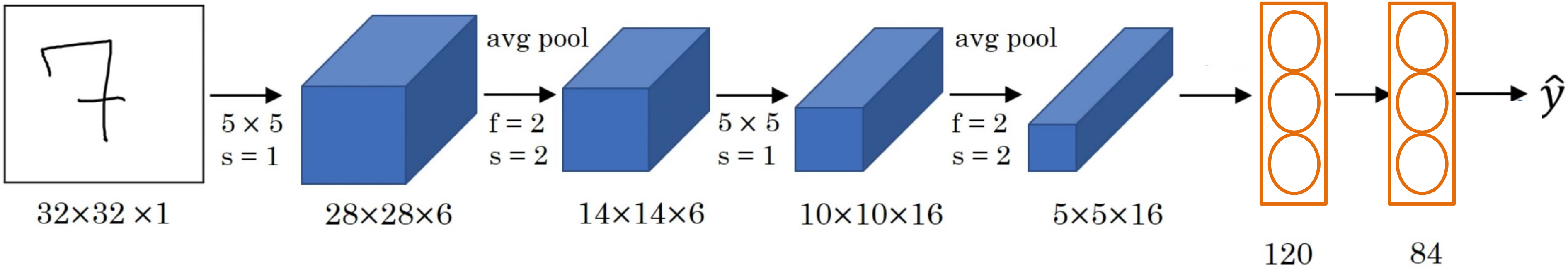
- Digit recognition: 10 classes



- Use of tanh/sigmoid activations → not common now!

LeNet

- Digit recognition: 10 classes

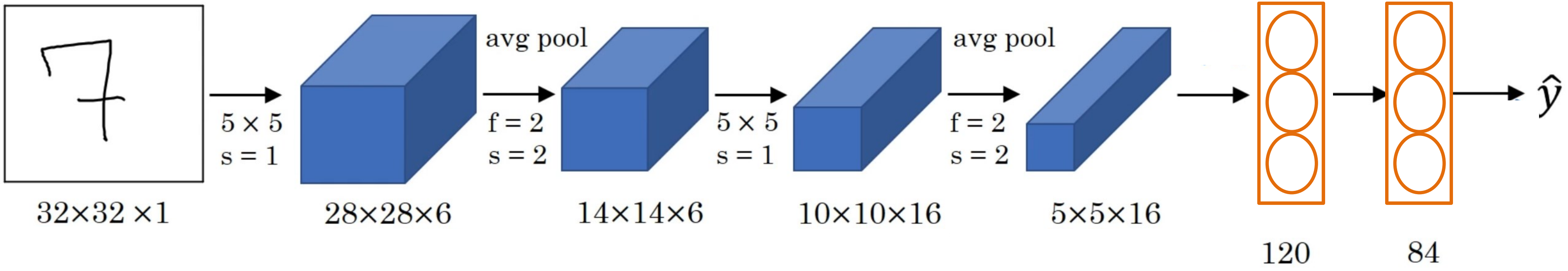


- Conv -> Pool -> Conv -> Pool -> Conv -> FC

LeNet

- Digit recognition: 10 classes

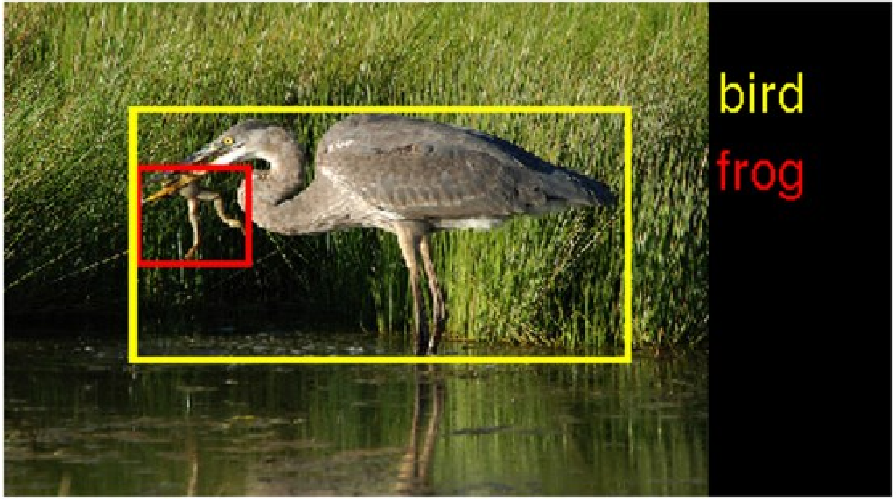
60k parameters

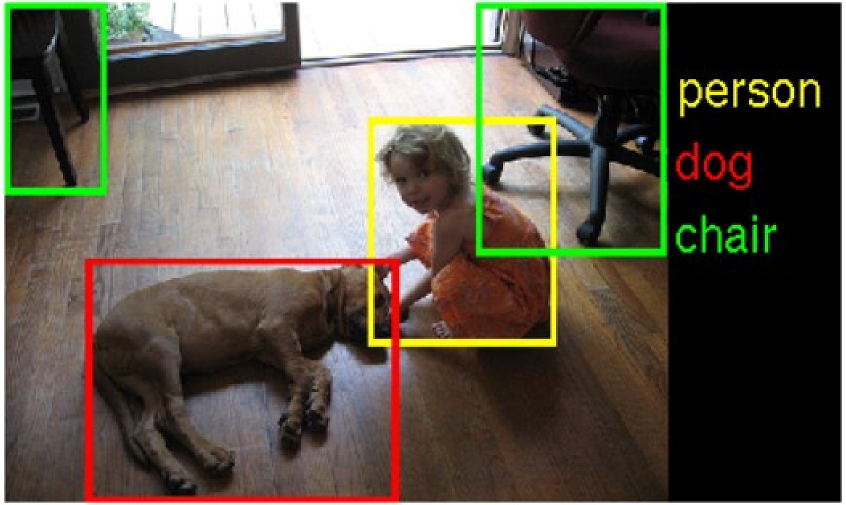


- Conv -> Pool -> Conv -> Pool -> Conv -> FC
- As we go deeper: Width, Height ↓ Number of Filters ↑

Test Benchmarks

- ImageNet Dataset:
  ImageNet Large Scale Visual Recognition Competition (ILSVRC)





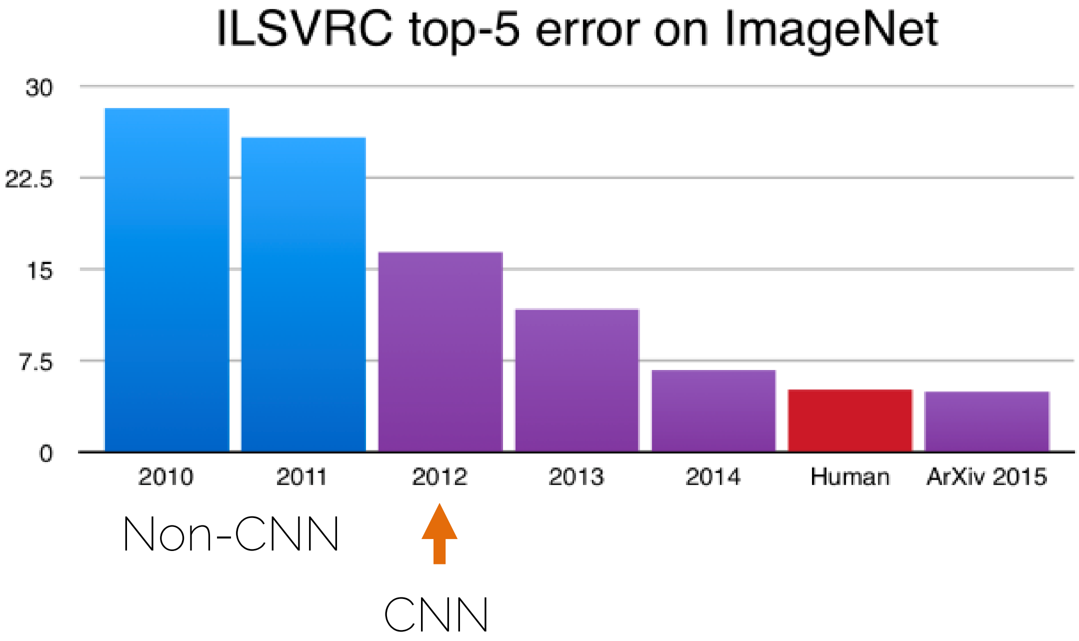
[Russakovsky et al., IJCV'15] "ImageNet Large Scale Visual Recognition Challenge."

Common Performance Metrics

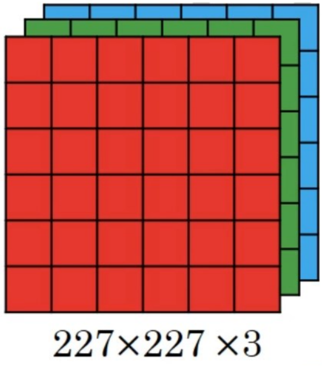
- **Top-1 score:** check if a sample's top class (i.e. the one with highest probability) is the same as its target label
- **Top-5 score:** check if your label is in your 5 first predictions (i.e. predictions with 5 highest probabilities)
- → **Top-5 error**: percentage of test samples for which the correct class was not in the top 5 predicted classes

