
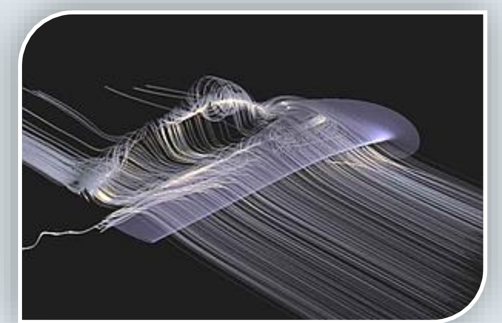

Visual Data Analytics
Flow Visualization II

Dr. Johannes Kehrer

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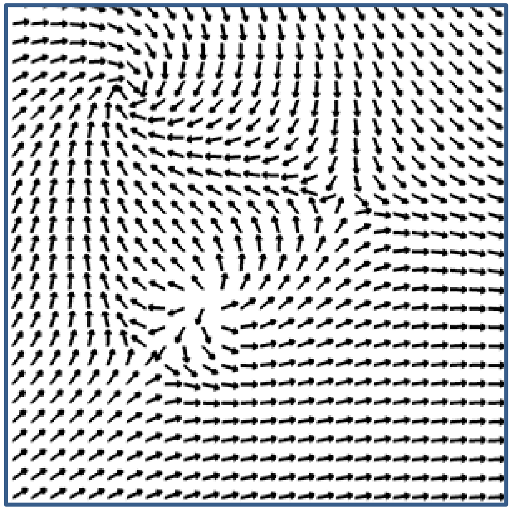
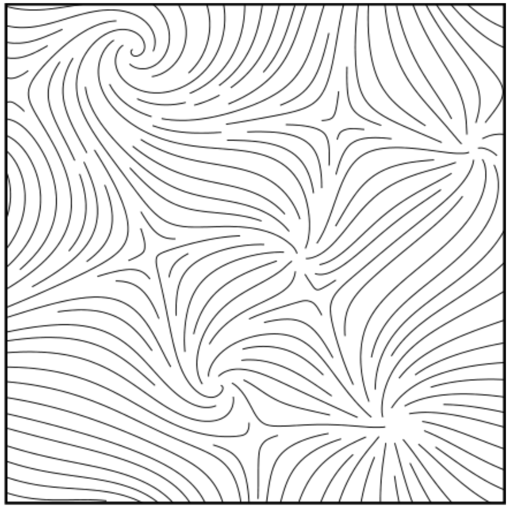
Flow visualization – Approaches



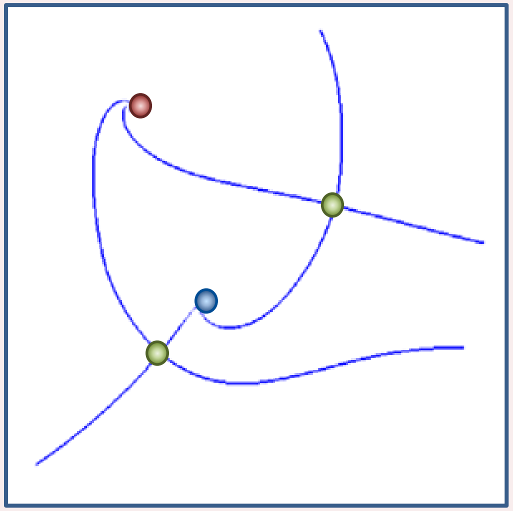
Direct flow visualization (arrows, color coding, …)

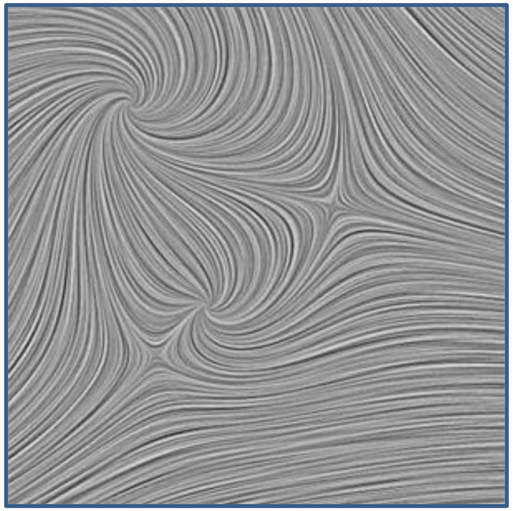


Geometric flow vis. (stream lines/surfaces, …)

Sparse (feature-based) vis.

- Global computation of flow features
- Vortices, shockwaves, vector field topology





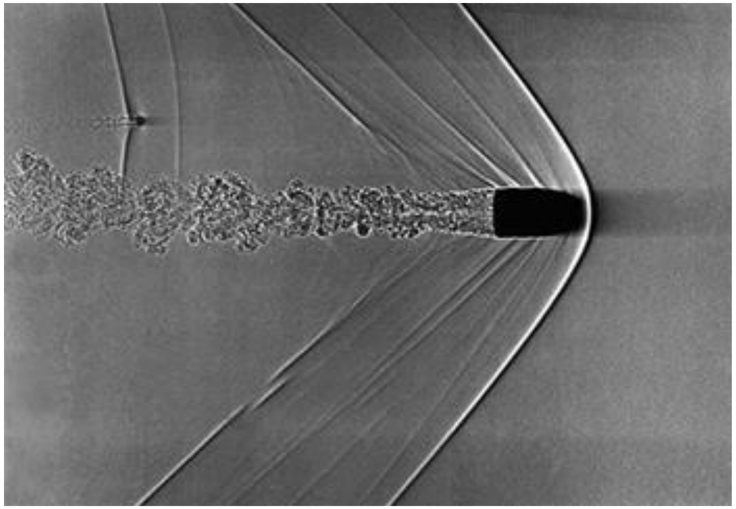
Dense (texture-based)

Flow features

SIEMENS
Ingenuity for life

- Vortices
  - One of most prominent features
  - Important in many applications (turbulent flows)
  - No formal, well accepted definition yet ("something swirling")

