

Christian B. Mendl, Irene López Gutiérrez, Keefe Huang

### Exercise 1.1 (Complex number arithmetic)

This exercise should refresh your knowledge and proficiency with complex numbers. Given  $a = 3 + 4i$  and  $b = 2 - i$ :

(a) Compute

- $a + b$
- $ab$  (product of  $a$  and  $b$ )
- $1/a$
- $a^*$  (complex conjugate of  $a$ )
- $|a|$  and  $\arg(a)$  (argument), such that  $a = |a| e^{i \arg(a)}$
- the Euclidean length of the vector  $\psi = \begin{pmatrix} a \\ b \end{pmatrix}$ , denoted  $\|\psi\|$

(b) Draw  $a$  in the complex plane, and interpret  $a^*$ ,  $|a|$  and  $\arg(a)$  geometrically.

(c) How can one construct  $a + b$  and  $ab$  geometrically in the complex plane?

### Solution

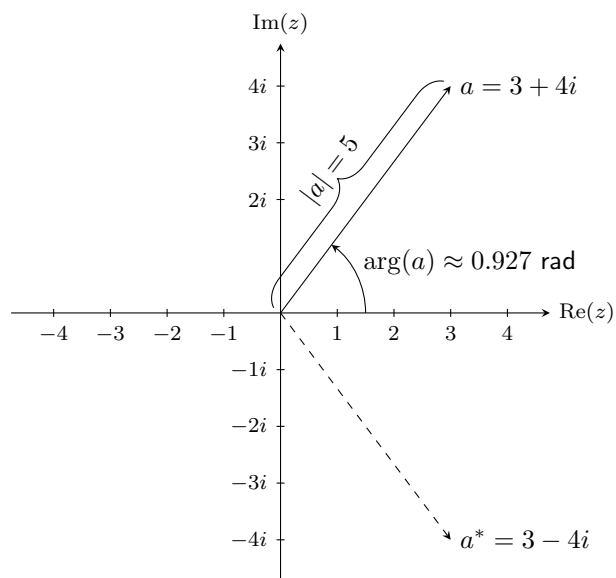
(a) Given  $a = 3 + 4i$  and  $b = 2 - i$ , the following expressions are equal to

- $a + b = 5 + 3i$
- $ab = 10 + 5i$
- $1/a = \frac{a^*}{|a|^2} = \frac{3}{25} - \frac{4}{25}i$
- $a^* = 3 - 4i$
- $|a| = \sqrt{aa^*} = \sqrt{\operatorname{Re}(a)^2 + \operatorname{Im}(a)^2} = 5$ ,  $\arg(a) = \arctan(4/3) \approx 0.927$  rad
- In general, the norm of a complex vector  $\psi = (\psi_1, \psi_2, \dots, \psi_n)$  is defined as

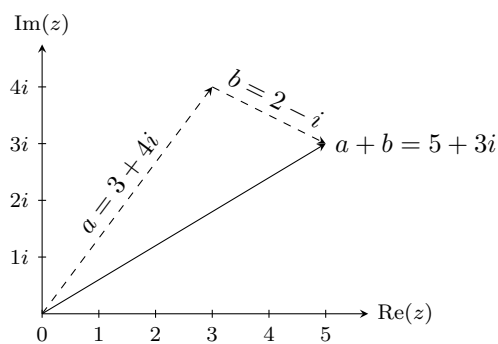
$$\|\psi\| = \sqrt{\sum_{i=1}^n |\psi_i|^2}.$$

Here  $\|\psi\| = \sqrt{|a|^2 + |b|^2} = \sqrt{25 + 5} = \sqrt{30}$ .

(b) Drawing  $a$  in the complex plane:



(c) Drawing  $a + b$  in the complex plane:



Drawing  $ab$  in the complex plane:

