CIT
Social Computing
Research Group

Technical
University
of Munich

Natural Language Processing
IN2361

Prof. Dr. Georg Groh

Chapter 4:
Naïve Bayes and
Sentiment Classification

- content is based on [1] and [2]
- certain elements (e.g. equations or tables) were taken over or taken over in a modified form from [1] or [2]
- citations of [1] and [2] or from [1] or [2] are omitted for legibility
- errors are fully in the responsibility of Georg Groh
- BIG thanks to Dan and James for a great book!

Repetition of Naïve Bayes from ML1-Class

$$\mathcal{D} = \{(x^{(1)}, y^{(1)}), (x^{(1)}, y^{(2)}), \ldots, (x^{(N)}, y^{(N)})\}$$

$$\begin{aligned}
p(\mathcal{D}|\Theta) &= \prod_{n=1}^{N} p(x^{(n)}, y^{(n)}|\theta, \pi) \\
&= \prod_{n=1}^{N} p(x^{(n)}|y^{(n)}, \theta) p(y^{(n)}|\pi) \\
&= \prod_{n=1}^{N} \prod_{v=1}^{V} p(x_v^{(n)}|y^{(n)}, \theta) p(y^{(n)}|\pi) \\
&= \prod_{n=1}^{N} \prod_{c=1}^{C} \prod_{v=1}^{V} p(x_v^{(n)}|\theta_{vc})^{\mathbb{1}(y^{(n)}=c)} \prod_{c'=1}^{C} \pi_{c'}^{\mathbb{1}(y^{(n)}=c')}
\end{aligned}$$

Repetition of Naïve Bayes from ML1-Class

$$\mathcal{D} = ((x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(N)}, y^{(N)}))$$

$$\overbrace{p(\mathcal{D}|\Theta)}^{\text{likelihood for model parameters}} = \prod_{n=1}^{N} p(x^{(n)}, y^{(n)}|\theta, \pi)$$

generative classifier

$$= \prod_{n=1}^{N} \overbrace{p(x^{(n)}|y^{(n)}, \theta)}^{\text{class conditional density} \atop \text{= class-likelihood}} \overbrace{p(y^{(n)}|\pi)}^{\text{class prior}}$$

"naïve" assumption

$$= \prod_{n=1}^{N} \prod_{v=1}^{V} p(x_v^{(n)}|y^{(n)}, \theta) p(y^{(n)}|\pi)$$

categorical class priors

$$= \prod_{n=1}^{N} \prod_{c=1}^{C} \prod_{v=1}^{V} p(x_v^{(n)}|\theta_{vc})^{\mathbb{1}(y^{(n)}=c)} \prod_{c'=1}^{C} \pi_{c'}^{\mathbb{1}(y^{(n)}=c')}$$

- **bag of words model**: position in document / word sequence does not matter in determining feature(s) for a word
- **naïve assumption**: conditional independence of features

feature vector $x^{(n)}$: e.g. V-dim. vector of term-frequencies
(notation in Jurafksy: $x^{(n)} := \mathrm{d}$ (a representation of a document))

Repetition of Naïve Bayes from ML1-Class

$$log\ p(\mathcal{D}|\Theta) = \sum_{c=1}^{C}\sum_{v=1}^{V}\sum_{\{n|y^{(n)}=c\}} log\ p(x_v^{(n)}|\theta_{vc}) + \sum_{c=1}^{C} N_c log\ \pi_c$$

- Simple MLE solution for π_c:

$$0 = \partial/\partial\pi_c log\ p(\mathcal{D}|\Theta)$$ + Lagrange multiplier $\Rightarrow$

$$\pi_c^{MLE} = \frac{N_c}{N}$$ as usual for a categorical class prior

Jurafsky notation:

$$\hat{P}(c) = \frac{N_c}{N_{doc}}$$

spam
----
total message

Repetition of Naïve Bayes from ML1-Class

$$log\ p(\mathcal{D}|\Theta) = \sum_{c=1}^{C}\sum_{v=1}^{V}\sum_{\{n|y^{(n)}=c\}} log\ p(x_v^{(n)}|\theta_{vc}) + \sum_{c=1}^{C} N_c log\ \pi_c$$

- Simple MLE solution for π_c:

$$0 = \partial/\partial\pi_c log\ p(\mathcal{D}|\Theta)$$ + Lagrange multiplier ⇨

$$\pi_c^{MLE} = \frac{N_c}{N}$$ as usual for a categorical class prior

Jurafsky notation:

$$\hat{P}(c) = \frac{N_c}{N_{doc}}$$

- Simple MLE solution for θ_{vc}:

Multinomial distribution: (Rolling a class-specific V-sided dice $M^{(n)}$ times to create a document $x^{(n)}$ of length $M^{(n)}$)

$$p(x^{(n)}|y^{(n)} = c,\theta) = Mu(x_1^{(n)},\ldots,x_V^{(n)}|\theta_{1c},\ldots,\theta_{Vc}) = \frac{M^{(n)}!}{\prod_{v=1}^{V} x_v^{(n)}!}\prod_{v=1}^{V} \overbrace{\theta_{vc}^{x_v^{(n)}}}^{p(x_v^{(n)}|\theta_{vc})}$$

Repetition of Naïve Bayes from ML1-Class

- Simple MLE solution for θ_{vc} (contd.):

(for better understanding (no change in model / results): conceptually concatenate all documents in class c into one big document $\widetilde{d}$ of length $\widetilde{N_c}$
→ words in $\widetilde{d}$ have multinomial distribution)

$0 = \partial/\partial\theta_{vc} log\ p(\mathcal{D}|\Theta)$ + Lagrange multiplier ⇨

$$\theta_{vc}^{MLE} = \frac{N_{vc}}{\widetilde{N_c}} = \frac{\sum_{\{n|x^{(n)}\in c\}} x_v^{(n)}}{\sum_{\{n|x^{(n)}\in c\}} M^{(n)}} = \frac{\sum_{\{n|x^{(n)}\in c\}} x_v^{(n)}}{\sum_{\{n|x^{(n)}\in c\}} \sum_{v=1}^{V} x_v^{(n)}} = \frac{N_{vc}}{\sum_{v=1}^{V} N_{vc}}$$

(as usual for a multinomial class conditional density)

Jurafsky notation (v = „i“):

$$\hat{P}(w_i|c) = \frac{count(w_i, c)}{\sum_{w\in V} count(w, c)}$$

Repetition of Naïve Bayes from ML1-Class

- Simple MAP solution for θ_{vc} using a Dirichlet prior:

$$p(\theta_c|D) \propto p(D|\theta_c) p(\theta_c|\alpha)$$

$$\propto \prod_{v=1}^{V} \theta_{vc}^{N_{vc}} \theta_{vc}^{\alpha_v - 1}$$

$$\propto \prod_{v=1}^{V} \theta_{vc}^{N_{vc}+\alpha_v - 1}$$

$$\propto Dir(\theta|(\alpha_1 + N_{1c}, \alpha_2 + N_{2c}, \ldots, \alpha_V + N_{Vc})$$

for prior vs posterior
https://agustinus.kristia.de/techblog/2017/01/01/mle-vs-map/

it basically if you prior is a MLE, you are calculating probabilty of something

and if you add your test result , evidence

you calculated posterior

$0 = \partial / \partial \theta_{vc} log\, p(\theta_c|\mathcal{D})$ + Lagrange multiplier $\Rightarrow$

$$\theta_{vc}^{MAP} = \frac{N_{vc} + \alpha_v - 1}{\widetilde{N}_c + (\sum_{v=1}^{V} \alpha_v) - V}$$

as usual for a Dirichlet posterior

Special MAP = Add One Smoothing

- Using weak prior $\alpha = (2,2,2,2, \ldots)$ we get Add-One-Smoothing (Laplace smoothing)

in Jurafsky notation:

$$\boxed{\hat{P}(w_i|c) = \frac{count(w_i, c) + 1}{\left(\sum_{w \in V} count(w, c)\right) + |V|}}$$

- Using the trained classifier (trained == model parameters have been determined) on a new unseen document x is simple:

$$argmax_c \;\; p(y = c|x, \Theta_{MAP/MLE})$$
$$= argmax_c \;\; p(x|y = c, \Theta_{MAP/MLE}) * p(y|\Theta_{MAP/MLE})$$

Very Simple Sentiment Classifiers

- For whole documents (e.g. a Facebook comment) or (better) individual sentences: classify into two (positive, negative) or three (positive, neutral, negative) classes of sentiment

- unknown words (words in test set but not in training set): ignore ☺

- stop words (I, at, in, can (?!),….): *maybe* remove them

	Cat	Documents
Training	-	just plain boring
	-	entirely predictable and lacks energy
	-	no surprises and very few laughs
	+	very powerful
	+	the most fun film of the summer
Test	?	predictable with no fun

Very Simple Sentiment Classifiers

- Instead of term-frequencies for words also possible: binary features (word present (=1) or not (=0))

  → generative process: instead of assuming rolling a V-sided dice M_n times to "create" a document, for each word toss a θ_{vc} coin to determine if present in the document.

  principal form of MLE solution for θ_{vc} does not change (it's a linear model)

  Binary Naïve Bayes: often works better for sentiment analysis

Four original documents:

- – it was pathetic the worst part was the boxing scenes
- – no plot twists or great scenes
- + and satire and great plot twists
- + great scenes great film

After per-document binarization:

- – it was pathetic the worst part boxing scenes
- – no plot twists or great scenes
- + and satire great plot twists
- + great scenes film

	NB Counts		Binary Counts	
	+	–	+	–
and	2	0	1	0
boxing	0	1	0	1
film	1	0	1	0
great	3	1	2	1
it	0	1	0	1
no	0	1	0	1
or	0	1	0	1
part	0	1	0	1
pathetic	0	1	0	1
plot	1	1	1	1
satire	1	0	1	0
scenes	1	2	1	2
the	0	2	0	1
twists	1	1	1	1
was	0	2	0	1
worst	0	1	0	1

Very Simple Sentiment Classifiers

- Dealing with Negations: prepend the prefix NOT to every word after a token of logical negation (n't, not, no, never) until the next punctuation mark → new words that indicate opposite sentiment

  ...didn't like this movie, but I... ⇨

  ...didn't NOT_like NOT_this NOT_movie , but I...

- Further possibility: work with only two features per document:
  number of words with (a priori) positive sentiment,
  number of words with (a priori) negative sentiment
  - (a priori) word sentiment: from sentiment lexica
    (e.g. LWIC (2007), MPQA (2005))

    $+$: *admirable, beautiful, confident, dazzling, ecstatic, favor, glee, great*

    $-$: *awful, bad, bias, catastrophe, cheat, deny, envious, foul, harsh, hate*

Naïve Bayes as Set of Class-Specific Unigram Language Models

- having learned the $\theta_{vc} = P(w_v|c)$, computing the probability of a sentence S in our model is easy:

$$P(S = (w_{v_1}, w_{v_2}, \ldots, w_{v_{|S|}})|\theta, c) = \prod_{i=1}^{|S|} P(w_{v_i}, |\theta, c) = \prod_{i=1}^{|S|} \theta_{v_i c} = \prod_{v=1}^{V} \theta_{vc}^{N_{vc}}$$

- example: for a two class sentiment classifier (using term-frequencies or binary word features) the class likelihoods for a sentence are

$$P(\text{"I love this fun film"}|+) = 0.1 \times 0.1 \times 0.01 \times 0.05 \times 0.1 = 0.0000005$$
$$P(\text{"I love this fun film"}|-) = 0.2 \times 0.001 \times 0.01 \times 0.005 \times 0.1 = .0000000010$$

assuming

w	P(w|+)	P(w|-)
I	0.1	0.2
love	0.1	0.001
this	0.01	0.01
fun	0.05	0.005
film	0.1	0.1
...	...	...