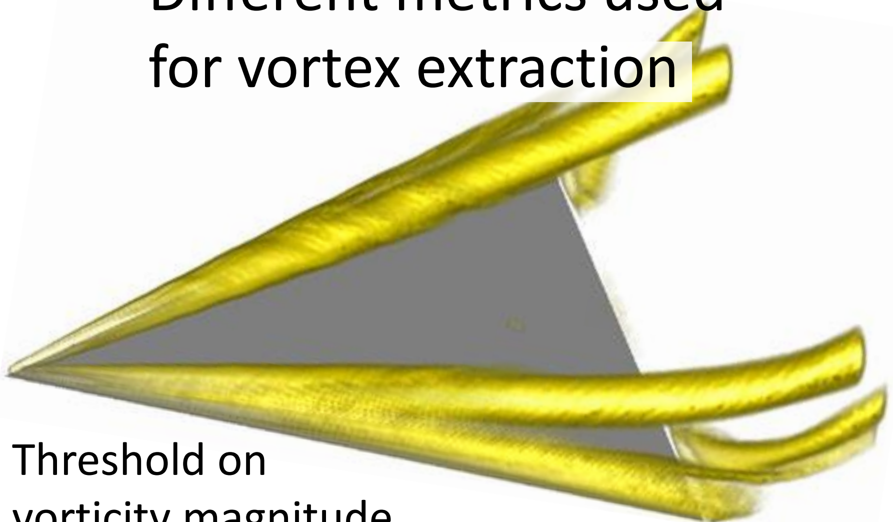
- Shock waves
  - Characterized by sharp discontinuities in flow attributes (pressure, velocity magnitude, …)



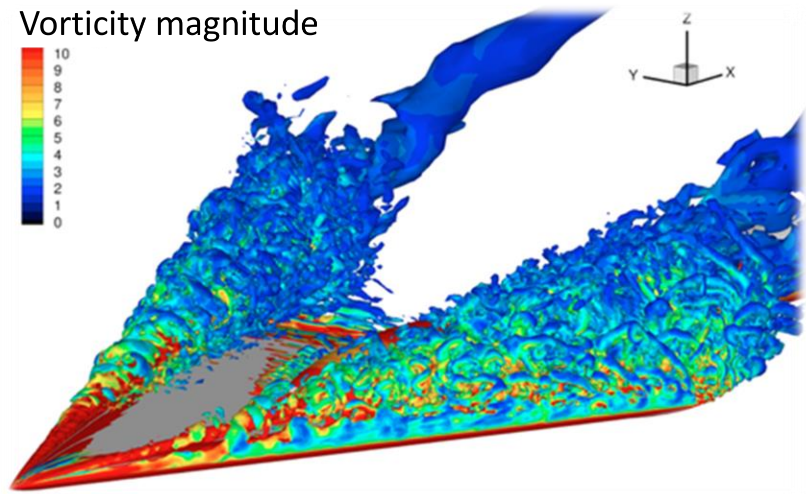
Bullet traveling through air at about 1.5 times sound speed

