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Tutorial 1 (Double-slit experiment)

With this tutorial we want to illustrate the counter-intuitiveness and strangeness of quantum mechanics. The content is not relevant for the final exam.

One of the most famous experiments that demonstrates the simultaneous particle-wave duality in quantum mechanics is Young's double-slit experiment. In the original experiment, the beam from a coherent light source (laser) illuminates a sheet with two thin parallel slits (Fig. 1).

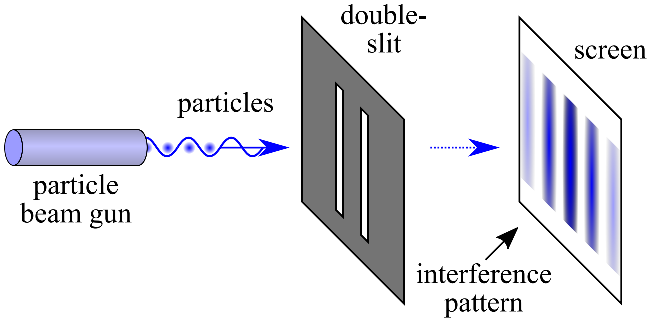

Figure 1: Double-slit experiment [1]

A screen is placed a distance away from the sheet and an interference pattern is observed on the screen. The pattern is attributed to the interference of the lightwave emitted from each slit, in which destructive and constructive interference result in a striped pattern. Surprisingly, interference is observed even when only a single particle (like a photon or electron) at a time is sent through.[2]

On the other hand, the interference pattern is destroyed when we determine which slit a particle traveled through.[3] This can be explained by the "wavefunction collapse" at the slit due to measurement.

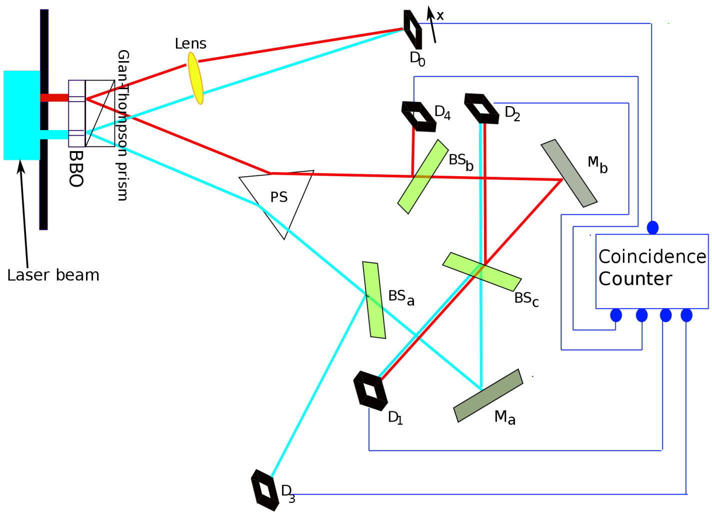

Figure 2: Experimental setup for the "delayed-choice quantum eraser" [4]

An even more counter-intuitive variant of the experiment is the "delayed-choice quantum eraser"[5], where the "which slit" information is obtained *after* the particle has already been detected on the screen (D_0 in Fig. 2).

[1] Image source: `https://en.wikipedia.org/wiki/Double-slit_experiment`

[2] R. Bach, D. Pope, S.-H. Liou, H. Batelaan: *Controlled double-slit electron diffraction.* New J. Phys. 15, 033018 (2013)

[3] S. Frabboni, G. C. Gazzadi, G. Pozzi: *Ion and electron beam nanofabrication of the which-way double-slit experiment in a transmission electron microscope.* Appl. Phys. Lett. 97, 263101 (2010)

[4] Image source: `https://en.wikipedia.org/wiki/Delayed-choice_quantum_eraser`

[5] Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih, M. O. Scully: *Delayed "choice" quantum eraser.* Phys. Rev. Lett. 84, 1 (2000); see also `https://en.wikipedia.org/wiki/Delayed-choice_quantum_eraser`

Exercise 1.1 (Complex number arithmetic)
This exercise should refresh your knowledge and proficiency with complex numbers. Given $a = 3 + 4i$ and $b = 2 - i$:

(a) Compute

- $a + b$
- ab (product of a and b)
- $1/a$
- a^* (complex conjugate of a)
- $|a|$ and $\arg(a)$ (argument), such that $a = |a|\, \mathrm{e}^{i \arg(a)}$
- the Euclidean length of the vector $\psi = \begin{pmatrix} a \\ b \end{pmatrix}$, denoted $\|\psi\|$

(b) Draw a in the complex plane, and interpret a^*, $|a|$ and $\arg(a)$ geometrically.

(c) How can one construct $a + b$ and ab geometrically in the complex plane?

Exercise 1.2 (Linear algebra basics)

(a) Compute (with "pen and paper") the matrix-vector product

$$\begin{pmatrix} 2 & -i & 5 \\ 3 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ i \\ -3 \end{pmatrix},$$

and the matrix-matrix product

$$\begin{pmatrix} -2 & 7 \\ 3 & 1+2i \end{pmatrix} \cdot \begin{pmatrix} 5 & -4 \\ 6i & 0 \end{pmatrix}.$$

(b) Find a 2×2 matrix which is not normal.
Hint: you can restrict your search to real-valued matrices.

(c) Show that the following matrix is normal, and compute its characteristic polynomial, eigenvalues and an eigenvector corresponding to one of the eigenvalues:

$$A = \begin{pmatrix} 0 & \frac{3}{5} & \frac{4}{5} \\ -\frac{3}{5} & 0 & 0 \\ -\frac{4}{5} & 0 & 0 \end{pmatrix}.$$

(d) Show that the following matrix is unitary (with $\theta \in \mathbb{R}$ a real parameter):

$$\begin{pmatrix} \cos(\theta) & i\sin(\theta) \\ i\sin(\theta) & \cos(\theta) \end{pmatrix}.$$

(e) Let $U \in \mathbb{C}^{n \times n}$ be a unitary matrix. Show that

$$|\det(U)| = 1,$$

where $|\cdot|$ denotes the absolute value.
Hint: consider $\det(U^\dagger U)$.