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Exercise 2.2 (Basic single qubit gates)
Imagine you are playing a game against an adversary. The game consists of multiple trials through which the adversary performs one of the following with equal probability:

1. They flip a coin and send you $|0\rangle$ or $|1\rangle$ depending on the outcome.

OR

2. They send you the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

Your goal is to decide which of the two they performed, and you win if you can decide correctly for $\frac{3}{4}$ of the trials on average.

(a) Before you make your guess (based on a quantum measurement on the qubit), you are allowed to perform **one** of the gates X, Y, Z or H. Compute the outputs you would obtain in each situation with each of these gates.

(b) Which of the gates would allow you to win the game? Explain your strategy.

Solution hints

(a) Applying Hadamard for case 2 gives $|0\rangle$.

(b) Strategy consists of applying a Hadamard gate, performing a standard basis measurement, and voting for case 2 in case of measuring 0. Winning probability $\frac{3}{4}$ can be computed by enumerating all possible cases.