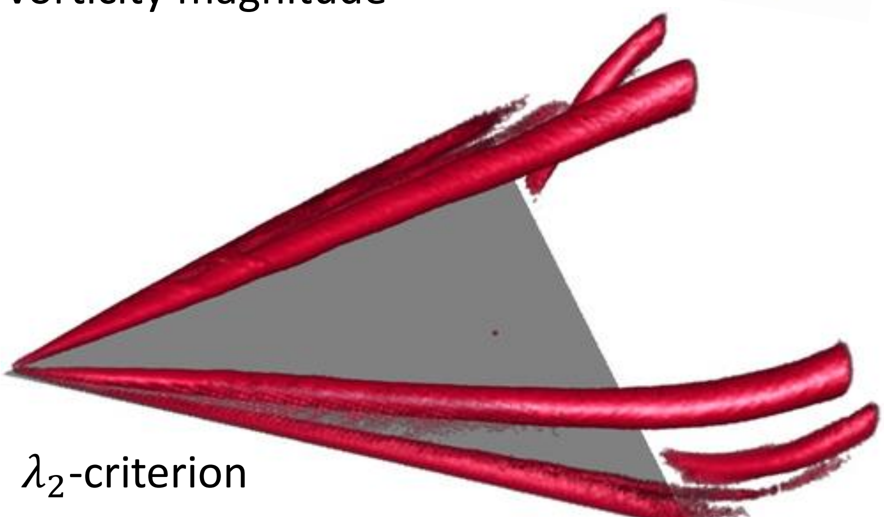
Flow features

- Vortices around delta wing
  - Different metrics used for vortex extraction

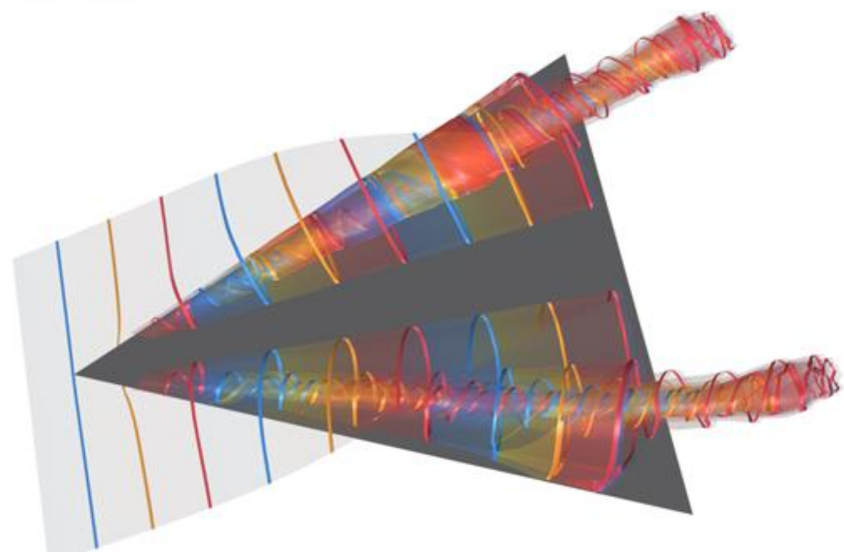


Threshold on vorticity magnitude





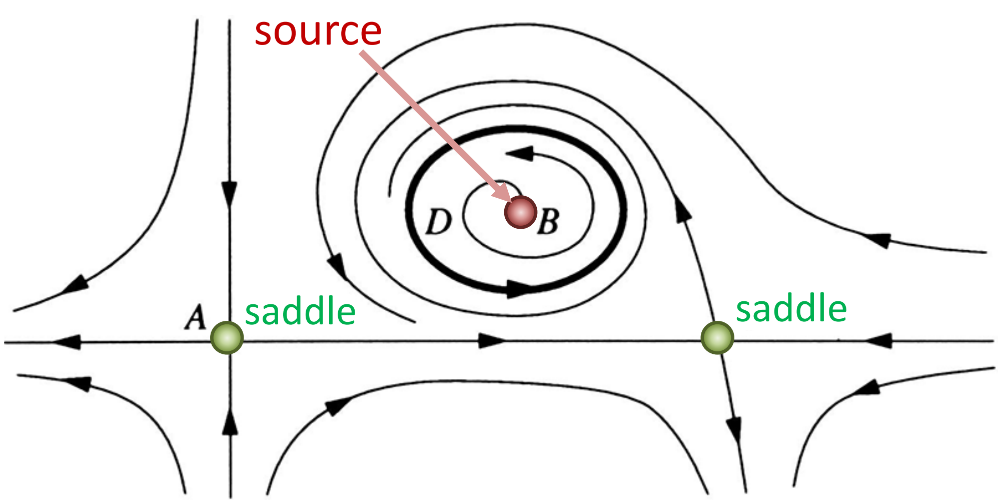
λ_2-criterion

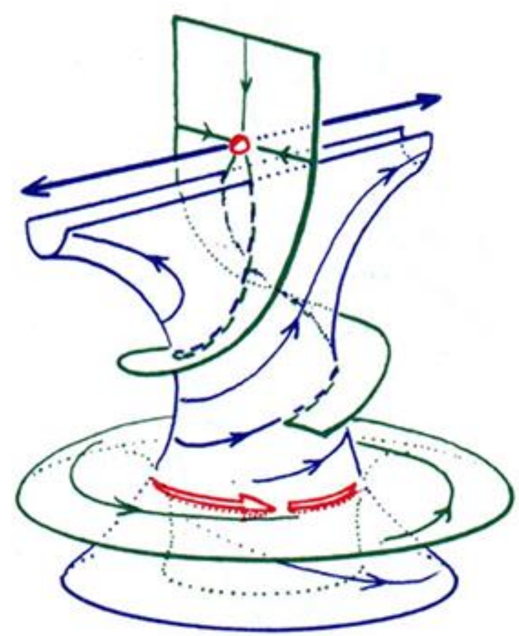


Vector field topology

SIEMENS
Ingenuity for life

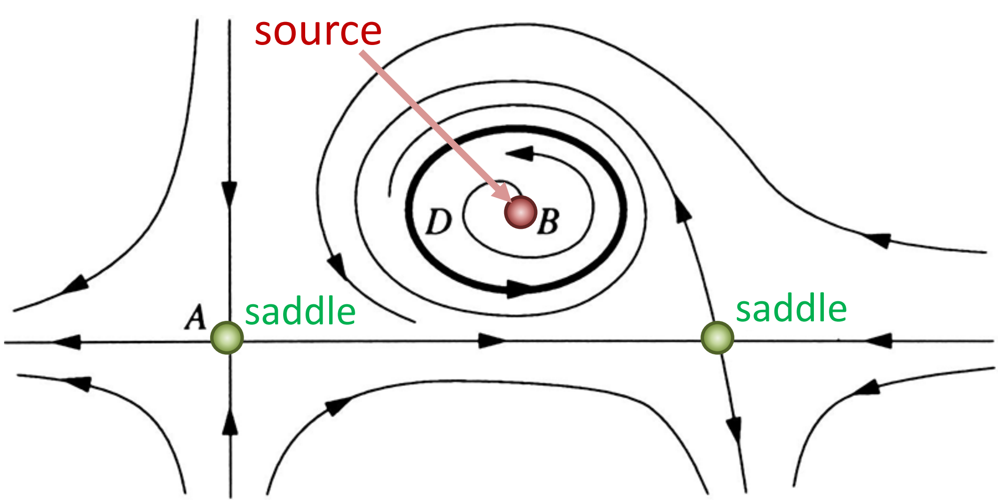
- Idea: Do not draw "all" stream lines, but only the "important" ones
- Show only topological skeleton
  - Connection of critical points
  - Characterization of global flow structures

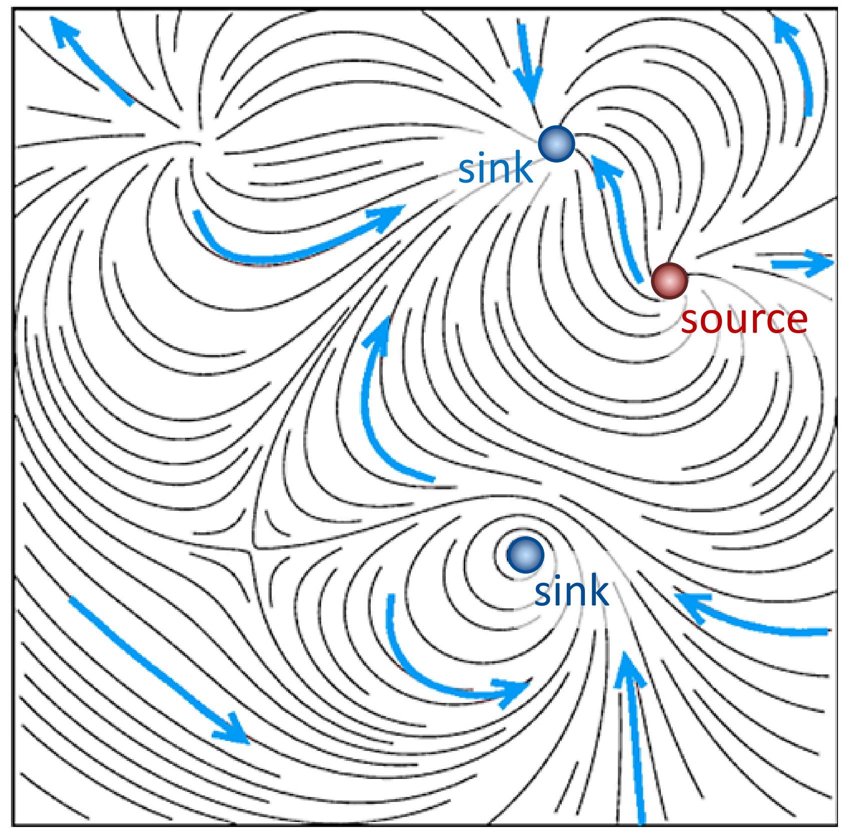




Vector field topology

- Critical points: singularities in vector field such that $\boldsymbol{v}(\boldsymbol{x}^*) = 0$
  - Points where magnitude of vector goes to zero and direction of vector is undefined
  - Stream lines reduced to single point

