
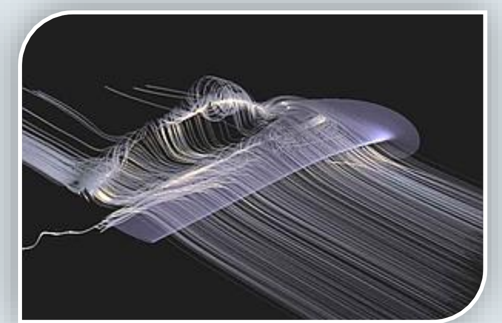

Visual Data Analytics
Data Reconstruction and Interpolation II

Dr. Johannes Kehrer

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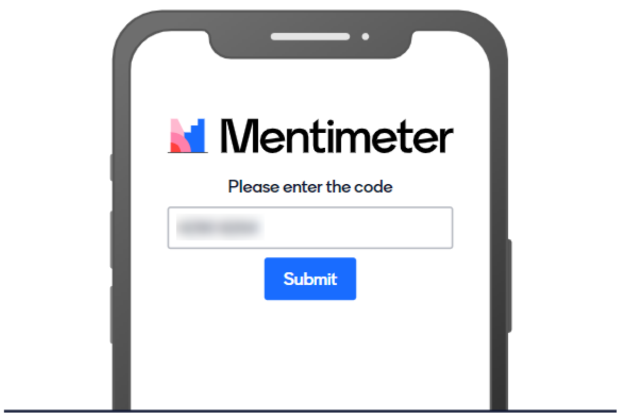

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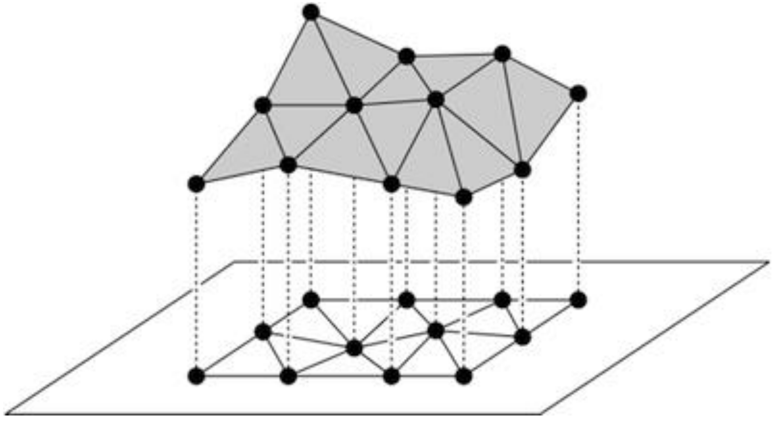
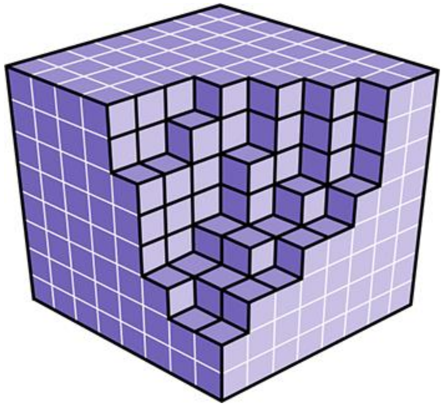
Overview

SIEMENS
Ingenuity for life

- Interpolation on grids / meshes
  - Barycentric interpolation
  - Bi-/trilinear interpolation
  - High-quality reconstruction





Tricubic vs. trilinear interpolation

Data interpolation

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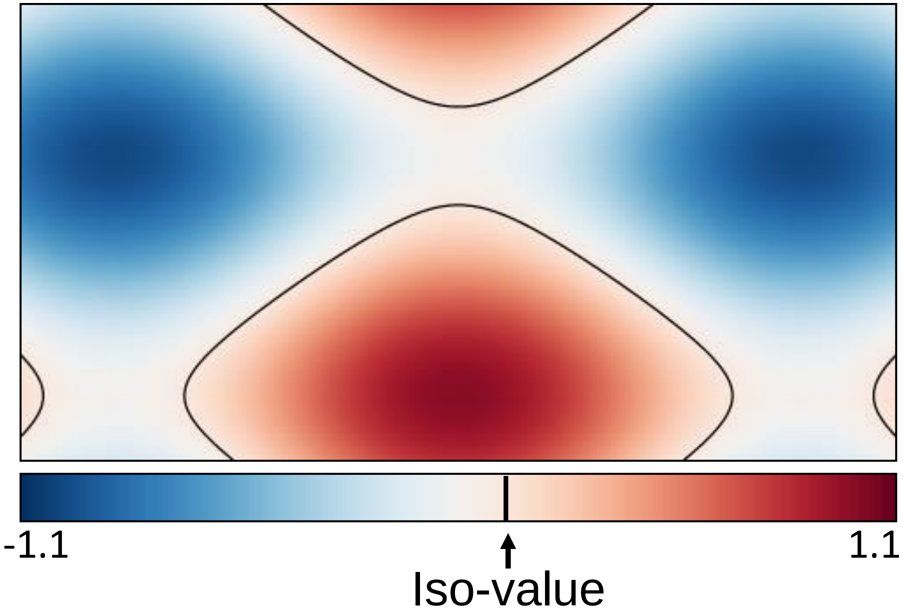
Ingenuity for life

- Why do we need a continuous representation of data given on grids / meshes
  - Better communication of spatial data distribution
  - Some techniques require a continuous representation (e.g., color mapping, iso-lines)



Isoline = all points having a certain value

Data interpolation

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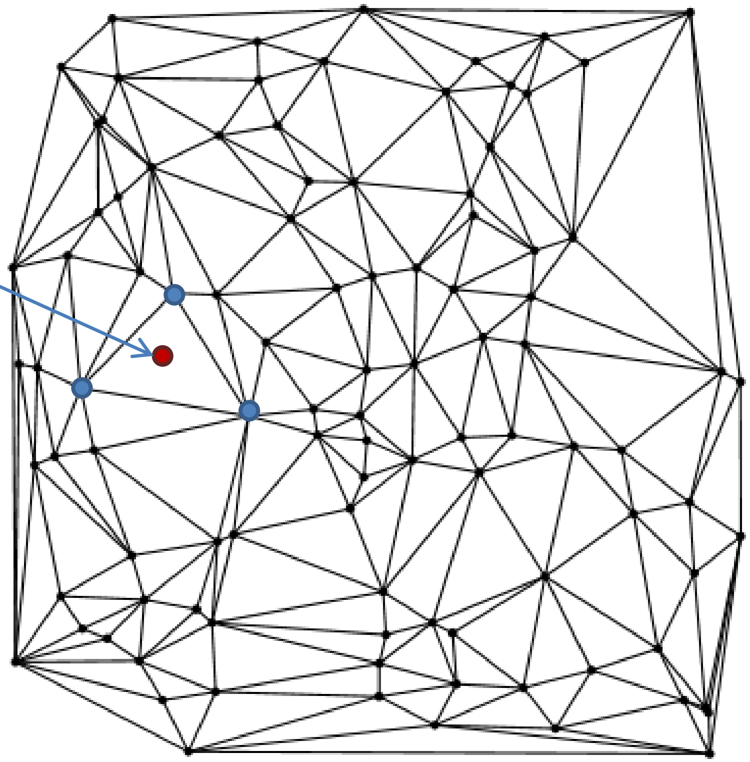
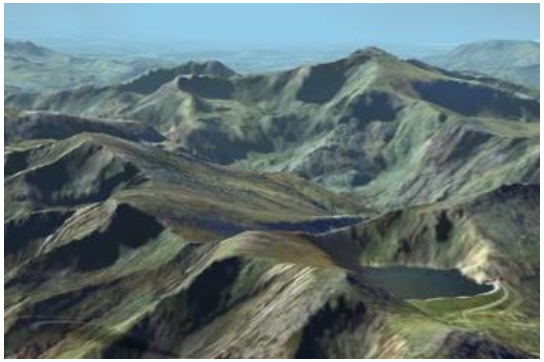
Ingenuity for life

- Try finding a piecewise (local) reconstruction function
  - Connect the points so that a triangulation is obtained
  - Interpolate locally within the triangles

Value obtained by only considering values at triangle corners





Data interpolation

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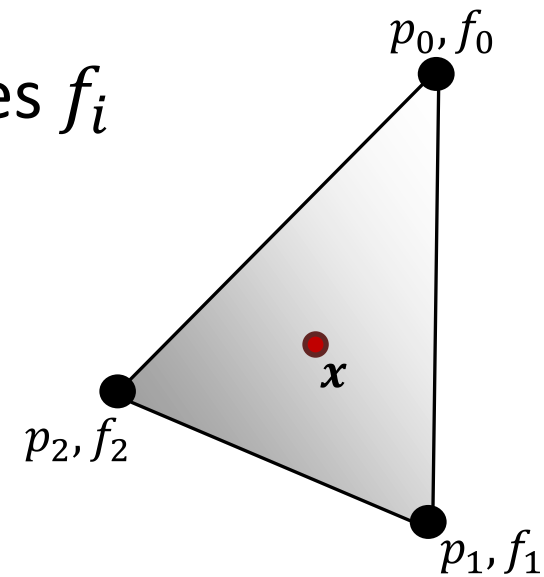
Ingenuity for life

- How to interpolate inside a triangle
  - The triangle lives in a (N = 2)D plane; it has $N + 1$ points (x_i, y_i) with values f_i
  - Can we find a function f that interpolates f_i at the points p_i , *i.e.*,

$$f(p_i) = f_i, \qquad i = 0, \dots, N$$

(interpolation constraint)

  - If so, then the value at any point $\boldsymbol{x}$ can be interpolated by evaluating $f(\boldsymbol{x})$



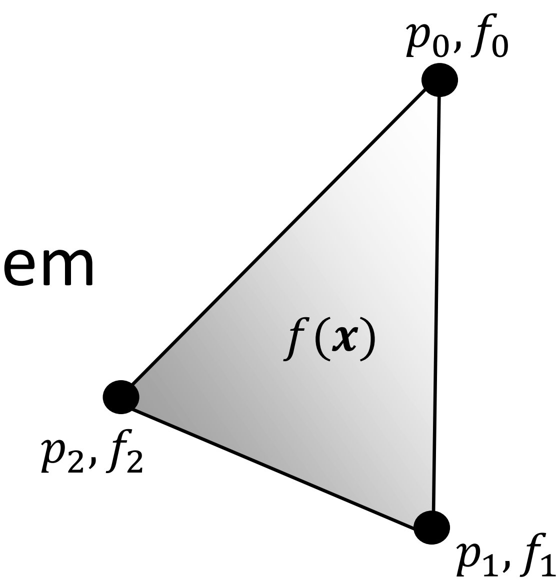
Data interpolation

- There is a unique linear function that satisfies the interpolation constraint
- A linear function can be written as

$$f(\boldsymbol{x}) = a + bx + cy$$

  – The unknown coefficients a, b, c can be obtained by solving the system

$$\begin{array}{l} p_0 \rightarrow \\ p_1 \rightarrow \\ p_2 \rightarrow \end{array} \quad \begin{bmatrix} 1 & x_0 & y_0 \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix}$$



Data interpolation

- Example

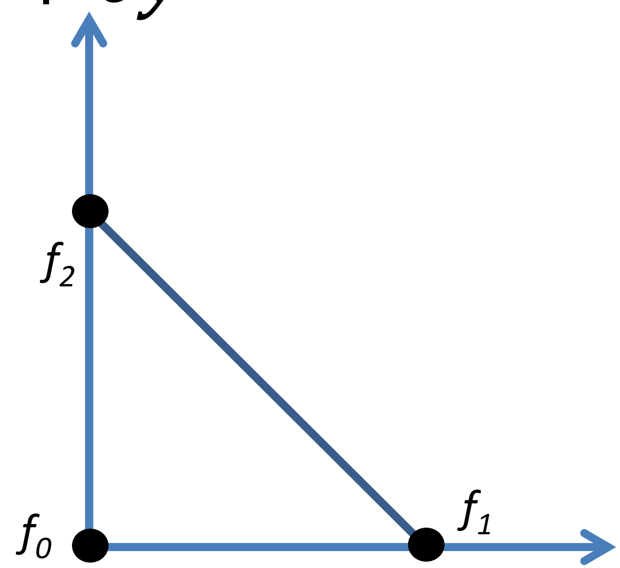
  $p_0 = (0,0),\ p_1 = (1,0),\ p_2 = (0,1)$ with
  $f_0 = 1,\ f_1 = 8,\ f_2 = 2$

- Obtain a, b, c in interpolation function

$$f(\boldsymbol{x}) = a + bx + cy$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$$

$a = 1$
$b = 7$
$c = 1$



Data interpolation

- Example

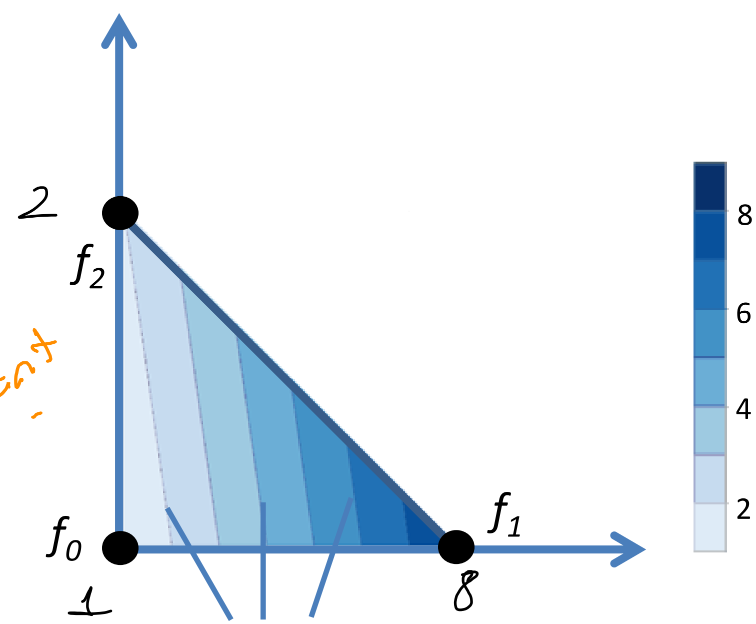
  $p_0 = (0,0),\ p_1 = (1,0),\ p_2 = (0,1)$ with
  $f_0 = 1,\ f_1 = 8,\ f_2 = 2$

- Obtain a, b, c by solving the system

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$$

$$a = 1, b = 7, c = 1$$
$$f(x,y) = 1 + 7x + 1y$$

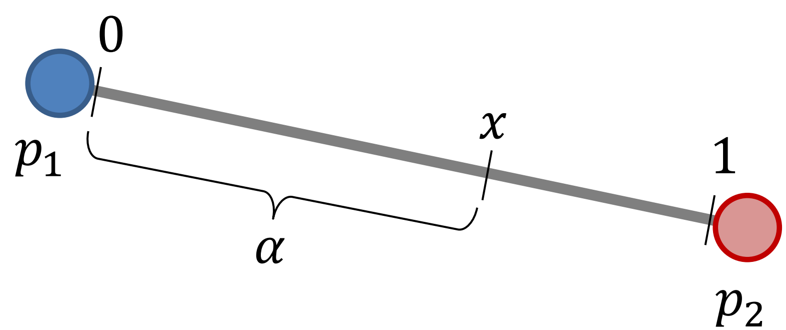
important



Notice that the isolines are straight lines

Data interpolation

- Barycentric interpolation
  - Another way to interpolate inside a triangle, which yields the same linear interpolation as before
  - But let's solve a simpler problem first



  - We want to define a color for every $\alpha \in [0, 1]$

Data interpolation

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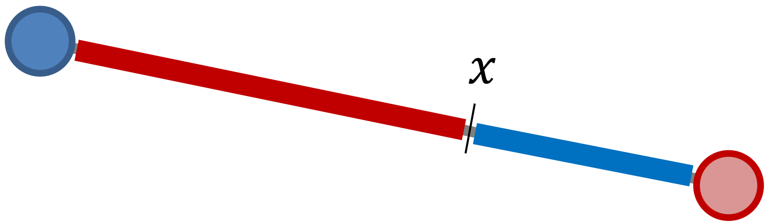
Ingenuity for life

- How do we come up with an equation?



