All Sample questions Solutions

Visual Data Analytics (Technische Universität München)

Lecture 2.1 – Visualization: Data sources, examples, visualization pipeline

Visualization Pipeline

a) Explain the **visualization pipeline**. What are the four stages?
The visualization pipeline is explaining the process from the data source to the final visualization which one can look at and analyze. It has four stages: 1. Data acquisition 2. Filtering/ Enhancement 3. Visualization mapping 4. Rendering

b) Explain the **data acquisition** stage. What are three general cases?
In the data acquisition phase, the data to be visualized is generated. This usually happens via a simulation (technical world), an experiment (real world) or from an artificial source (Painting, video, database).

c) Explain the **filtering/enhancement** stage. Give at least two examples.
In the filtering/ enhancement stage data is being prepared for further processing, for example converted to another format, resampled to a new or any grid, missing values are being interpolated, unnecessary data deleted, noises reduced, etc. Characteristic properties: gradients, curvature, extrema, segmentation.

d) Explain the **visualization mapping** stage. Give at least two examples.
The filtered data is mapped to a renderable representation. How do we need to represent the data to understand correlations? A scalar field could be transformed to an isosurface, a Vector field into vectors at certain locations, a 2D field into a height field...

e) In which stage of the visualization pipeline happens **resampling** to a regular grid?
Filtering/data enhancement.

f) In which stage of the visualization pipeline are the **viewpoint and lighting parameters** specified?
Rendering.

g) In which stage of the visualization pipeline happen **lighting and shading**?
Rendering.

h) In which stage of the visualization pipeline are **colors** assigned to every voxel?
Visualization mapping.

i) In which stage of the visualization pipeline happen **smoothing and noise suppression**?
Filtering/data enhancement.

Lecture 2.2 – Data Representation

Data Representation

a) Discuss **independent vs. dependent variables** in data. Give at least two examples each.
Independent variables are variables representing the domain. E.g.: x and y direction, time.
Dependent variables are the measured values at certain points in time and/or space, so they depend on the independent variables. E.g.: temperature, pressure, velocity, etc.

b) What are the independent and dependent variables in a **3D spatial curve** $\varphi: \mathbb{R} \to \mathbb{R}^3$?
Independent variables: spatial curve φ in 1D domain $\mathbb{R}$.
Dependent variables: what the curve represents in 3D, for example a location in spatial domain $\mathbb{R}^3$.

c) What are the independent and dependent variables in a **3D vector field**?
Independent variables: 3D space.
Dependent variables: 3D vector field (i.e. vectors at each point of the 3D space defined by the independent variables.

d) What type of attribute are the following: **categorical, ordinal, or quantitative**?
   a. Type of cheese (e.g., Swiss, Brie)
   Categorical.
   b. Tire pressure (e.g., 2.3 bar, 2.5 bar)

Quantitative.

c. First name (e.g., Alice, Bob)
Categorical.

d. Unemployment rate (e.g., 6%, 10%)
Quantitative.

e. T-Shirt sizes (e.g., medium, large)
Ordinal.

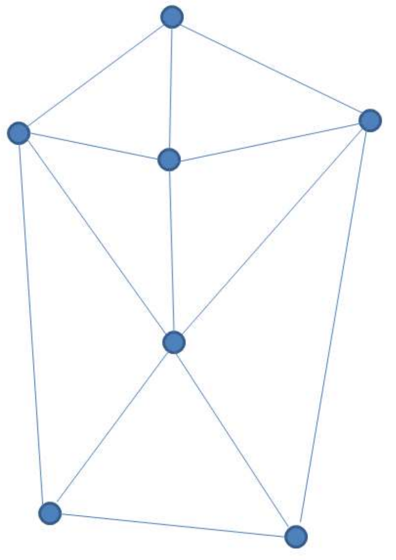
e) Draw an illustration of a **Cartesian grid**. Describe how such a grid is different from a **regular grid**? Which information needs to be specified explicitly for such a grid?
A cartesian grid is orthogonal, equidistant in all directions, and structured. We need to specify the starting location and the number of vertices in each direction.
A regular grid is also orthogonal and structured, the nodes are equidistant but for different directions the node distance might be different, i.e. $dx \neq dy$. Therefore the node distance must be explicitly specified, in addition to the information required for a cartesian grid.

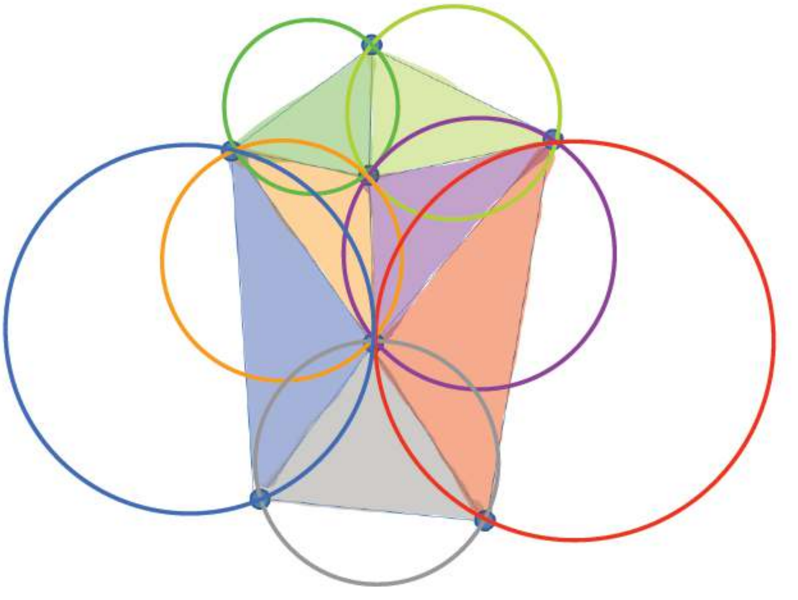
f) What is a **curvilinear grid**? How is it characterized? How is it different from an unstructured grid? Which information needs to be specified explicitly for such a grid?
One direction of the gridlines follows a certain shape, which makes the cells non-orthogonal. Each grid point's position must be explicitly specified, however there is an implicit neighborhood relationship (unlike for an unstructured grid, where we also need explicit neighborhood information).

Lecture 3 – Data Reconstruction and Interpolation Part 1

Data Interpolation

a) For the triangulation shown below, proof or disproof that this triangulation is a **Delaunay triangulation**. Your proof should be geometrically, meaning that you either illustrate the Delaunay property in the figure or illustrate that this property is violated.