AlexNet

- Cut ImageNet error down in half

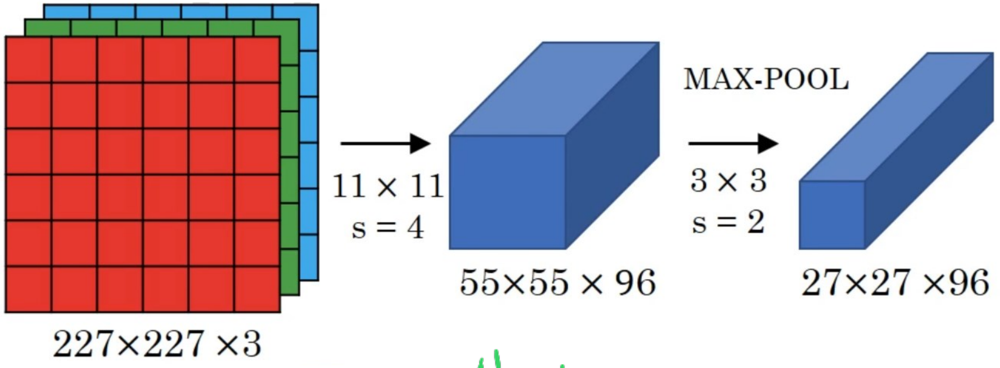


AlexNet



[Krizhevsky et al. NIPS'12] AlexNet

AlexNet

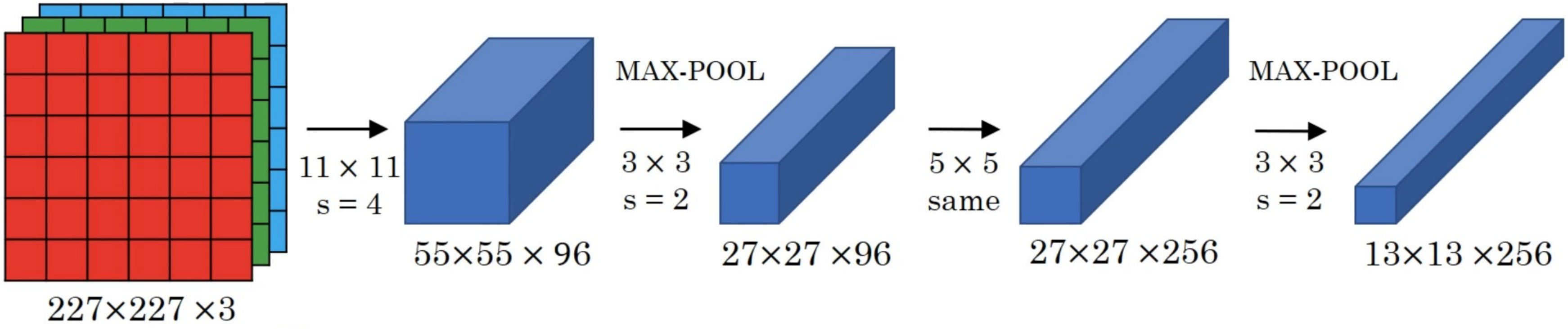


$$\frac{227-11}{4}+1 = \boxed{55}$$

$$\frac{55-3}{2}+1 = \boxed{27}$$

[Krizhevsky et al. NIPS'12] AlexNet

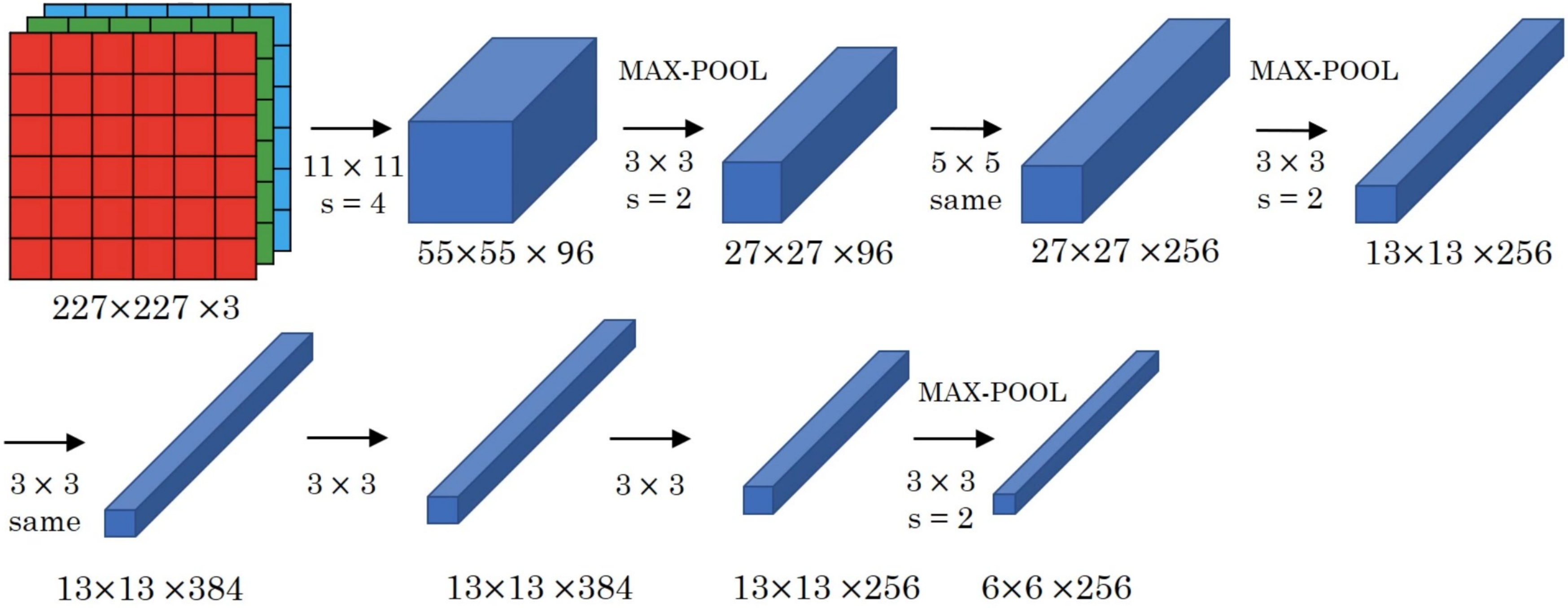
AlexNet



- Use of same convolutions
- As with LeNet: Width, Height ↓ Number of Filters ↑

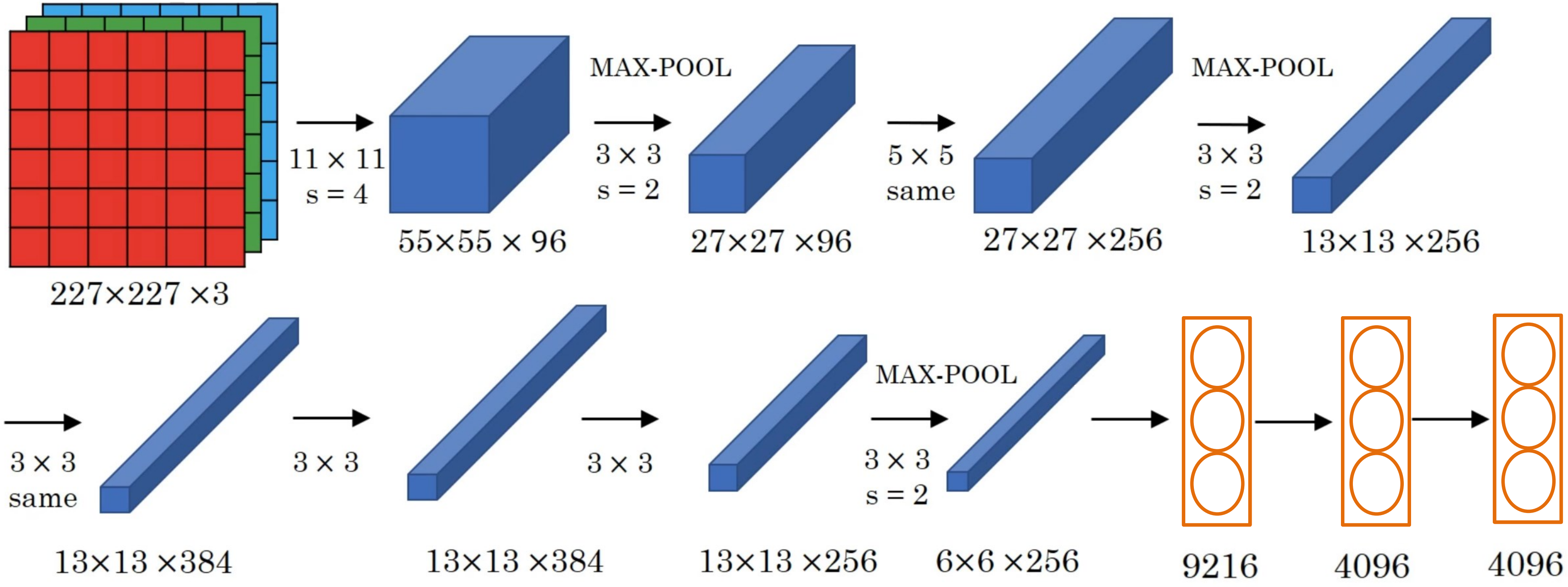
[Krizhevsky et al. NIPS'12] AlexNet

AlexNet



[Krizhevsky et al. NIPS'12] AlexNet

AlexNet



- Softmax for 1000 classes

[Krizhevsky et al. NIPS'12] AlexNet

AlexNet

- Similar to LeNet but much bigger (~1000 times)
- Use of ReLU instead of tanh/sigmoid

$\boxed{\text{60M parameters}}$

[Krizhevsky et al. NIPS'12] AlexNet

VGGNet

- Striving for simplicity

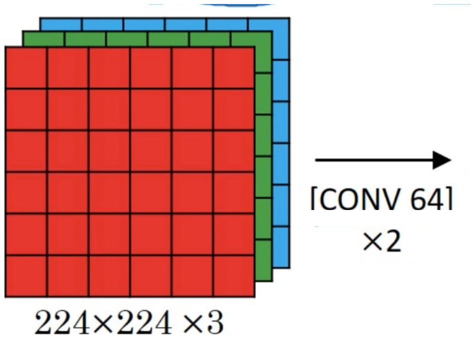
- CONV = 3x3 filters with stride 1, same convolutions

- MAXPOOL = 2x2 filters with stride 2

[Simonyan and Zisserman ICLR'15] VGGNet

VGGNet

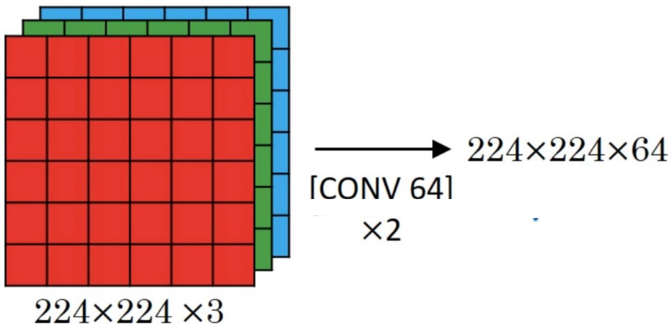
Conv=3x3,s=1,same
Maxpool=2x2,s=2



[Simonyan and Zisserman ICLR'15] VGGNet

VGGNet

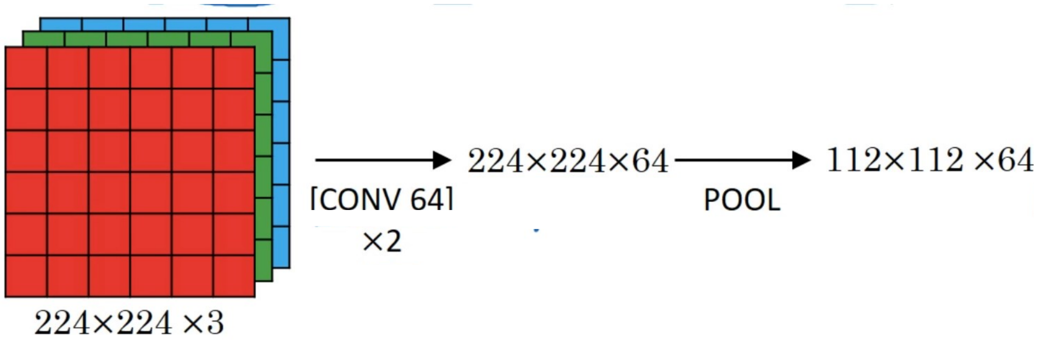
Conv=3x3,s=1,same
Maxpool=2x2,s=2



[Simonyan and Zisserman ICLR'15] VGGNet

VGGNet

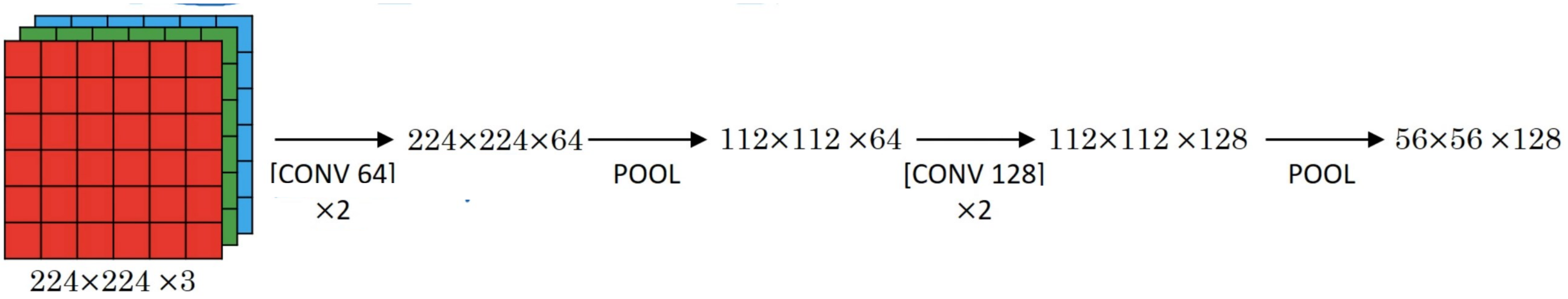
Conv=3x3,s=1,same
Maxpool=2x2,s=2



[Simonyan and Zisserman ICLR'15] VGGNet

VGGNet

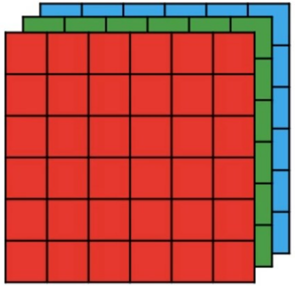
Conv=3x3,s=1,same
Maxpool=2x2,s=2



[Simonyan and Zisserman ICLR'15] VGGNet

VGGNet

Conv=3x3,s=1,same
Maxpool=2x2,s=2



$224\times224\times3$ $\xrightarrow{\text{[CONV 64]} \times 2}$ $224\times224\times64$ $\xrightarrow{\text{POOL}}$ $112\times112\times64$ $\xrightarrow{\text{[CONV 128]} \times 2}$ $112\times112\times128$ $\xrightarrow{\text{POOL}}$ $56\times56\times128$

$\xrightarrow{\text{[CONV 256]} \times 3}$ $56\times56\times256$ $\xrightarrow{\text{POOL}}$ $28\times28\times256$ $\xrightarrow{\text{[CONV 512]} \times 3}$ $28\times28\times512$ $\xrightarrow{\text{POOL}}$ $14\times14\times512$

$\xrightarrow{\text{[CONV 512]} \times 3}$ $14\times14\times512$ $\xrightarrow{\text{POOL}}$ $7\times7\times512$ $\longrightarrow$ FC 4096 $\longrightarrow$ FC 4096 $\longrightarrow$ Softmax 1000

[Simonyan and Zisserman ICLR'15] VGGNet

VGGNet

- Conv -> Pool -> Conv -> Pool -> Conv -> FC
- As we go deeper: Width, Height ↓ Number of Filters ↑
- Called VGG-16: 16 layers that have weights

  $\boxed{\text{138M parameters}}$
- Large but simplicity makes it appealing

[Simonyan and Zisserman ICLR'15] VGGNet

VGGNet

- A lot of architectures were analyzed

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224×224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

[Simonyan and Zisserman 2014]

Table 2: **Number of parameters** (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

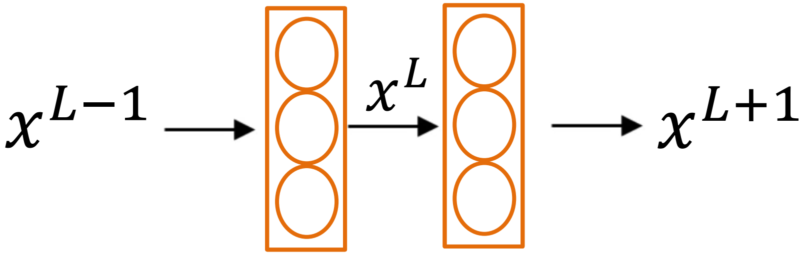
Skip Connections

The Problem of Depth

- As we add more and more layers, training becomes harder
- Vanishing and exploding gradients
- How can we train very deep nets?