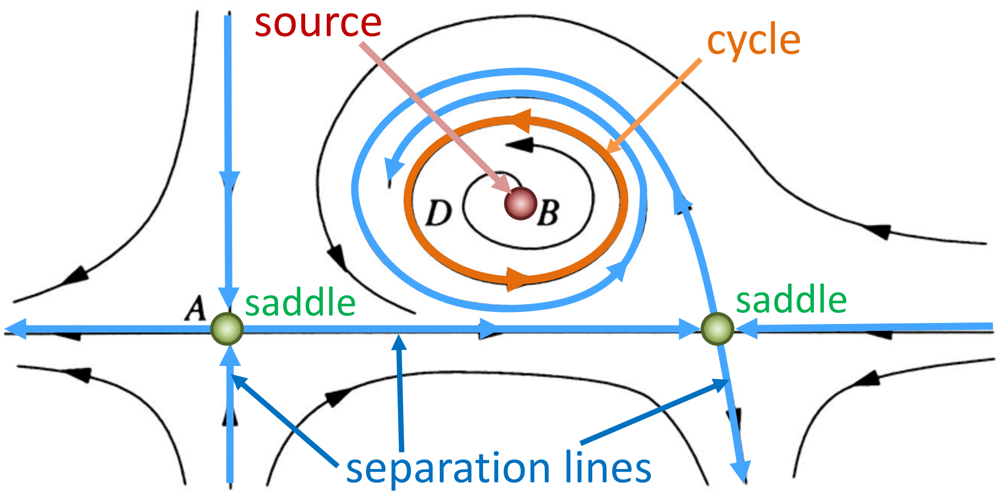


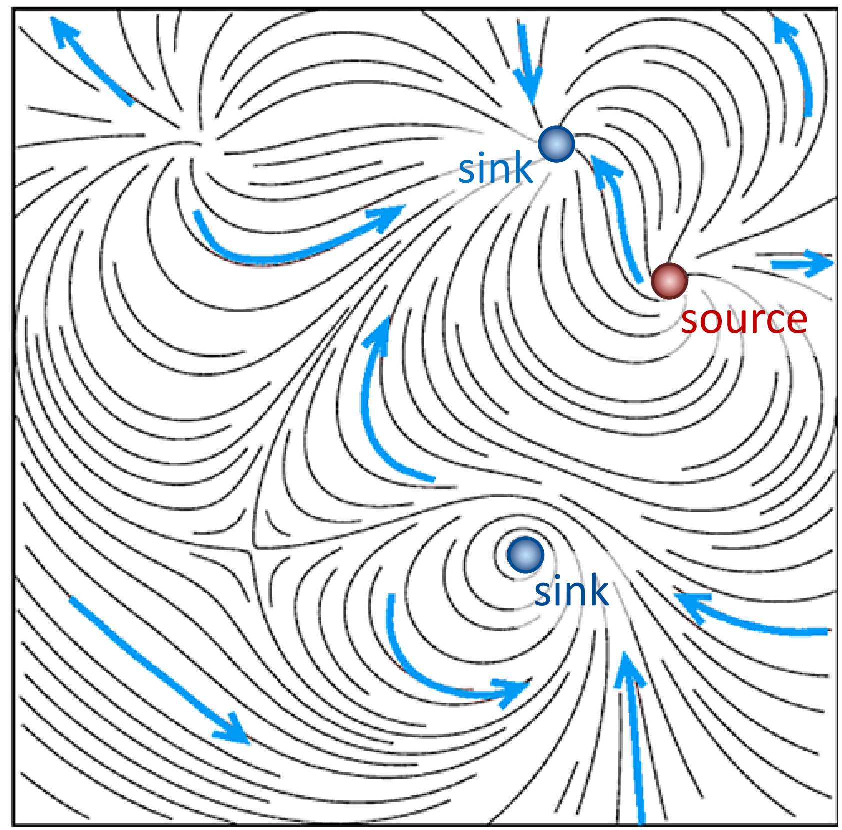


Vector field topology

SIEMENS
Ingenuity for life

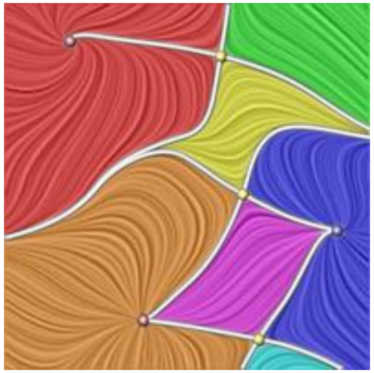
- Critical points: singularities in vector field such that $\boldsymbol{v}(\boldsymbol{x}^*) = 0$
  - Points where magnitude of vector goes to zero and direction of vector is undefined
  - Stream lines reduced to single point
  - Type of critical point determines flow pattern around it