Classifiers: Error / Performance Measures: Confusion Matrix

		Actual class	
		Cat	Non-cat
Predicted class	Cat	5 True Positives	2 False Positives
	Non-cat	3 False Negatives	17 True Negatives

		Actual class		
		Cat	Dog	Rabbit
Predicted class	Cat	5	2	0
	Dog	3	3	2
	Rabbit	0	1	11

Classifiers: Error / Performance Measures: Two Classes

Accuracy:

$$ACC = \frac{TP + TN}{TP + FP + FN + TN}$$

		Actual	
		y=1	y=0
Predicted	y=1	TP	FP type I error
	y=0	FN type II error	TN

Precision (positive predictive value):

$$PREC = \frac{TP}{TP + FP}$$

Recall (sensitivity, true positive rate):

$$REC = \frac{TP}{TP + FN}$$

Specifity (true negative rate):

$$TNR = \frac{TN}{FP + TN}$$

False Negative Rate (miss rate):

$$FNR = \frac{FN}{TP + FN}$$

False Positive Rate (fall out):

$$FPR = \frac{FP}{FP + TN}$$

F1 Score (harmonic mean of Recall and Precision):

$$\mathrm{F1} = \frac{2 * PREC * REC}{PREC + REC}$$

Classifiers: Error / Performance Measures: >2 Classes

- usually: use average of one vs rest (macro averaging): example:

Accuracy:

$$ACC = \frac{1}{C}\sum_{c=1}^{C} \frac{TP_c + TN_c}{TP_c + FP_c + FN_c + TN_c} = \frac{1}{C}\sum_{c=1}^{C} ACC_c$$

- possible: weighted approach (e.g. using class-priors / inverse class priors to emphasize importance of frequent / infrequent classes): example:

Accuracy:

$$ACC = \sum_{c=1}^{C} \pi_c \, ACC_c$$

- micro-averaging μ vs macro-averaging M example:

Precision$_\mu$

$$PREC = \frac{\sum_{c=1}^{C} TP_c}{\sum_{c=1}^{C} TP_c + FP_c}$$

Precision$_M$:

$$PREC = \frac{1}{C}\sum_{c=1}^{C} \frac{TP_c}{TP_c + FP_c} = \frac{1}{C}\sum_{c=1}^{C} PREC_c$$

Classifiers: Error / Performance Measures: >2 Classes

example

		gold labels urgent	*gold labels* normal	*gold labels* spam	
system output	urgent	8	10	1	**precisionu**= $\frac{8}{8+10+1}$
system output	normal	5	60	50	**precisionn**= $\frac{60}{5+60+50}$
system output	spam	3	30	200	**precisions**= $\frac{200}{3+30+200}$
		recallu = $\frac{8}{8+5+3}$	**recalln** = $\frac{60}{10+60+30}$	**recalls** = $\frac{200}{1+50+200}$	