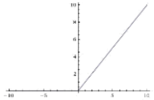
Residual Block

- Two layers



Input $\longrightarrow$ $W^L x^{L-1} + b^L \longrightarrow x^L = f(W^L x^{L-1} + b^L) \longrightarrow$

Linear

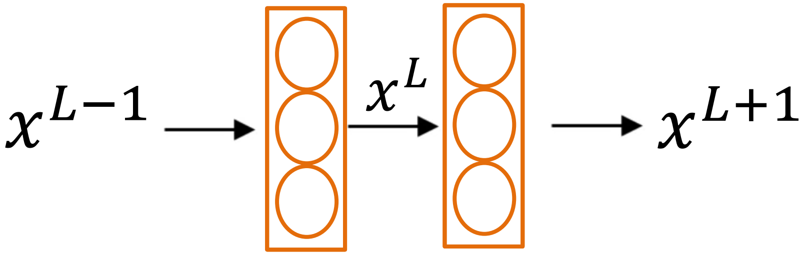


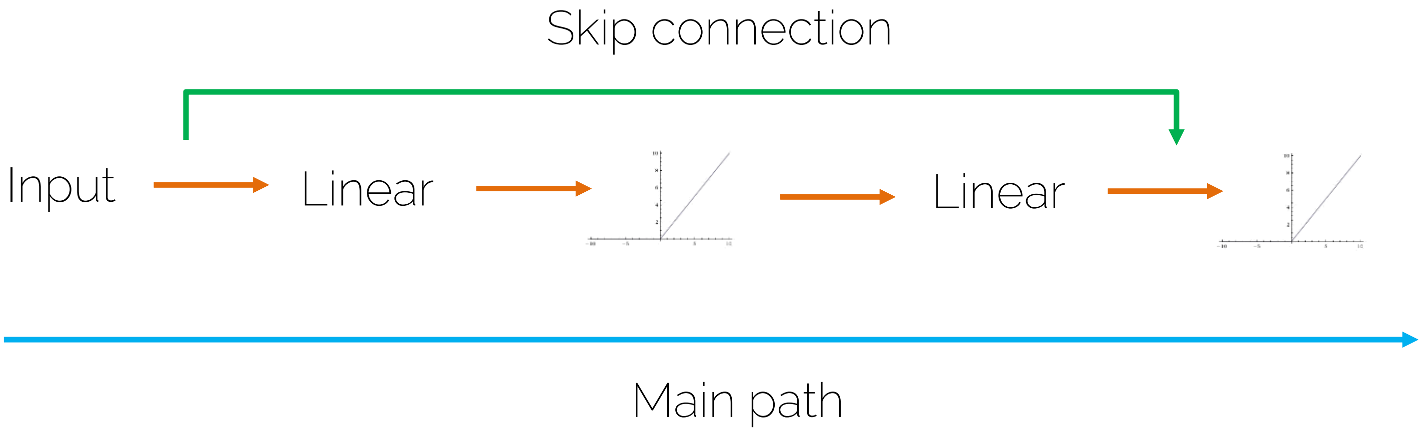
Non-linearity

$\longrightarrow x^{L+1} = f(W^{L+1} x^L + b^{L+1})$

Residual Block

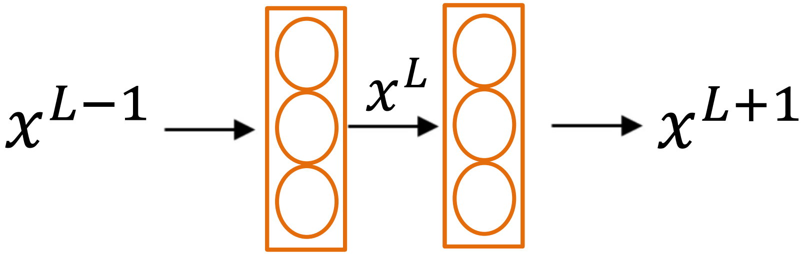
- Two layers



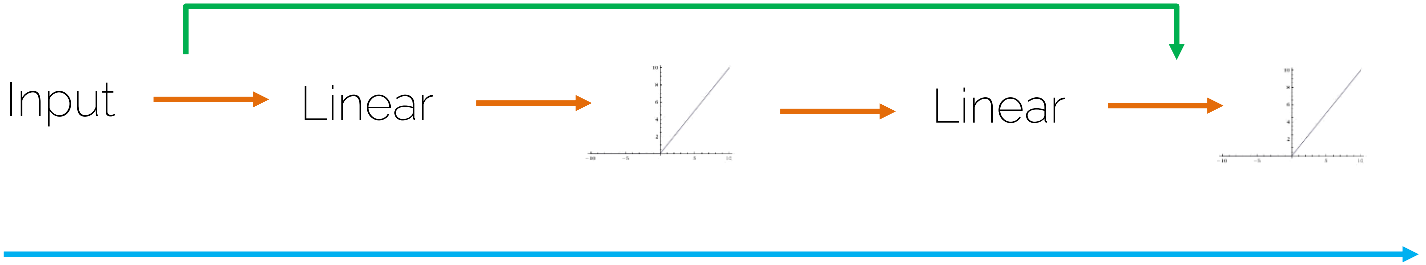


Residual Block

- Two layers



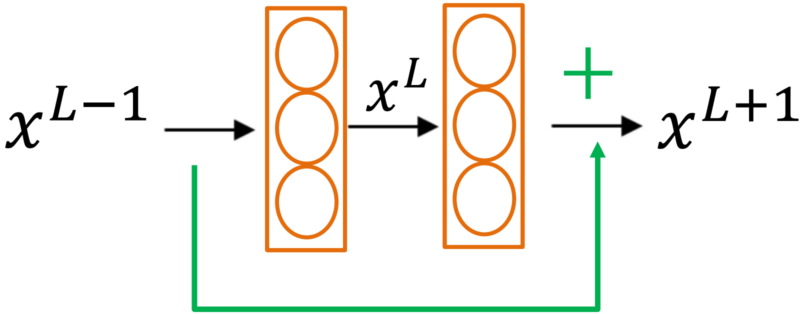
$$x^{L+1} = f(W^{L+1}x^{L} + b^{L+1} + x^{L-1})$$



$$x^{L+1} = f(W^{L+1}x^{L} + b^{L+1})$$

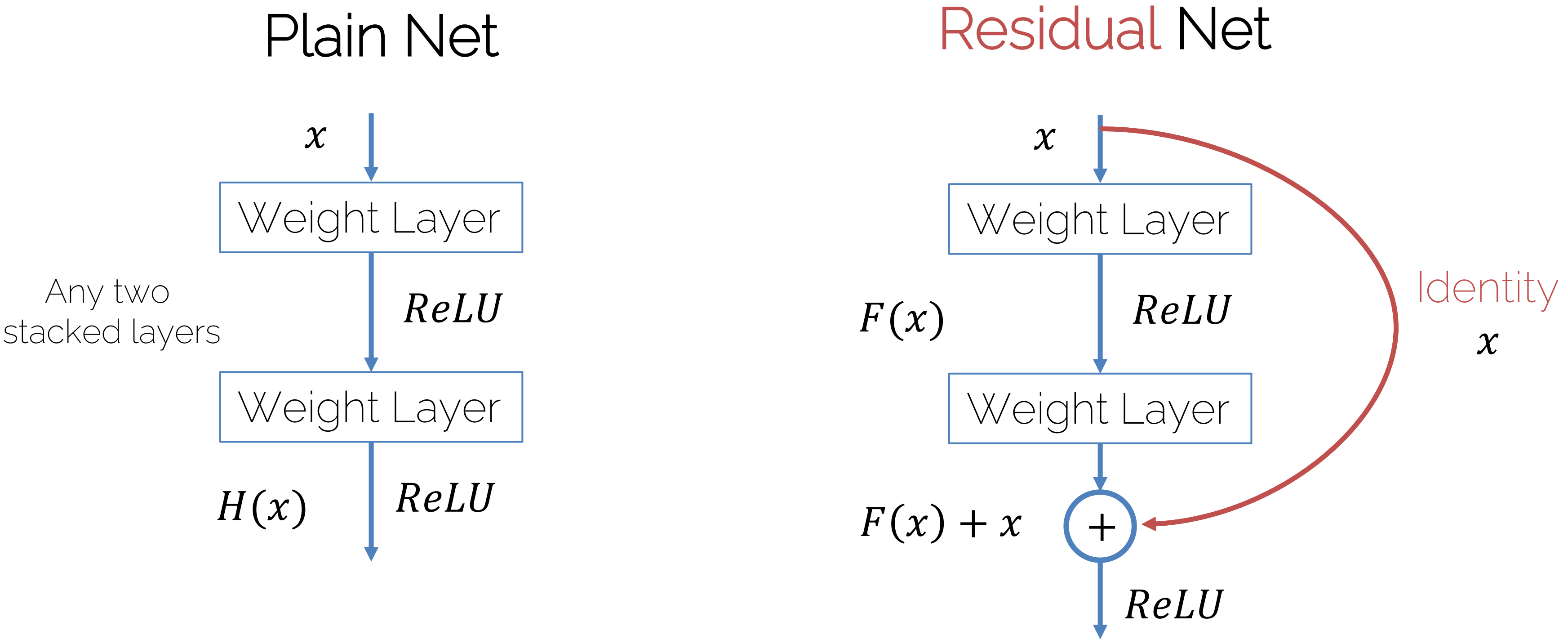
Residual Block

- Two layers



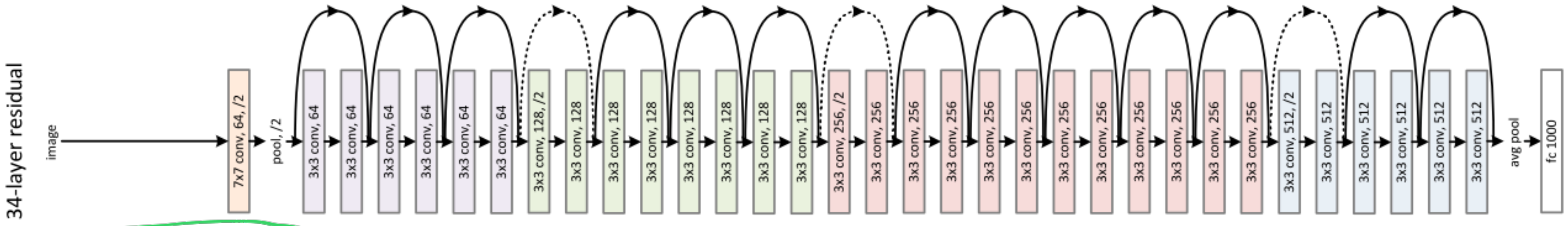
- Usually use a same convolution since we need same dimensions
- Otherwise we need to convert the dimensions with a matrix of learned weights or zero padding

ResNet Block



[He et al. CVPR'16] ResNet

ResNet



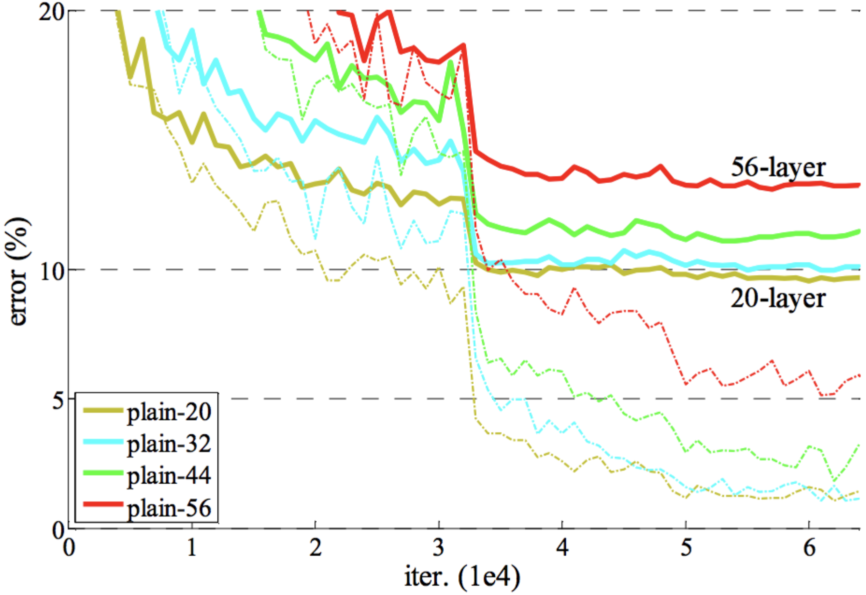
- Xavier/2 initialization
- SGD + Momentum (0.9)
- Learning rate 0.1, divided by 10 when plateau
- Mini-batch size 256
- Weight decay of 1e-5
- No dropout