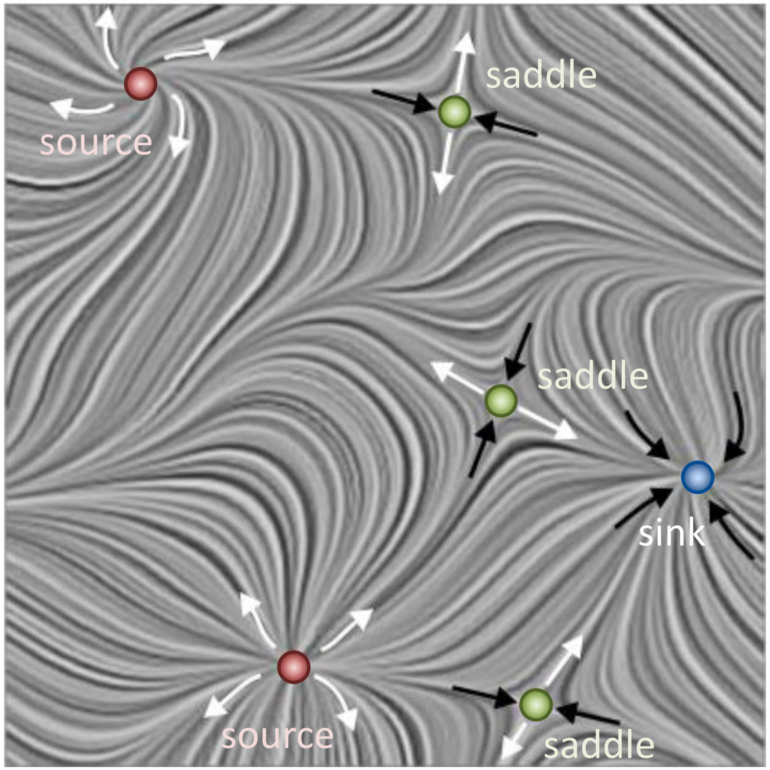




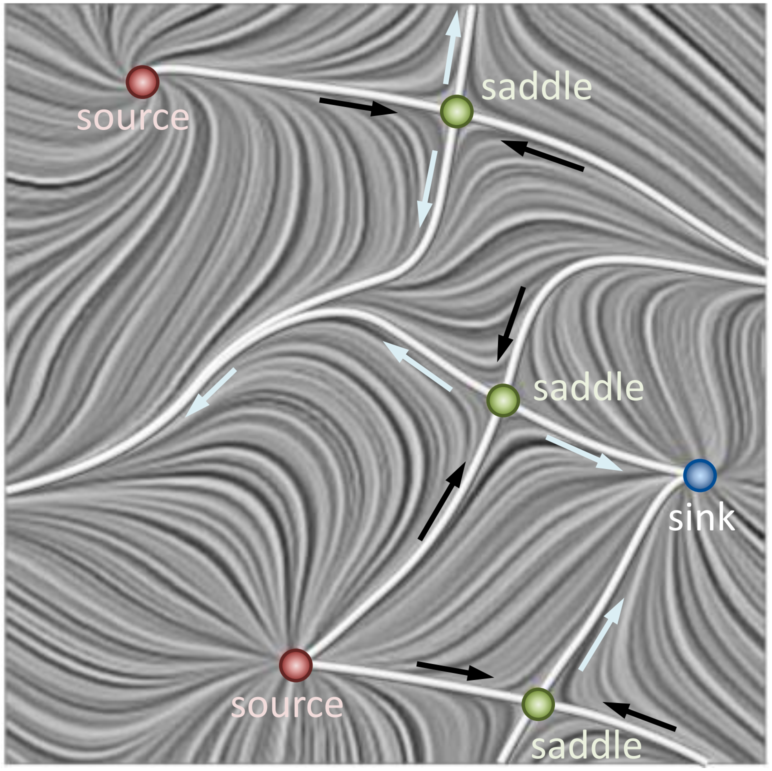
Vector field topology

- Topological skeleton / graph
  - Nodes: critical points
  - Edges: separation lines and cycles
  - Flow divided into regions with similar properties





Critical points



Critical points + separation lines

[Weinkauf]

Vector field topology (2D)

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Ingenuity for life

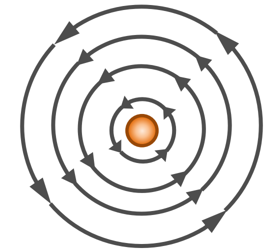
- How to find critical points $\boldsymbol{x}^*$?
  - Points where $\boldsymbol{v}(\boldsymbol{x}^*) = 0$
- How to classify critical points $\boldsymbol{x}^*$?
  - Jacobian matrix $\mathbf{J} = \begin{pmatrix} \frac{\partial}{\partial x}\boldsymbol{v}_x & \frac{\partial}{\partial y}\boldsymbol{v}_x \\ \frac{\partial}{\partial x}\boldsymbol{v}_y & \frac{\partial}{\partial y}\boldsymbol{v}_y \end{pmatrix}$ governs the behavior near $\boldsymbol{x}^*$
  - For each $\boldsymbol{x}^*$, calculate eigenvalues λ_1, λ_2 of $\mathbf{J}$

$$\mathbf{J}\boldsymbol{u} = \lambda\boldsymbol{u}$$

$\mathbf{J}$ ... Jacobian matrix
$\boldsymbol{u}$ ... eigenvector (non-zero)
λ ... eigenvalue

Vector field topology (2D)

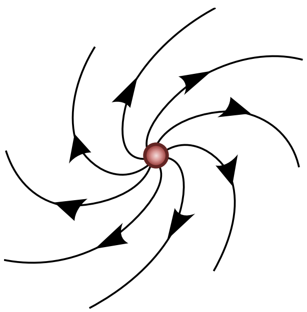
- Classify critical points by eigenvalue analysis of $\mathbf{J}$
  - λ_1, λ_2 both positive → local repulsion (source)
  - λ_1, λ_2 both negative → local attraction (sink)
  - $\lambda_1 \lambda_2 < 0$ → saddle point
  - λ_1, λ_2 both complex → rotation around $\boldsymbol{x}^*$



Center

$\mathrm{Im}(\lambda_{1,2}) \neq 0$

$\mathrm{Re}(\lambda_{1,2}) = 0$

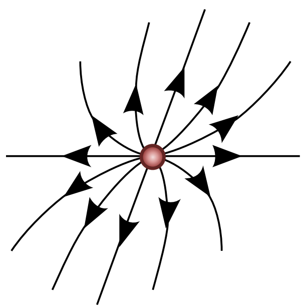


Circulating source
(repelling focus)

$\mathrm{Im}(\lambda_{1,2}) \neq 0$

$\mathrm{Re}(\lambda_{1,2}) > 0$

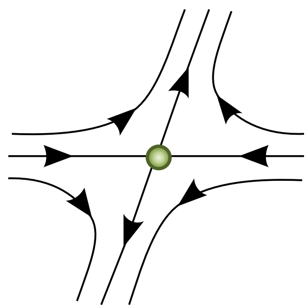
$\lambda_{1,2} = a \pm bi$



Noncirculating source
(repelling node)

