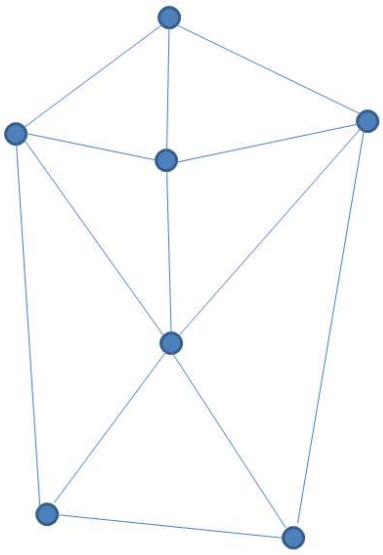
All Sample questions Solutions-pages-3

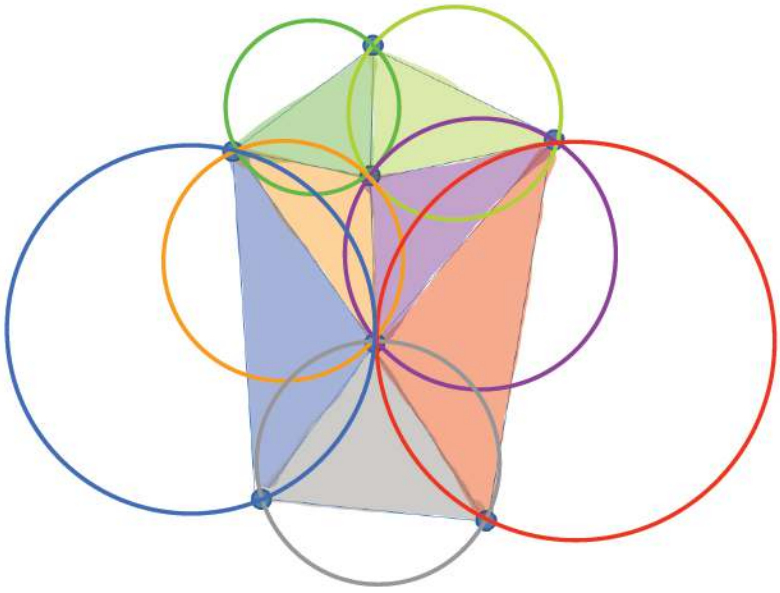
Visual Data Analytics (Technische Universität München)

Lecture 3 – Data Reconstruction and Interpolation Part 1

Data Interpolation

a) For the triangulation shown below, proof or disproof that this triangulation is a **Delaunay triangulation**. Your proof should be geometrically, meaning that you either illustrate the Delaunay property in the figure or illustrate that this property is violated.





None of the circumcircles contain another point than the 3 that define it, therefore the Delaunay property is never violated for this triangulation. Thus, we can say that this triangulation is a Delaunay triangulation.

b) An interpolation function $f(x) = \sum_{i=1}^{N} \varphi(\|p_i - x\|)$ is a weighted sum of N **radial functions** $\varphi(r) = e^{-r^2}$ where $\|p_i - x\|$ is the distance between the points p_i and x. Compute the **weights** w_i such that the function $f(p_i)$ interpolates the data points $p_1 = 1, p_2 = 3, p_3 = 3.5$ with corresponding scalar values $f_1 = 1, f_2 = 0, f_3 = \frac{1}{4}$.

The table below shows approximate values for $\varphi(r)$ with respect to different distances r.

r	0	0.5	1	1.5	2	2.5	3
$\varphi(r)$	1	$\frac{4}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0	0	0

$$W = A^{-1} F \Leftrightarrow \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} \varphi\|p_1-p_1\| & \varphi\|p_2-p_1\| & \varphi\|p_3-p_1\| \\ \varphi\|p_1-p_2\| & \varphi\|p_2-p_2\| & \varphi\|p_3-p_2\| \\ \varphi\|p_1-p_3\| & \varphi\|p_2-p_3\| & \varphi\|p_3-p_3\| \end{pmatrix}^{-1} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} \varphi(0) & \varphi(2) & \varphi(2.5) \\ \varphi(2) & \varphi(0) & \varphi(0.5) \\ \varphi(2.5) & \varphi(0.5) & \varphi(0) \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 1/4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 4/5 \\ 0 & 4/5 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 1/4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{25}{9} & -\frac{20}{9} \\ 0 & -\frac{20}{9} & \frac{25}{9} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{5}{9} \\ \frac{25}{36} \end{pmatrix}$$

$p_1 = 1,\ p_2 = 3,\ p_3 = 3.5$

$$\therefore w_1 = 1, w_2 = -\frac{5}{9} \approx -0.556, w_3 = \frac{25}{36} \approx 0.694$$

c) Given the **interpolation function** $f(x)$ from the previous assignment, compute the interpolated value at point x=2.

$$f(x) = \sum_{i=1}^{N} w_i \varphi(\|p_i - x\|)$$

$$\therefore f(2) = w_1 \varphi(\|p_1 - 2\|) + w_2 \varphi(\|p_2 - 2\|) + w_3 \varphi(\|p_3 - 2\|) = 1\,\varphi(1) - \frac{5}{9}\varphi(1) + \frac{25}{36}\varphi(1.5) = 1 \cdot \frac{2}{5} - \frac{5}{9} \cdot \frac{2}{5} + \frac{25}{36} \cdot \frac{1}{10} = \frac{89}{360}$$

$$f(x = 2) = \frac{89}{360} \approx 0.247$$