None of the circumcircles contain another point than the 3 that define it, therefore the Delaunay property is never violated for this triangulation. Thus, we can say that this triangulation is a Delaunay triangulation.

b) An interpolation function $f(x) = \sum_{i=1}^{N} \varphi(\|p_i - x\|)$ is a weighted sum of N **radial functions** $\varphi(r) = e^{-r^2}$ where $\|p_i - x\|$ is the distance between the points p_i and x. Compute the **weights** w_i such that the function $f(p_i)$ interpolates the data points $p_1 = 1, p_2 = 3, p_3 = 3.5$ with corresponding scalar values $f_1 = 1, f_2 = 0, f_3 = \frac{1}{4}$.

The table below shows approximate values for $\varphi(r)$ with respect to different distances r.

r	0	0.5	1	1.5	2	2.5	3
$\varphi(r)$	1	$\frac{4}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0	0	0

$$W = A^{-1}F \Leftrightarrow \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} \varphi\|p_1-p_1\| & \varphi\|p_2-p_1\| & \varphi\|p_3-p_1\| \\ \varphi\|p_1-p_2\| & \varphi\|p_2-p_2\| & \varphi\|p_3-p_2\| \\ \varphi\|p_1-p_3\| & \varphi\|p_2-p_3\| & \varphi\|p_3-p_3\| \end{pmatrix}^{-1} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} \varphi(0) & \varphi(2) & \varphi(2.5) \\ \varphi(2) & \varphi(0) & \varphi(0.5) \\ \varphi(2.5) & \varphi(0.5) & \varphi(0) \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 1/4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 4/5 \\ 0 & 4/5 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 1/4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{25}{9} & -\frac{20}{9} \\ 0 & -\frac{20}{9} & \frac{25}{9} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{5}{9} \\ \frac{25}{36} \end{pmatrix}$$

$\det = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$

$p_1 = 1,\ p_2 = 3,\ p_3 = 3.5$

$$\therefore w_1 = 1, w_2 = -\frac{5}{9} \approx -0.556, w_3 = \frac{25}{36} \approx 0.694$$

c) Given the **interpolation function** $f(x)$ from the previous assignment, compute the interpolated value at point x=2.

$$f(x) = \sum_{i=1}^{N} w_i \varphi(\|p_i - x\|)$$

$$\therefore f(2) = w_1 \varphi(\|p_1 - 2\|) + w_2 \varphi(\|p_2 - 2\|) + w_3 \varphi(\|p_3 - 2\|) = 1\varphi(1) - \frac{5}{9}\varphi(1) + \frac{25}{36}\varphi(1.5) = 1 \cdot \frac{2}{5} - \frac{5}{9} \cdot \frac{2}{5} + \frac{25}{36} \cdot \frac{1}{10} = \frac{89}{360}$$

$$f(x = 2) = \frac{89}{360} \approx 0.247$$

Lecture 4 – Data Reconstruction and Interpolation Part 2

Data Interpolation

a) For a tetrahedron with vertices $A = (0,0,0)$, $B = (1,0,0)$, $C = (0,1,0)$, $D = (0,0,1)$ and the corresponding scalar values f_A, f_B, f_C, f_D, the **linear interpolation function** $f(x, y, z) = 1 - x + 2y - 2z$ is given. Compute the concrete scalar values at the four vertices.

$f(x,y,z) = 1 - x + 2y - 2z$

$f_A = f(0,0,0) = 1$ $\quad f_C = f(0,1,0) = 1 + 2\cdot 1 = 3$

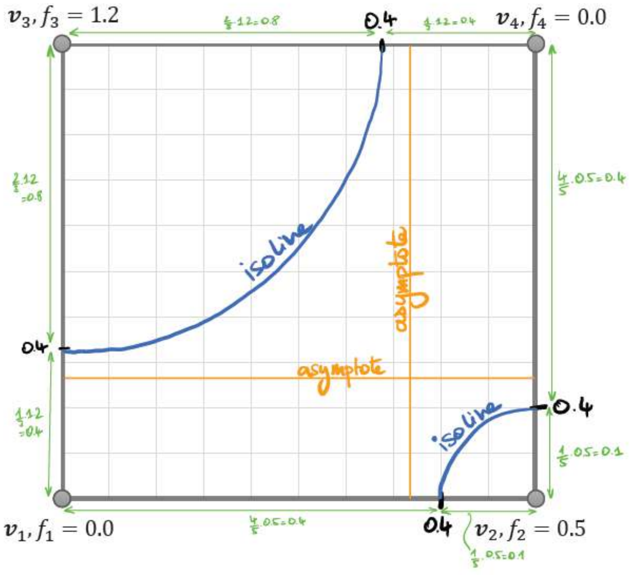
$f_B = f(1,0,0) = 1 - 1 = 0$ $\quad f_D = f(0,0,1) = 1 - 2\cdot 1 = -1$

b) For the tetrahedron given in the previous assignment, assume that the scalar values inside the tetrahedron are interpolated via **barycentric interpolation**. Compute the gradient of the interpolated scalar field at the points $P = (0.5, 0.25, 0.5)$ and $Q = (0, 0, 0)$.

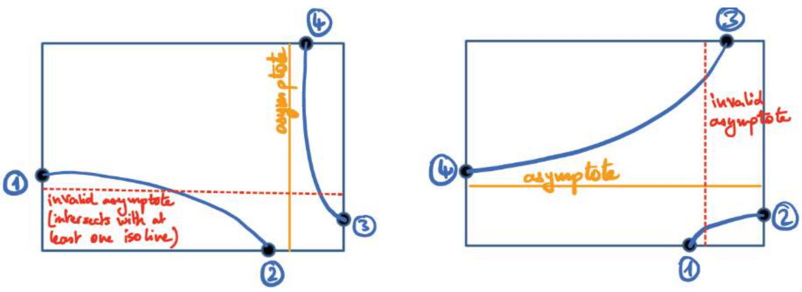
$f(x,y,z) = 1 - x + 2y - 2z \quad \therefore \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$

The gradient is independent of the point's (x, y, z) coordinates, therefore the gradient at P and Q is the same, i.e. $\begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$.

c) Given is the following quadrilateral cell with its four vertices $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}, \boldsymbol{v_4}$ and the corresponding scalar values f_1, f_2, f_3, f_4. A grid is shown for orientation purpose, i.e., it does not affect the interpolation.
Draw the **iso-contour** (bold) for the iso-value 0.4 into the same illustration, i.e., all points in the interior of the cell which have a value equal to 0.4. Bi-linear interpolation is assumed for interpolation.

