Disclaimer: Below you find some example questions, which should help you prepare for the exam. Note, however, the actual questions at the exam can be very different and can cover all material presented in the lecture!

Data Interpolation

a) For a tetrahedron with vertices A = (0,0,0), B = (1,0,0), C = (0,1,0), D = (0,0,1) and the corresponding scalar values f_A, f_B, f_C, f_D, the linear interpolation function $f(x,y,z) = 1 - x + 2 \cdot y - 2 \cdot z$ is given.
Compute the concrete scalar values at the four vertices.

$f_A = 1$ $\quad f_D = 1 - 2 = -1$

$f_B = 0 +$

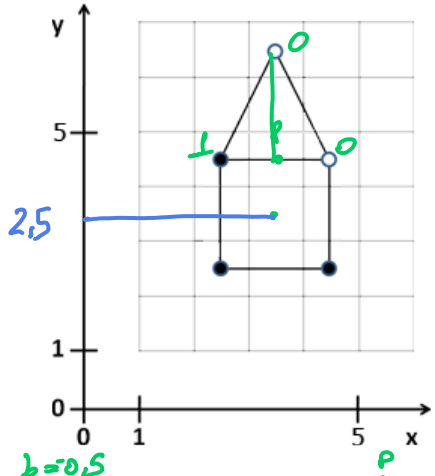
$f_C = 1 + 2 = 3$

b) For the tetrahedron given in the previous assignment, assume that the scalar values inside the tetrahedron are interpolated via barycentric interpolation. Compute the gradient of the interpolated scalar field at the points P = (0.5, 0.25, 0.5) and Q = (0, 0, 0).

Gradient is independant from coordinates

$f(x,y,z) = 1 - x + 2y - 2z \Rightarrow \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$

c) In the figure below, a 2D Cartesian grid is shown. It has a constant spacing of 1 between adjacent grid points along either axis. The origin of this grid is at (1, 1). A second grid is shown, which consists of one triangle with vertices at (2.5, 4.5), (3.5, 6.5), (4.5, 4.5) and one quadrilateral with vertices (2.5, 2.5), (2.5, 4.5), (4.5, 4.5), (4.5, 2.5). At the vertices of the second grid, scalar values are given. These values are equal to 1 at vertices marked with a filled circle and 0 at vertices marked with a non-filled circle.



$\begin{bmatrix} 1 & 3,5 & 6,5 \\ 1 & 2,5 & 4,5 \\ 1 & 4,5 & 4,5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$a + 3,5b + 6,5c = 0$

$a + 2,5b + 4,5c = 1$

$a + 4,5b + 4,5c = 0$

$2b = 1 \quad b = 0,5$

$(0, 0,5, 0,5)$

$3,5 = 2,5 \cdot \alpha_1 + 3,5 \cdot \alpha_2 + 4,5 \cdot \alpha_3$

$2,5 = 4,5 \cdot \alpha_1 + 6,5 \cdot \alpha_2 + 4,5 \alpha_3$

$1 = \alpha_1 + \alpha_2 + \alpha_3$

$-1 = 2\alpha_1 + 3\alpha_2$

$-2 = 2\alpha_2 \quad \alpha_2 = -1$

$\alpha_1 = 1$

$\alpha_3 = 0$

Compute the barycentric coordinates of the points (3.5, 4.5) and (3.5, 2.5) with respect to the triangle.

For the triangle, determine the coefficients *a, b, c* of the linear interpolation function $f(x,y) = a + b \cdot x + c \cdot y$ which interpolates the scalar values at the triangle vertices.

For the quadrilateral, determine the coefficients *a*, *b*, *c*, *d* of the bi-linear interpolation function $f(x,y) = a + b \cdot x + c \cdot y + d \cdot x \cdot y$ which interpolates the scalar values at the vertices of the quadrilateral.