ResNet-152:
60M parameters

[He et al. CVPR'16] ResNet

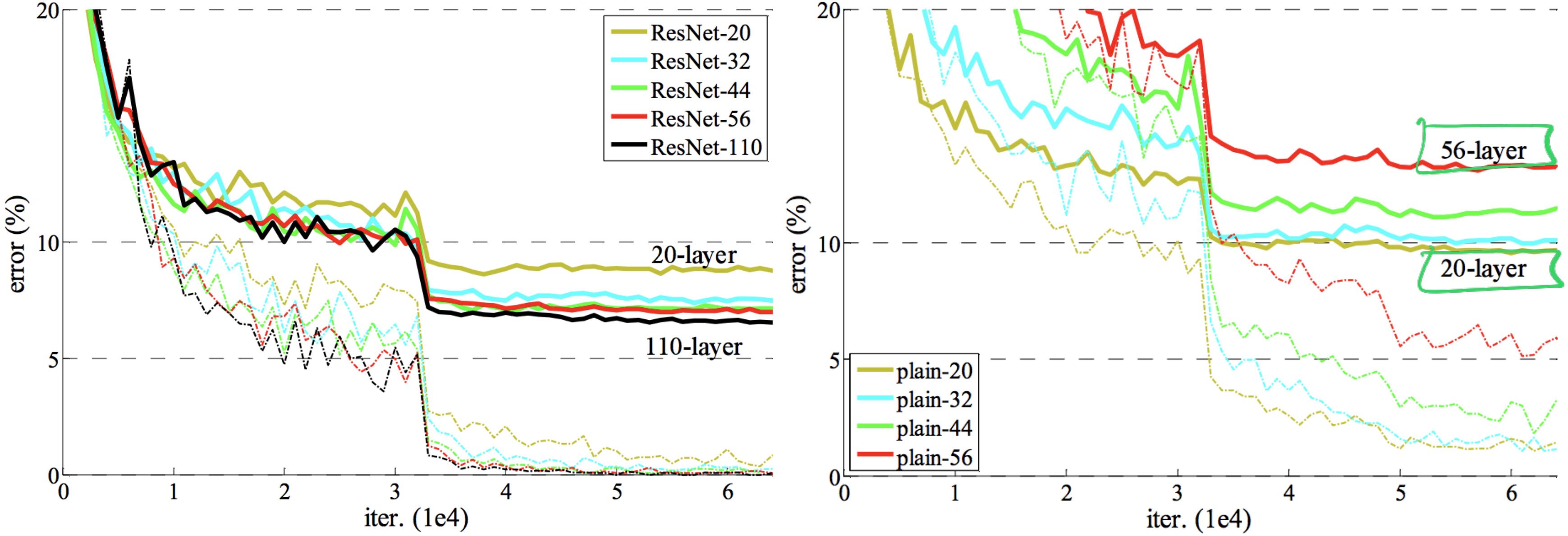
ResNet

- If we make the network deeper, at some point performance starts to degrade

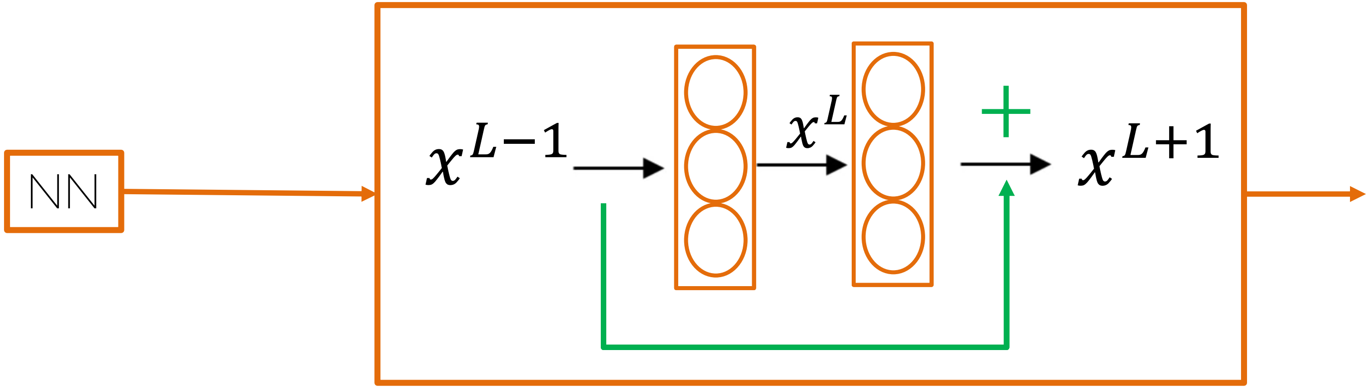


ResNet

- If we make the network deeper, at some point performance starts to degrade -> Residual connections fixed this

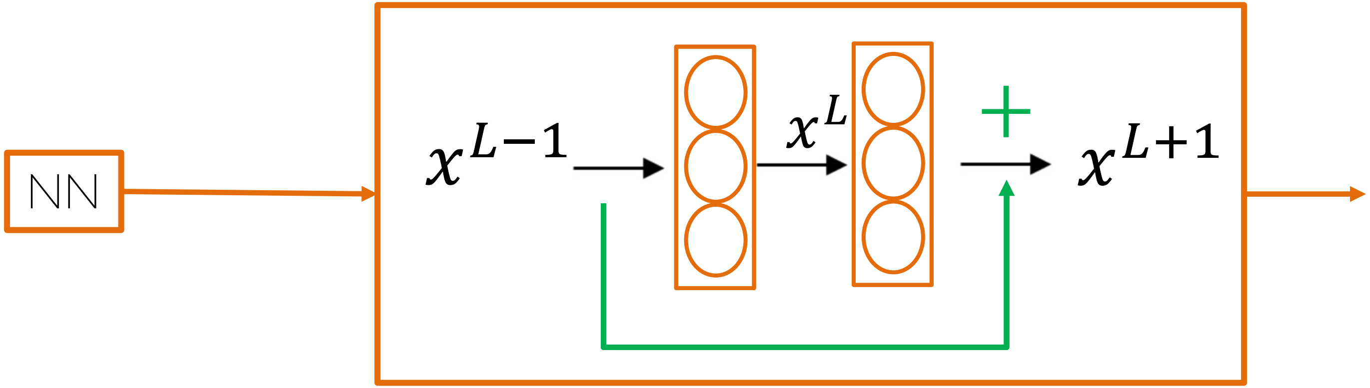


Why do ResNets Work?



- How is this block really affecting me?

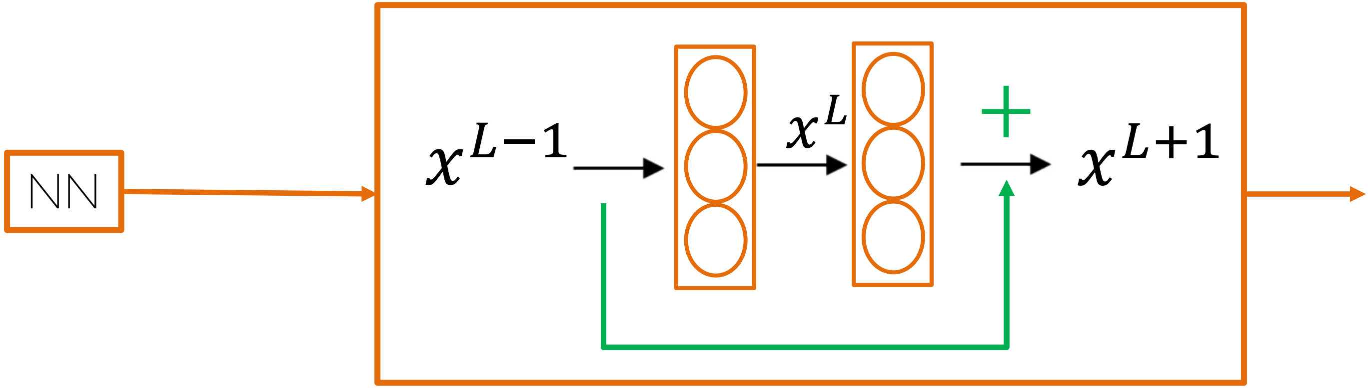
Why do ResNets Work?



$$x^{L+1} = f(\underbrace{W^{L+1}}_{\sim\text{zero}}x^{L} + \underbrace{b^{L+1}}_{\sim\text{zero}} + x^{L-1})$$

$$x^{L+1} = f(x^{L-1})$$

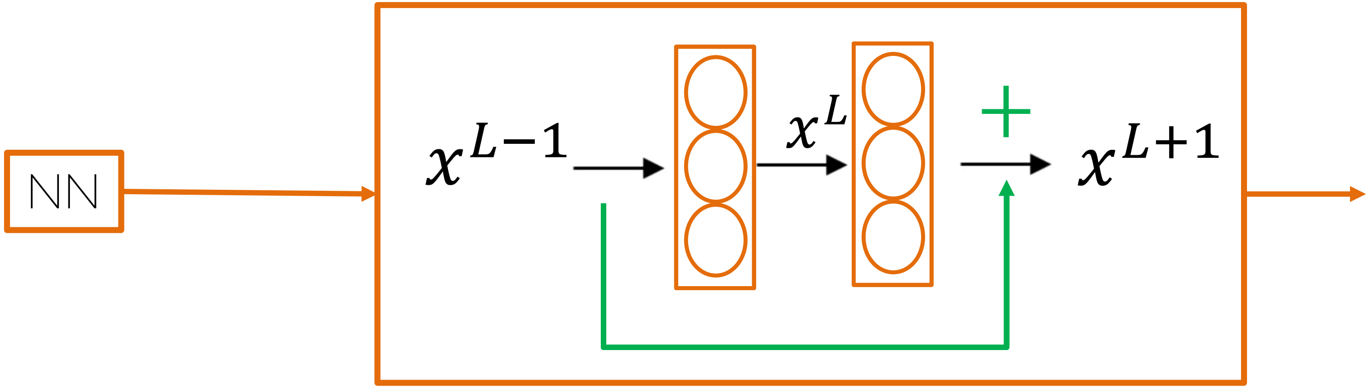
Why do ResNets Work?



- We kept the same values and added a non-linearity

$$x^{L+1} = f(x^{L-1})$$

Why do ResNets Work?



- The identity is easy for the residual block to learn
- Guaranteed it will not hurt performance, can only improve

1x1 Convolutions

Recall: Convolutions on Images

Image 5x5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 3x3

0	-1	0
-1	5	-1
0	-1	0

Output 3x3

6		

$$5 \cdot 3 + (-1) \cdot 3 + (-1) \cdot 2 + (-1) \cdot 0 + (-1) \cdot 4 =$$
$$15 - 9 = 6$$

1x1 Convolution

Image 5x5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

Kernel 1x1

2

What is the output size?

$$\frac{N-1}{1}+1=\underline{N}$$

1x1 Convolution

Image 5x5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

-10				

Kernel 1x1

2

$$-5 * 2 = -10$$

1x1 Convolution

Image 5x5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

-10	6	4	-10	6
8	6	4	2	-6
2	0	6	6	10
-4	0	2	8	8
10	12	14	18	-2

Kernel 1x1

2

$$-1 * 2 = -2$$

1x1 Convolution

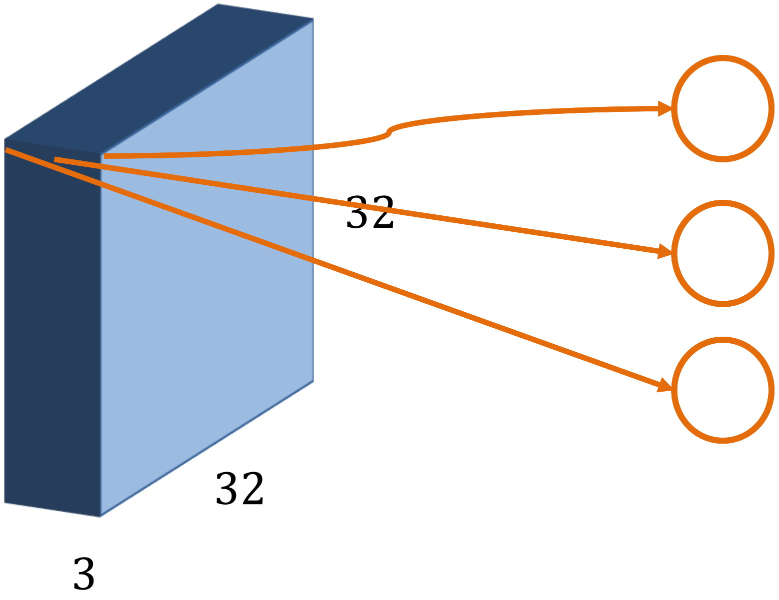
Image 5x5

-5	3	2	-5	3
4	3	2	1	-3
1	0	3	3	5
-2	0	1	4	4
5	6	7	9	-1

-10	6	4	-10	6
8	6	4	2	-6
2	0	6	6	10
-4	0	2	8	8
10	12	14	18	-2

- 1x1 kernel: keeps the dimensions and scales input

1x1 Convolution

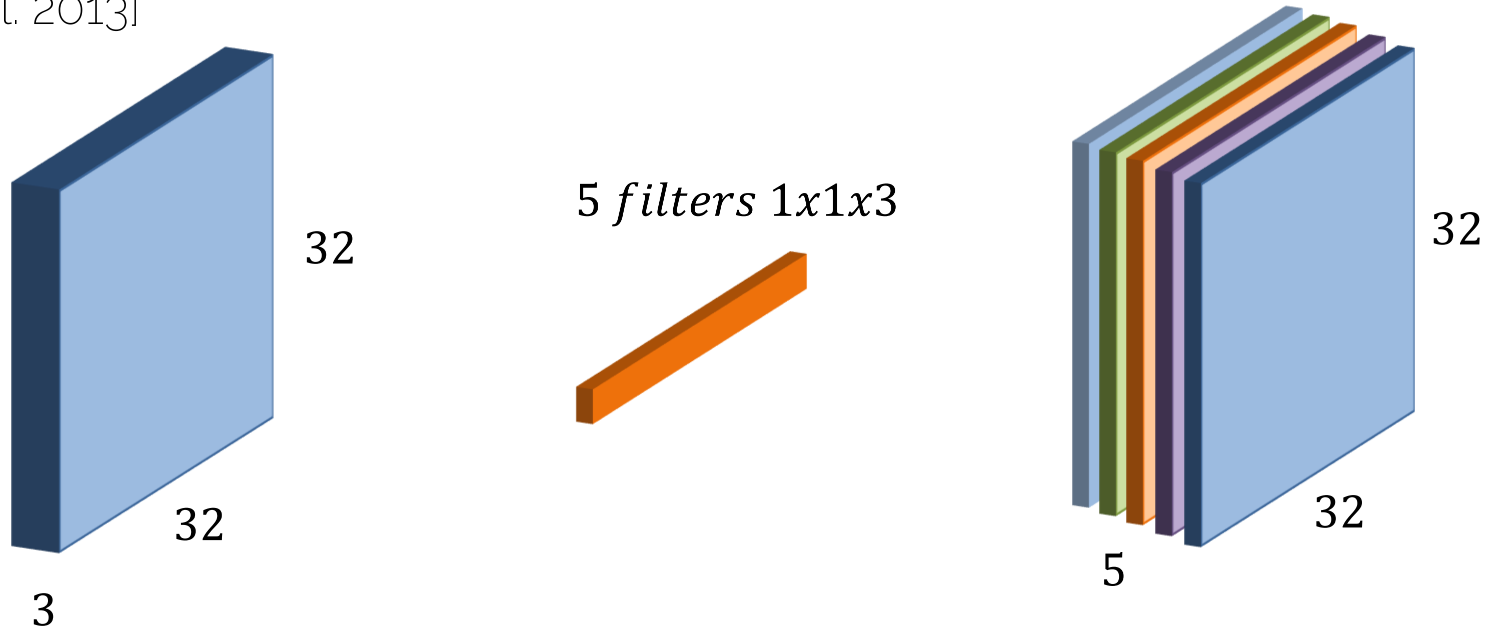


1 output

- Same as having a 3 neuron fully connected layer

1x1 Convolution

[Li et al. 2013]