Class 1: Urgent

	true urgent	true not
system urgent	8	11
system not	8	340

$$\text{precision} = \frac{8}{8+11} = .42$$

Class 2: Normal

	true normal	true not
system normal	60	55
system not	40	212

$$\text{precision} = \frac{60}{60+55} = .52$$

Class 3: Spam

	true spam	true not
system spam	200	33
system not	51	83

$$\text{precision} = \frac{200}{200+33} = .86$$

Pooled

	true yes	true no
system yes	268	99
system no	99	635

$$\text{microaverage precision} = \frac{268}{268+99} = .73$$

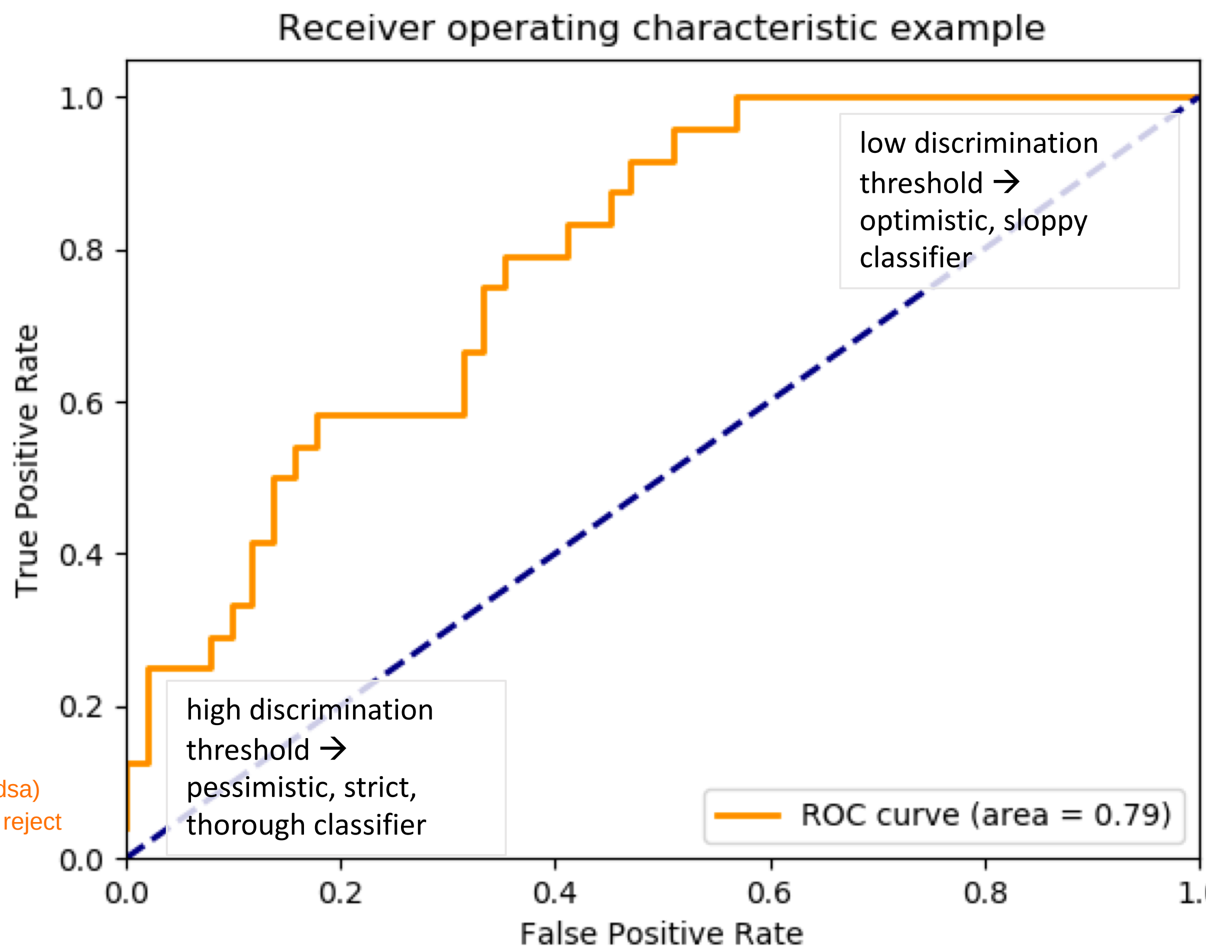
$$\text{macroaverage precision} = \frac{.42+.52+.86}{3} = .60$$

Classifiers: Error / Performance Measures: ROC / AUC

		Actual	
		y=1	y=0
Predic-ted	y=1	TP	FP
	y=0	FN	TN

Recall (sensitivity, true positive rate):

$$REC = \frac{TP}{TP + FN}$$



the blue says prob of given x is a class C1
also red says prob of given x is a class C2
The green are says ((((i didnt understand adsjlljdsaldsa)
maybe it is a posteria region like if your are not sure reject

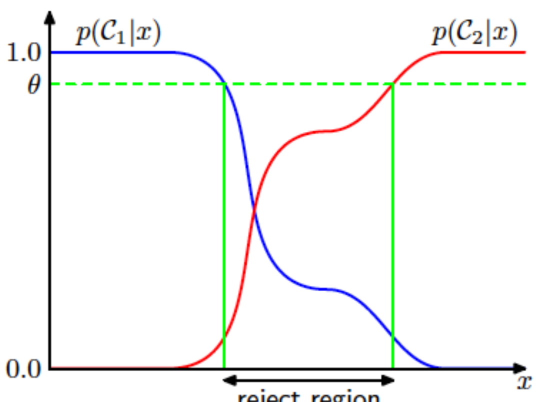


Figure 1.26, Bishop

False Positive Rate (fall out):

$$FPR = \frac{FP}{FP + TN}$$

		Actual	
		y=1	y=0
Predic-ted	y=1	TP	FP
	y=0	FN	TN

Statistical Significance Testing

- Compare two classifiers A and B: use F1 score on test set x, yielding e.g. the result $\delta(x) = F1(A) - F1(B) > 0$. ("A better than B")
  But: is result statistically significant?

we have 2 classifier, one has slightly higher f1 score than other
Can we say the first one better than second one ?

