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Tutorial 7 (Experimental quantum teleportation)
In this tutorial, we will discuss one of the first lab experiments[1] for quantum teleportation, which uses the polarization and "spatial path information" of photons as logical qubits. Quantum teleportation requires the following ingredients, which are challenging to realize experimentally:

1. the creation and distribution of entangled qubits,
2. measurement in the Bell basis.

The experimental setup is illustrated in the following Figure 1. It uses non-linear crystal excitations and beam splitters to generate entanglement.

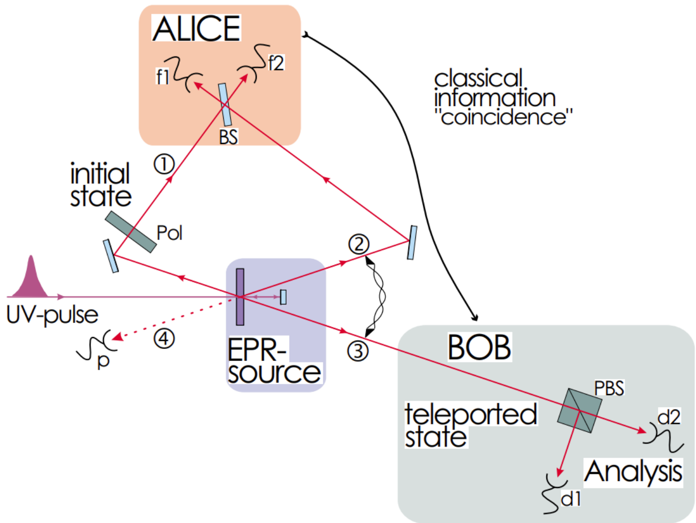


Figure 1: Quantum teleportation setup [1]

Discuss the various steps of the quantum teleportation process.

Solution The process can be divided into 4 steps:

1. An entangled pair is generated by exciting a non-linear crystal (like barium-borate) to produce an *polarity-entangled* pair of photons (labeled (2) and (3)) in the Bell state
$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right).$$
2. These photons are distributed to Alice and Bob.
3. The retroflection of the laser is used to generate a secondary entangled pair, one of which is discarded (4). The other (1) is initialized to an arbitrary polarization ($u, v \in \mathbb{C}$):
$$|\psi\rangle = u\,|0\rangle + v\,|1\rangle.$$
This is the state which Alice tries to teleport to Bob.

[1] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger: *Experimental quantum teleportation*. Nature 390, 575 (1997); see also: T. Jennewein, G. Weihs, A. Zeilinger: *Photon statistics and quantum teleportation experiments*. J. Phys. Soc. Jpn. 72, 168–173 (2003)

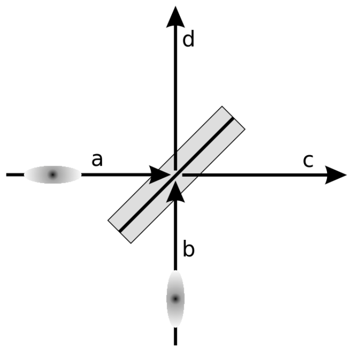
We can now represent the overall three-qubit state as follows:

$$
\begin{aligned}
|\psi\rangle\,|\beta_{11}\rangle &= \frac{1}{\sqrt{2}}(u\,|0\rangle + v\,|1\rangle)(|01\rangle - |10\rangle) \\
&= -\frac{1}{2\sqrt{2}}\Big(-2u\,|001\rangle + 2u\,|010\rangle - 2v\,|101\rangle + 2v\,|110\rangle \Big) \\
&= -\frac{1}{2\sqrt{2}}\Big((|00\rangle + |11\rangle)\,(v\,|0\rangle - u\,|1\rangle) \\
&\qquad\qquad (|00\rangle - |11\rangle)\,(-v\,|0\rangle - u\,|1\rangle) \\
&\qquad\qquad (|01\rangle + |10\rangle)\,(u\,|0\rangle - v\,|1\rangle) \\
&\qquad\qquad (|01\rangle - |10\rangle)\,(u\,|0\rangle + v\,|1\rangle) \Big) \\
&= -\frac{1}{2}\Big(|\beta_{00}\rangle\, iY\,|\psi\rangle - |\beta_{10}\rangle\, X\,|\psi\rangle + |\beta_{01}\rangle\, Z\,|\psi\rangle + |\beta_{11}\rangle\,|\psi\rangle \Big).
\end{aligned}
$$

(Note that the gates which Alice applies in the quantum teleportation circuit perform a base change between the Bell basis and standard basis, and hence Alice's overall operation can be interpreted as measurement with respect to the Bell basis.)

4. In the experimental realization, only the Bell state $|\beta_{11}\rangle$ of Alice's photons (1) and (2) is probed for. Note that $|\beta_{11}\rangle$ is distinct from the other Bell states in the sense that it is antisymmetric, i.e., it flips its sign when swapping both particles, while the other three Bell states are symmetric. According to the above representation, detecting $|\beta_{11}\rangle$ for (1) and (2) will collapse Bob's qubit into $|\psi\rangle$.

   This measurement step is performed by combining the photons via a 50-50 beam splitter. This means that there is an equal possibility that incoming photons either pass through or are reflected by the beam splitter.



The action of a beam splitter on a single photon is:

$$
|a\rangle \overset{\text{BS}}{\mapsto} \frac{1}{\sqrt{2}}(|c\rangle + i\,|d\rangle), \qquad |b\rangle \overset{\text{BS}}{\mapsto} \frac{1}{\sqrt{2}}(|d\rangle + i\,|c\rangle),
$$

introducing a phase of i in the case of reflection.

Here, we examine the *spatial* coupling of two indistinguishable photons, a and b, along the outgoing paths c and d. The trick consists of differentiating (e.g., using photodetectors) between the case that

(i) both photons emerge along path c or both along d, or

(ii) the photons emerge at different paths, i.e., one photon travels along c and the other along d.

We can use this experimental protocol to determine whether the incident photons are in a symmetric or antisymmetric superposition. Namely for a symmetric input:

$$
\begin{aligned}
\frac{1}{\sqrt{2}}(|ab\rangle + |ba\rangle) &\overset{\text{BS}}{\mapsto} \frac{1}{2\sqrt{2}}\Big((|c\rangle + i\,|d\rangle)(|d\rangle + i\,|c\rangle) + (|d\rangle + i\,|c\rangle)(|c\rangle + i\,|d\rangle) \Big) \\
&= \frac{1}{2\sqrt{2}}\Big(|cd\rangle + i\,|cc\rangle + i\,|dd\rangle - |dc\rangle + |dc\rangle + i\,|dd\rangle + i\,|cc\rangle - |cd\rangle \Big) \\
&= \frac{i}{\sqrt{2}}(|cc\rangle + |dd\rangle),
\end{aligned}
$$

that is, either both photons emerge in c or both in d (case (i)).

Analogously for an antisymmetric input state:

$$\frac{1}{\sqrt{2}}(|ab\rangle - |ba\rangle) \overset{\text{BS}}{\mapsto} \cdots = \frac{1}{\sqrt{2}}(|cd\rangle - |dc\rangle),$$

which is a superposition of $|cd\rangle$ and $|dc\rangle$ and means that the photons travel along different paths (case (ii)).

Verification of the teleportation information is performed with a *polarizing beam splitter* (PBS) that only reflects a particular polarization and allows the rest to pass through. This is done by tuning qubit (1) to a particular polarity and adjusting the PBS to reflect that particular polarity. Note that Alice still has to classically transmit the information whether case (ii) occurred to Bob.

Remark: The experimental details of this tutorial are not relevant for the final exam.