Exercise 1.1

a) • $a+b = 3+4i+2-i = \boxed{5+3i}$

• $ab = 6-3i+8i+4 = \boxed{10+5i}$

• $1/a = \dfrac{1}{3+4i} = \dfrac{3-4i}{9+16} = \boxed{\dfrac{3-4i}{25}}$

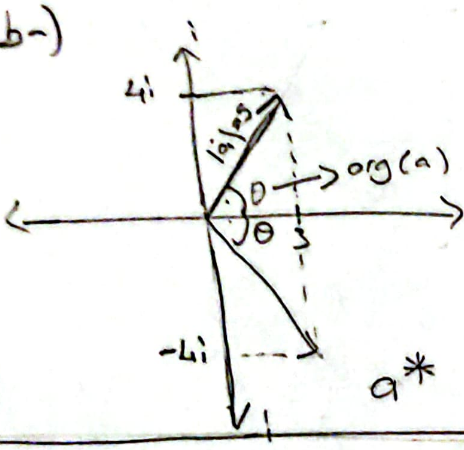
• $a^* = \boxed{3-4i}$

• $|a| = \sqrt{9+16} = \boxed{5}$ $\qquad \boxed{a = 5 \cdot e^{i \cdot (53.13^\circ)}}$

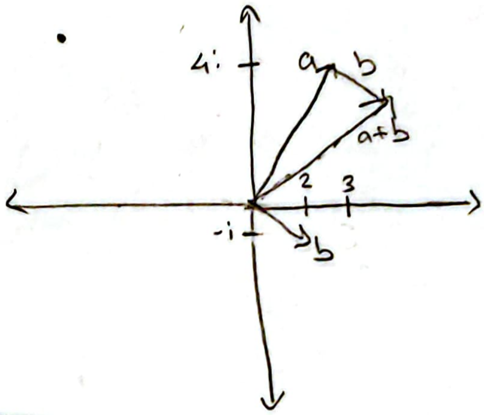
$\arg(a) = \arg(3+4i)$
$= \boxed{53.13^\circ}$

• $\|\psi\| = \left\| \begin{pmatrix} 3+4i \\ 2-i \end{pmatrix} \right\| = \sqrt{9+16+5} = \boxed{\sqrt{30}}$

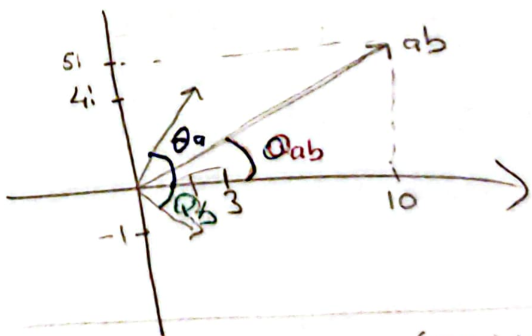
b-)



c-) •



• $ab = 10+5i$



$ab = |a| \cdot |b| \cdot e^{(\arg(a)+\arg(b))}$

Exercise 1.2

a) $\begin{pmatrix} 2 & -i & 5 \\ 3 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ i \\ -3 \end{pmatrix} = \boxed{\begin{pmatrix} -6 \\ 9 \end{pmatrix}}$

$\begin{pmatrix} -2 & 7 \\ 3 & 1+2i \end{pmatrix} \cdot \begin{pmatrix} 5 & -4 \\ 6i & 0 \end{pmatrix}$

$= \begin{pmatrix} -10+42i & 8+0 \\ 15+6i-12 & -12+0 \end{pmatrix} = \begin{pmatrix} -10+42i & 8 \\ 6i+3 & -12 \end{pmatrix}$

b-) Normal matrices can be defined as

$A \cdot A^* = A^* A$ where A is complex squared matrix.

As we are searching normal matrices in real number, we can say $A^* = A^T$, so final formula should be

$A \cdot A^T = A^T \cdot A$

$\Rightarrow$ $A \cdot A^T \neq A^T \cdot A$

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \neq \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\begin{pmatrix} 5 & 11 \\ 11 & 25 \end{pmatrix} \neq \begin{pmatrix} 10 & 14 \\ 14 & 20 \end{pmatrix}$

c-) $A \cdot A^T = A^T \cdot A$ ✓

$$\begin{pmatrix} 0 & 0{,}6 & 0{,}8 \\ -0{,}6 & 0 & 0 \\ -0{,}8 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -0{,}6 & -0{,}8 \\ 0{,}6 & 0 & 0 \\ 0{,}8 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0{,}36 & 0{,}48 \\ 0 & 0{,}48 & 0{,}64 \end{pmatrix} = \begin{pmatrix} 0 & -0{,}6 & -0{,}8 \\ 0{,}6 & 0 & 0 \\ 0{,}8 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0{,}6 & 0{,}8 \\ -0{,}6 & 0 & 0 \\ -0{,}8 & 0 & 0 \end{pmatrix}$$

✓ ✓

• Chracteristic polynomial $p(\lambda) = \det(A - \lambda I)$

$$p(\lambda) = \begin{vmatrix} -\lambda & 0{,}6 & 0{,}8 \\ -0{,}6 & -\lambda & 0 \\ -0{,}8 & 0 & -\lambda \end{vmatrix} \begin{matrix} -\lambda & 0{,}6 \\ -0{,}6 & -\lambda \\ -0{,}8 & 0 \end{matrix} = -\lambda^3 + 0 + 0 - (0{,}36\lambda + 0{,}64\lambda)$$

$$\boxed{p(\lambda) = -\lambda^3 - \lambda}$$

For eigen volues

$$-\lambda^3 - \lambda = 0 \Rightarrow \boxed{\lambda_1 = 0} \qquad \boxed{\lambda_2 = i \qquad \lambda_3 = -i}$$

$$-\lambda \cdot (\lambda^2 + 1) = 0$$

$$\begin{bmatrix} 0 & 0{,}6 & 0{,}8 \\ -0{,}6 & 0 & 0 \\ -0{,}8 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$0{,}6y + 0{,}8z = 0$ $\qquad 0{,}3y = -0{,}4z$

$\left.\begin{matrix} -0{,}6x = 0 \\ -0{,}8x = 0 \end{matrix}\right\} x = 0 \qquad \begin{matrix} y = -4 \\ z = 3 \end{matrix}$

$\Rightarrow$ $\lambda_1 = 0$ Eigen vector $\begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$

d-) $A^+ A = I$

$$\begin{pmatrix} \cos(\theta) & -i\sin\theta \\ -i\sin(\theta) & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + \sin^2\theta & i\cos\theta\sin\theta - i\cos\theta\sin\theta \\ -i\cos\theta\sin\theta + i\cos\theta\sin\theta & \cos^2\theta + \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

e-) $U^+ \cdot U = I$ ~~det (U~~

$\det(U^+ \cdot U) = \det(I)$

$\det(U^+) \cdot \det(U) = 1$

We know that $\det(U^+) = \det(U)$

$|\det^2(U)| = 1$

$|\det(U)| = 1$