- Null Hypothesis H_0:
  "result 'A better than B' was achieved by chance" =
  "typically, A is not better" = $\delta(x) \leq 0$

  i didnt understand :d

  → if X is random variable over all conceivable test-sets, under H_0 (== small $\delta(x)$) we expect $P(\delta(X) > \delta(x)|H_0)$ to be large (sic!) and for a large $\delta(x)$ (== $\neg H_0 = H_1$) we expect $P(\delta(X) > \delta(x)|H_0)$ to be small
  → reject H_0 ←→ p-Value $\mathrm{P}(\delta(X) > \delta(x)|H_0) < 0.05$.

- Performance measures are rarely normally distributed → paired T-test not applicable; furthermore: test data is scarce → use bootstrap testing (can be applied to any performance measure)

Bootstrap Testing

- two classes: for each element of x, we can have four cases:
  A and B are right (AB), A is right and B wrong (A~~B~~),
  A is wrong and B right (~~A~~B), A and B are wrong (~~AB~~)

	1	2	3	4	5	6	7	8	9	10	A%	B%	$\delta()$
x	AB	A~~B~~	AB	~~A~~B	A~~B~~	~~A~~B	A~~B~~	AB	~~AB~~	A~~B~~	.70	.50	.20
$x^{*(1)}$	A~~B~~	AB	A~~B~~	~~A~~B	~~A~~B	A~~B~~	~~A~~B	AB	~~AB~~	AB	.60	.60	.00
$x^{*(2)}$	A~~B~~	AB	~~AB~~	~~A~~B	~~A~~B	AB	~~A~~B	A~~B~~	AB	AB	.60	.70	-.10
...													
$x^{*(b)}$													

assuming accuracy instead of F1 for the sake of simplicity

Since these $x^*(i)$ (with the same size as the original x) are sampled from x with replacement, $\mathrm{E}[X] \approx \delta(x)$, i.e. $\delta(x)$ is the average (expected) performance advantage. Therefore, H_0 expects to see a lot of the $x^*(i)$ to have $\delta(x^*(i)) > \delta(x)$

Figure 6.8 The bootstrap: Examples of b pseudo test sets being created from an initial true test set x. Each pseudo test set is created by sampling $n = 10$ times with replacement; thus an individual sample is a single cell, a document with its gold label and the correct or incorrect performance of classifiers A and B.

then apply ($1.65\sigma \approx$) 2σ-rule <-> one sided Bootstrap t-Test: if p-value(x) is not low enough, we conclude that $\delta(x)$ was not "extreme" enough ("too few $\delta(x^*(i)) > 2\delta(x)$") to warrant the rejection of H_0
(for an explanation of the Bootstrapping technique see Berg-Kirkpatrick et al. (2012) and B. Efron and R. Tibshirani. 1993. An introduction to the bootstrap. Chapman & Hall/CRC) The Bootstrapping algorithm presented here implicitly accounts for / neglects the missing division by the standard error $\sigma/\sqrt{b}$
Kirkpatrick et al. (2012) state that instead of checking „are enough $\delta(x^*(i)) > 2\delta(x)$" you could also check for "are enough $\delta(x^*(i)) < 0$?" to reject H_0

function BOOTSTRAP(x, b) **returns** *p-value*(x)

Calculate $\delta(x)$
for $i = 1$ **to** b **do**
  for $j = 1$ **to** n **do** # Draw a bootstrap sample $x^{*(i)}$ of size n
    Select a member of x at random and add it to $x^{*(i)}$
  Calculate $\delta(x^{*(i)})$
for each $x^{*(i)}$
  $s \leftarrow s + 1$ **if** $\delta(x^{*(i)}) > 2\delta(x)$
p-value(x) $\approx \frac{s}{b}$
return p-value(x)

Feature Selection

- "per feature" feature selection (unlike PCA etc.): compare decision trees: rank features according to their discriminative power:
  - Entropy-based (Information Gain)

$$
\begin{aligned}
G(w) = \quad & -\sum_{i=1}^{C} P(c_i) \log P(c_i) \\
& + P(w) \sum_{i=1}^{C} P(c_i|w) \log P(c_i|w) \\
& + P(\bar{w}) \sum_{i=1}^{C} P(c_i|\bar{w}) \log P(c_i|\bar{w})
\end{aligned}
$$