- As always we use more convolutional filters

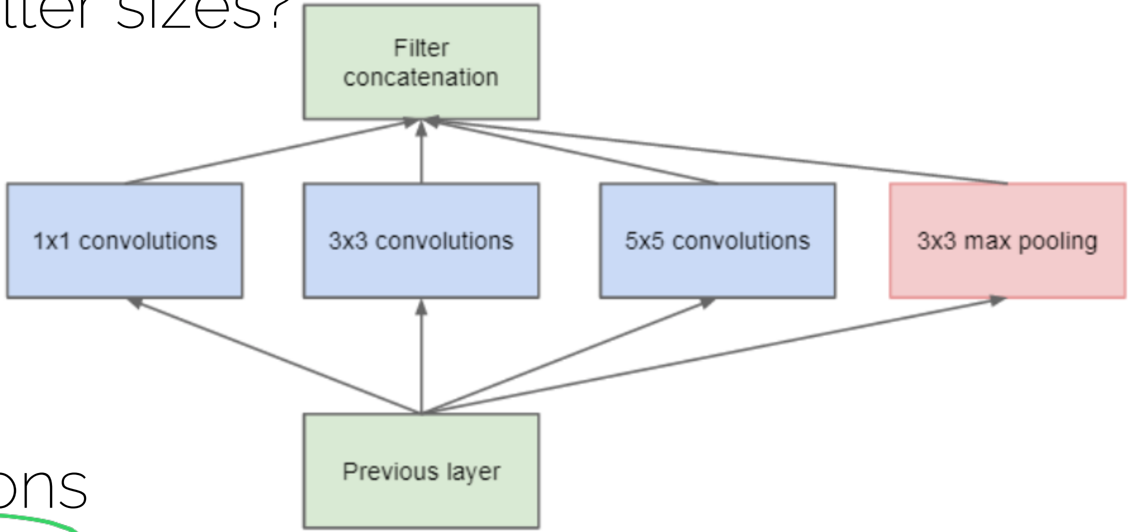
Using 1x1 Convolutions

- Use it to shrink the number of channels
- Further adds a non-linearity → one can learn more complex functions

Inception Layer

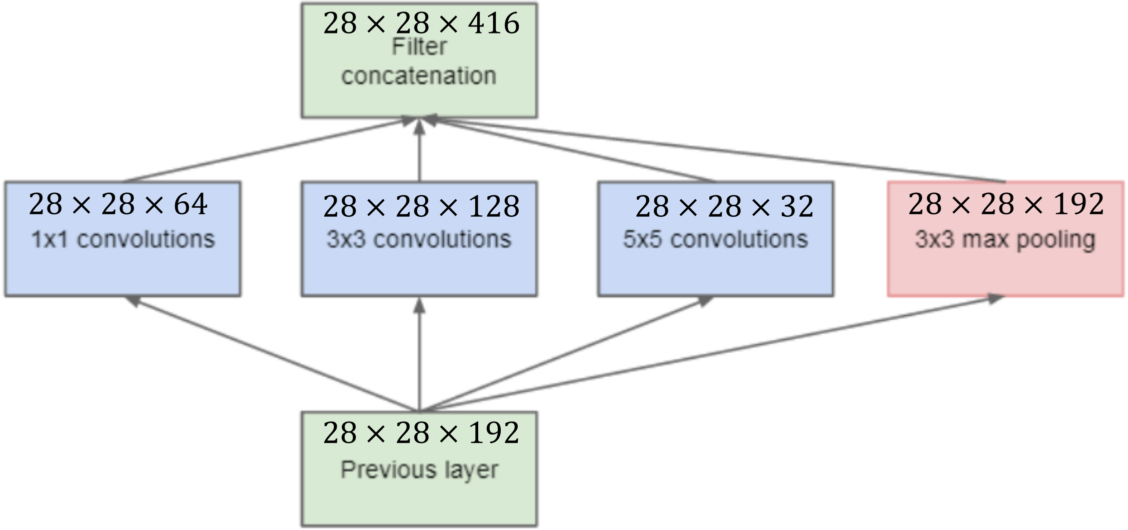
Inception Layer

- Tired of choosing filter sizes?
- Use them all!
- All same convolutions
- 3x3 max pooling is with stride 1



Inception Layer

- Possible size of the output



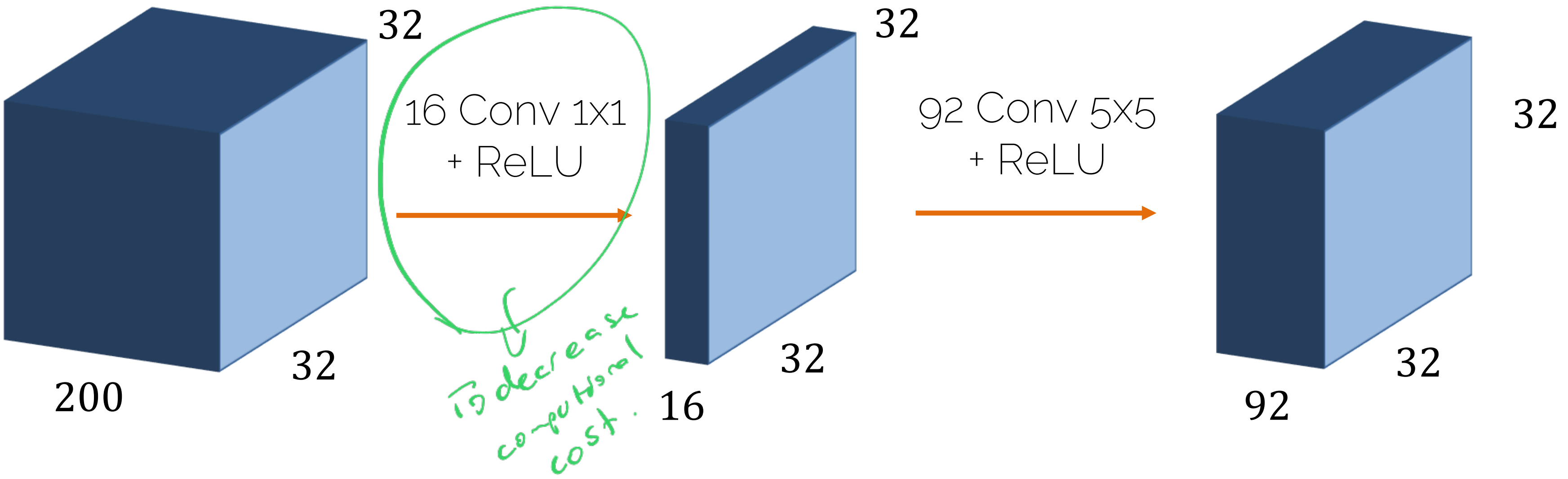
- Not sustainable!

Inception Layer: Computational Cost

Multiplications: 5x5x200 x 32x32x92 ~ 470 million

1 value of the output volume

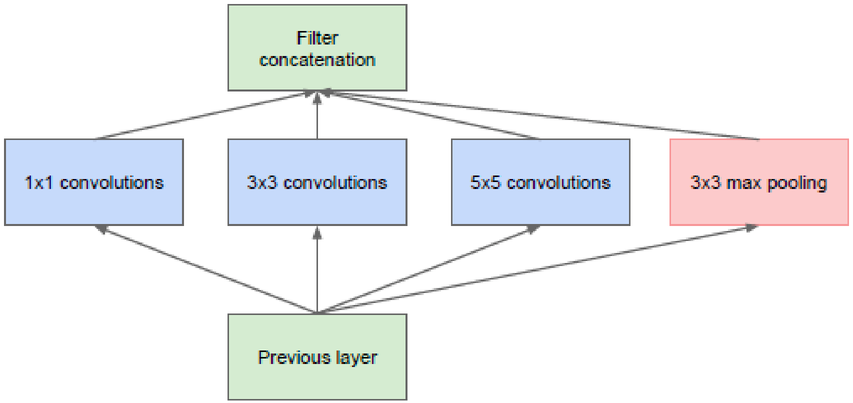
Inception Layer: Computational Cost



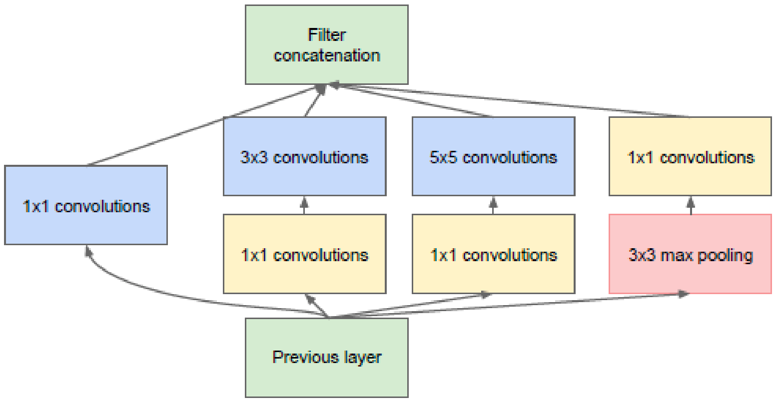
Multiplications: 1x1x200x32x32x16 5x5x16x32x32x92 ~ 40 million

