Exercise 8.1 solution

(a)

We insert the representation of $|0\rangle$ and $|1\rangle$ in terms of the new basis:

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} (\alpha\, |a\rangle + \beta\, |b\rangle) \otimes (\gamma\, |a\rangle + \delta\, |b\rangle) - \frac{1}{\sqrt{2}} (\gamma\, |a\rangle + \delta\, |b\rangle) \otimes (\alpha\, |a\rangle + \beta\, |b\rangle) =$$

$$\frac{1}{\sqrt{2}} (\alpha\,\gamma\, |aa\rangle + \alpha\,\delta\, |ab\rangle + \beta\,\gamma\, |ba\rangle + \beta\,\delta\, |bb\rangle) -$$

$$\frac{1}{\sqrt{2}} (\alpha\,\gamma\, |aa\rangle + \beta\,\gamma\, |ab\rangle + \alpha\,\delta\, |ba\rangle + \beta\,\delta\, |bb\rangle) =$$

$$\frac{1}{\sqrt{2}} (\alpha\,\delta\, |ab\rangle - \beta\,\gamma\, |ab\rangle + \beta\,\gamma\, |ba\rangle - \alpha\,\delta\, |ba\rangle) = (\alpha\,\delta - \beta\,\gamma) \frac{|ab\rangle - |ba\rangle}{\sqrt{2}}$$

(b)

```
Q = PauliMatrix[3];
% // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

```
R = PauliMatrix[1];
% // MatrixForm
```

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{S = \frac{-PauliMatrix[3] - PauliMatrix[1]}{\sqrt{2}};}$$

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

```
(* S is equal to the negative Hadamard matrix *)
Norm[S + HadamardMatrix[2]]
```

0

$$\mathbf{T = \frac{PauliMatrix[3] - PauliMatrix[1]}{\sqrt{2}};}$$

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

```
(* vector representation of spin singlet quantum state *)
```

$$\psi = \frac{1}{\sqrt{2}} \{0, 1, -1, 0\};$$

```
% // MatrixForm
```

$$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Now computing average values (for the homework you should compute this with “pen and paper”):

```
KroneckerProduct[Q, S] // MatrixForm
Conjugate[ψ].KroneckerProduct[Q, S].ψ
```

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}$$

```
KroneckerProduct[R, S] // MatrixForm
Conjugate[ψ].KroneckerProduct[R, S].ψ
```

$$\begin{pmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}$$

```
KroneckerProduct[R, T] // MatrixForm
Conjugate[ψ].KroneckerProduct[R, T].ψ
```

$$\begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}$$

```
KroneckerProduct[Q, T] // MatrixForm
Conjugate[ψ].KroneckerProduct[Q, T].ψ
```

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$-\frac{1}{\sqrt{2}}$$