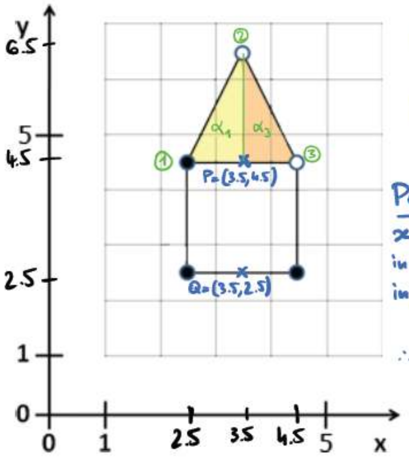


d) In the figure below, a quadrilateral cell is shown. Scalar values are given at the cell corners and **bi-linear interpolation** of these values is used to reconstruct scalar values across the cell. Is it possible that at the four edge points marked by dots the same scalar value is reconstructed? Explain your answer.
It is impossible because the asymptotes cannot be drawn without intersecting with at least one hyperbolic arch.



e) In the figure below, a 2D Cartesian grid is shown. It has a constant spacing of 1 between adjacent grid points along either axis. The origin of this grid is at $(1, 1)$. A second grid is shown, which consists of one triangle with vertices at $(2.5, 4.5)$, $(3.5, 6.5)$, $(4.5, 4.5)$ and one quadrilateral with vertices $(2.5, 2.5)$, $(2.5, 4.5)$, $(4.5, 4.5)$, $(4.5, 2.5)$. At the vertices of the second grid, scalar values are given. These values are equal to 1 at vertices marked with a filled circle and 0 at vertices marked with a non-filled circle.

Compute the **barycentric coordinates** of the points $P = (3.5, 4.5)$ and $Q = (3.5, 2.5)$ with respect to the triangle.



Point P:

$\alpha_2 = 0$

$\alpha_1 = \alpha_3 = \frac{1}{2}$

Point Q:

$x = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3$

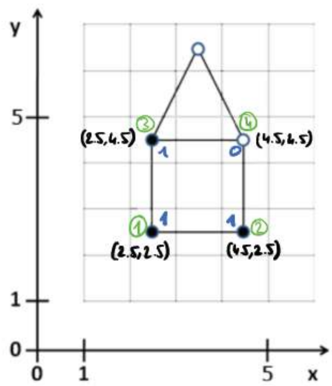
$$\begin{cases} \text{in } x: & \alpha_1 \cdot 2.5 + \alpha_2 \cdot 3.5 + \alpha_3 \cdot 4.5 = 3.5 \\ \text{in } y: & \alpha_1 \cdot 4.5 + \alpha_2 \cdot 6.5 + \alpha_3 \cdot 4.5 = 2.5 \\ & \alpha_1 + \alpha_2 + \alpha_3 = 1 \end{cases}$$

$\therefore$ solve system and find that

$\alpha_1 = \alpha_3 = 1$, $\alpha_2 = -1$

Solve system:
https://www.wolframalpha.com/input?i=systems+of+equations+calculator&assumption=%7B%22F%22%2C+%22SolveSystemOf3EquationsCalculator%22%2C+%22equation1%22%7D+-%3E%222.5+x+%2B+3.5+y+%2B+4.5+z+%3D+3.5%22&assumption=%22FSelect%22+-%3E+%7B%7B%22SolveSystemOf3EquationsCalculator%22%7D%7D&assumption=%7B%22F%22%2C+%22SolveSystemOf3EquationsCalculator%22%2C+%22equation2%22%7D+-%3E%224.5+x+%2B+6.5+y+%2B+4.5+z+%3D+2.5%22&assumption=%7B%22F%22%2C+%22SolveSystemOf3EquationsCalculator%22%2C+%22equation3%22%7D+-%3E%22x+%2B+y+%2B+z+%3D+1%22

For the quadrilateral, determine the coefficients a, b, c, d of the **bi-linear interpolation function** $f(x, y) = a + b \cdot x + c \cdot y + d \cdot x \cdot y$ which interpolates the scalar values at the vertices of the quadrilateral.



$f(x,y) = a + bx + cy + dxy$

① $f(2.5, 2.5) = a + 2.5b + 2.5c + 6.25d = 1$

② $f(4.5, 2.5) = a + 4.5b + 2.5c + 11.25d = 1$

③ $f(2.5, 4.5) = a + 2.5b + 4.5c + 11.25d = 1$

④ $f(4.5, 4.5) = a + 4.5b + 4.5c + 20.25d = 0$

Solving the system,

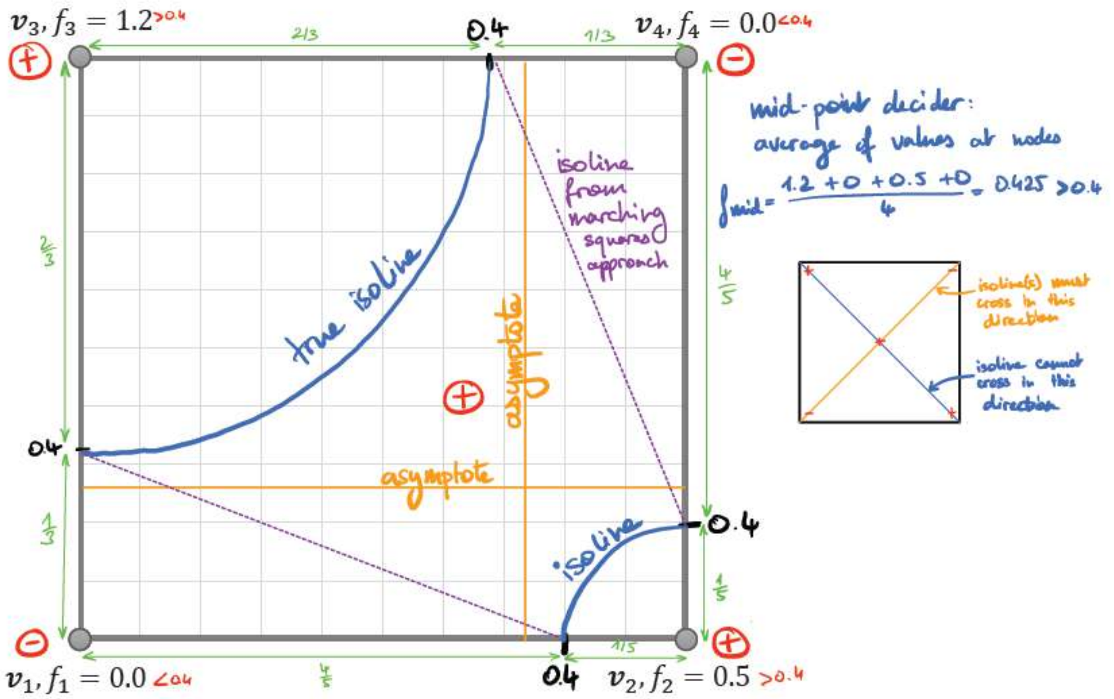
$a = -\frac{9}{16}$, $b = \frac{5}{8}$, $c = \frac{5}{8}$, $d = -\frac{1}{4}$