  - GINI Index

$$G(w) = 1 - \sum_{i=1}^{C} P(c_i|w)^2$$

- or select skew directions in feature space: e.g. via PCA, pPCA, Factor Analysis, SVD etc. (compare Murphy chapter 12)

Repetition Statistics: Two-sided T-Test, known σ_0 [4]

$$X_i \sim \mathcal{N}(\mu, \sigma_0^2) \xrightarrow[A \sim \mathcal{N}(\mu,\sigma_0^2) \to B = aA + c \sim \mathcal{N}(a\mu + c, a^2\sigma_0^2)]{\sum_{i=1}^{N} X_i \sim \mathcal{N}(N\mu, N\sigma_0^2)} \overline{X}_N = \frac{1}{N}\sum_{i=1}^{N} X_i \sim \mathcal{N}(\mu, \frac{1}{N}\sigma_0^2)$$

$$\longrightarrow \quad \frac{\overline{X}_N - \mu}{\sqrt{\frac{\sigma_0^2}{N}}} \sim \mathcal{N}(0,1)$$

$$\longrightarrow \quad P(\frac{\overline{X}_N - \mu}{\sqrt{\frac{\sigma_0^2}{N}}} \in [-1.96 = u_{0.025} = u_{\alpha/2}, 1.96 = u_{0.975} = u_{1-\alpha/2}]) = 0.95$$

$$\longrightarrow \quad P(-1.96 \leq \frac{\overline{X}_N - \mu}{\sqrt{\frac{\sigma_0^2}{N}}} \leq 1.96) = 0.95$$

$$P(\overline{X}_N - 1.96\frac{\sigma_0}{\sqrt{N}} \leq \mu \leq \overline{X}_N + 1.96\frac{\sigma_0}{\sqrt{N}}) = 0.95$$

so the confidence interval for μ given the empirical result of the point estimator $\overline{X}_N$ is

$$\mu \in [\overline{X}_N - 1.96\frac{\sigma_0}{\sqrt{N}}, \overline{X}_N + 1.96\frac{\sigma_0}{\sqrt{N}})]$$

if $H_0 : \mu = \mu_0$ were true we would need to have

$$\mu_0 - 1.96\frac{\sigma_0}{\sqrt{N}} \leq \overline{X}_N \leq \mu_0 + 1.96\frac{\sigma_0}{\sqrt{N}}$$

$$so \quad reject \quad H_0 : \mu = \mu_0 \quad if \quad \overline{X}_N \leq \mu_0 - 1.96\frac{\sigma_0}{\sqrt{N}}$$

$$or \quad \overline{X}_N \geq \mu_0 + 1.96\frac{\sigma_0}{\sqrt{N}}$$

Repetition Statistics: Two-sided T-Test, unknown σ_0 [4]

$$X_i \sim \mathcal{N}(\mu, \sigma_0^2) \qquad \overline{V}_N = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X}_N)^2$$

$$\longrightarrow \quad T_N = \frac{\overline{X}_N - \mu}{\sqrt{\frac{\overline{V}_N}{N}}} \sim St_{N-1} \quad \longrightarrow$$

$$\longrightarrow \quad P(T_N \in [t_{N-1,0.025} = t_{N-1,\alpha/2}, t_{N-1,0.975} = t_{N-1,\alpha/2}]) = 0.95$$

$$\longrightarrow \quad P\left(t_{N-1,0.025} \le T_N = \frac{\overline{X}_N - \mu}{\sqrt{\frac{\overline{V}_N}{N}}} \le t_{N-1,0.975}\right) = 0.95$$

$$\longrightarrow \quad P\left(\overline{X}_N - t_{N-1,0.025}\sqrt{\frac{\overline{V}_N}{N}} \le \mu \le \overline{X}_N + t_{N-1,0.975}\sqrt{\frac{\overline{V}_N}{N}}\right) = 0.95$$

$$reject \quad H_0 : \mu = \mu_0 \quad if \quad \overline{X}_N \le \mu_0 - t_{N-1,0.025}\sqrt{\frac{\overline{V}_N}{N}} \quad or \quad \overline{X}_N \ge \mu_0 + t_{N-1,0.975}\sqrt{\frac{\overline{V}_N}{N}}$$