$\mathrm{Im}(\lambda_{1,2}) = 0$

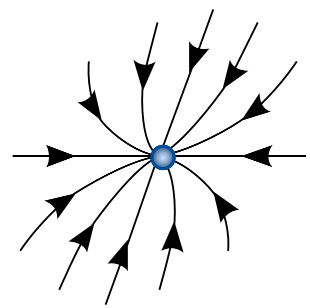
$\mathrm{Re}(\lambda_{1,2}) > 0$



Saddle point

$\mathrm{Im}(\lambda_{1,2}) = 0$

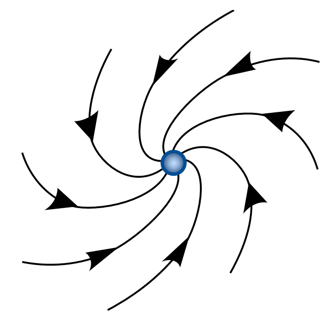
$\lambda_1 \lambda_2 < 0$



Noncirculating sink
(attracting node)

$\mathrm{Im}(\lambda_{1,2}) = 0$

$\mathrm{Re}(\lambda_{1,2}) < 0$



Circulating sink
(attracting focus)

$\mathrm{Im}(\lambda_{1,2}) \neq 0$

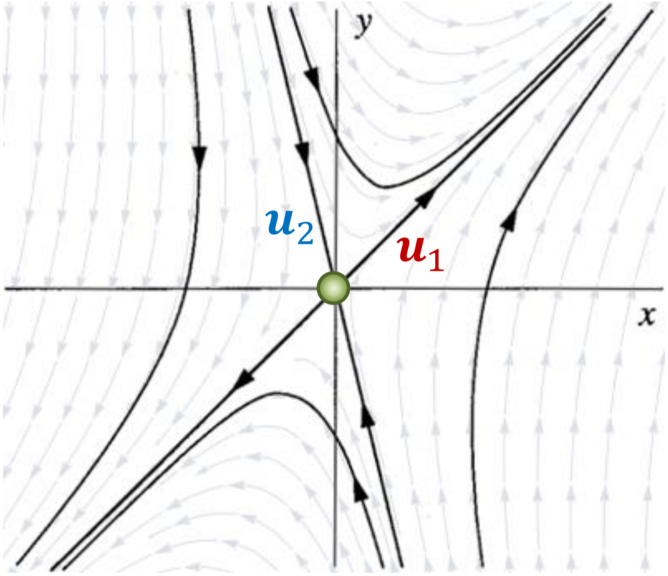
$\mathrm{Re}(\lambda_{1,2}) < 0$

$\lambda_{1,2} = a \pm bi$

Vector field topology (2D)

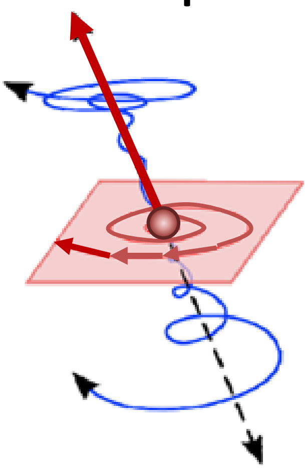
SIEMENS
Ingenuity for life

- Mapping to graphical primitives: streamlines
  - Start streamlines close to critical points
  - Initial direction along the eigenvectors $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ (forward+backward integration)
- End particle tracing at
  - Other "real" critical points
  - Interior boundaries
  - Boundaries of computational domain



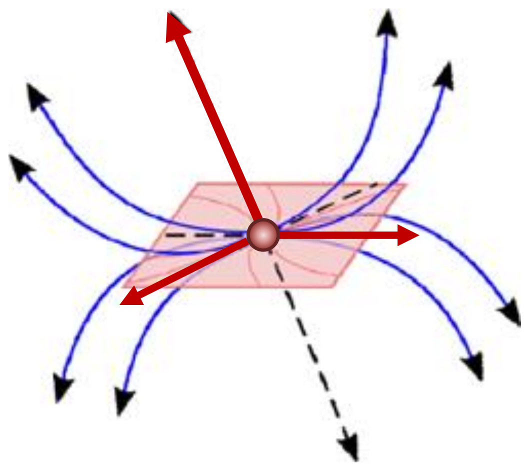
Vector field topology (3D)

- Critical points in 3D
  - More complicated
  - Line and surface separatices exist
- Examples



Spiral source

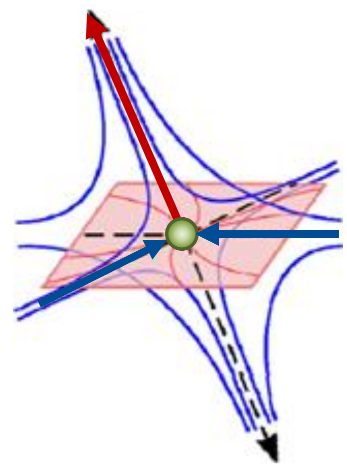
$\mathrm{Im}(\lambda_1) = 0,$
$\mathrm{Im}(\lambda_{2,3}) \neq 0$
$\mathrm{Re}(\lambda_{1,2,3}) > 0$