Solve system:
https://www.wolframalpha.com/input?i=systems+of+equations+calculator&assumption=%7B%22F%22%2C+%22SolveSystemOf4EquationsCalculator%22%2C+%22equation1%22%7D+-%3E%22a+%2B+2.5b+%2B+2.5c+%2B+6.25d+%3D+1%22&assumption=%22FSelect%22+-%3E+%7B%7B%22SolveSystemOf4EquationsCalculator%22%7D%7D&assumption=%7B%22F%22%2C+%22SolveSystemOf4EquationsCalculator%22%2C+%22equation2%22%7D+-%3E%22a+%2B+4.5b+%2B+2.5c+%2B+11.25d+%3D+1%22&assumption=%7B%22F%22%2C+%22SolveSystemOf4EquationsCalculator%22%2C+%22equation3%22%7D+-%3E%22a+%2B+2.5b+%2B+4.5c+%2B+11.25d+%3D+1%22&assumption=%7B%22F%22%2C+%22SolveSystemOf4EquationsCalculator%22%2C+%22equation4%22%7D+-%3E%22a+%2B+4.5b+%2B+4.5c+%2B+20.25d+%3D+0%22

Lecture 5 – Isolines & Isosurfaces

Isolines & Isosurfaces

a) Given is the following quadrilateral cell with its four vertices $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}, \boldsymbol{v_4}$ and the corresponding scalar values f_1, f_2, f_3, f_4. A grid is shown for orientation purpose, i.e., it does not affect the interpolation.
Draw the approximated **iso-lines** (dashed) for the iso-value 0.4 into the illustration using the **marching squares algorithm**. Use the **mid-point decider** for ambiguous cases.
Also, draw the true iso-contour (bold) for the iso-value 0.4 into the same illustration, i.e., all points in the interior of the cell which have a value equal to 0.4. Bi-linear interpolation is assumed for interpolation.



b) Name the three components of the **Phong illumination model**.
Ambient light, specular reflector, diffuse reflector.

c) How can a **perfect mirror** be simulated via the Phong illumination model?
If the shininess coefficient $n \to \infty$.

d) Let L be an incoming light ray and M a **diffusely reflecting material**. Complement the illustration on the right in Figure 7 according to the illustration on the left. Hint: Take into account the direction and strength of the reflection.

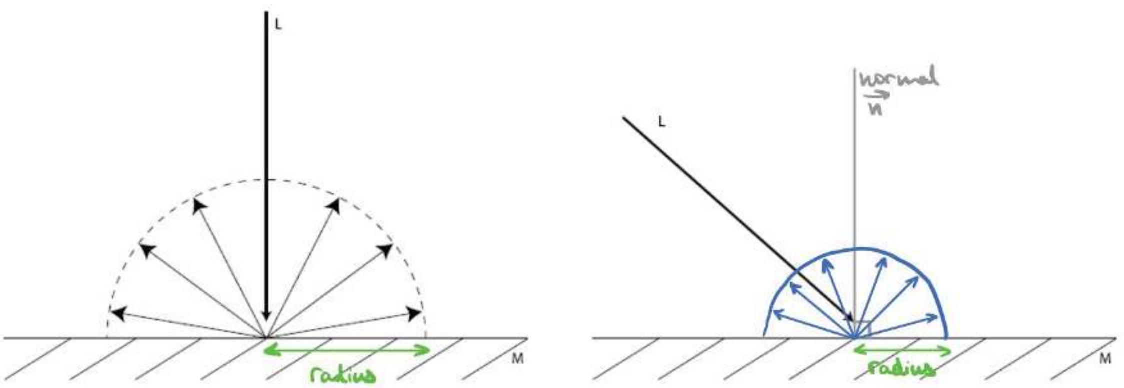


Figure 7

The radius is smaller in the RHS of Figure 7 because the reflection is weaker.

e) Let L be an incoming light ray and M a **specular reflecting material**. The viewer is positioned at the light source and is looking along L. Complement Figure 8 according to Figure 7.

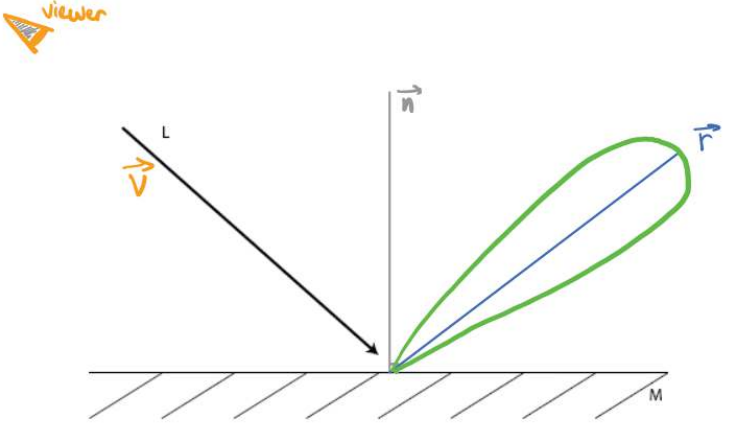
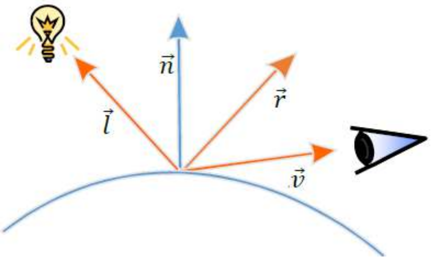


Figure 8

f) Below is an illustration of the **Phong illumination model**. All indicated vectors are normalized (i.e., their length is one). Answer whether the following statements are true or false:



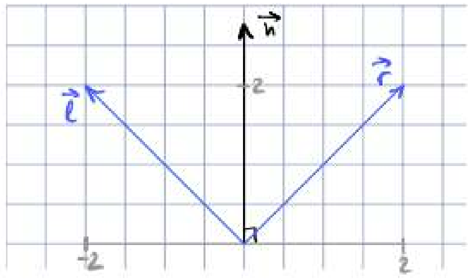
$\vec{n}$ ... Normal vector of the surface
$\vec{l}$ ... Vector pointing to the light source
$\vec{r}$ ... Reflected light ray
$\vec{v}$ ... Vector pointing to the viewer (view vector)

The specular reflection is based on the scalar/dot product of $\vec{\boldsymbol{n}}$ and $\vec{\boldsymbol{v}}$.	False: depends on $\vec{\boldsymbol{r}}$ and $\vec{\boldsymbol{v}}$, not $\vec{\boldsymbol{n}}$.
The diffuse reflection is based on the scalar/dot product of $\vec{\boldsymbol{n}}$ and $\vec{\boldsymbol{l}}$.	True.
The ambient light is based on the scalar/dot product of $\vec{\boldsymbol{l}}$ and $\vec{\boldsymbol{r}}$.	False: ambient light is always constant.
The specular reflection is independent of the view vector $\vec{\boldsymbol{v}}$.	False: depends on angle between on $\vec{r}$ and $\vec{v}$.

g) Compute the **specular reflection** at a surface point $P = (3,1,-2)^T$ using the Phong lighting model. The normal at the point is $\vec{n} = (0,1,0)^T$. Moreover, the specular reflection coefficient of the surface is $k_s = 0.5$ and the specular exponent is $n = 2$. The position of a point light source is $L_{pos} = (1,3,-1)^T$ and the camera position is at $E_{pos} = (5,2,-2)^T$. Both the point light source and the surface color are white with RGB-values $(1,1,1)$.