The closer x is to the red point, the more red we want

The closer x is to the blue point, the more blue we want

↓

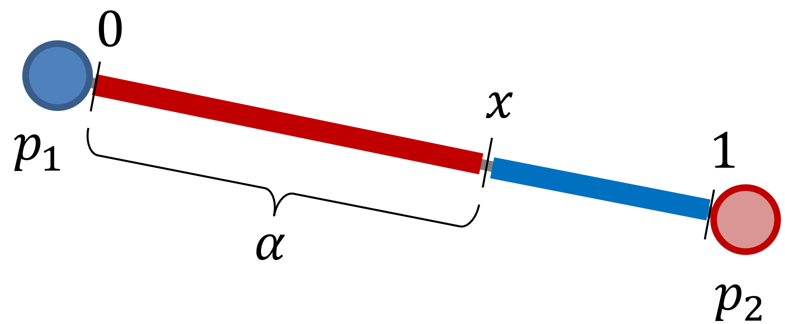
Percentage red = (length of red segment) / (total length)

Percentage blue = (length of blue segment) / (total length)

Data interpolation

SIEMENS Ingenuity for life

- How do we come up with an equation?



The closer x is to the red point, the more red we want

The closer x is to the blue point, the more blue we want

↓

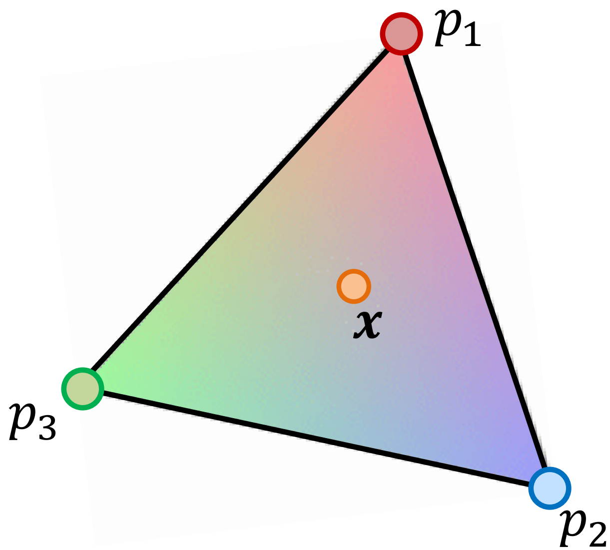
Percentage red $= \alpha$ → Normalized.

Percentage blue $= 1 - \alpha$

$$f(\alpha) = (1 - \alpha) \cdot p_1 + \alpha \cdot p_2$$

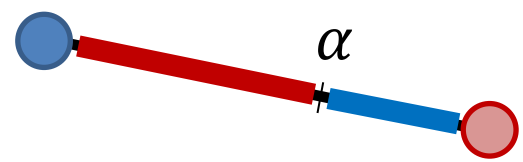
Data interpolation

- Barycentric interpolation
  – Now what about triangles?



What's the interpolated value at the point $\boldsymbol{x}$?

In 1D we used ratios of lengths



Data interpolation

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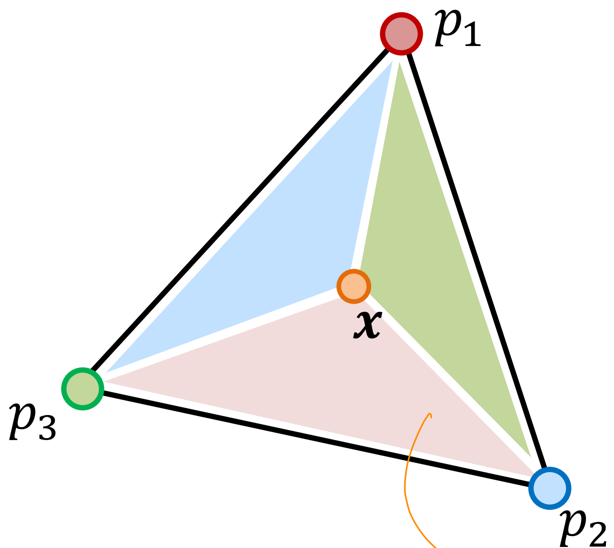
- Barycentric interpolation
  - Now what about triangles?



What about ratios of 2D areas?

Sizes of this triangle represent how much red I want. percentage of red.

Data interpolation

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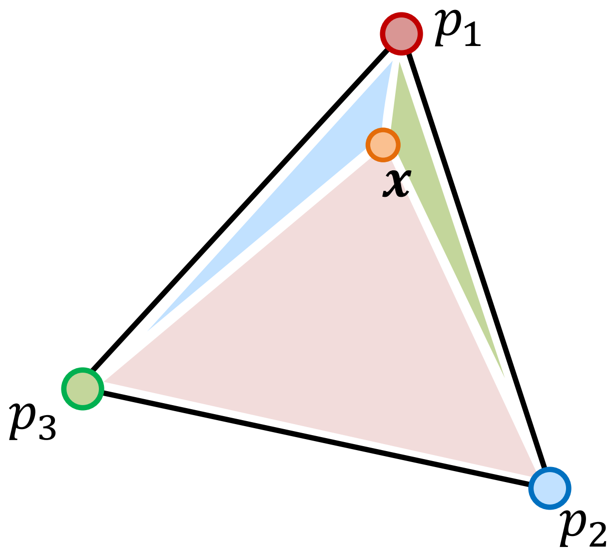
Ingenuity for life

- Barycentric interpolation
  - Now what about triangles?



As $\boldsymbol{x}$ approaches the red point, the red area (for example) covers more of the triangle

Data interpolation

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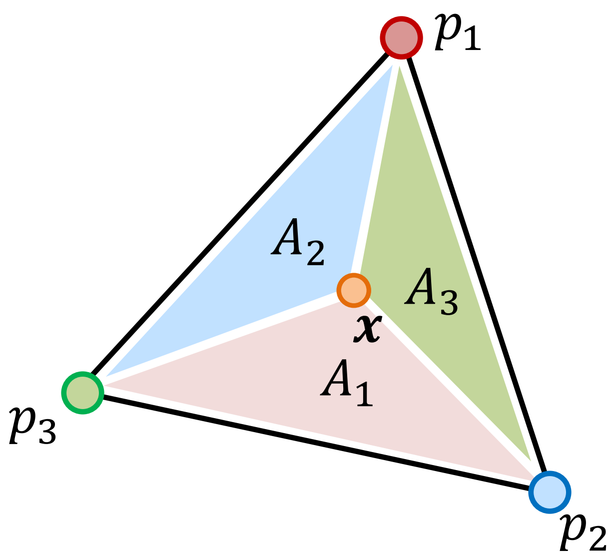
- Barycentric interpolation
  - Just like before:

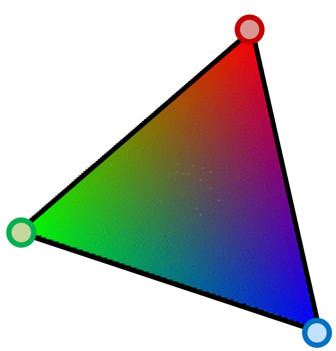


$\alpha_1 = {A_1}/{A}$ percentage red

$\alpha_2 = {A_2}/{A}$ percentage blue

$\alpha_3 = {A_3}/{A}$ percentage green

A … area of whole triangle



$$\boldsymbol{x} = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

Data interpolation

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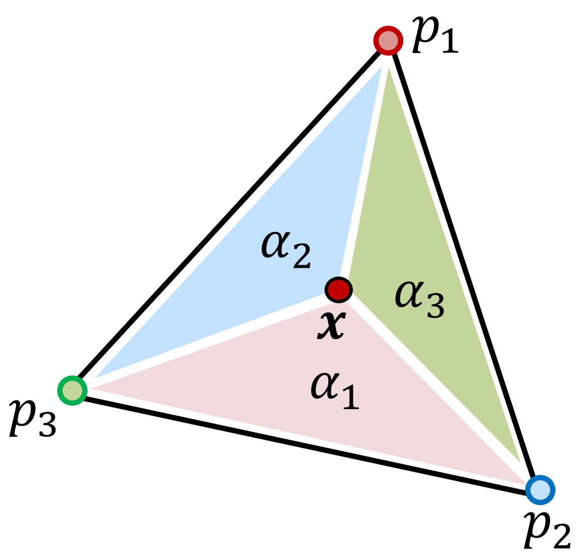
Ingenuity for life



$$\alpha_1 = \frac{\text{area}(\Delta \boldsymbol{x} p_2 p_3)}{\text{area}(\nabla p_1 p_2 p_3)}$$

$$\alpha_2 = \frac{\text{area}(\Delta p_1 \boldsymbol{x} p_3)}{\text{area}(\nabla p_1 p_2 p_3)}$$

$$\alpha_3 = \frac{\text{area}(\Delta p_1 p_2 \boldsymbol{x})}{\text{area}(\nabla p_1 p_2 p_3)}$$

Barycentric interpolation

$$\boldsymbol{x} = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

Inside triangle criteria

$$0 \le \alpha_1 \le 1$$

$$0 \le \alpha_2 \le 1$$

$$0 \le \alpha_3 \le 1$$

Data interpolation

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Ingenuity for life

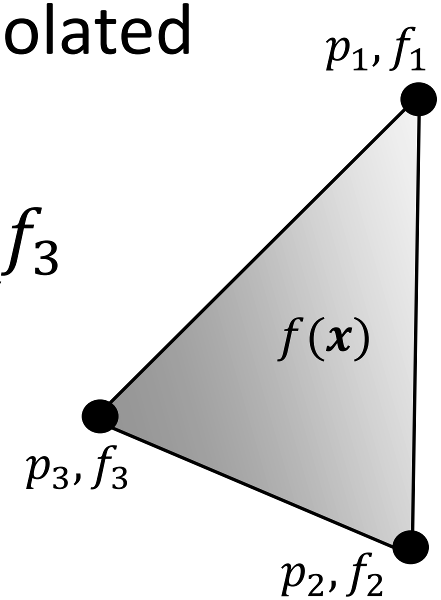
- Barycentric interpolation
  - Every point $\boldsymbol{x}$ in a triangle can be written as a barycentric combination of the vertices p_i:

$$\boldsymbol{x} = \sum_i \alpha_i p_i \quad \text{with} \quad \sum_i \alpha_i = 1$$

(α_i … barycentric coordinates)

  - If α_i are known, then $f(\boldsymbol{x})$ can be interpolated from values f_i at the vertices via

$$f(\boldsymbol{x}) = \alpha_1 f_1 + \alpha_2 f_2 + \underbrace{(1 - \alpha_1 - \alpha_2)}_{\alpha_3} f_3$$



Data interpolation

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Example: Given a triangle with vertices $p_1 = (0.5, 2.5)$, $p_2 = (1.5, 4.5)$ and $p_3 = (2.5, 2.5)$. Compute the barycentric coordinates of the points $P = (1.5, 2.5)$ and $Q = (1.5, 0.5)$ with respect to the triangle.

The area of p2 is 0 because P in the bottom of the triangle, $a_2 = 0$

$a_1 = 0.5 \quad a_3 = 0.5$

So for the Q value we need to calculate areas

Firstly, we put x values

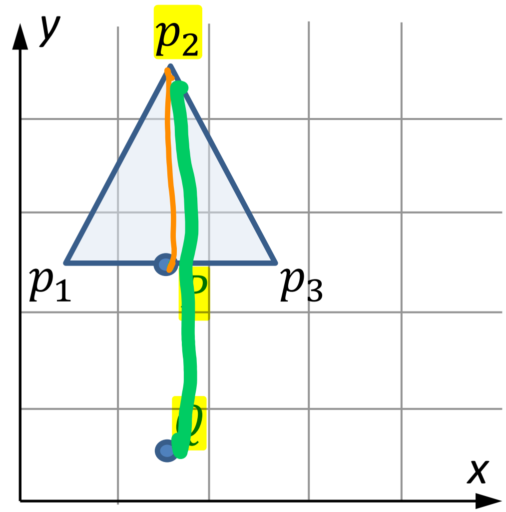
$1,5 = 0,5\,a_1 + 1.5\,a_2 + 2.5\,a_3 \quad (0.5)$

$0,5 = 2,5\,a_1 + 4.5\,a_2 + 2.5\,a_3$

$1 = a_1 + a_2 + a_3 \quad (2.5)$

$-1 = 2a_1 + 3a_2$

$2a_2 = -2 \quad a_2 = \boxed{-1} \qquad a_1 = 1 \quad a_3 = 1$



http://kehrer.bplaced.net/lecture/barycentric.html

Data interpolation

Example: Given a triangle with vertices $p_1 = (0.5, 2.5)$, $p_2 = (1.5, 4.5)$ and $p_3 = (2.5, 2.5)$. Compute the barycentric coordinates of the points $P = (1.5, 2.5)$ and $Q = (1.5, 0.5)$ with respect to the triangle.

Point P:

$$\alpha_2 = 0 \ \rightarrow \alpha_1 = \alpha_3 = 0.5$$

Point Q:

I: $1.5 = 0.5\,\alpha_1 + 1.5\,\alpha_2 + 2.5\,\alpha_3$ ← x coordinates

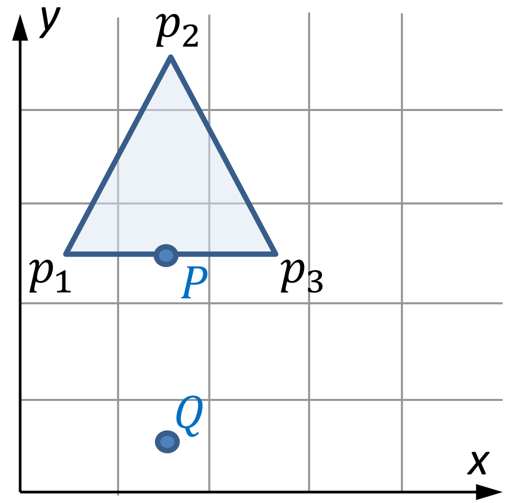
II: $0.5 = 2.5\,\alpha_1 + 4.5\,\alpha_2 + 2.5\,\alpha_3$ ← y coordinates

III: $1 = \alpha_1 + \alpha_2 + \alpha_3$

$I - II$: $1 = -2\alpha_1 - 3\alpha_2$

$II - 2.5\,III$: $-2 = 2\alpha_2$ $\rightarrow \alpha_2 = -1$

$\alpha_2 \rightarrow I'$: $-2 = -2\alpha_1$ $\rightarrow \alpha_1 = 1$ $\rightarrow \alpha_3 = 1$



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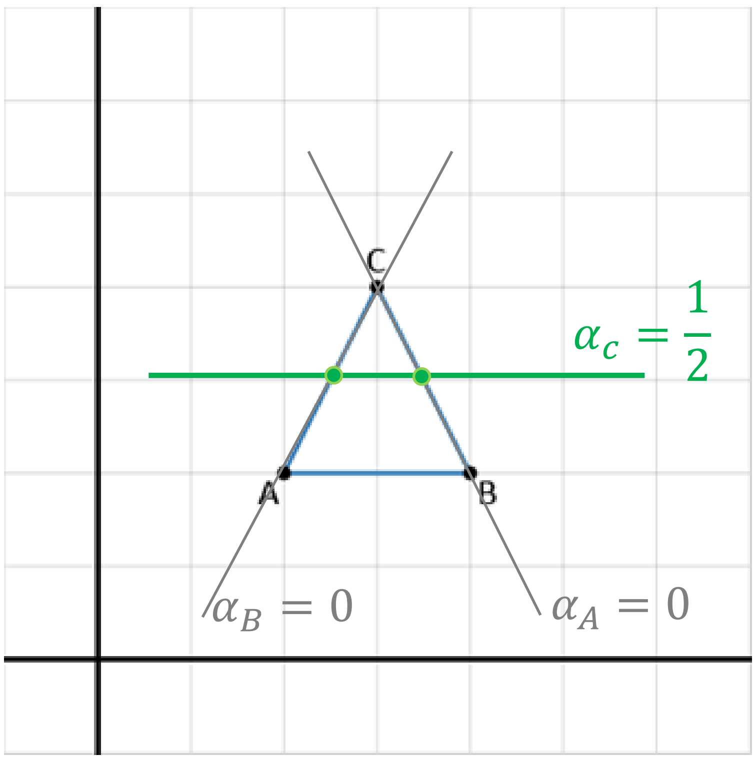
Barycentric coordinates

Example: Given a triangle with vertices (2, 2), (4, 2), and (3, 4). Draw the iso-contours along which the barycentric coordinate α_c corresponding to point C has the value $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{3}{2}$, respectively.