Noncirculating source

$\mathrm{Im}(\lambda_{1,2,3}) = 0$

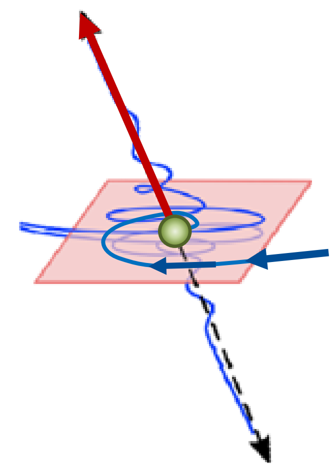
$\mathrm{Re}(\lambda_{1,2,3}) > 0$



Noncirculating saddle

$\mathrm{Im}(\lambda_{1,2,3}) = 0$

$\mathrm{Re}(\lambda_1) > 0,$
$\mathrm{Re}(\lambda_{2,3}) < 0$



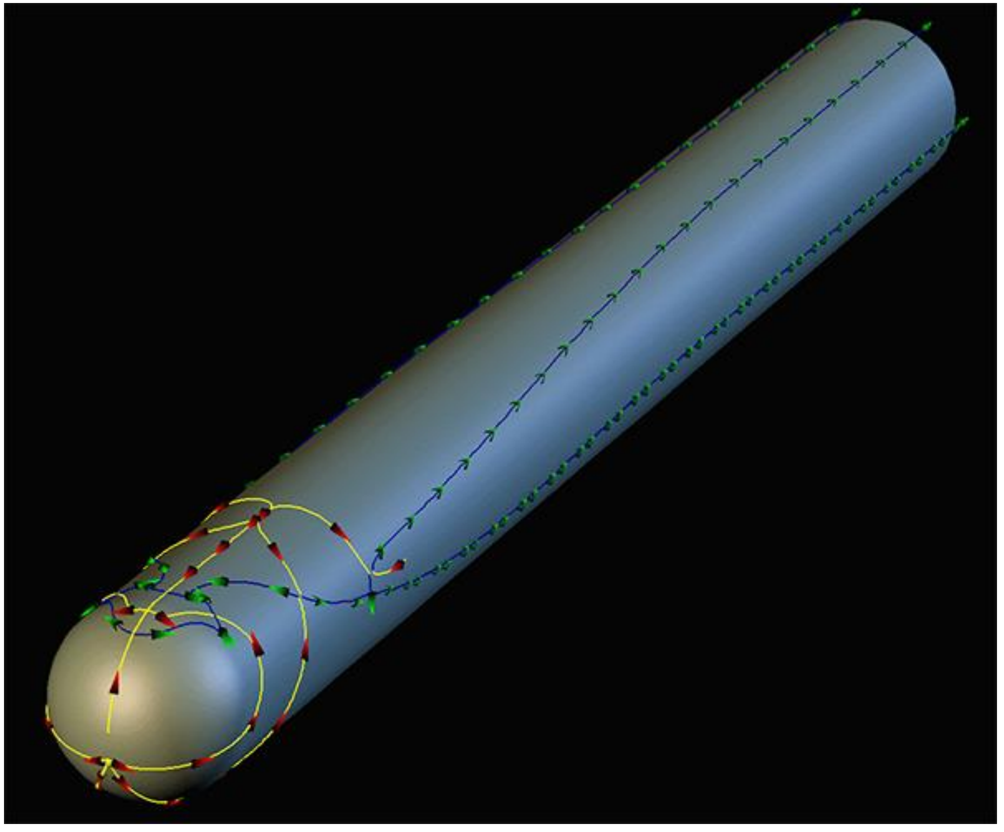
Spiral saddle

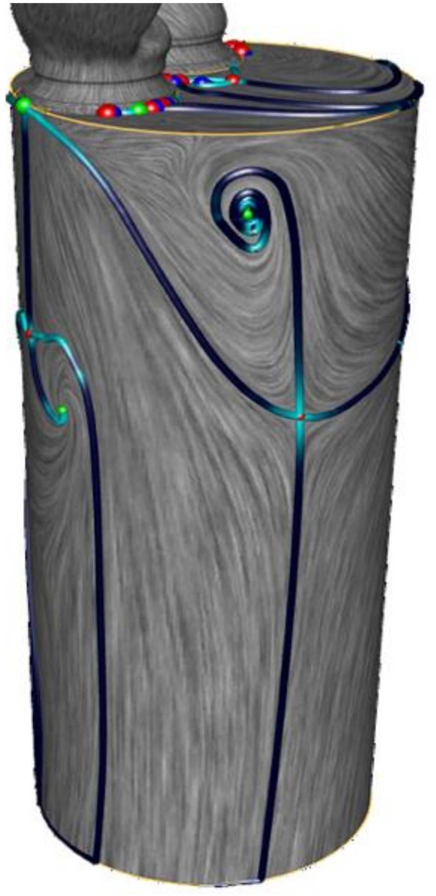
$\mathrm{Im}(\lambda_1) = 0,$
$\mathrm{Im}(\lambda_{2,3}) \neq 0$
$\mathrm{Re}(\lambda_1) > 0,$
$\mathrm{Re}(\lambda_{2,3}) < 0$

[Peikert & Sadlo 10]

Vector field topology (3D)

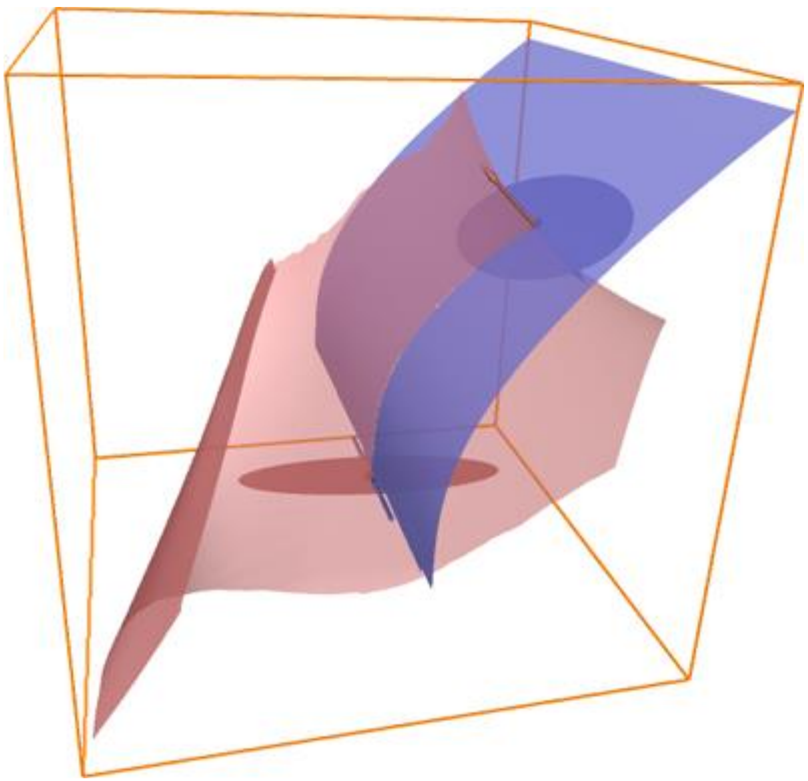
- Topology on surfaces
  - Critical points + separation lines applied to projections of vector field onto polygonal surface