Hint: You can compute the reflected light ray as $\vec{r} = 2(\vec{n} \cdot \vec{l})\vec{n} - \vec{l}$, where $\vec{n}$ and $\vec{l}$ need to be normalized.

$$C = \underbrace{k_s}_{\text{specular reflection coef.}} \cdot \overset{\text{color}}{O_c} \cdot (\underset{\text{reflection}}{\vec{r}} \cdot \overset{\text{viewing}}{\vec{v}})^{\overset{\text{shininess factor}}{n}}$$



Light vector:

$$\underset{L_{pos}}{\begin{pmatrix}1\\3\\-1\end{pmatrix}} - \underset{P}{\begin{pmatrix}3\\1\\-2\end{pmatrix}} = \begin{pmatrix}-2\\2\\1\end{pmatrix} \quad \sqrt{(-2)^2+(2)^2+(1)^2} = 3 \quad \therefore \vec{l} = \underbrace{\frac{1}{3}}_{\text{normalisation factor}}\begin{pmatrix}-2\\2\\1\end{pmatrix}$$

Reflection vector:

$$\vec{r} = \underbrace{\frac{1}{3}}\begin{pmatrix}+2\\2\\-1\end{pmatrix}$$

Check:

$\vec{r} = 2(\vec{n} \cdot \vec{l})\vec{n} - \vec{l}$ with $\vec{n}$ and $\vec{l}$ normalised:

here, $\vec{n} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $\quad |\vec{l}| = \sqrt{2^2+2^2+1^2} = 3 \quad \therefore$ take $\vec{l} = \frac{1}{3}\begin{pmatrix}-2\\2\\1\end{pmatrix}$

$$\Rightarrow \vec{r} = 2\left[\begin{pmatrix}0\\1\\0\end{pmatrix}\frac{1}{3}\begin{pmatrix}-2\\2\\1\end{pmatrix}\right]\begin{pmatrix}0\\1\\0\end{pmatrix} - \frac{1}{3}\begin{pmatrix}-2\\2\\1\end{pmatrix} = \frac{2}{3}[2]\begin{pmatrix}0\\1\\0\end{pmatrix} - \frac{1}{3}\begin{pmatrix}-2\\2\\1\end{pmatrix}$$

$$= \frac{4}{3}\begin{pmatrix}0\\1\\0\end{pmatrix} - \frac{1}{3}\begin{pmatrix}-2\\2\\1\end{pmatrix} = \frac{1}{3}\begin{pmatrix}2\\2\\-1\end{pmatrix}$$

View vector:

$$\underset{E_{pos}}{\begin{pmatrix}5\\2\\-2\end{pmatrix}} - \underset{P}{\begin{pmatrix}3\\1\\-2\end{pmatrix}} = \begin{pmatrix}2\\1\\0\end{pmatrix} \quad \sqrt{2^2+1^2+0^2} = \sqrt{5} \quad \therefore \vec{v} = \underbrace{\frac{1}{\sqrt{5}}}_{\text{normalisation factor}}\begin{pmatrix}2\\1\\0\end{pmatrix}$$

Specular reflection:

$$C = k_s\, O_c\, (\vec{r}\cdot\vec{v})^n = 0.5 \cdot O_c \cdot \left[\frac{1}{3}\begin{pmatrix}2\\2\\-1\end{pmatrix} \cdot \frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\\0\end{pmatrix}\right]^2 = \frac{1}{2} O_c \left[\frac{1}{3\sqrt{5}}(6)\right]^2 = \frac{1}{2} O_c \frac{4}{5} = \frac{2}{5} O_c$$

$$\therefore C = \frac{2}{5}\underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\text{RGB values}} = \begin{pmatrix}0.4\\0.4\\0.4\end{pmatrix}$$

h) In the four images below, a sphere is rendered using the Phong illumination model. Four different specular exponents (2, 4, 16, and 64) were used to create the specular reflection. Write the **specular exponent** that was used to create the rendering below each image.

64	16	4	2

From $C = k_s \cdot C_p \cdot O_d \cos^n \varphi$, larger $n \rightarrow$ smaller $\cos^n \varphi \rightarrow$ smaller C, thus the larger the exponent (=shininess factor), the smaller the specular highlight.

Lecture 6 – Direct Volume Visualization

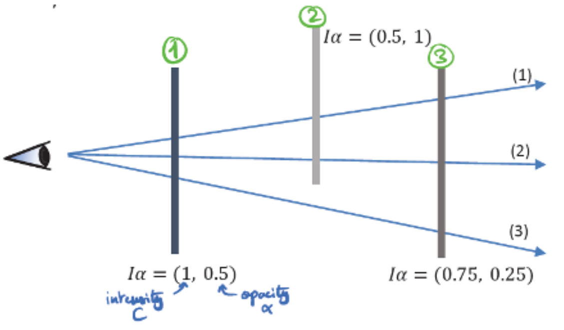
Volume Rendering

a) Name an algorithm commonly used in **indirect volume visualization**.
Marching Cubes (MC) algorithm.

Why is it considered to be “**indirect**” compared to “**direct**” **volume rendering**?
In indirect volume rendering, we must first construct an intermediate representation of the data, for example a surface, so technically it’s not a volume but a 3D surface now.

b) Which **optical/visual properties** are assigned by a transfer function in volume rendering?
Opacity and color.

c) A scene consisting of 3 objects (vertical lines) with different intensities (I) and opacities (α) is shown. The α-value (second component) represents the object’s opacity, where 0 = ´completely transparent´ and 1 = ´completely opaque´. The 3 objects are ordered as shown.



For the three rays starting at the viewpoint, determine the intensity that is seen along these rays using:

(1) front-to-back $\boldsymbol{\alpha}$**-compositing** for the upper ray.

$C_{in} = 0$, $\alpha_{in} = 0$

$C_{out} = C_{in} + (1-\alpha_{in})\alpha C$

$\alpha_{out} = \alpha_{in} + (1-\alpha_{in})\alpha$

At first object 1:
$C_{in} = 0$, $\alpha_{in} = 0$, $C = 1$, $\alpha = 0.5$
$\begin{cases} C_{out} = 0 + (1-0)\, 0.5 \cdot 1 = 0.5 \\ \alpha_{out} = 0 + (1-0)\, 0.5 = 0.5 \end{cases}$

At second object 2:
$C_{in} = 0.5$, $\alpha_{in} = 0.5$, $C = 0.5$, $\alpha = 1$
$\begin{cases} C_{out} = 0.5 + (1-0.5)\, 1 \cdot 0.5 = 0.75 \\ \alpha_{out} = 0.5 + (1-0.5)\, 1 = 1 \end{cases}$

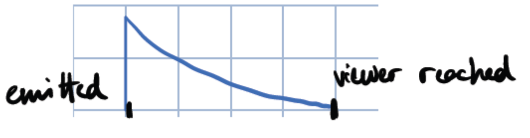
At third object 3:
$C_{in} = 0.75$, $\alpha_{in} = 1$, $C = 0.75$, $\alpha = 0.25$
$\begin{cases} C_{out} = 0.75 + (1-1)\, 0.25 \cdot 0.75 = 0.75 \\ \alpha_{out} = 1 + (1-1)\, 0.25 = 1 \end{cases}$

(2) the compositing scheme **Average** for the middle ray.
Average of intensity (C) values: $avg = \frac{1+0.5+0.75}{3} = 0.75.$

(3) the compositing scheme **Maximum** for the lower ray.
Careful: the lower ray does <u>not</u> pass through the 2nd object!
Maximum of intensity (C) values: $\max(1, 0.75) = 1.$

Specify exactly how the intensities are combined in either case.

d) Light/color has been emitted at a point along the viewing ray. How is it diminished (due to **absorption**) in a homogeneous, semi-transparent medium? Draw a typical curve.



e) How do you get **values along the viewing ray** (from volume data)?
Perform a numerical approximation of the volume rendering integral → Volume resampled and interpolated at equidistant intervals along the ray (integral as sum over samples).

f) Which **compositing schemes** do you know (for combining values along the viewing ray)?
α-compositing, Averaging, Maximum intensity projection

Lecture 7 – Flow Visualization Part 1

Vector Field Visualization (part I)

a) Calculate the **Jacobian matrix J** of the 3D vector field $\boldsymbol{v}(x,y,z) = (y^2 - 1, x - y, xz)^T$.

$$J = \nabla v(x,y,z) = \begin{pmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{pmatrix} = \begin{pmatrix} 0 & 2y & 0 \\ 1 & -1 & 0 \\ z & 0 & x \end{pmatrix}$$

Wolfram Alpha:
https://www.wolframalpha.com/input/?i=Jacobian+of+%28y%5E2-1%2C+x-y%2C+x*z%29+with+respect+to%28x%2Cy%2Cz%29

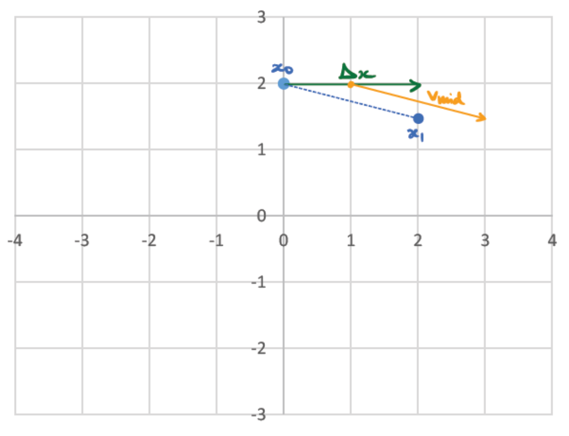
b) Calculate the **Jacobian matrix**, **divergence** and **curl** of the 3D vector field $\boldsymbol{v}(x,y,z) = (cos(xy), sin(x), z)^T$.

$$J = \begin{pmatrix} -y\sin(xy) & -x\sin(xy) & 0 \\ \cos(x) & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\operatorname{div} v(x,y,z) = \nabla \cdot v(x,y,z) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 1 - y\sin(xy)$$

$$\operatorname{curl} v(x,y,z) = \nabla \times v(x,y,z) = \begin{pmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{pmatrix} = \begin{pmatrix} 0-0 \\ 0-0 \\ \cos(x) + x\sin(xy) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \cos(x) + x\sin(xy) \end{pmatrix}$$

c) Given a 2D vector field $\boldsymbol{v}(x,y) = \left(y, -\frac{x}{2}\right)^T$. Compute the next point x_1 of a streamline with seed point $x_0 = (0,2)^T$ using the **midpoint integration method** (also known as **Runge-Kutta of 2nd order**) with a step size $\Delta t = 1$. Draw the resulting point and the used vector(s) in the illustration below (don't forget to annotate them). Specify exactly how the point and vectors are calculated.



$$v(x,y) = \begin{pmatrix} y \\ -x/2 \end{pmatrix}$$

$$\Delta x = v(0,2) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$v_{mid} = v(1,2) = \begin{pmatrix} 2 \\ -1/2 \end{pmatrix}$$

$$x_1 = x_0 + \Delta t \cdot v_{mid} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + 1\begin{pmatrix} 2 \\ -1/2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

d) In Figure 6, a vector field at four different times is shown. The grid vertex marked by a dot is selected as initial position for particles that are seeded into the flow. Assume that a particle can only move diagonally, vertically, or horizontally (depending on the vector at the particles current position), and that it always moves from the current grid point to the next grid point in the respective direction.

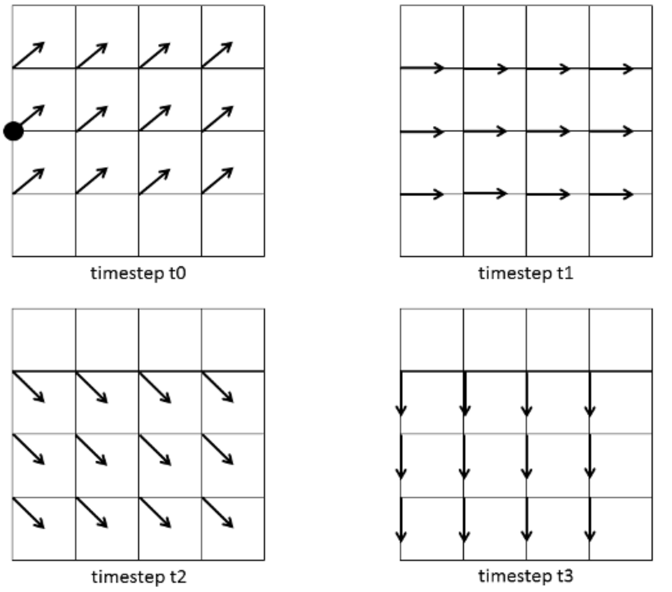
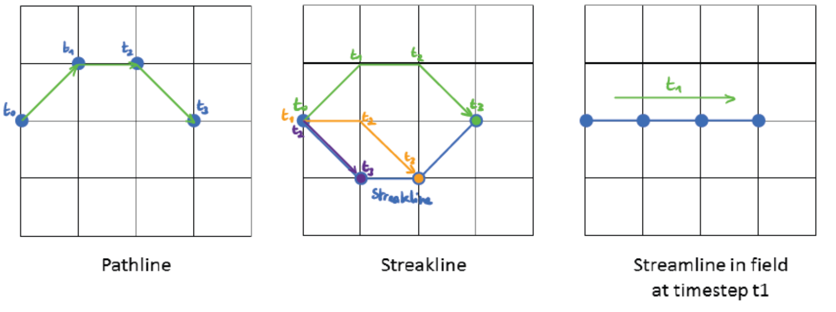