$$\alpha_A + \alpha_B + \alpha_c = 1$$



$$\alpha_c = \frac{1}{2}, \qquad \alpha_A = 0 \rightarrow \alpha_B = \frac{1}{2}$$

$$\alpha_B = 0 \rightarrow \alpha_A = \frac{1}{2}$$

So we assume that

$\alpha_A = 0$, the $\alpha_B = 1/2$

For finding exact cordinate,

$x = 4 \times \frac{1}{2} + 3 \cdot \frac{1}{2} = 3.5$

$y = 2 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2} = 3$

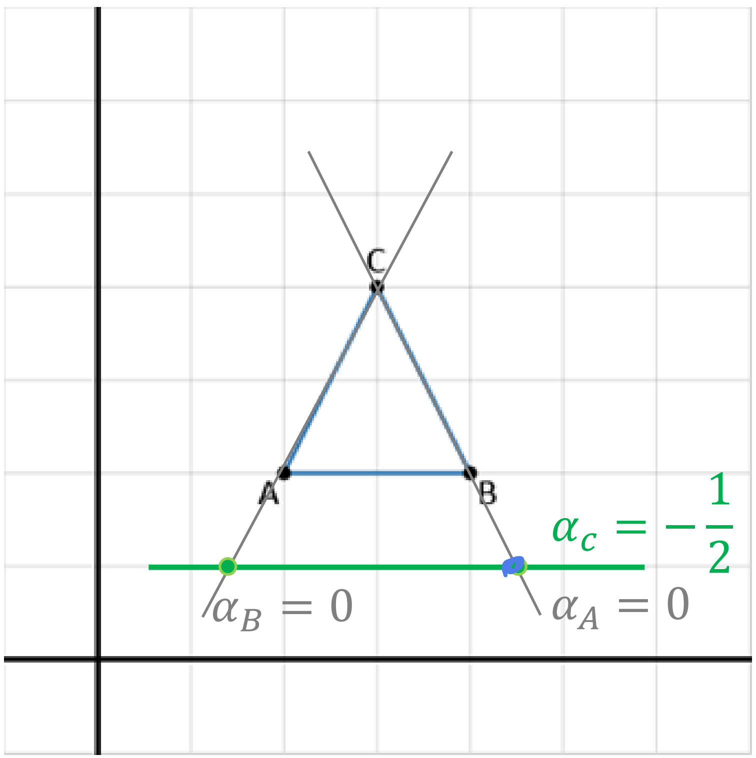
Barycentric coordinates

Example: Given a triangle with vertices (2, 2), (4, 2), and (3, 4). Draw the iso-contours along which the barycentric coordinate α_c corresponding to point C has the value $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{3}{2}$, respectively.

$$\alpha_A + \alpha_B + \alpha_c = 1$$

$$\alpha_c = -\frac{1}{2}, \qquad \alpha_A = 0 \rightarrow \alpha_B = \frac{3}{2}$$

$$\alpha_B = 0 \rightarrow \alpha_A = \frac{3}{2}$$



Barycentric coordinates

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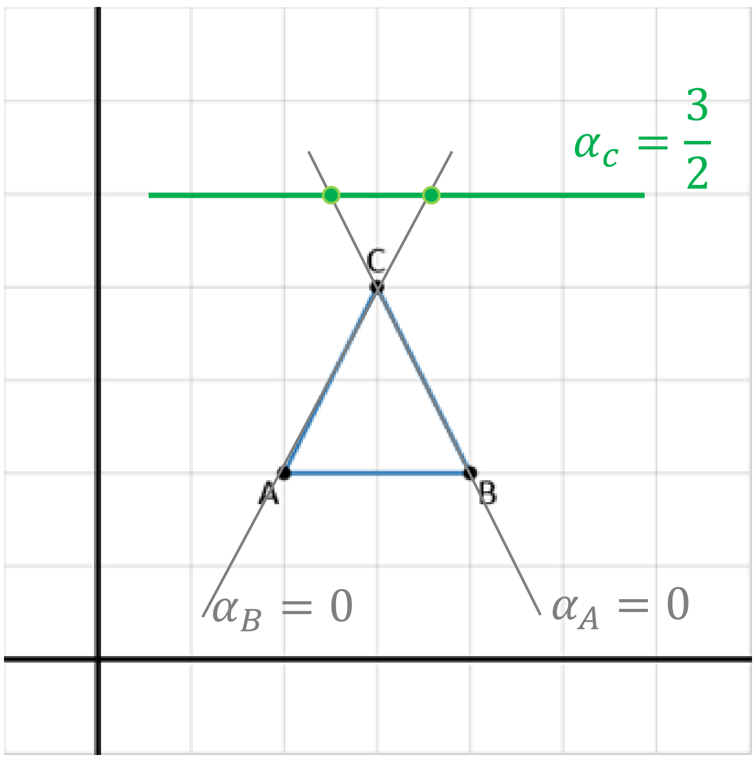
$$\alpha_A + \alpha_B + \alpha_c = 1$$



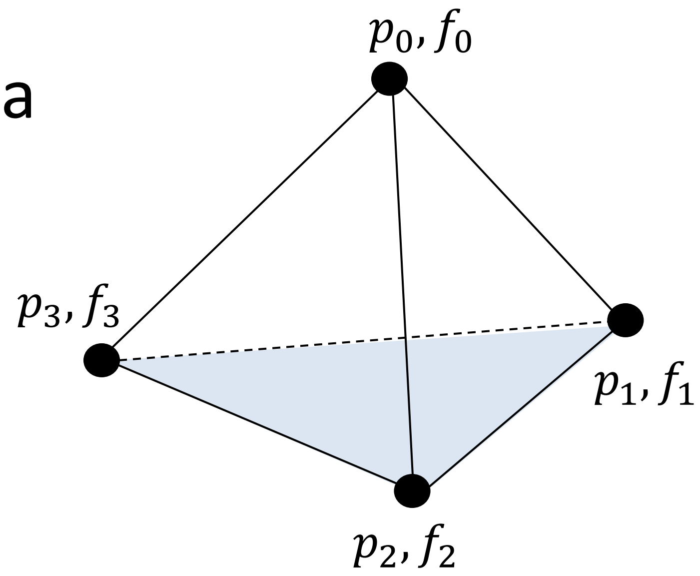
$$\alpha_c = \frac{3}{2}, \qquad \alpha_A = 0 \rightarrow \alpha_B = -\frac{1}{2}$$

$$\alpha_B = 0 \rightarrow \alpha_A = -\frac{1}{2}$$

Data interpolation

- Interpolation of scalar values in a tetrahedron
  - A unique linear interpolation function $f(\boldsymbol{x}) = a + bx + cy + dz$ exists which interpolates the scalar values at the vertices
  - Solve for coefficients a, b, c, d via

$$\begin{bmatrix} 1 & x_0 & y_0 & z_0 \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix}$$



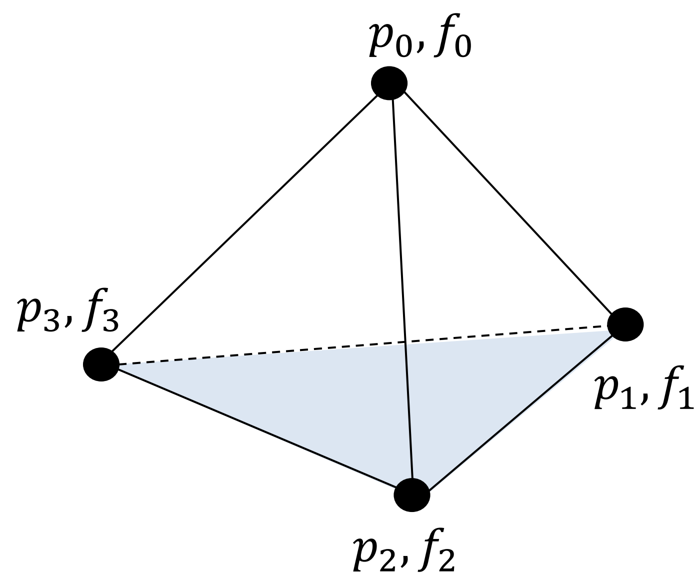
Data interpolation

- How to get the gradient inside the tetrahedron?
  - Given the linear interpolation function

$$f(\boldsymbol{x}) = a + bx + cy + dz$$

  - The gradient of the interpolated scalar field can be obtained by

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} = \begin{pmatrix} b \\ c \\ d \end{pmatrix}$$



  - The gradient is constant within the tetrahedron

Data interpolation

SIEMENS
Ingenuity for life

Example: For the tetrahedron with vertices A = (0,0,0), B = (1,0,0), C = (0,1,0), D = (0,0,1), compute the linear interpolation function $f(x, y, z) = a + b \cdot x + c \cdot y + d \cdot z$ which interpolates the scalar values $f_A = 1, f_B = 0, f_C = 0, f_D = 1$ at the corresponding vertices.

$f_A = 1 = a$

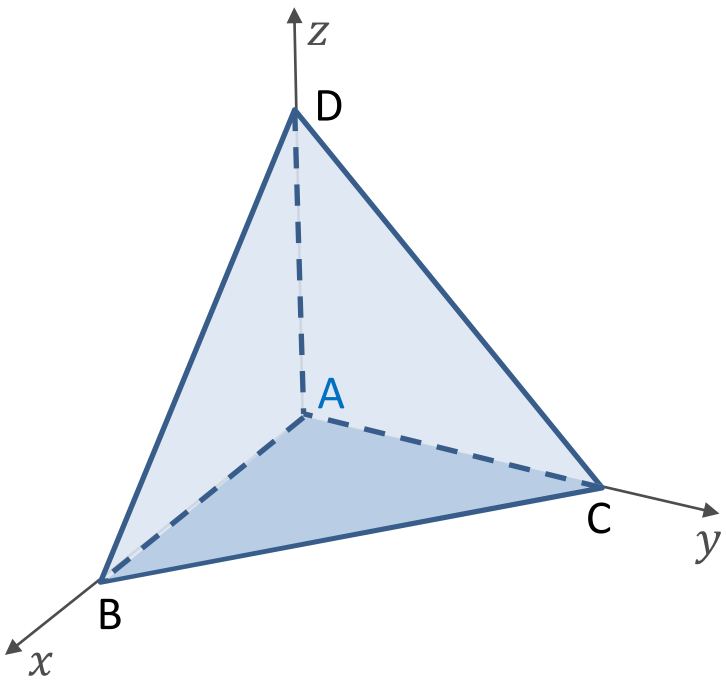
$f_B = 0 = a + b \rightarrow -1$

$f_C = 0 = a + c \rightarrow -1$

$f_d = 1 = a + d \rightarrow 0$

$f(x,y,z) = 1 - x - y$

$\nabla f = (-1, -1, 0)^T$



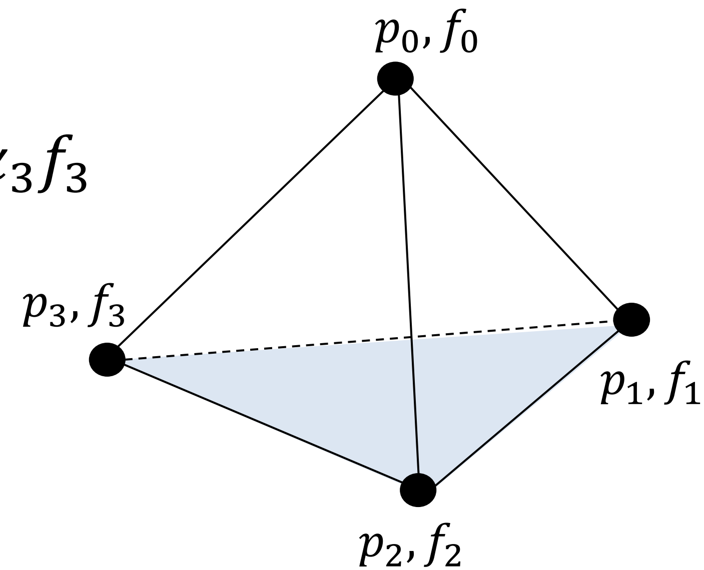
Compute the gradient of the interpolated scalar field at the center of the tetrahedron.

Data interpolation

- Barycentric interpolation in 3D
  - Scalar values can be interpolated by means of barycentric coordinates:

$$\boldsymbol{x} = \alpha_0 p_0 + \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3$$
$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$$

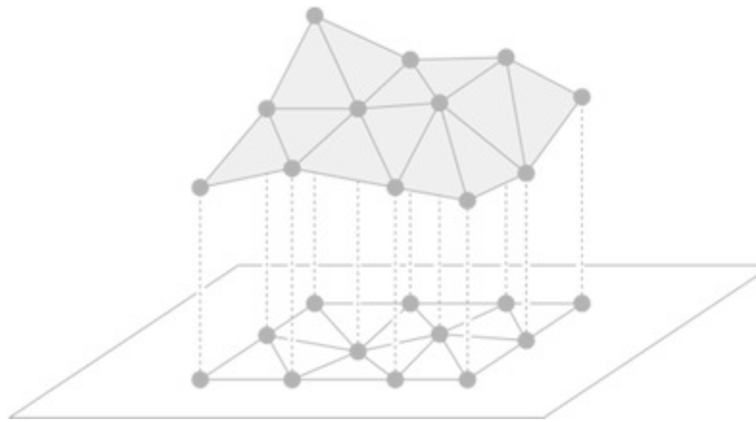
$$\rightarrow f(x, y, z) = \alpha_0 f_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3$$

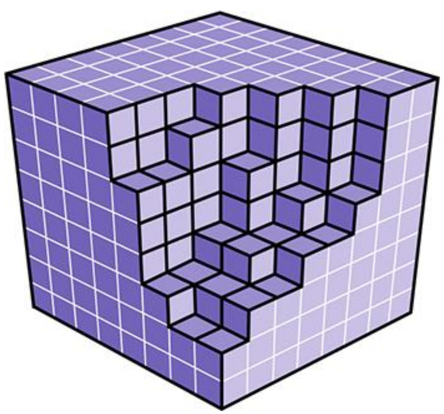


Overview

SIEMENS
Ingenuity for life

- Interpolation on grids / meshes
  - Barycentric interpolation
  - Bi-/trilinear interpolation
  - High-quality reconstruction



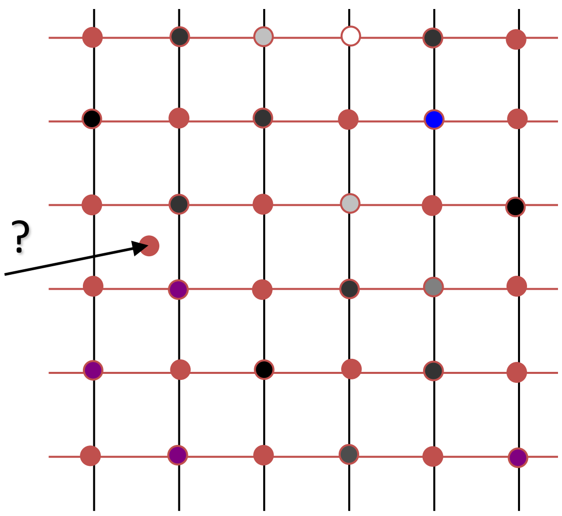


Tricubic vs. trilinear interpolation

Interpolation on grids

SIEMENS
Ingenuity for life

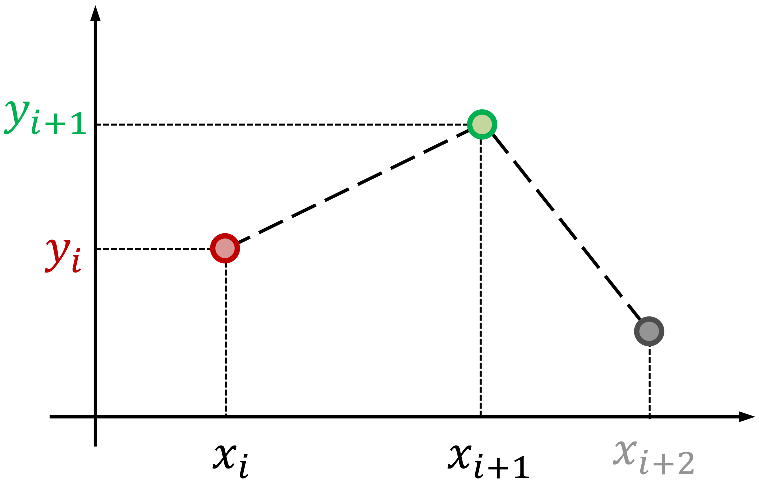
- **Problem**: assume data values are given only at vertices of a Cartesian grid
- How can we hide the underlying grid structure, i.e., how can we get a continuous data distribution over the spatial domain?
- This can be done via interpolation!