Repetition Statistics: One-sided T-Test, known σ_0 [4]

$$X_i \sim \mathcal{N}(\mu, \sigma_0^2) \xrightarrow[A \sim \mathcal{N}(\mu,\sigma_0^2) \to B = aA + c \sim \mathcal{N}(a\mu + c, a^2\sigma_0^2)]{\sum_{i=1}^{N} X_i \sim \mathcal{N}(N\mu, N\sigma_0^2)} \overline{X}_N = \frac{1}{N}\sum_{i=1}^{N} X_i \sim \mathcal{N}(\mu, \frac{1}{N}\sigma_0^2)$$

$$\longrightarrow \quad \frac{\overline{X}_N - \mu}{\sqrt{\frac{\sigma_0^2}{N}}} \sim \mathcal{N}(0,1)$$

$$\longrightarrow \quad P(\frac{\overline{X}_N - \mu}{\sqrt{\frac{\sigma_0^2}{N}}} \in [-\infty, 1.645 = u_{0.95} = u_{1-\alpha}]) = 0.95$$

$$\longrightarrow \quad P(\frac{\overline{X}_N - \mu}{\sqrt{\frac{\sigma_0^2}{N}}} \le 1.65) = 0.95$$

$$P(\mu \ge \overline{X}_N - 1.65\frac{\sigma_0}{\sqrt{N}}) = 0.95$$

so the confidence interval for μ given the empirical result of the point estimator $\overline{X}_N$ is

$$\mu \in [\overline{X}_N - 1.65\frac{\sigma_0}{\sqrt{N}}), \infty]$$

if $H_0 : \mu \le \mu_0$ were true we would need to have

$$\overline{X}_N \le \mu_0 + 1.65\frac{\sigma_0}{\sqrt{N}}$$

$$so \quad reject \qquad H_0 : \mu \le \mu_0 \qquad if \qquad \overline{X}_N \ge \mu_0 + 1.65\frac{\sigma_0}{\sqrt{N}}$$

Repetition Statistics: One-sided T-Test, unknown σ_0 [4]

$$X_i \sim \mathcal{N}(\mu, \sigma_0^2) \qquad \overline{V}_N = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X}_N)^2$$

$$\longrightarrow \quad T_N = \frac{\overline{X}_N - \mu}{\sqrt{\frac{\overline{V}_N}{N}}} \sim St_{N-1} \quad \longrightarrow$$

$$\longrightarrow \quad P(T_N \in [-\infty, t_{N-1,0.95} = t_{N-1,\alpha}]) = 0.95$$

$$\longrightarrow \quad P(T_N = \frac{\overline{X}_N - \mu}{\sqrt{\frac{\overline{V}_N}{N}}} \leq t_{N-1,0.95}) = 0.95$$

$$\longrightarrow \quad P(\mu \geq \overline{X}_N - t_{N-1,0.95}\sqrt{\frac{\overline{V}_N}{N}}) = 0.95 \quad \longrightarrow$$

$$reject \qquad H_0 : \mu \leq \mu_0 \qquad if \qquad \overline{X}_N \geq \mu_0 + t_{N-1,0.975}\sqrt{\frac{\overline{V}_N}{N}}$$

Bibliography

(1) Dan Jurafsky and James Martin: Speech and Language Processing (3rd ed. draft, version Jan, 2022); Online: https://web.stanford.edu/~jurafsky/slp3/ (URL, Oct 2022) (this slideset is especially based on chapter 3)

(2) Powerpoint slides from Dan Jurafsky and James Martin: Speech and Language Processing (3rd ed. draft); Online: https://web.stanford.edu/~jurafsky/slp3/ (URL, Oct 2022)

(3) K. Murphy: Machine Learning – a Probabilistic Perspective, MIT Press 2012 (especially section 3.5)

(4) Hübner: Stochastik, Vieweg, 2003, chapter 10

Recommendations for Studying

- minimal approach:

  work with the slides and understand their contents! Think beyond instead of merely memorizing the contents

- standard approach:

  minimal approach + read the corresponding pages in Jurafsky [1]

- interested students

  == standard approach