Reduction of multiplications by 1/10

Inception Layer



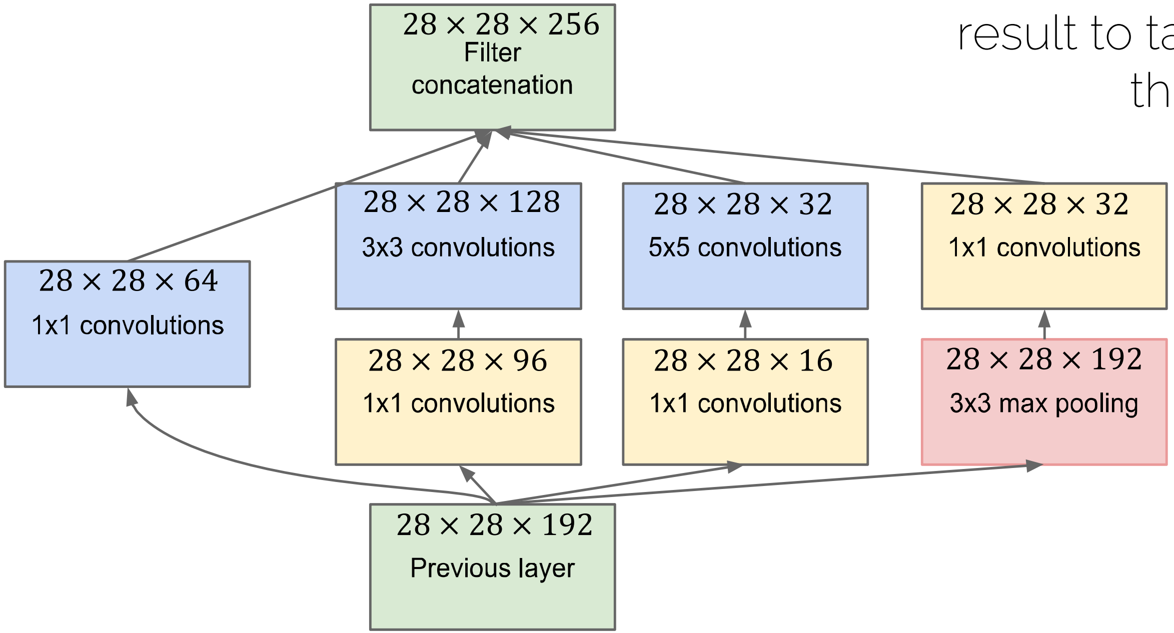
(a) Inception module, naïve version



(b) Inception module with dimensionality reduction

[Szegedy et al CVPR'15] GoogLeNet

Inception Layer: Dimensions

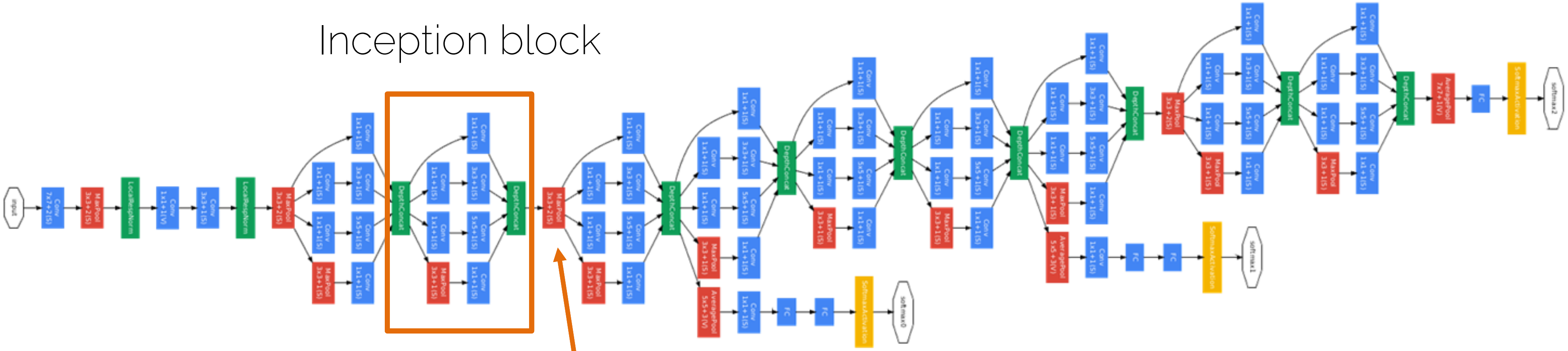


We do not want max pool result to take up almost all the output

[Szegedy et al CVPR'15] GoogLeNet

GoogLeNet: Using the Inception Layer

Inception block



Extra max pool layers to reduce dimensionality

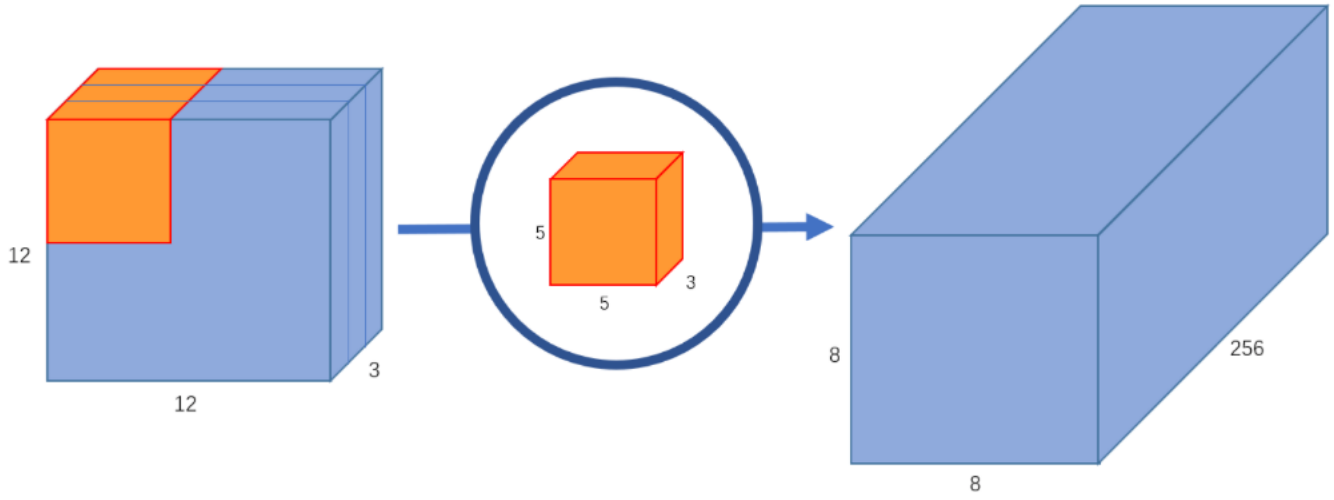
[Szegedy et al CVPR'15] GoogLeNet

Xception Net

- “Extreme version of Inception”: applying (modified) **Depthwise Separable Convolutions** instead of normal convolutions
- 36 conv layers, structured into several modules with **skip connections**
- outperforms Inception Net V3

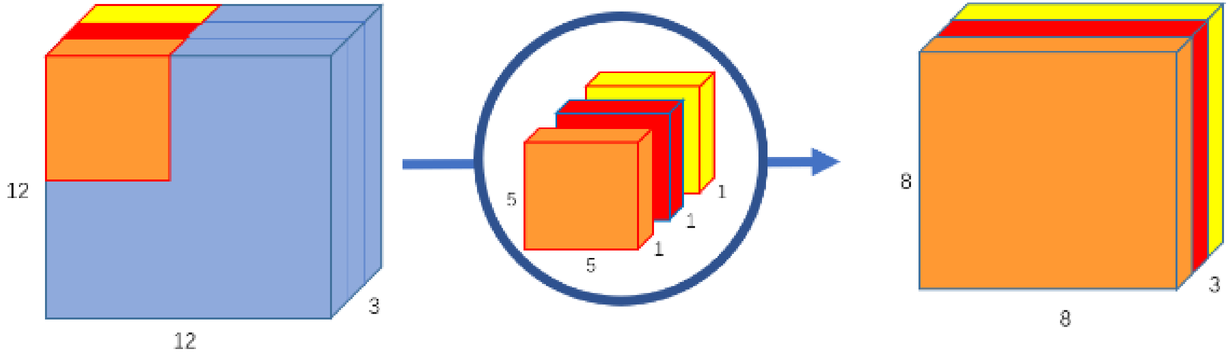
[Chollet CVPR'17] Xception

Depth-wise separable convolutions



Normal convolutions act on all channels.

Depth-wise separable convolutions

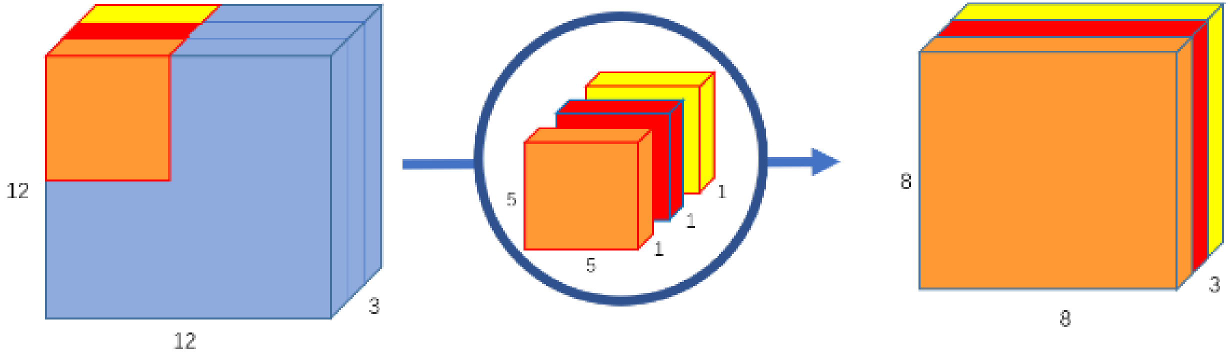


Filters are applied only at certain depths of the features.
Normal convolutions have groups set to 1, the convolutions used in this image have groups set to 3.

class`torch.nn.Conv2d`(*in_channels*, *out_channels*, *kernel_size*, *stride=1*, *padding=0*, *groups=3*)

class`torch.nn.ConvTranspose2d`(*in_channels*, *out_channels*, *kernel_size*, *stride=1*, *padding=0*, *groups=3*)

Depth-wise separable convolutions

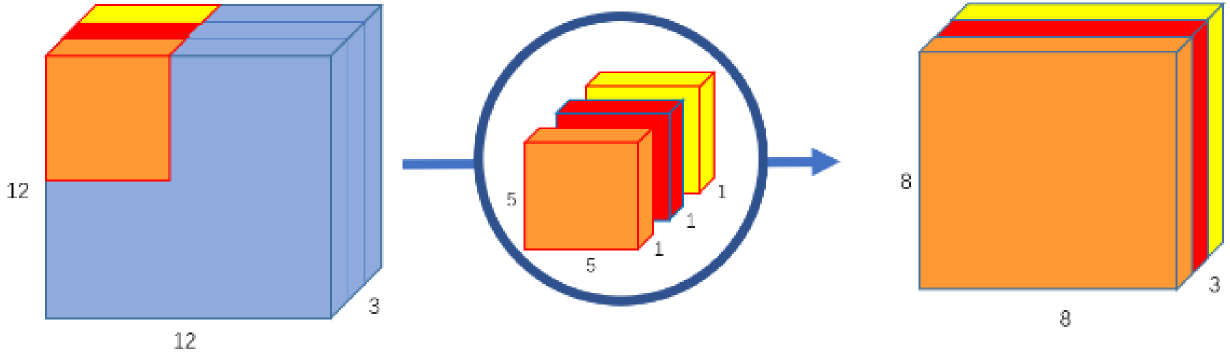


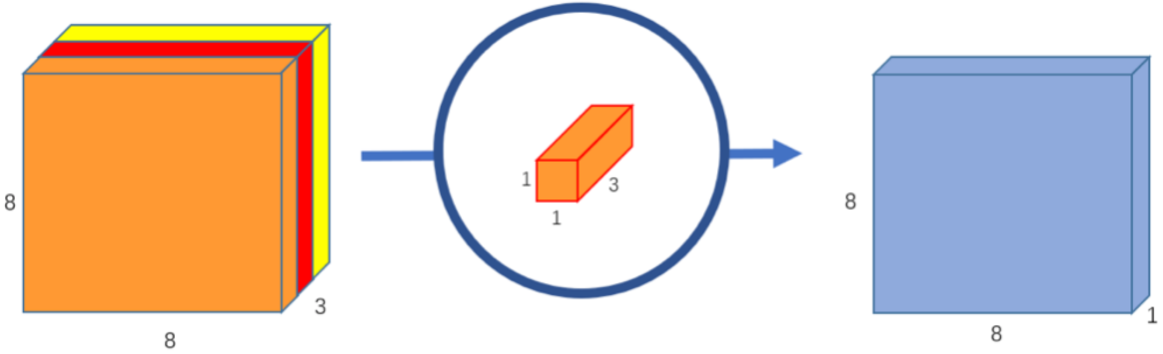
But the depth size is always the same!

class `torch.nn.Conv2d`(*in_channels, out_channels, kernel_size, stride=1, padding=0, groups=3*)

class `torch.nn.ConvTranspose2d`(*in_channels, out_channels, kernel_size, stride=1, padding=0, groups=3*)

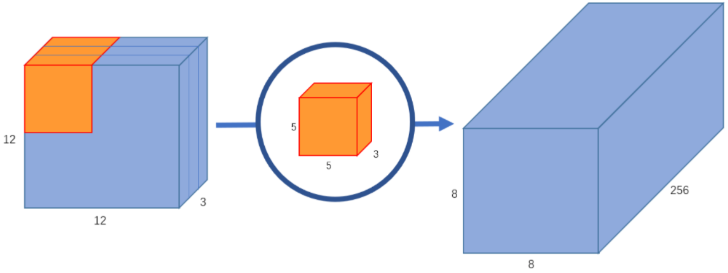
Depth-wise separable convolutions





Solution:
1x1 convs!

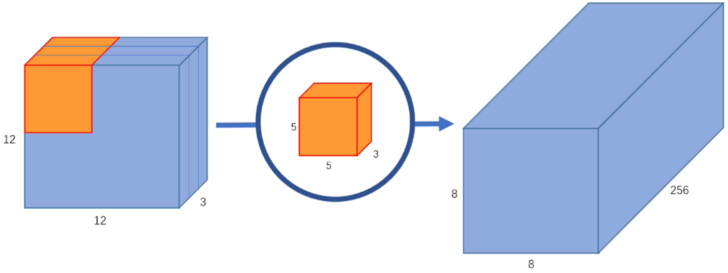
But why?



Original convolution
256 kernels of size 5x5x3

Multiplications:
256x5x5x3 x (8x8 locations) = 1.228.800

But why?



Original convolution
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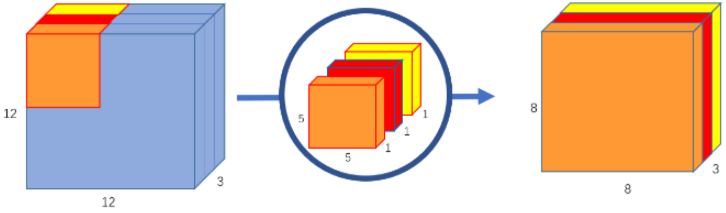
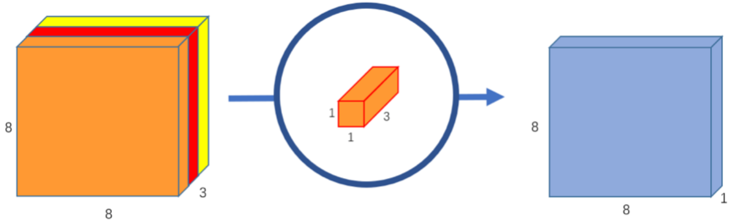
256x5x5x3 x (8x8 locations) = 1.228.800



Depth-wise convolution
3 kernels of size 5x5x1

Multiplications:
5x5x3 x (8x8 locations) = 4800

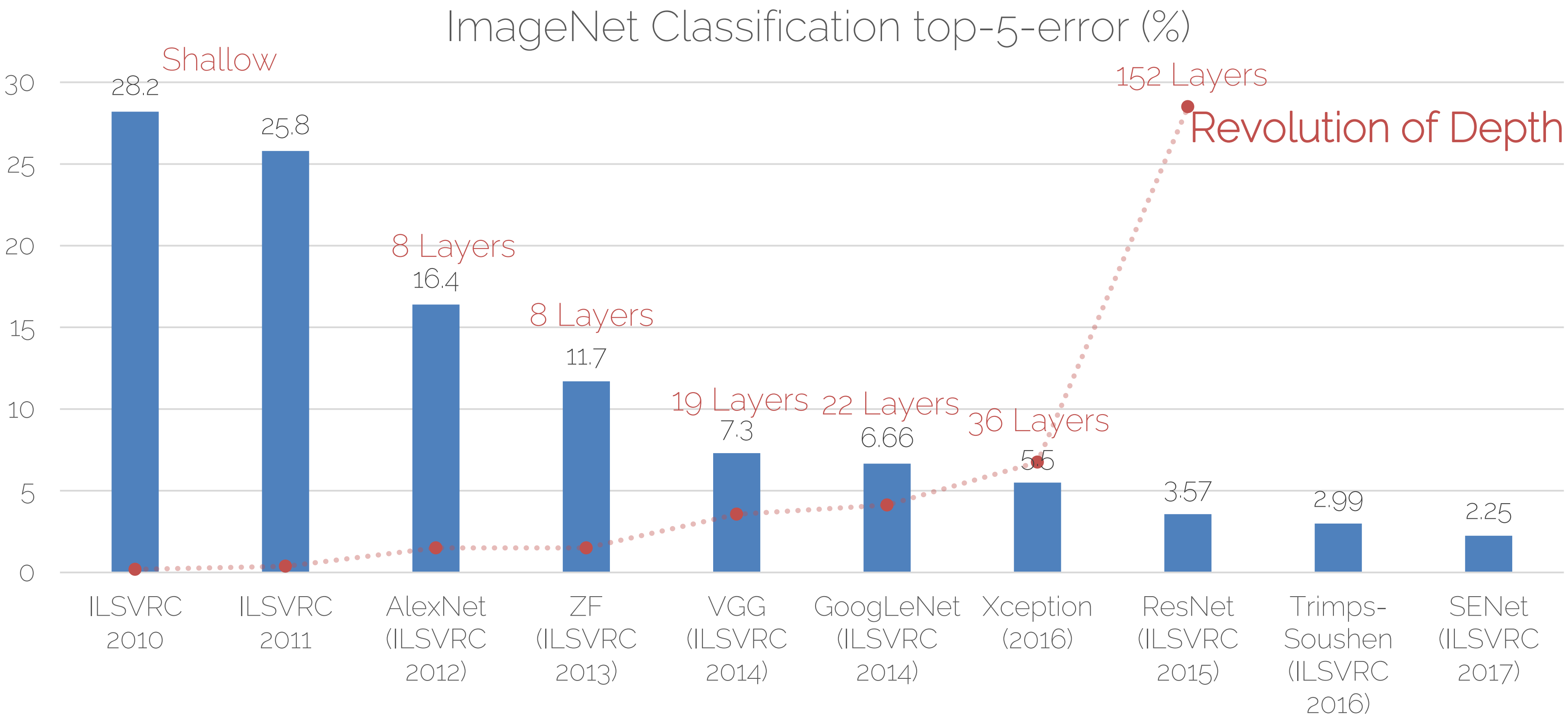


1x1 convolution
256 kernels of size 1x1x3

Multiplications:
256x1x1x3x (8x8 locations) = 49152

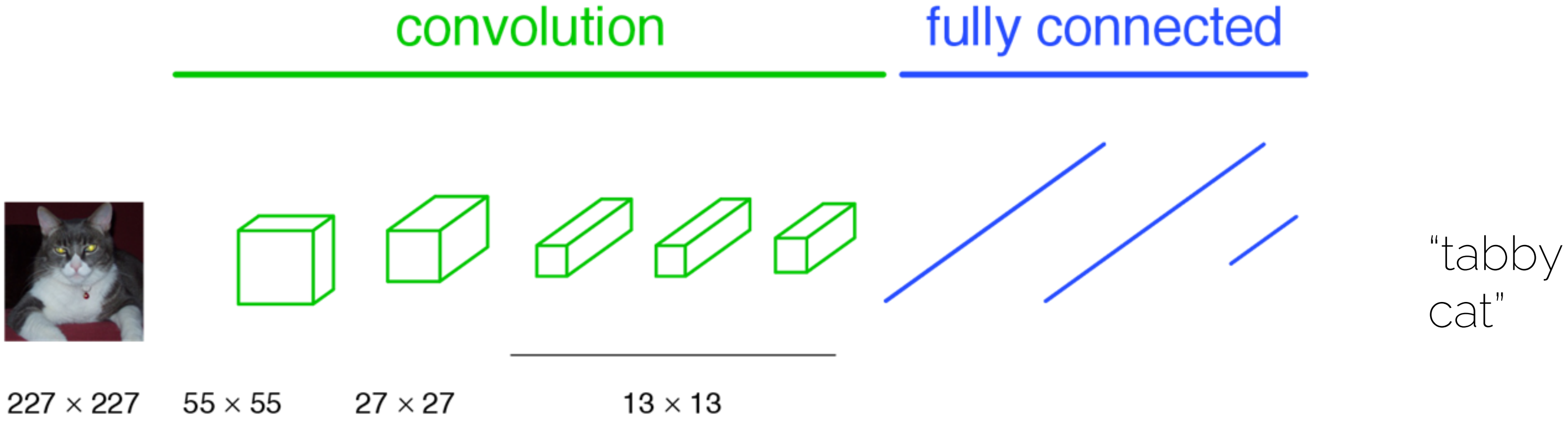
Less computation!

ImageNet Benchmark

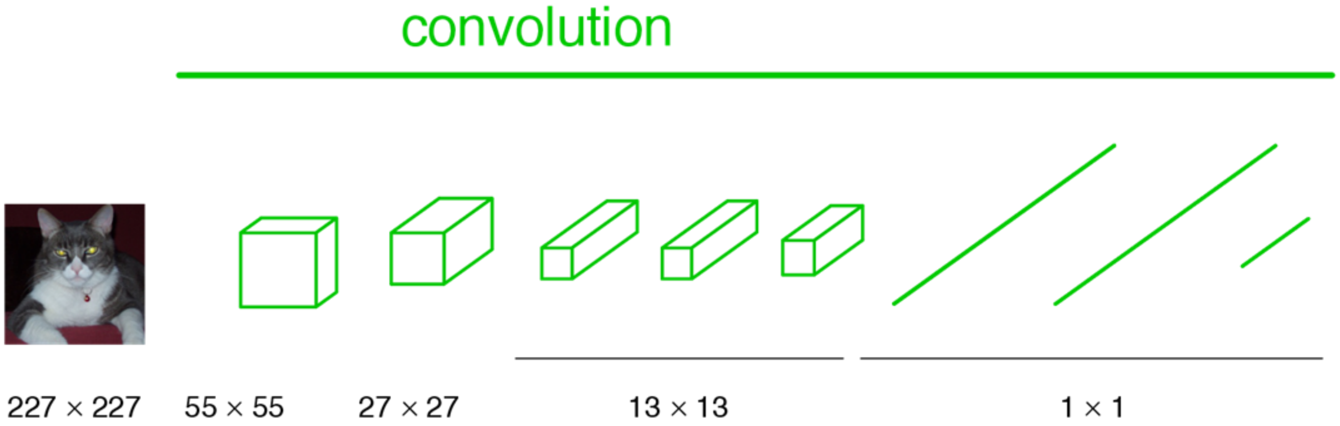


Fully Convolutional Network

Classification Network

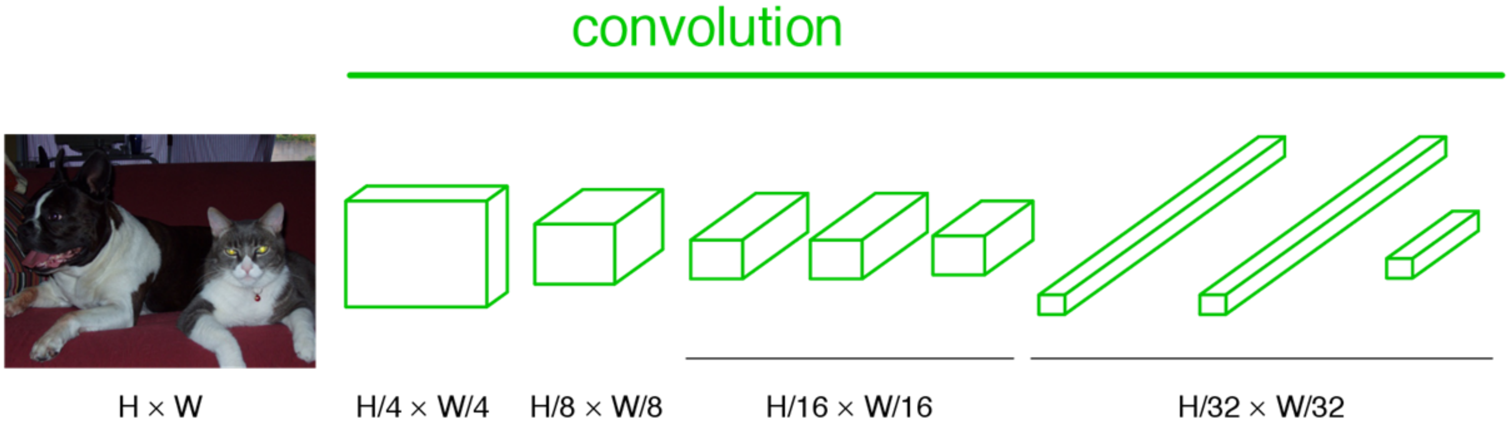


FCN: Becoming Fully Convolutional

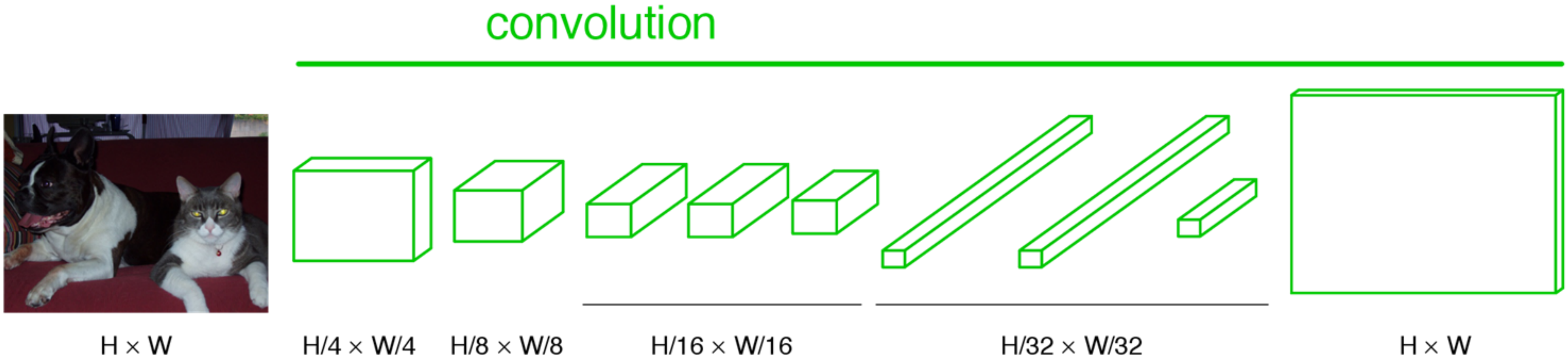


Convert fully connected layers to convolutional layers!

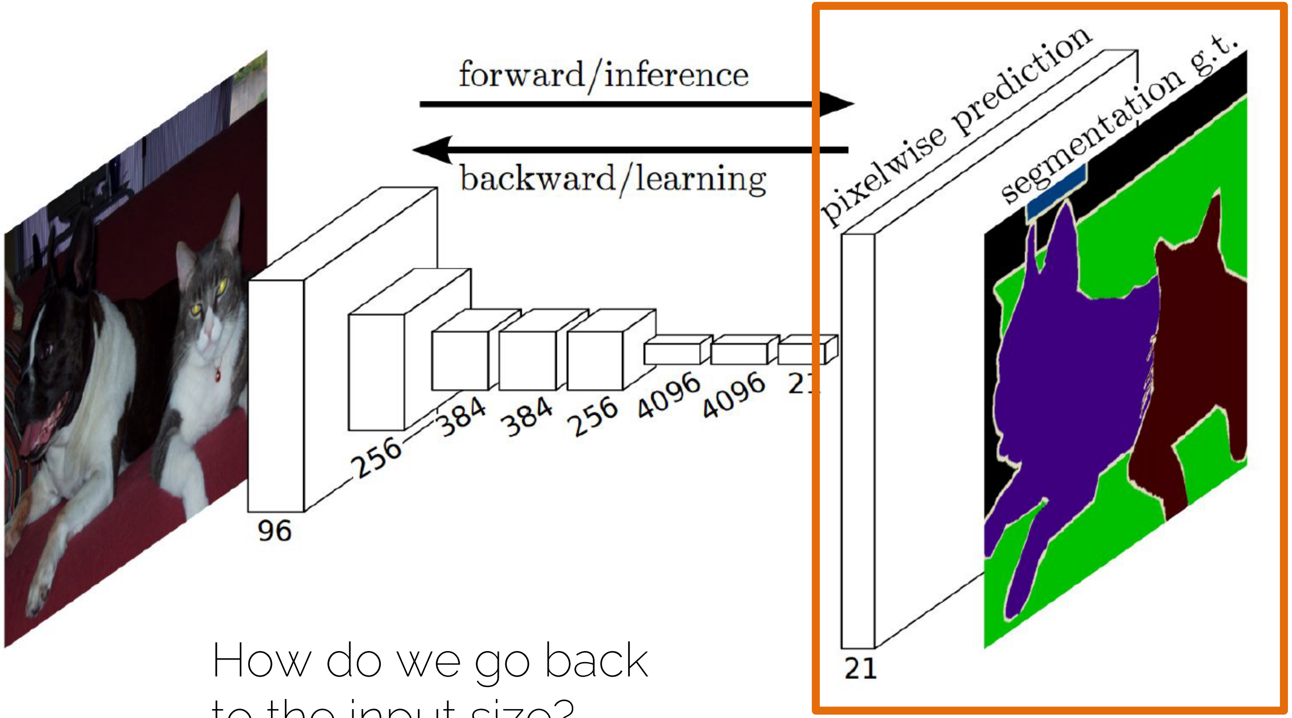
FCN: Becoming Fully Convolutional



FCN: Upsampling Output



Semantic Segmentation (FCN)