Interpolation on grids

SIEMENS
Ingenuity for life

- Piecewise linear interpolation
  - Simplest approach (except for piece-wise constant interpolation)
  - Data points: $(x_1,y_1),\ldots,(x_N,y_N)$



Interpolation on grids

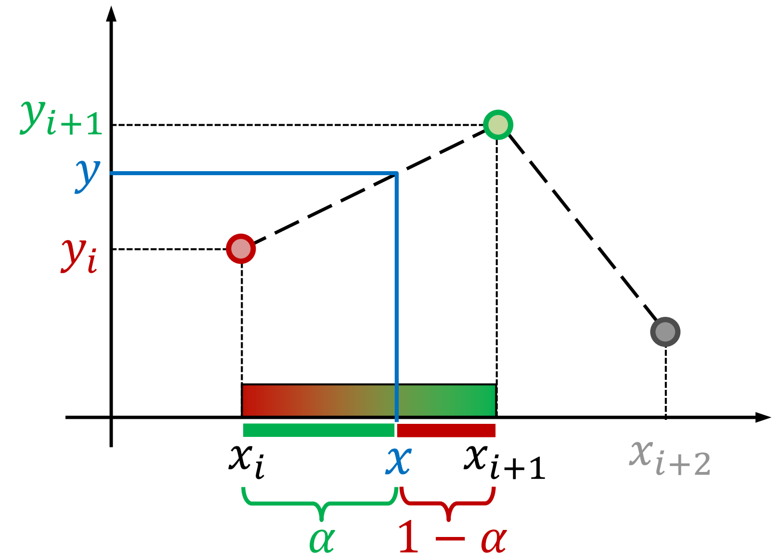
SIEMENS
Ingenuity for life

- Piecewise linear interpolation
  - Simplest approach (except for piece-wise constant interpolation)
  - Data points: $(x_1,y_1),\ldots,(x_N,y_N)$
  - For any point x with

$$x_i \le x \le x_{i+1}$$

evaluate $f(x) = (1-\alpha)y_i + \alpha y_{i+1}$

where $\alpha = \frac{x-x_i}{x_{i+1}-x_i} \in [0,1]$

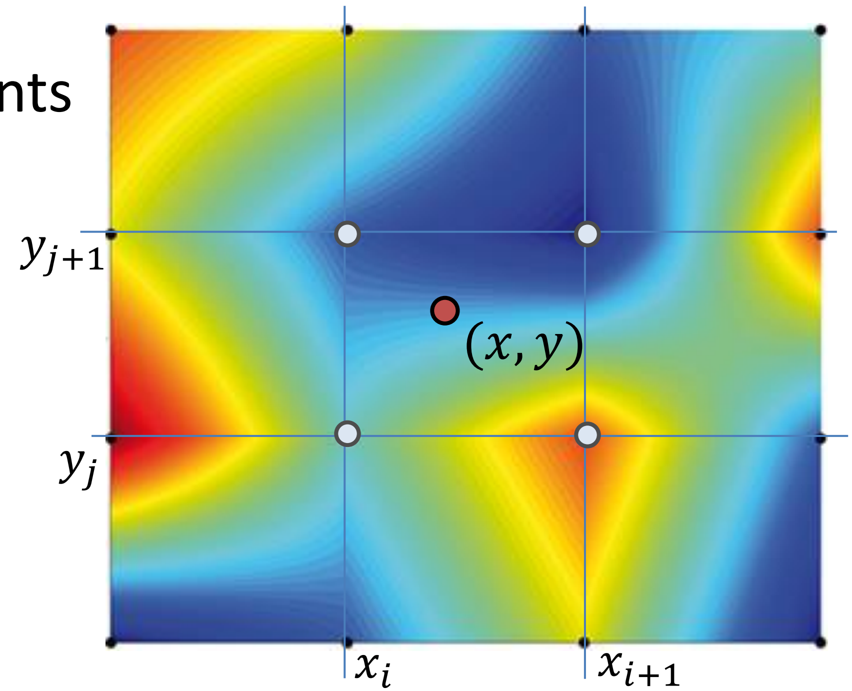


The closer x is to the green point, the more green we want

Interpolation on grids

SIEMENS
Ingenuity for life

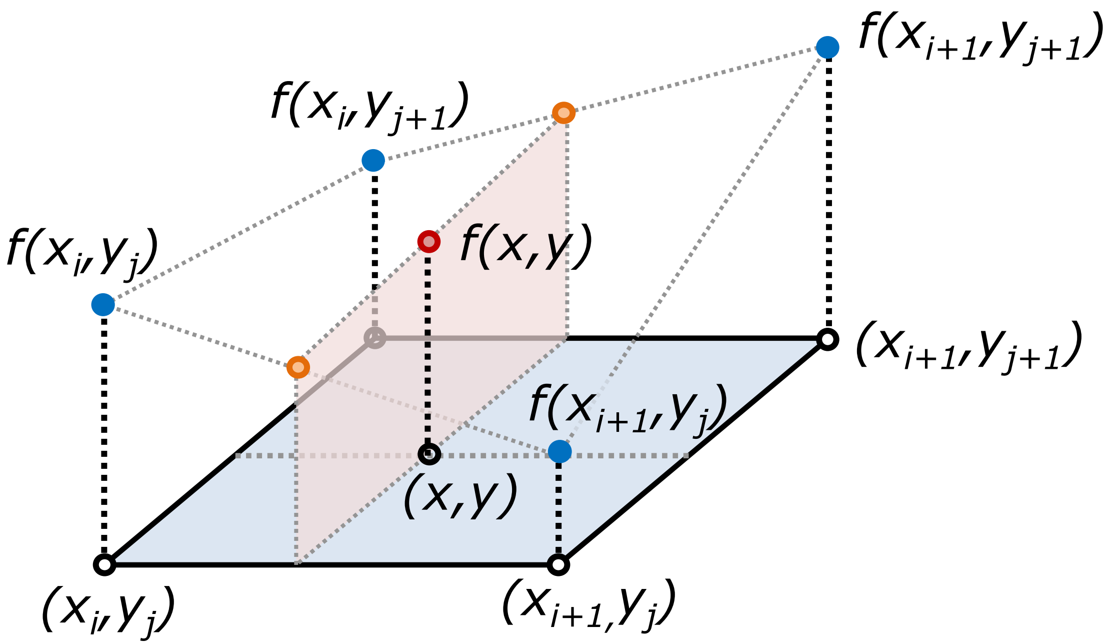
- Linear interpolation
  - C^0 continuity at segment boundaries
    - Tangents don't match at segment transition
  - Easily extendible to 2D
    - 2D cell consisting of 4 data points $(x_i, y_j), \ldots, (x_{i+1}, y_{j+1})$ with scalar values $f_{k,l} = f(x_k, y_l)$
    - Bilinear interpolation of points (x, y) with $x_i \leq x \leq x_{i+1}$ and $y_j \leq y \leq y_{j+1}$



Interpolation on grids

SIEMENS
Ingenuity for life

- Bilinear interpolation on a rectangle



Interpolation on grids

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Ingenuity for life

- Bilinear interpolation on a rectangle

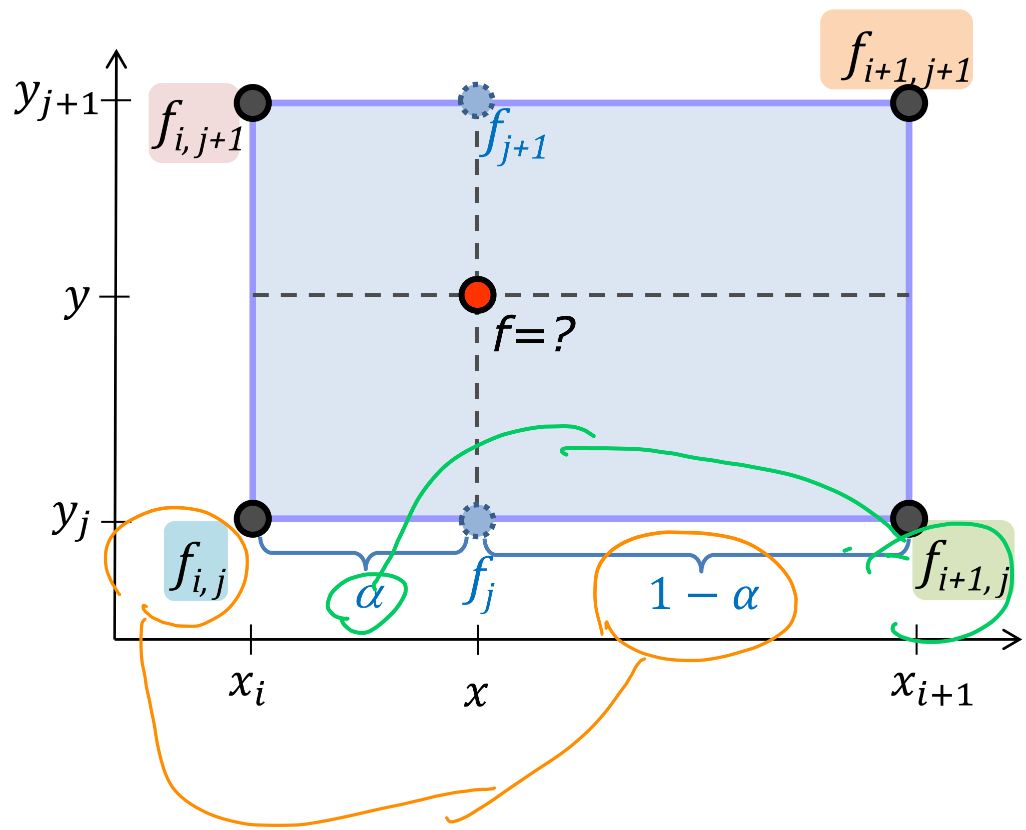
$$f(\alpha,\beta) = ?$$

Interpolate horizontally

$$f_j = (1-\alpha)\, f_{i,j} + \alpha\, f_{i+1,j}$$

$$f_{j+1} = (1-\alpha)\, f_{i,j+1} + \alpha\, f_{i+1,j+1}$$

$$\alpha = \frac{x - x_i}{x_{i+1} - x_i}$$

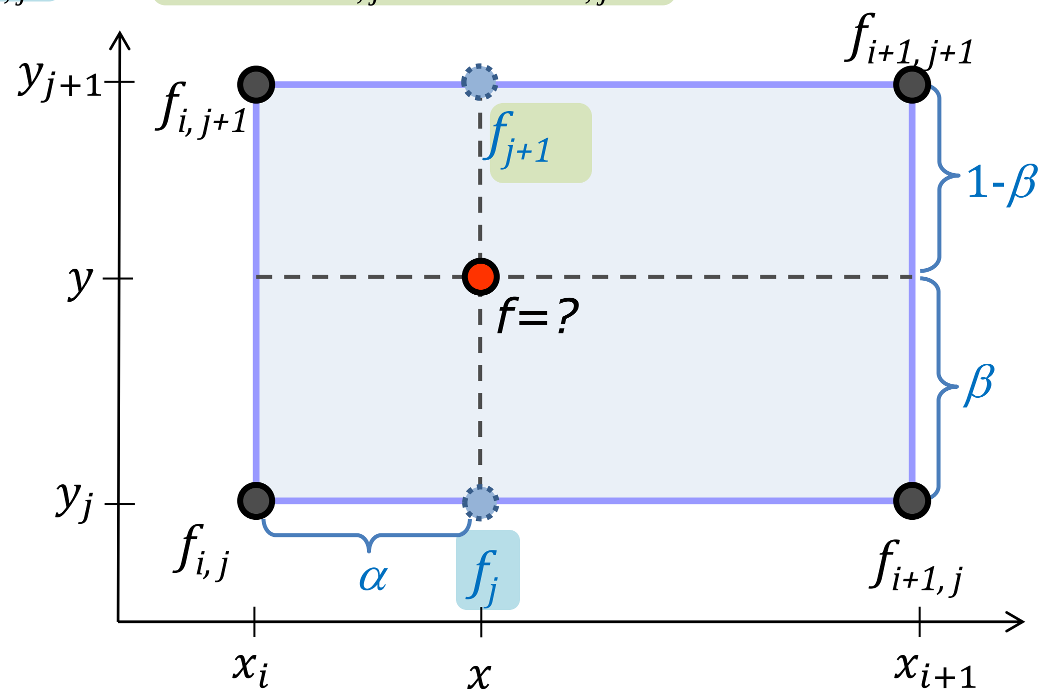


Interpolation on grids

SIEMENS
Ingenuity for life

- Bilinear interpolation on a rectangle

$$f(\alpha,\beta) = (1-\beta)\left[(1-\alpha)\, f_{i,j} + \alpha\, f_{i+1,j}\right] + \beta\left[(1-\alpha)\, f_{i,j+1} + \alpha\, f_{i+1,j+1}\right]$$
$$= (1-\beta)\, f_j + \beta\, f_{j+1}$$



with

$$f_j = (1-\alpha)\, f_{i,j} + \alpha\, f_{i+1,j}$$
$$f_{j+1} = (1-\alpha)\, f_{i,j+1} + \alpha\, f_{i+1,j+1}$$

and local coordinates

$$\alpha = \frac{x - x_i}{x_{i+1} - x_i}, \qquad \beta = \frac{y - y_i}{y_{i+1} - y_i}, \qquad \alpha, \beta \in [0,1]$$

Interpolation on grids

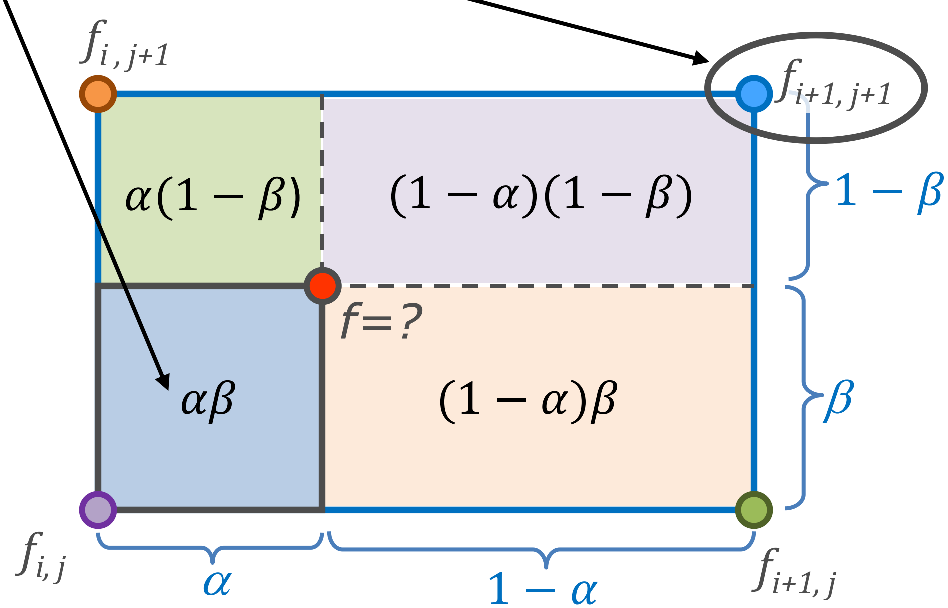
SIEMENS
Ingenuity for life

- Geometric interpretation of bilinear interpolation

$$f(\alpha,\beta) = (1-\beta)[(1-\alpha) f_{i,j} + \alpha f_{i+1,j}] + \beta [(1-\alpha) f_{i,j+1} + \alpha f_{i+1,j+1}]$$