Figure 6

Illustrate in the grids given below:

1. the **pathline** of a particle released into the flow at the marked position at time t0,
2. the **streakline** of particles released into the flow at the marked position at time t0, and
3. the **streamline** of a particle released into the flow at the marked position in the fixed vector field at time t1.



e) Which **characteristic line** approaches do you know for ***unsteady/time-varying*** **flow**? What happens if you apply them to steady flows?
Streaklines, pathlines → applied to steady state flow they look the same, in fact then they are called Streamlines.

f) Answer whether the following statements are true or false.

The **Jacobian matrix** at a point in a constant 3D vector field has non-zero elements on the main diagonal.	False: constant vector field → $\frac{\partial v_{x_i}}{\partial x_j} = 0 \ \forall i, j$ → elements of $\boldsymbol{J}$ are 0.
If the Jacobian matrix at every point in a 3D vector field is the identity matrix, then the vector field is divergence free.	False: divergence is sum of diagonal elements ∴ $div I_3 = 1 + 1 + 1 = 3$
The **divergence** at every point in a 3D vector field is a scalar value.	True: div is always a scalar value.
Streamlines in a steady 3D vector field never cross.	True.
Path lines in a time-varying 2D vector field never cross.	False: the vector field is time-varying, therefore unsteady).

g) Give two examples for **direct flow visualization**.
Vector field, Glyphs, color coding

h) What challenges does **arrow-based direct flow visualization** have?
There can be a spatial Ambiguity when representing value with the length of the vectors (1D Object in 3D). Also, the results seem disordered/ hard to interpret if magnitude of velocity varies significantly and changes rapidly. Additionally, there can occur inherent occlusion effects.

i) Give two examples for **geometric** (integration-based) **flow visualization**. How do these techniques relate to direct flow visualization?
Streamlines, Streak lines. They are computed via vector field integration over time, meaning they are solutions to initial value problems in a vector field.

j) What is **Runge-Kutta integration** (2nd/4th order)? What's the advantage over Euler integration?
Runge-Kutta is an integration method / Explicit ODE solver scheme and can be used to compute the characteristic lines in a vector field. Compared to the explicit Euler method it has a higher order (Euler 1st order, Runge-Kutta 2nd or even 4th order) which means it is much more accurate than the explicit Euler at the same time step size.

k) **Characteristic lines** are **tangential** to the flow. What does that mean?
That means that the tangent at every point on the line shows the direction of the flow field at that point.

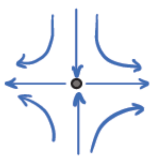
Lecture 8 – Flow Visualization Part 2

Vector Field Visualization (part II)

a) A **critical point** in a 2D vector field $\boldsymbol{v}(x,y)\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a point (x,y) where $\boldsymbol{v}(x,y) = (0,0)^T$.
How many critical points does the vector field $\boldsymbol{v}(x,y) = (y^2 - 1, x - y)^T$ have, and where are these points located?

$$v(x,y) = \begin{pmatrix} y^2-1 \\ x-y \end{pmatrix} \quad \left.\begin{cases} y^2-1=0 \Rightarrow y=\pm 1 \\ x-y=0 \Rightarrow x=y=\pm 1 \end{cases}\right\} \rightarrow \text{2 critical points: } \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

b) Below are three examples of **critical points** together with the **eigenvalues** λ_1, λ_2 of the respective Jacobi matrices. Classify each critical point according to the eigenvalues and sketch the typical flow around it.

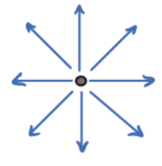


$\lambda_1 = 2 > 0$
$\lambda_2 = -1 < 0$

Type: Saddle point

$\lambda_1 = \overset{<0}{-1} + i$
$\lambda_2 = \underset{<0}{-1} - i$

Type: sink



$\left.\begin{matrix} \lambda_1 = 1 \\ \lambda_2 = 3 \end{matrix}\right\} > 0$

Type: attractive node

c) Answer whether the following statements are true or false.

LIC is a local method for visualizing a vector field.	False: LIC is global method, gives flow information ∀ point in the vector field.
The larger the extent of the convolution kernel used in LIC, the lower is the correlation between adjacent intensity values along a streamline.	False: larger kernel → blurrier noise textures along the streamlines → adjacent intensity values along a streamline are more correlated.
LIC images show high correlation between the intensity values at adjacent streamlines.	False: in LIC, for neighboring streamlines, correlation is very low because the texture is blurred along the streamline, but not between neighboring streamlines.
LIC is restricted to 2D vector fields.	False: feasible for 3D, but there are limitations, such as occlusion effects.
The convolution kernel used in LIC must be symmetric.	False: e.g. oriented LIC: anisotropic convolution kernel to show the direction of the flow in the LIC.

Lecture 9 – Visualization Mapping Part 1

Visualization Mapping

a) What is the **pop-out effect / pre-attentive processing**? How can it be used?
The pop out effect is an effect resulting from automatic and parallel detection of basic features in a visual information (pre-attentive processing, 200-250msec). It works on many individual channels. It can be used to very fast identify/understand specific information. For example, if all sevens in a field of numbers have a different color than the rest, then the viewer can identify and count these numbers very fast.

b) Sort the following **visual channels** according to how accurately humans can compare them starting with the highest accuracy: *2D area – length – curvature – angle/slope*.
length - angle/slope - 2D area - curvature

c) What is the difference between **separable and integral visual channels**?
Separable visual channels combine two visual channels one representation that are differentiable (color and position), whereas integral visual channels combine different versions of a visual channel (e.g., combine color red and green or combine width and length in an ellipse to represent 2 information in one) So it's hard to read out the individual channels once they are mixed.

d) Name an example for **fully separable / integral visual channels**.
Color and Position are fully separable visual channels. Height and width are integral visual channels for ellipses.

e) Which **visual channel(s)** can be used in a bar chart? For what types of data?
Length and color. Quantitative and Categorical Data

f) From a **perceptual point of view**, what works better: Bar charts or pie charts? Why?
Bar charts normally give a better perception of how much difference there is between each category being represented, because length and position a visual channel that is being perceived more accurately than angle for a human.