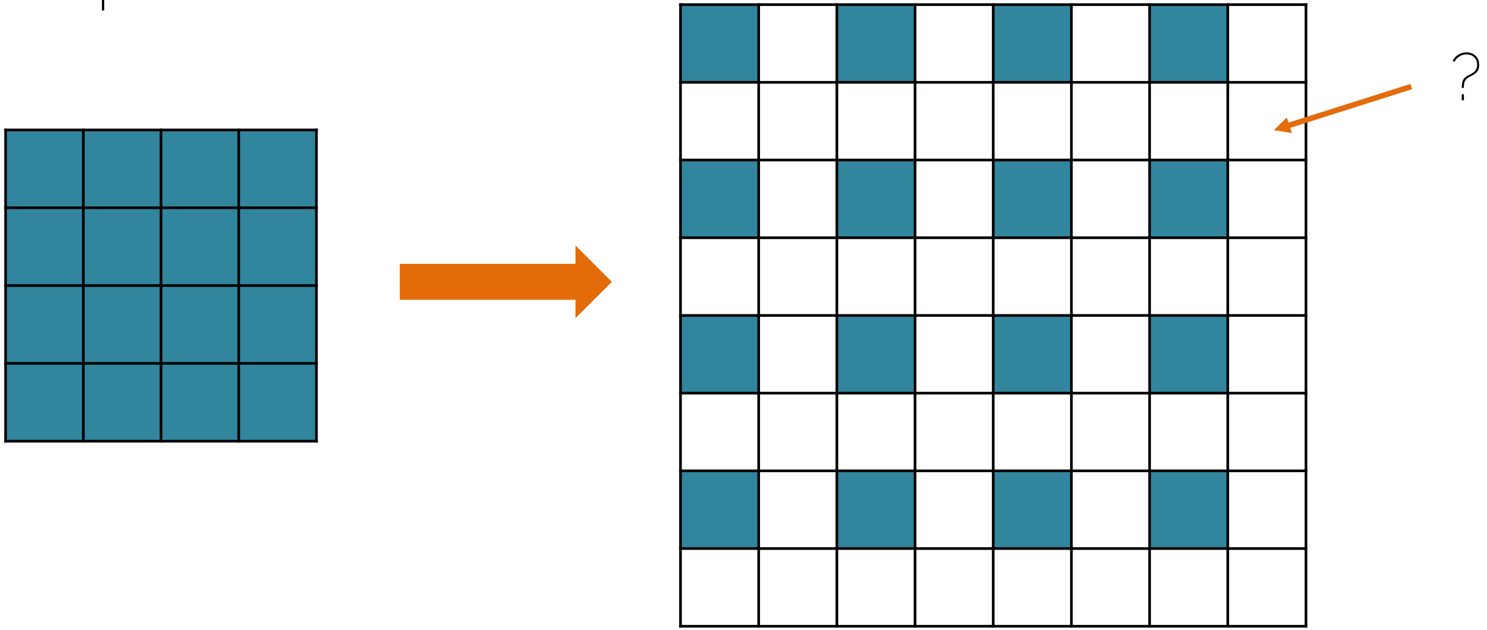


How do we go back
to the input size?

[Long and Shelhamer. 15] FCN

Types of Upsampling

- 1. Interpolation



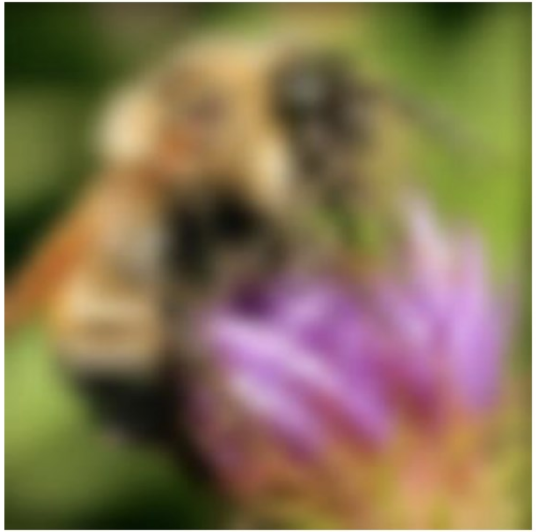
Types of Upsampling

- 1. Interpolation

Original image  x 10

Nearest neighbor interpolation

Bilinear interpolation



Bicubic interpolation

Image: Michael Guerzhoy

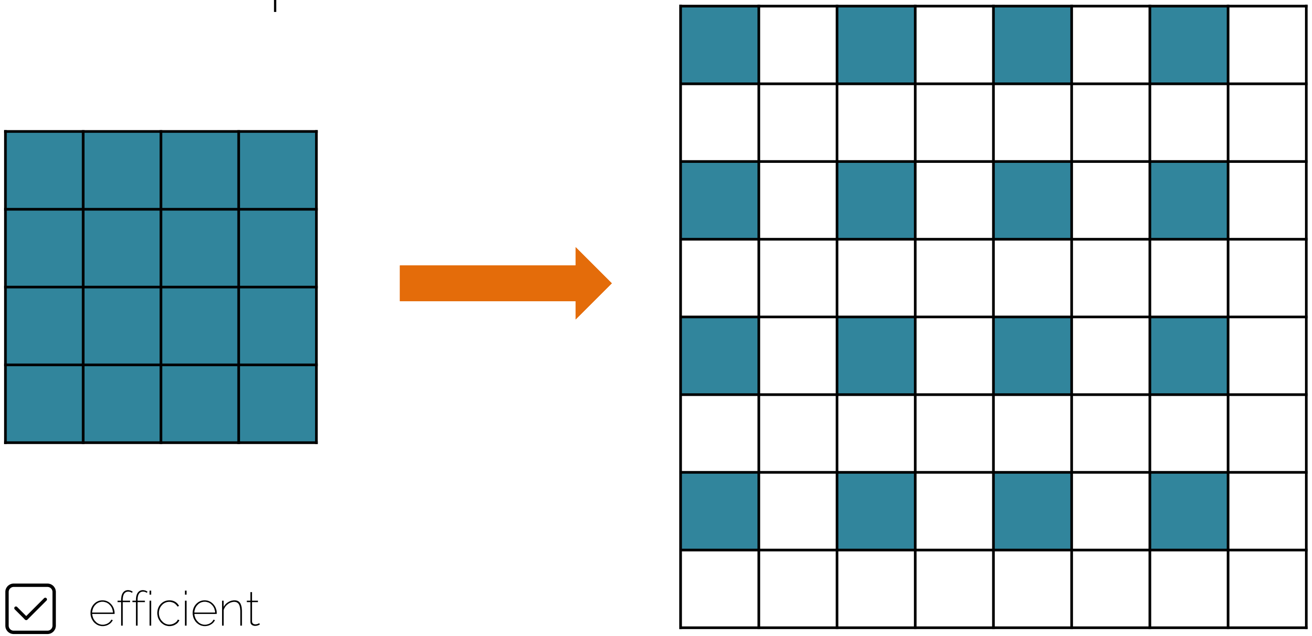
Types of Upsampling

- 1. Interpolation

    ☑ Few artifacts

Types of Upsampling

- 2. Transposed conv



+ CONVS

☑ efficient

[A. Dosovitskiy, TPAMI 2017] "Learning to Generate Chairs, Tables and Cars with Convolutional Networks"

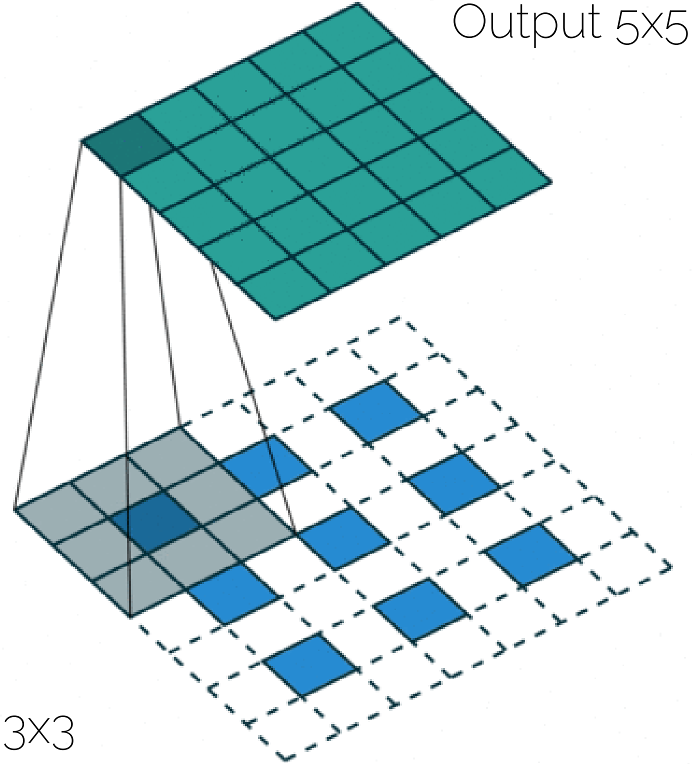
Types of Upsampling

- 2. Transposed convolution

  - Unpooling
  - Convolution filter (learned)

  - Also called up-convolution (**never** deconvolution)

Output 5x5



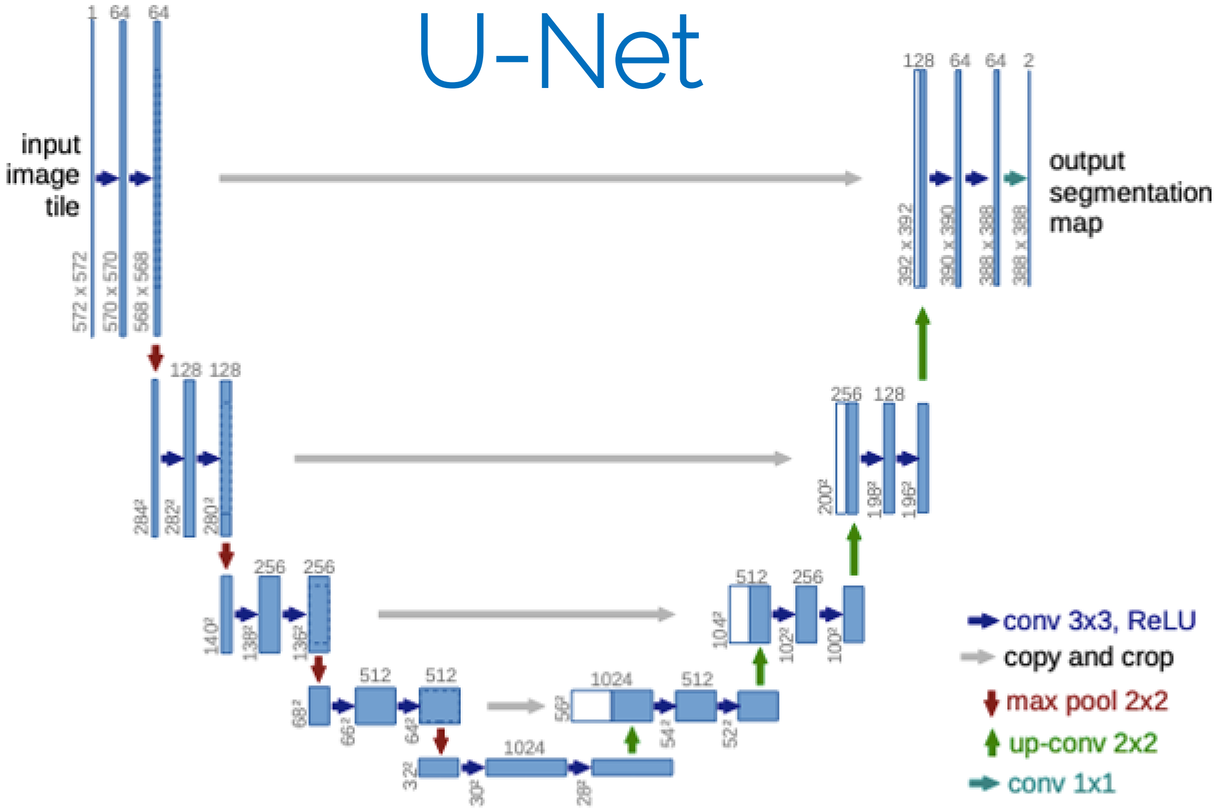
Input 3x3

Refined Outputs

- If one does a cascade of unpooling + conv operations, we get to the encoder-decoder architecture

- Even more refined: Autoencoders with skip connections (aka U-Net)

U-Net



U-Net architecture: Each blue box is a multichannel feature map. Number of channels denoted at the top of the box . Dimensions at the top of the box. White boxes are the copied feature maps.

[Ronneberger et al. MICCAI'15] U-Net

U-Net: Encoder

Left side: **Contraction Path** (Encoder)

- Captures context of the image
- Follows typical architecture of a CNN:
  - Repeated application of 2 unpadded 3x3 convolutions
  - Each followed by ReLU activation
  - 2x2 maxpooling operation with stride 2 for downsampling
  - At each downsampling step, # of channels is doubled

→ as before: Height, Width ↓, Depth: ↑

[Ronneberger et al. MICCAI'15] U-Net

U-Net: Decoder

Right Side: **Expansion Path** (Decoder):

- Upsampling to recover spatial locations for assigning class labels to each pixel
  - 2x2 up-convolution that halves number of input channels
  - **Skip Connections**: outputs of up-convolutions are concatenated with feature maps from encoder
  - Followed by 2 ordinary 3x3 convs
  - final layer: 1x1 conv to map 64 channels to # classes

- Height, Width: ↑ , Depth: ↓

[Ronneberger et al. MICCAI'15] U-Net

See you next time!

References

We highly recommend to read through these papers!

- AlexNet [Krizhevsky et al. 2012]
- VGGNet [Simonyan & Zisserman 2014]
- ResNet [He et al. 2015]
- GoogLeNet [Szegedy et al. 2014]
- Xception [Chollet 2016]
- Fast R-CNN [Girshick 2015]
- U-Net [Ronneberger et al. 2015]
- EfficientNet [Tan & Le 2019]