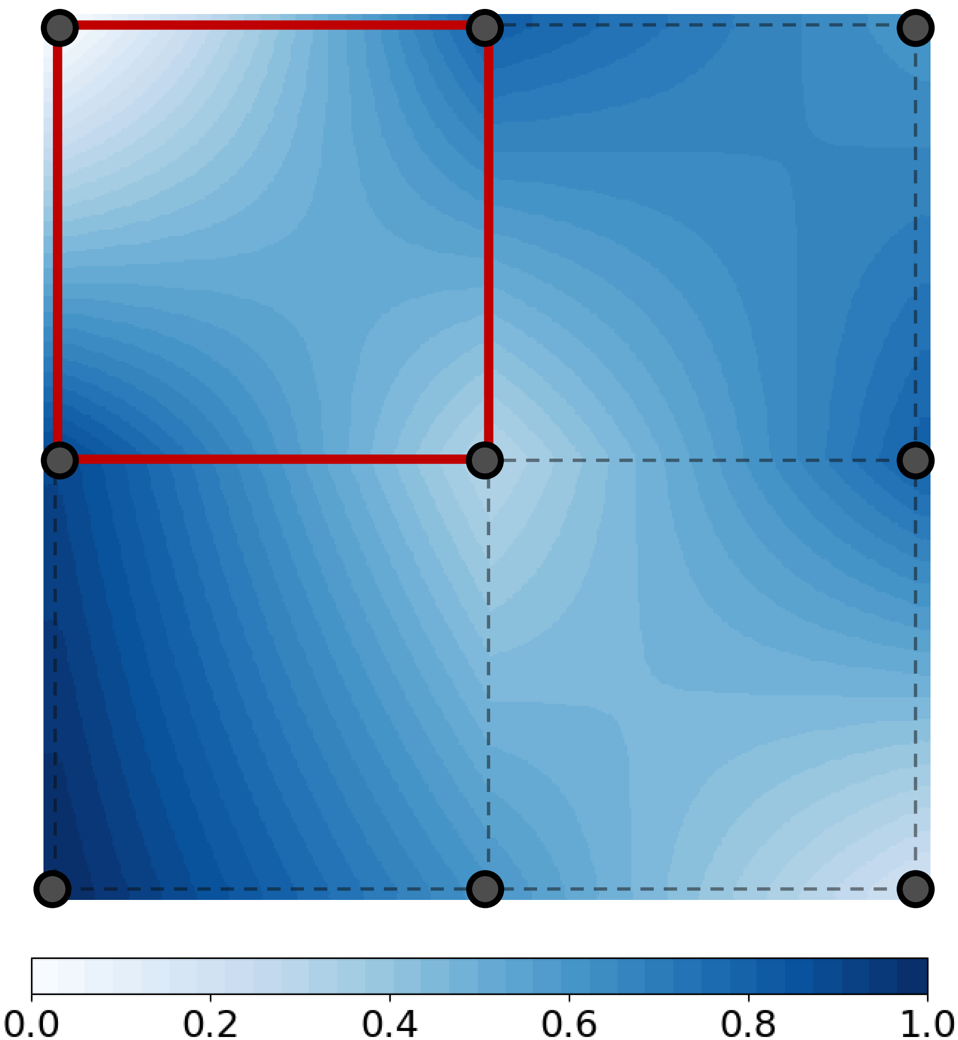
$$= (1-\alpha)(1-\beta) f_{i,j} + \alpha(1-\beta) f_{i+1,j} + (1-\alpha)\beta f_{i,j+1} + \alpha\beta f_{i+1,j+1}$$

  - Opposite points are weighted by local areas



Interpolation on grids

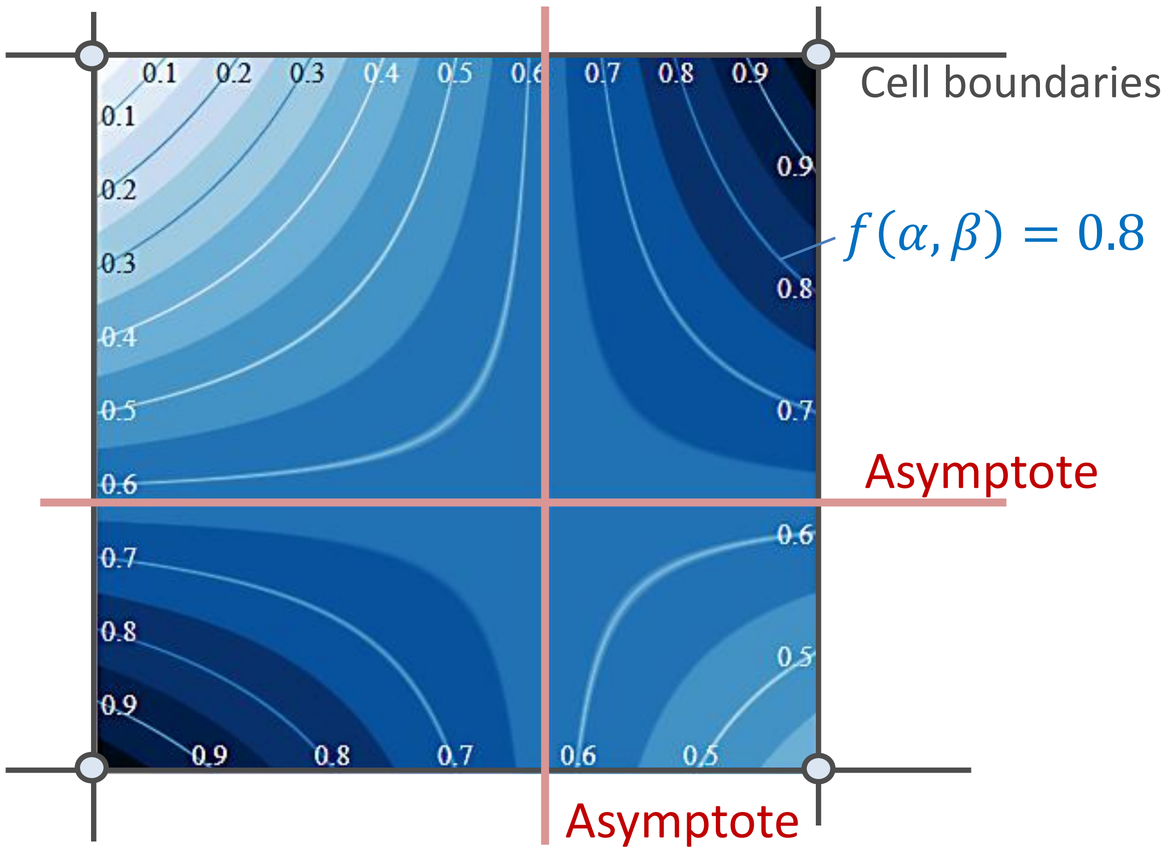
- Example
  - Data values given at 9 grid vertices
  - Bilinear interpolation used within grid cells



Interpolation on grids

SIEMENS
Ingenuity for life

- When bilinear interpolation is used, isolines within a cell are hyperbolas
  - **Isoline:** curve on which all points have a certain value

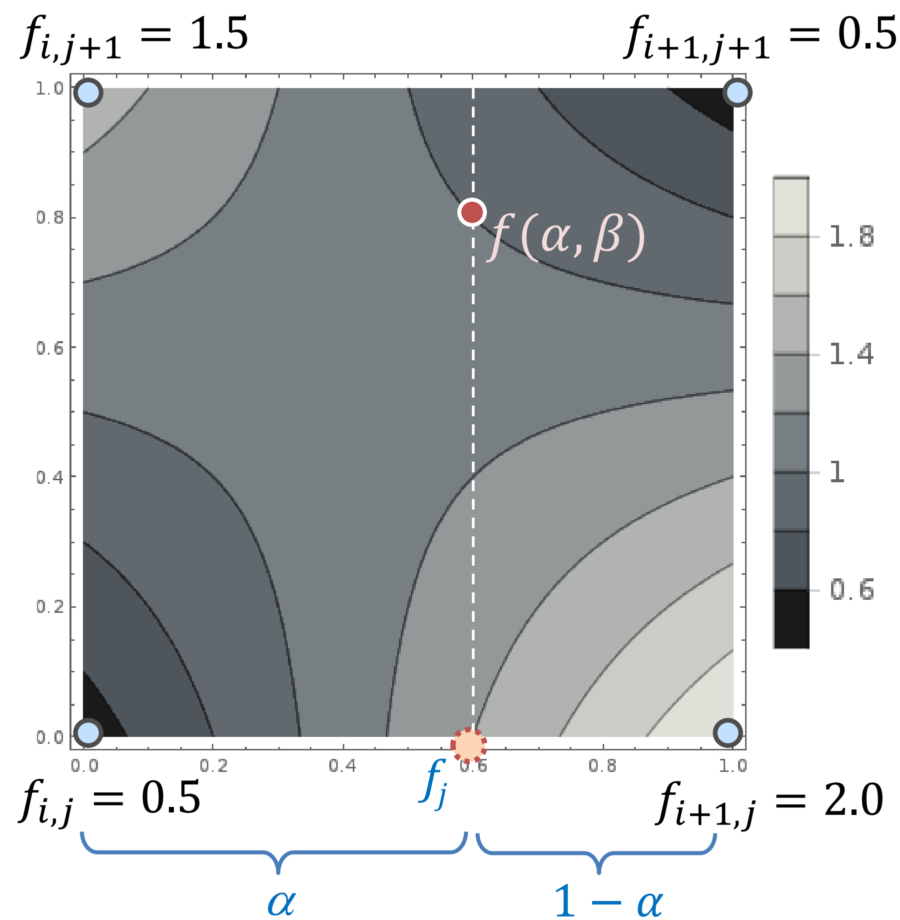


multivis.net/lecture/bilinear.html

Interpolation on grids

- How to evaluate the isolines?

$$\begin{aligned} f_j &= \alpha\, f_{i+1,j} + (1-\alpha) f_{i,j} \\ &= f_{i,j} + \left(f_{i+1,j} - f_{i,j}\right)\alpha \\ &= 0.5 + 1.5\alpha \end{aligned}$$



multivis.net/lecture/bilinear.html

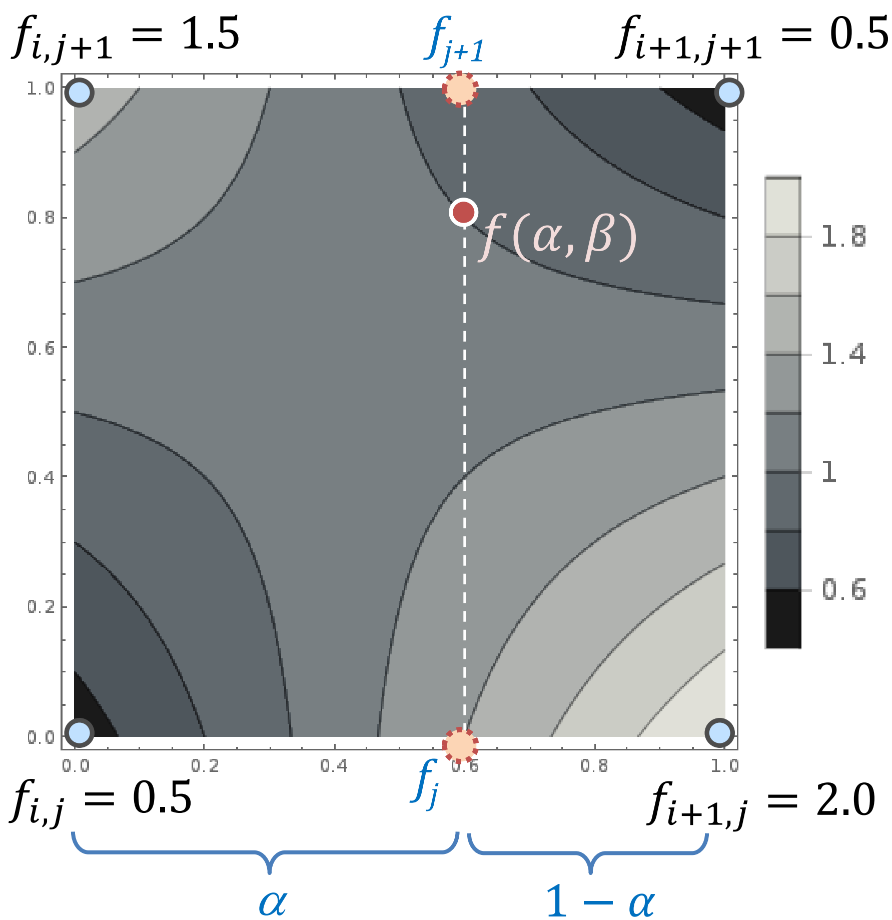
Interpolation on grids

SIEMENS
Ingenuity for life

- How to evaluate the isolines?

$$f_j = \alpha\, f_{i+1,j} + (1-\alpha) f_{i,j}$$
$$= f_{i,j} + \left(f_{i+1,j} - f_{i,j}\right)\alpha$$
$$= 0.5 + 1.5\alpha$$

$$f_{j+1} = f_{i,j+1} + \left(f_{i+1,j+1} - f_{i,j+1}\right)\alpha$$
$$= 1.5 - \alpha$$



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Interpolation on grids

SIEMENS
Ingenuity for life

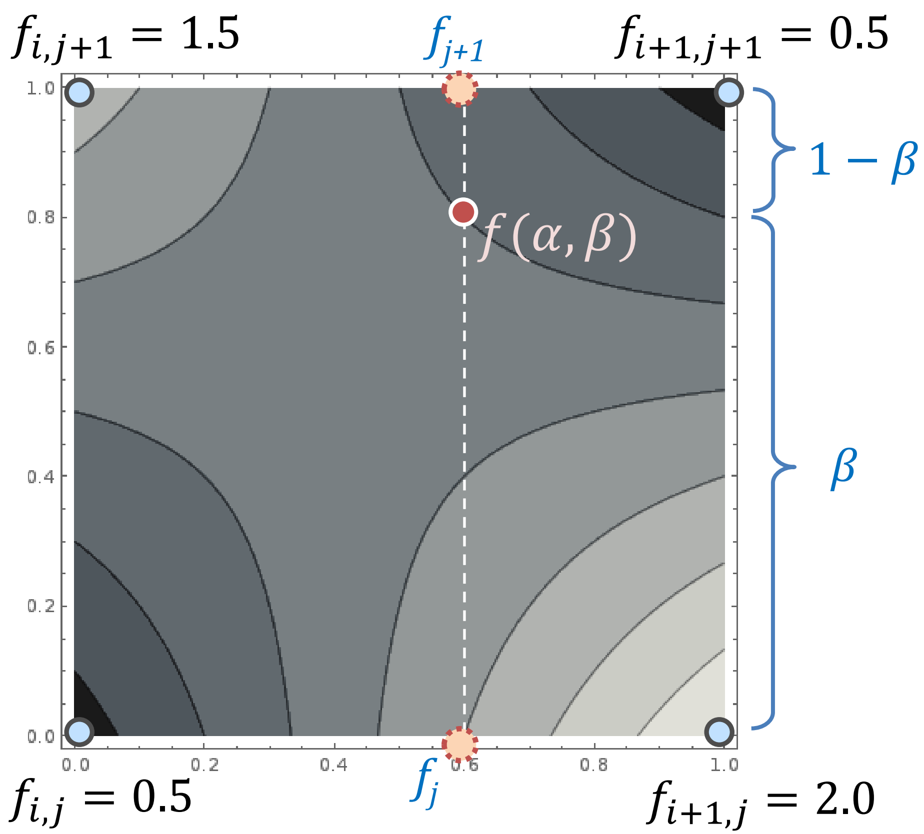
- How to evaluate the isolines?

$$f_j = \alpha\, f_{i+1,j} + (1-\alpha) f_{i,j}$$
$$= f_{i,j} + \left(f_{i+1,j} - f_{i,j}\right)\alpha$$
$$= 0.5 + 1.5\alpha$$
$$f_{j+1} = f_{i,j+1} + \left(f_{i+1,j+1} - f_{i,j+1}\right)\alpha$$
$$= 1.5 - \alpha$$

$$f(\alpha,\beta) = f_j + \left(f_{j+1} - f_j\right)\beta$$
$$= (0.5 + 1.5\alpha) + \left(1.5 - \alpha - (0.5 + 1.5\alpha)\right)\beta$$

$$f(\alpha,\beta) = 0.5 + 1.5\alpha + \beta - 2.5\alpha\beta$$

Bi-linear interpolation function defining the scalar value at each point (α,β) within the cell

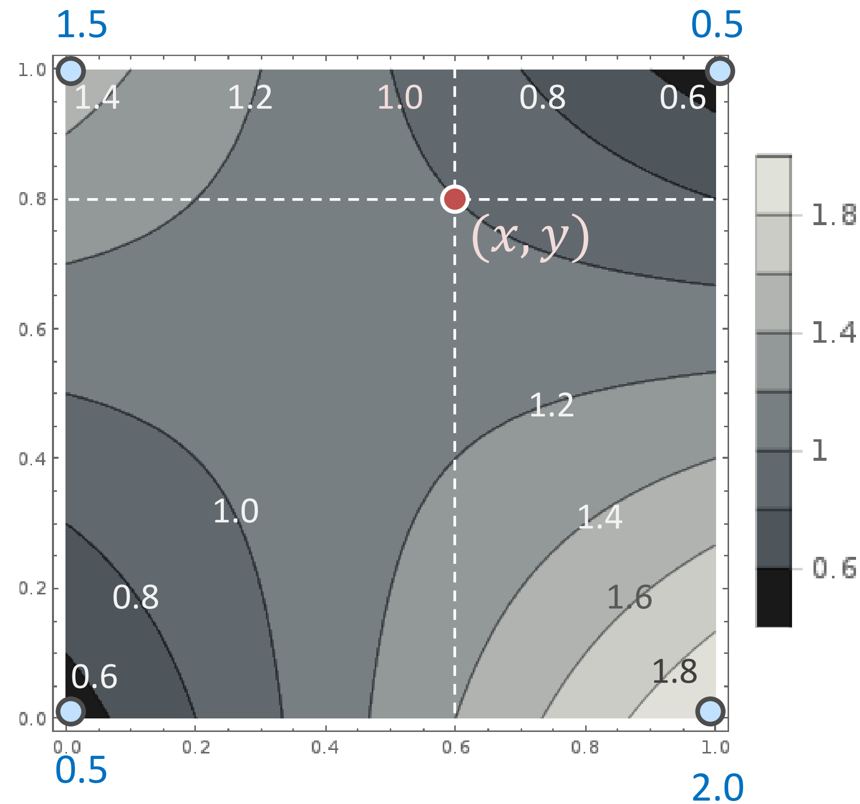


multivis.net/lecture/bilinear.html

Interpolation on grids

- How to evaluate the isolines?
  - Compute y-coordinate of a point (x, y) with $x = 0.6$ which is on the iso-contour $f(x, y) = 1$

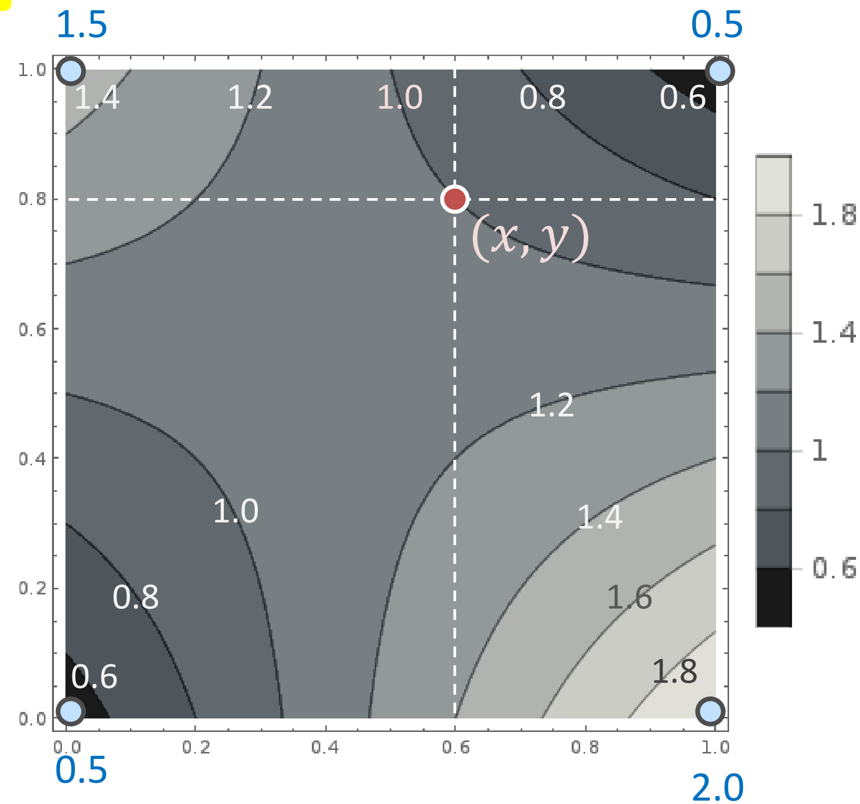
$$f(x, y) = 0.5 + 1.5x + y - 2.5xy$$



Interpolation on grids

SIEMENS
Ingenuity for life

- How to evaluate the isolines?
  - Compute y-coordinate of a point (x, y) with $x = 0.6$ which is on the iso-contour $f(x, y) = 1$



$$f(x, y) = 0.5 + 1.5x + y - 2.5xy$$

$$1 = 0.5 + 1.5 \cdot 0.6 + y - 2.5 \cdot 0.6 \cdot y$$

$$1 = 1.4 - 0.5y$$

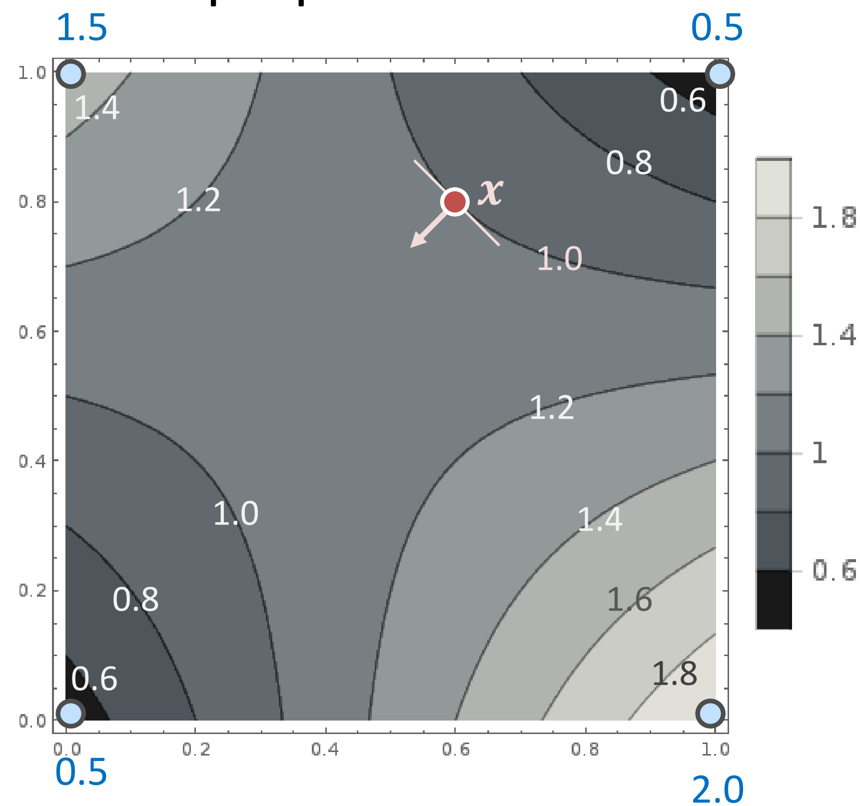
The coordinates of the point are $(0.6, 0.8)$

Interpolation on grids

SIEMENS
Ingenuity for life

- What is the normal at a point $\boldsymbol{x}$ on an iso-surface?
  - It is the gradient at this point, which is perpendicular to the tangent of the iso-surface
  - Gradient points into direction of steepest ascent of f

$$\nabla f(\boldsymbol{x}) = \left(\frac{\partial}{\partial x} f(\boldsymbol{x}),\ \frac{\partial}{\partial y} f(\boldsymbol{x}) \right)$$



Interpolation on grids

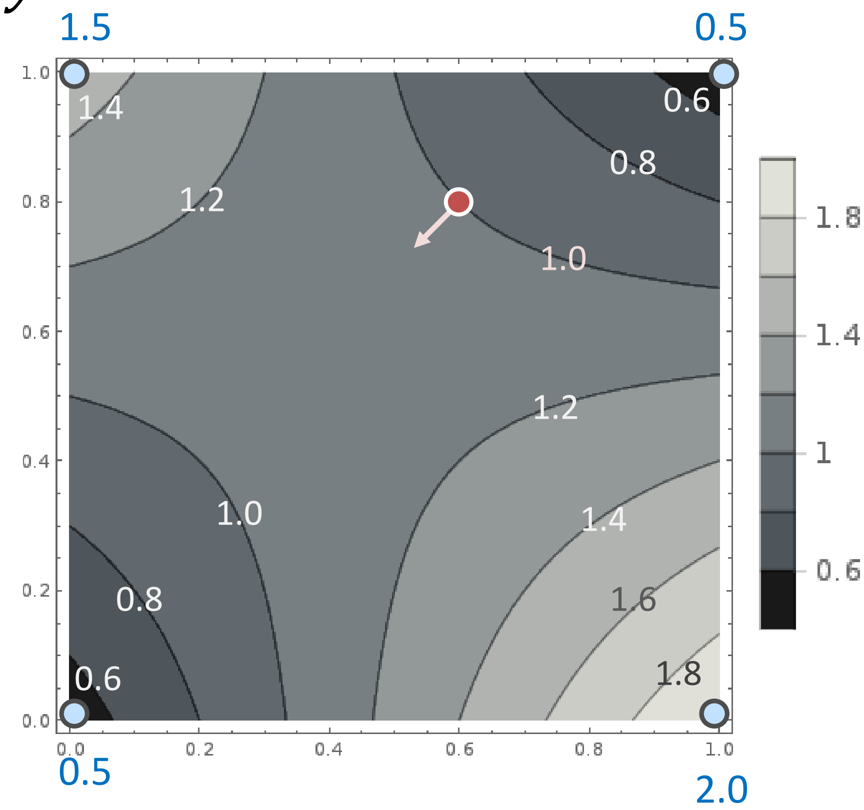
- What is the normal at point $(0.6, 0.8)$ on the iso-surface?

$$f(x, y) = 0.5 + 1.5x + y - 2.5xy$$

$$\frac{\partial f}{\partial x} = 1.5 - 2.5y$$

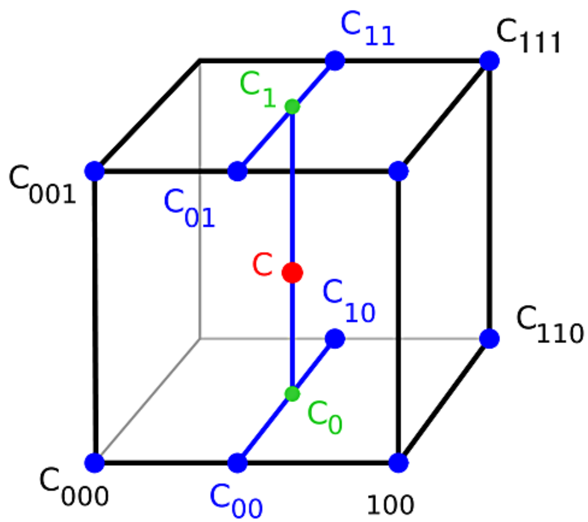
$$\frac{\partial f}{\partial y} = 1 - 2.5x$$

Gradient at $(0.6, 0.8)$: $\begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$



Interpolation on grids

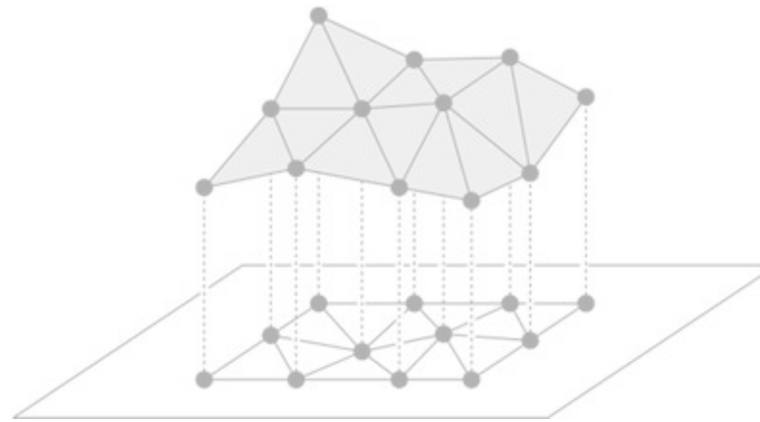
- In 3D we use trilinear interpolation:

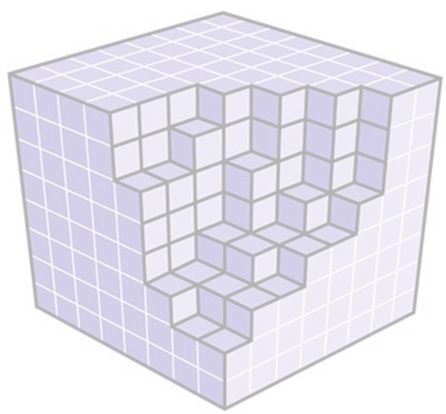


  – Apply linear interpolation of the initial data along the edges to obtain C_{00} , C_{01} , C_{10} , C_{11}
  – Interpolate linearly between C_{00} and C_{01} , and between C_{10} and C_{11} to obtain C_0 and C_1
  – Finally, interpolate between C_1 and C_0 to obtain C.

Overview

- Interpolation on grids / meshes
  - Barycentric interpolation
  - Bi-/trilinear interpolation
  - High-quality reconstruction

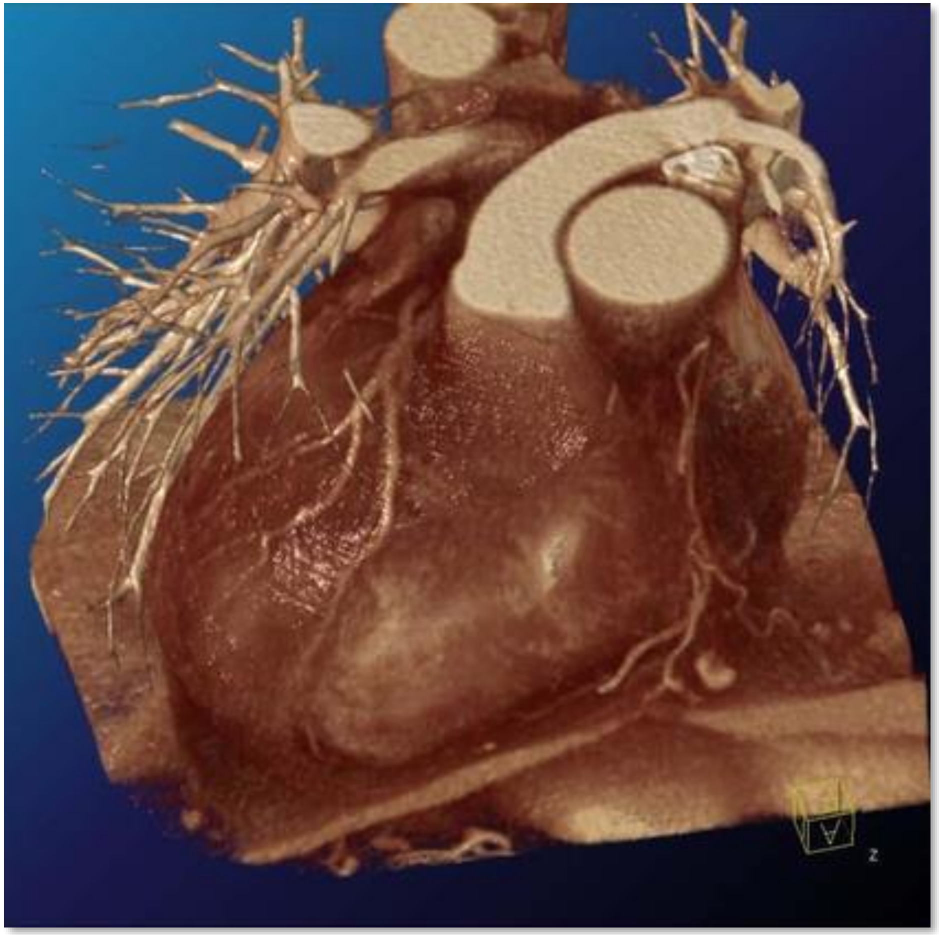




Tricubic vs. trilinear interpolation

Interpolation on grids

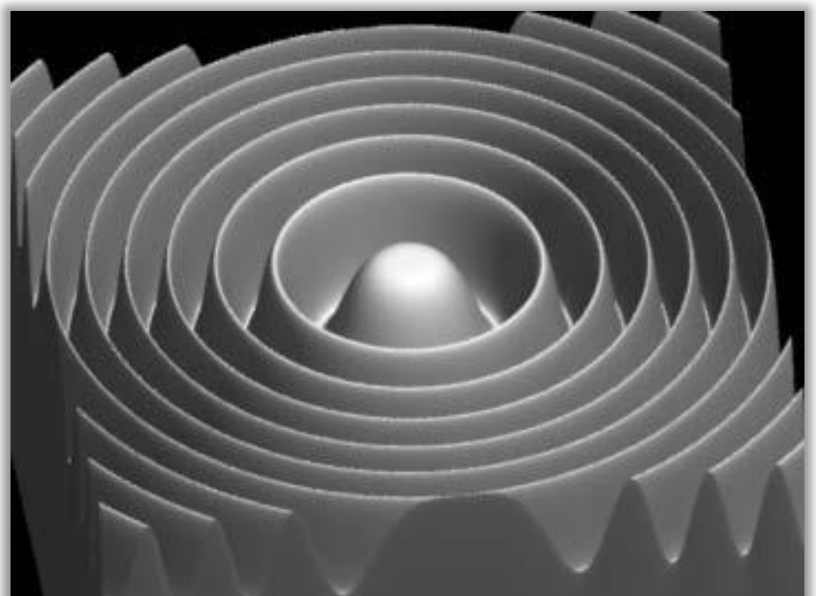
- This is what we want…



…but sometimes this is what we get

Interpolation on grids

- Higher-order reconstruction/interpolation
  - Required if very high quality is needed
  - Usually tested on Marschner-Lobb function
    - High amount of its energy is near its Nyquist frequency
    - Very demanding test for accurate reconstruction



[Marschner & Lobb 94]

High-quality Reconstruction

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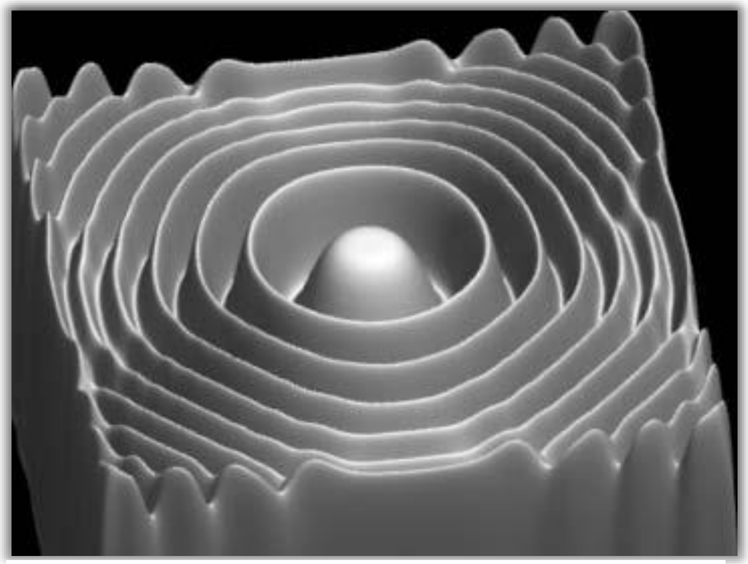
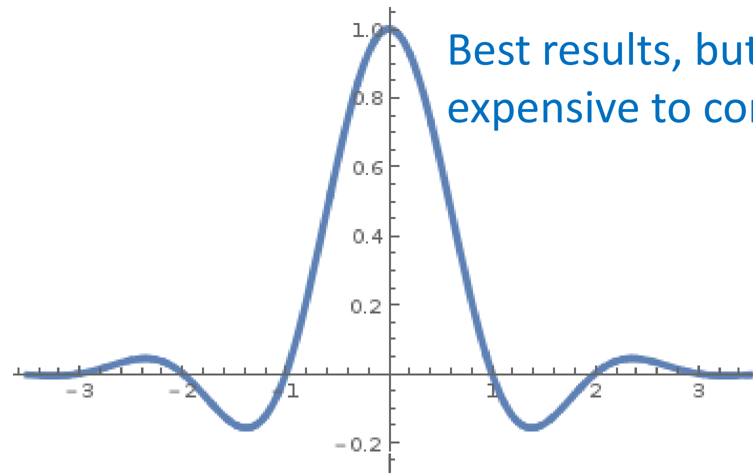
Ingenuity for life

- Sinc function



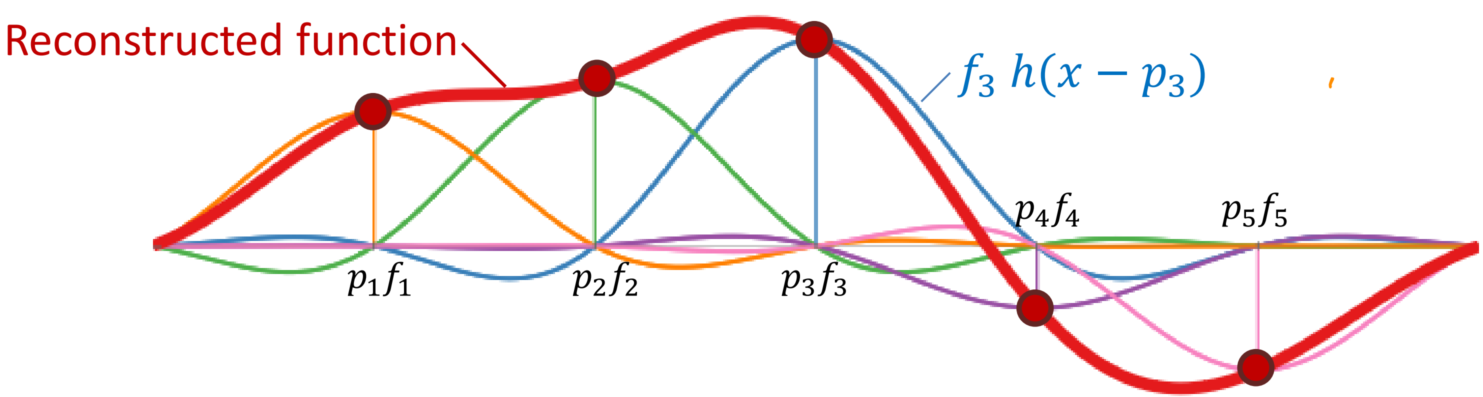
[Marschner & Lobb 94]

Windowed sinc (Hann window)



Best results, but very expensive to compute

"Optimal" reconstruction filter



multivis.net/lecture/interpolation.html

High-quality Reconstruction

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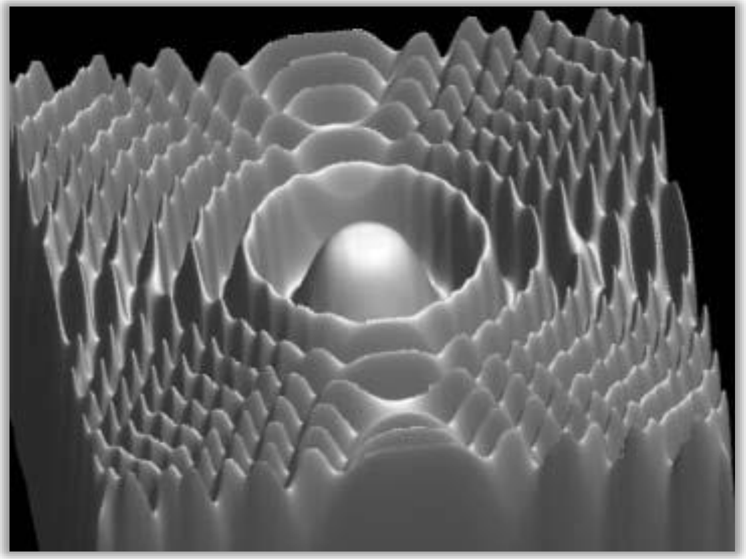
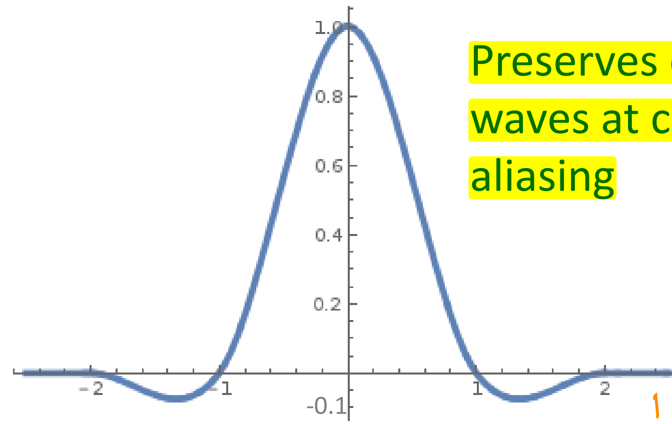
- Bicubic interpolation (Catmull-Rom spline)



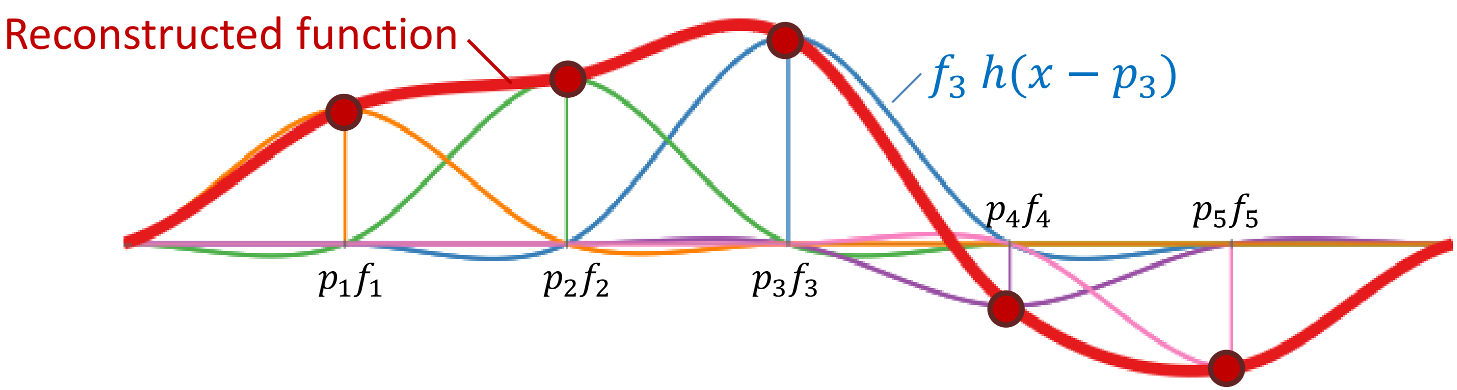
[Marschner & Lobb 94]

Reconstructed Marschner-Lobb function



Preserves depth of waves at cost of aliasing

Interpolation: 1 at center and 0 at integers



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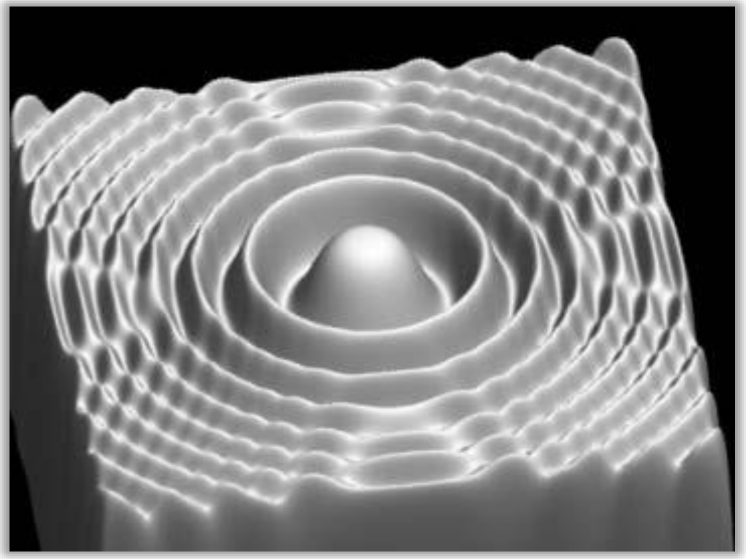
High-quality Reconstruction

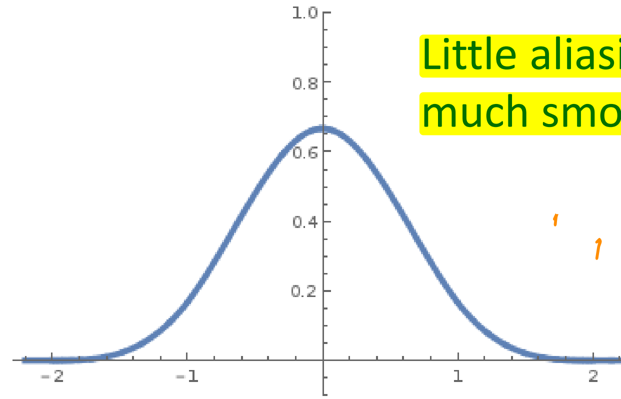
SIEMENS
Ingenuity for life

- Cubic B-spline (with smoothing)



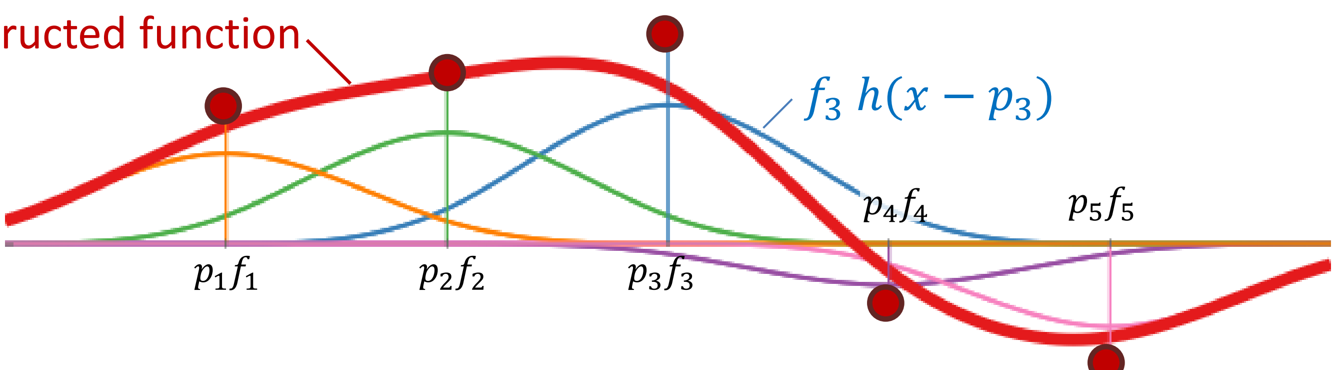
[Marschner & Lobb 94]

Reconstructed Marschner-Lobb function



Little aliasing, but much smoothing

Smoothing: $^2/_3$ at center and $^1/_6$ at ± 1



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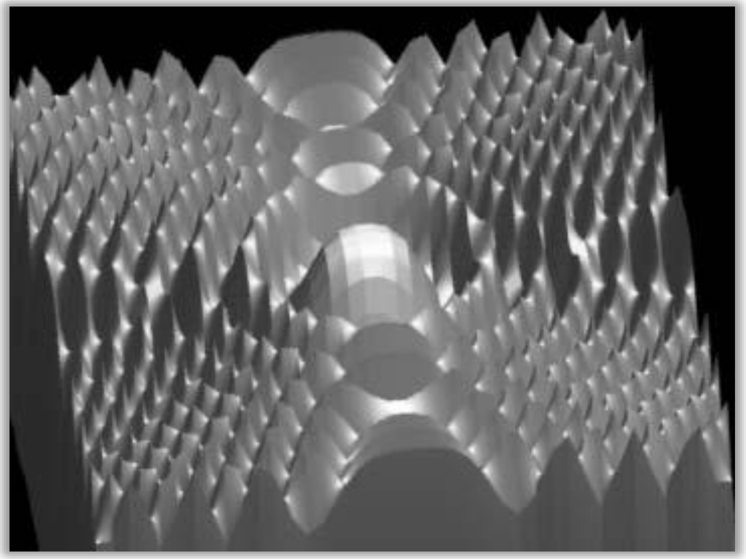
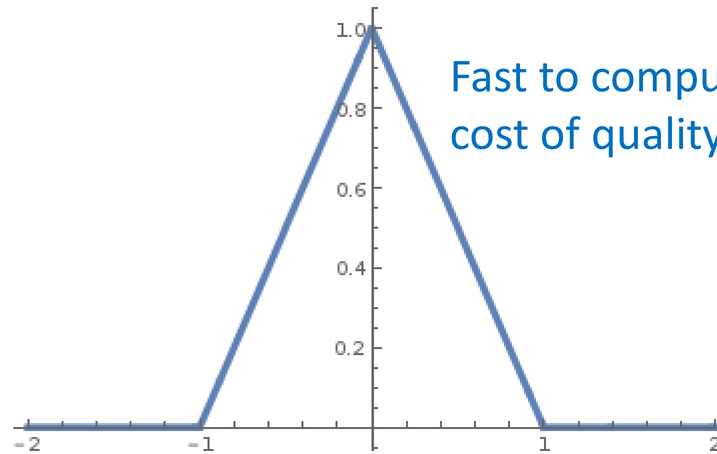
High-quality Reconstruction

- Trilinear interpolation (Tent)



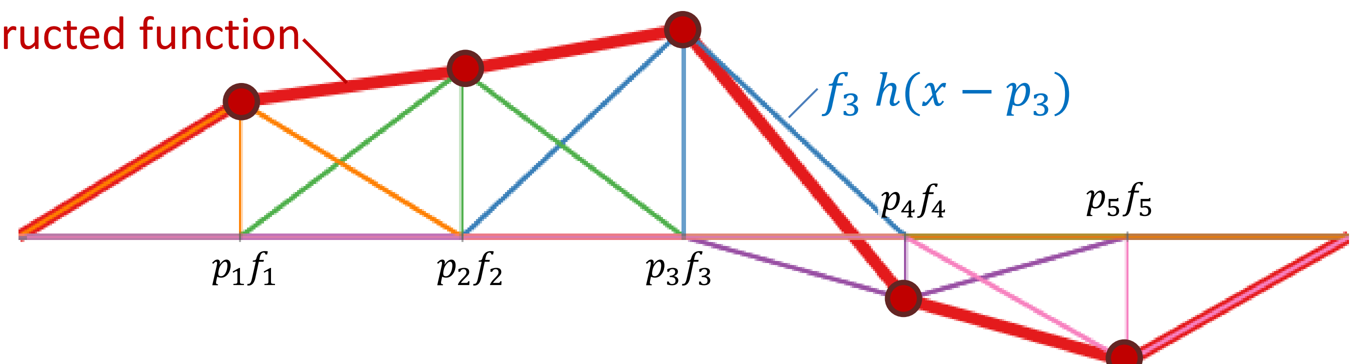
Reconstructed Marschner-Lobb function



Fast to compute at cost of quality

Interpolation: 1 at center and 0 at integers

Reconstructed function



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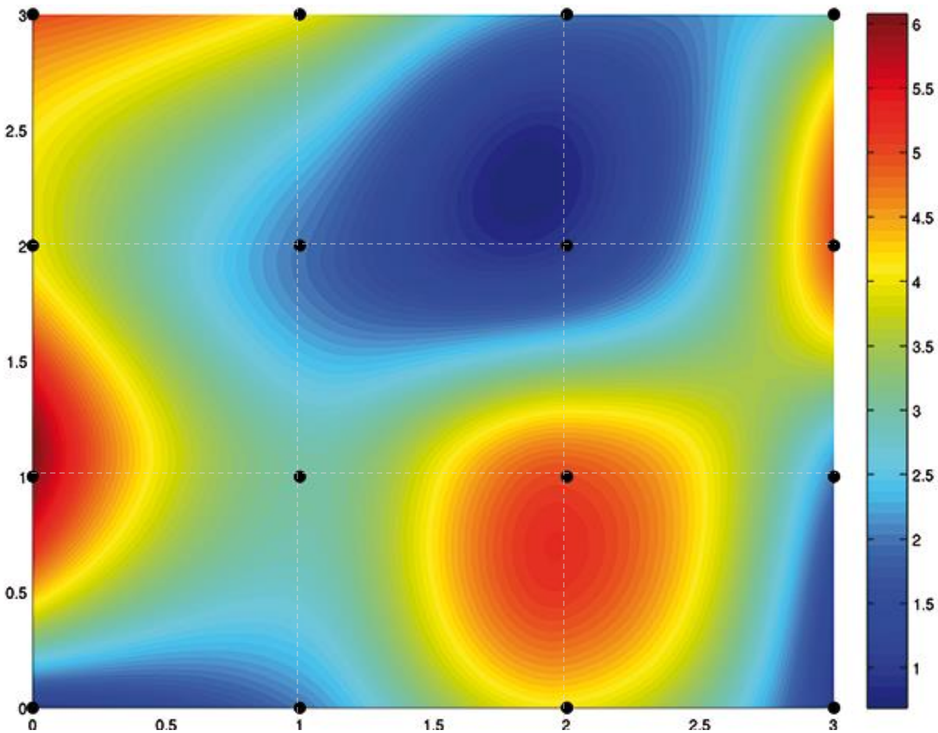
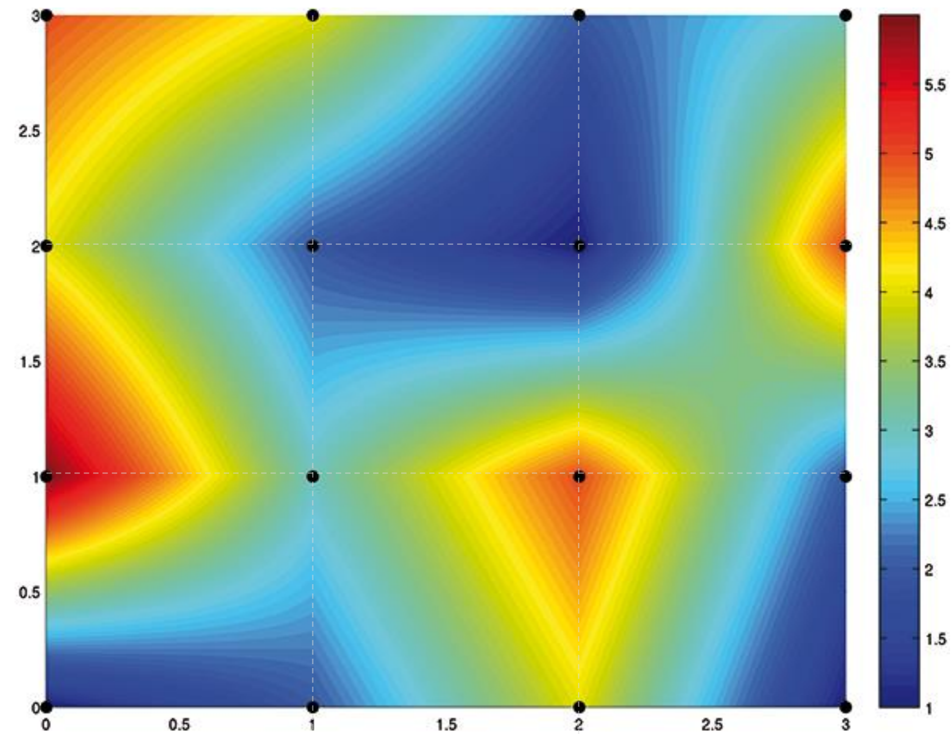
Interpolation on grids

- 2D interpolation with different reconstruction filters





Bicubic interpolation (default in Photoshop)
C^1 continuous → tangents match
First derivative
at segment transition

Second derivative

Cubic B-spline (C^2 continuous
→ also curvature matches)
Eğrilik

Bilinear interpolation
(C^0 continuous → values match
at segment transition)

No derivative

[Wikipedia]

Interpolation on grids

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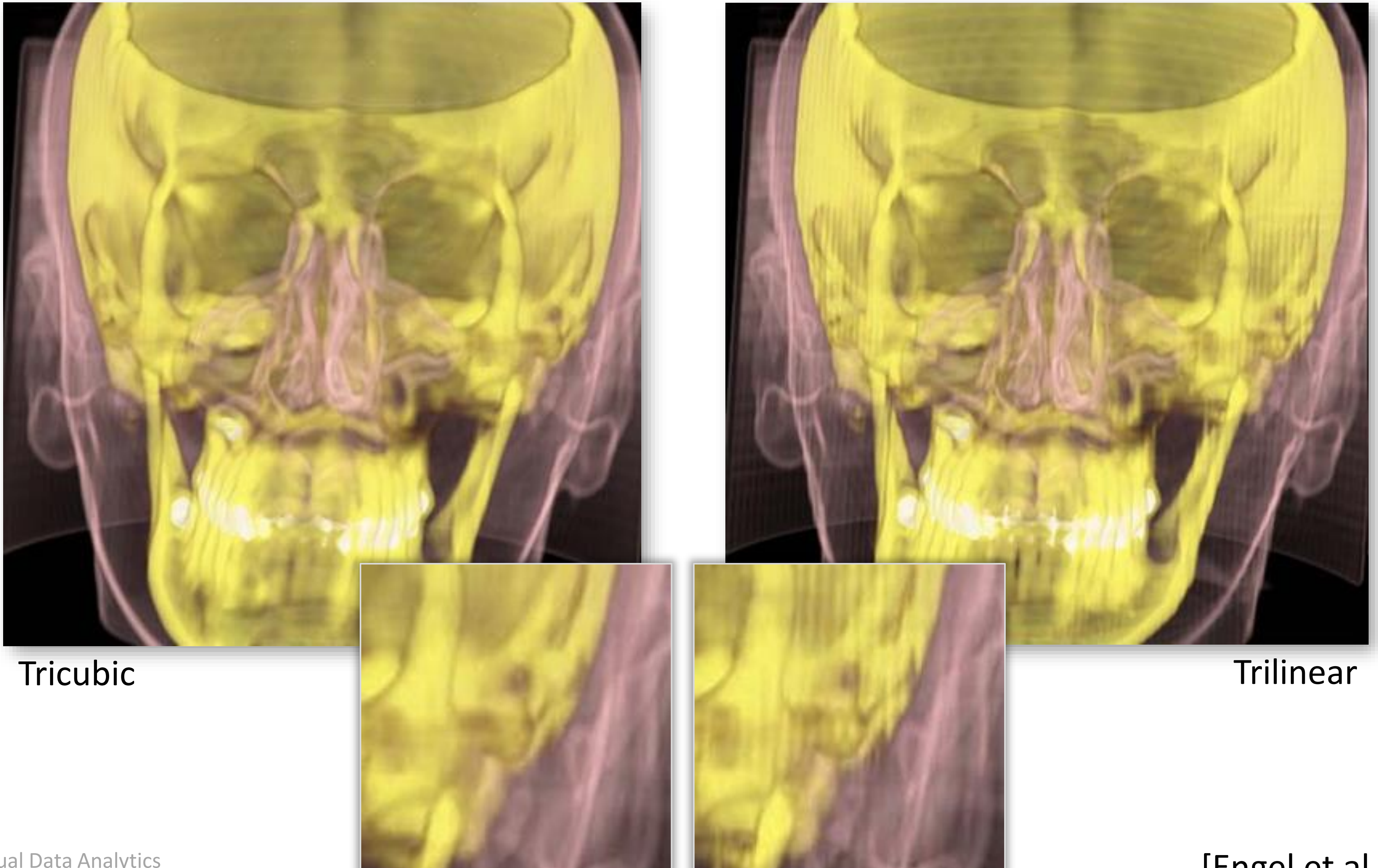
Ingenuity for life

- Volume rendering with different reconstruction filters



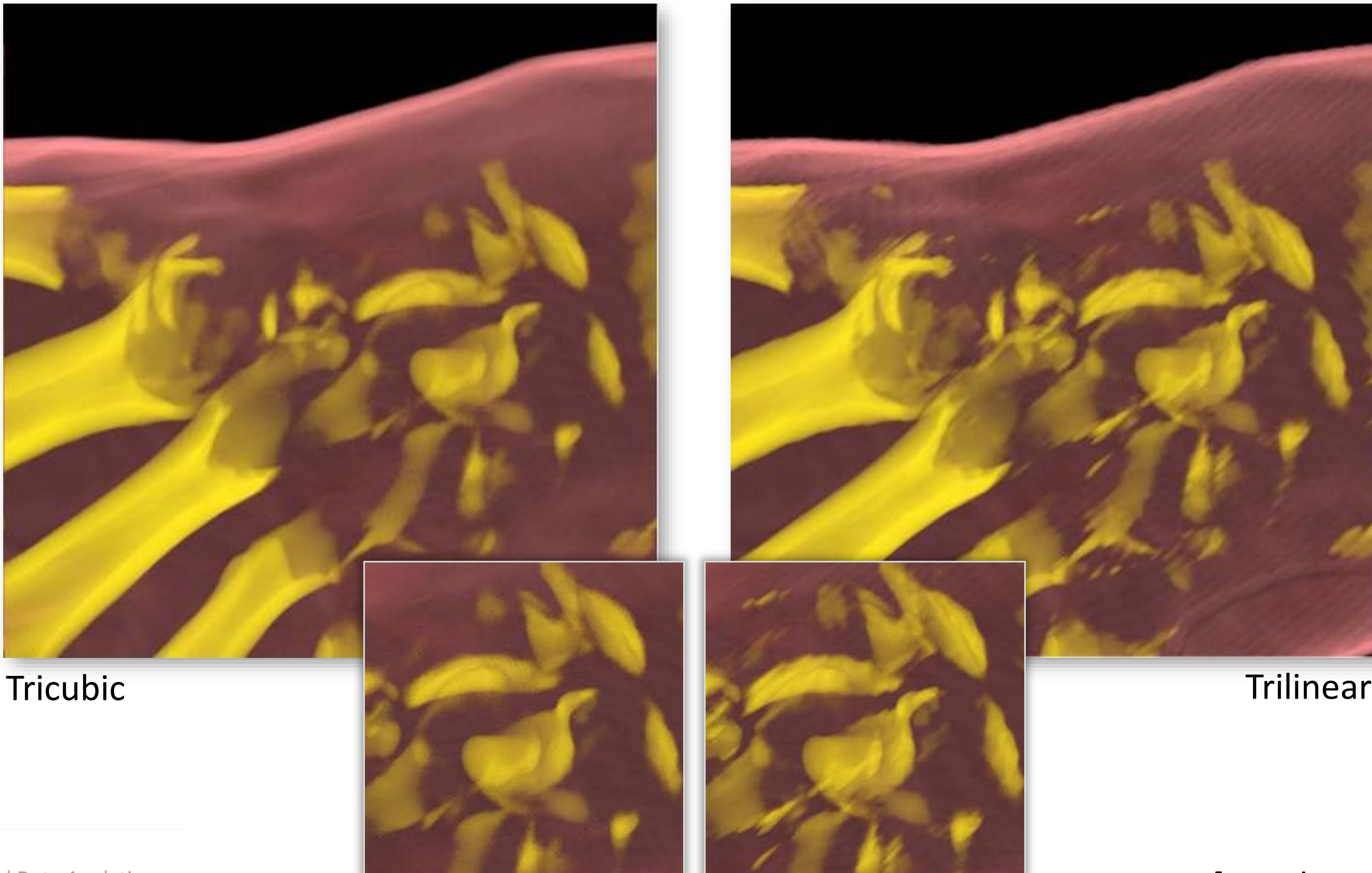
Tricubic

Trilinear

[Engel et al. 06]

Interpolation on grids

- Volume rendering with different reconstruction filters

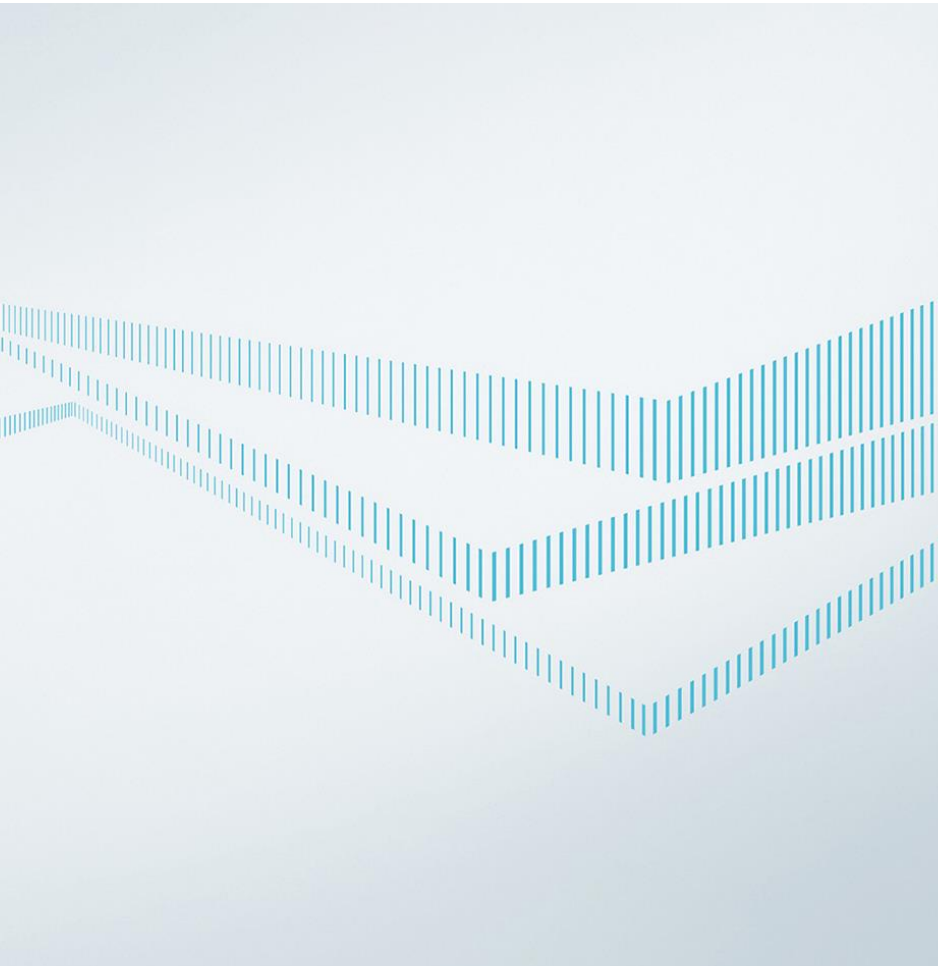


Tricubic

Trilinear

[Engel et al. 06]

Contact information



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