g) How do **Parallel sets** work? What kind of data can be shown?
Parallel sets show quantitative data w.r.t. multiple categorical attributes. It shows their connection and the proportions of how they are distributed.

h) How does the **ThemeRiver** work? Which visual channels are used for which type(s) of data?
Area, shape and color are used to visualize thematic changes in a collection of data over time. E.g., which movie was how successful for how long.

i) How do **Horizon Graphs** work? How can you read out values at a position?
A horizon graph reduces vertical space without losing precision, it splits a normal graph into vertical bands and collapses the color bands to show values in less vertical space, optionally negative values can be mirrored. For reading out exact values, one needs to know the height of each band and measure the height of the graph at that location. The height of the graph divided by the height of the band needs to be multiplied with the delta value of the color. (I.e., if the color represents values from 10 to 20it, the multiplier is 20-10=10) if the color range does not represent values from zero to x then the underlying magnitude needs to be added (In the previous example ten needs to be added, because it starts at 10).

Lecture 10 – Visualization Mapping Part 2

Visualization Mapping

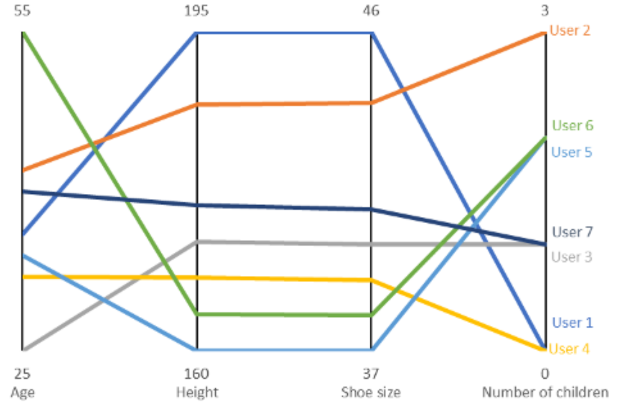
a) Which **visual channels** can be used in a **scatterplot** besides position?
Color, size, shape, position

b) How does a **scatterplot matrix** work? How can you see correlations?
It shows all possible combinations of attributes, where each row/column is one attribute. It gives an overview of correlation and patterns between data attributes. Correlations can easily be shown by **Brushing** (=marking) a specific data subset, and via **Linking** the brushed data is shown in the other graphs.

c) How does **linking and brushing** work?
Brushing: Mark interesting data subset
Linking: enhance/highlight brushed data in linked views

d) Draw a visualization of the following data using **parallel coordinates**.

	Age	Height	Shoe size	Number of children
User 1	36	195	46	0
User 2	42	187	44	3
User 3	25	172	40	1
User 4	32	168	39	0
User 5	34	160	37	2
User 6	55	164	38	2
User 7	40	176	41	1



e) What does it mean when the lines between two axes in a **parallel coordinates**' visualization meet in a point?
Their relationship is inverse/ negative correlation

f) What are **glyphs**? For which type of data are they typically used?
Glyphs are small independent visual objects that depict attributes of data records. The Data attributes are represented by different visual channels (e.g., shape, color, size and orientation). The visual channels should be separable, not integral. They are mainly used for multivariate data.

g) How do **star glyphs**/stick figures work? How do they encode the data?
A star is composed of equally spaced spikes originating from center → Length of spikes represent value of the respective attribute; the lines' extremities are connected.
Stick figures are 2D figures with limbs→ Data encoded by (length, line thickness, angle between lines). Often used as an assembly of many to encode texture patterns.

h) What are the advantages/disadvantages of a **rainbow color map**?
Rainbow colormap is perceptually unordered, for ordering data this is not super useful. Also compared to displaying the luminance, in a rainbow colormap it can happen that details get lost. Another disadvantage is that it is not linearly grading - so linear changes in data are not perceived as such by the viewer. However, the different colors help to emphasize certain areas specifically.

i) What does it mean, when a visual channel (e.g., color) is ***perceptually linear/ordered***?

Ordering of data should is represented by ordering of colors. If this ordering happens with equal distances, then it is linearly ordered.

j) What are the characteristics of a **sequential / diverging color map**?
Sequential is if it is the same color but lighter to darker, diverging is if in the middle it is very light and gets darker to the top and to the bottom it also gets darker, but with another color.

Lecture 11 – Visual Analysis of Scientific data

Visual Analysis of Scientific Data

a) What is the main goal of **visual exploration**?
Gain insight, verify hypotheses, presentation/explanation help

b) Which three major areas/**concepts** are combined in **visual analysis/analytics**?
Scientific Visualization (Flow, Volume), Information visualization and Visual Analytics

c) Give examples how **multivariate data** can be encoded in a spatial context?
Volume Rendering, Glyphs

d) What are challenges when fusing **multi-modal data** stemming from different data sources?
Each data source can have a different grid and different resolution → co-registration and normalization necessary as well as a technique to show different volumes in one representation.

e) **Visual data fusion** intermixes data in a single visualization using a common frame of reference. Give at least two general approaches.
1. Layering techniques (e.g., glyphs, color, transparency)
    - Different layers that must be differentiable
    - Colormap enhancement
2. Glyphs
3. Texture and colors

f) What are three general approaches for **comparative visualization** (according to the taxonomy of Gleicher et al. 2011)?
Juxtaposition, Overlay, Explicit encoding

g) What is **focus + context visualization**? Explain the general approach. How is it different from an overview + detail visualization?
It seamlessly integrates focus/ context in single visualization, uses distortion to have more space for the focus → We can keep the context, without cropping away data outside of zoomed data. The overview and detail view is a spatially separate overview / detail (e.g., juxtaposed views), which means the user has to switch attention between representations, which is not necessary in focus + context.

h) Give at least three examples of visual channels (graphical resources) that can be used for **focus - context discrimination**.
Opacity, Frequency/Blurring, Position, Color

i) Give two examples for **focus + context visualization** techniques which use **spatial distortion**.
Spacial distortion of map, chess game with unnecessary figures in the background

j) What is the main idea in **clustering**? Is clustering a **supervised or unsupervised** method?
Clustering is unsupervised. The main idea of clustering is to group data that have a certain maximal distance to each other to one group

k) What is the main idea in **dimensionality reduction**? Name one example method? How does it work?
The idea of dimensionality reduction is to derive a low dimensional target space from high-dimensional measured space, it is useful when the interesting information cannot be extracted/measured directly from the measured space -> True dimensionality of dataset is assumed to be smaller than dimensionality of measurement

l) **Principal component analysis** transforms data from a cartesian coordinate system into another coordinate system. Why is it then still considered a dimensionality reduction method?
Reduce the data without meaning/small variants and new axes don't have a meaning anymore.