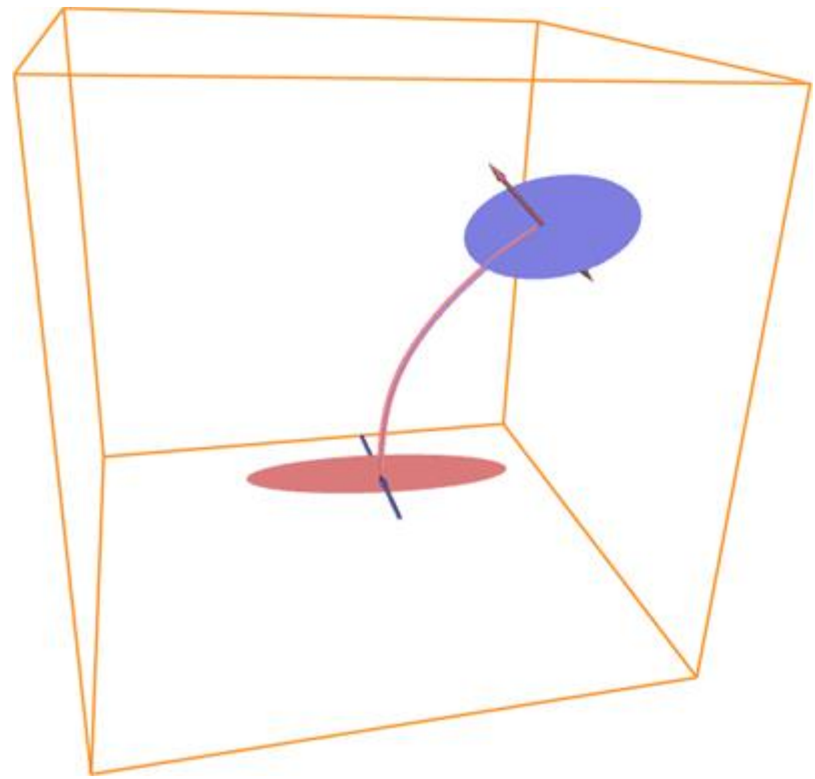


Vector field topology (3D)

- Saddle connectors in 3D



Separation surfaces of two saddles

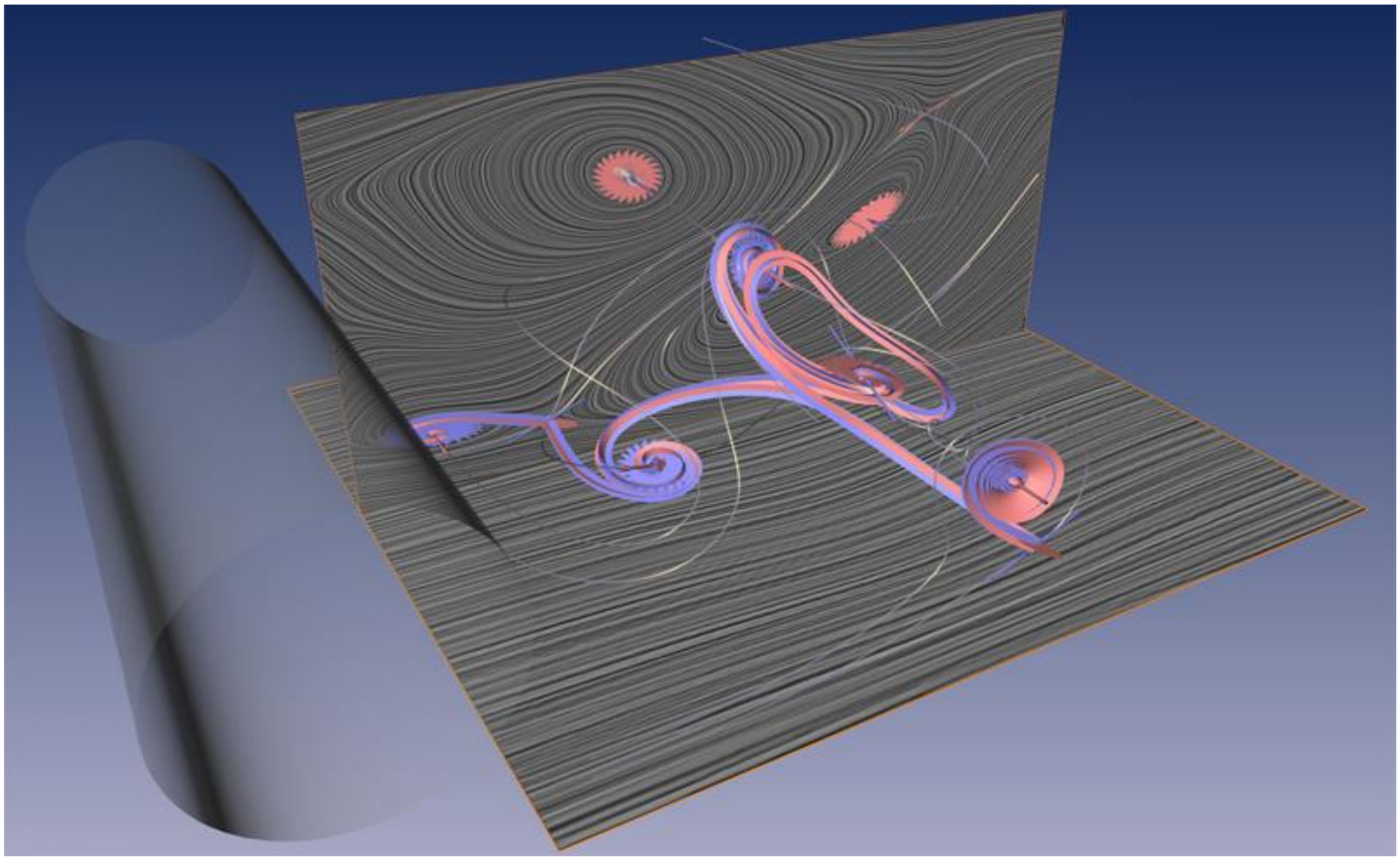


The intersection of the separation surfaces is the saddle connector

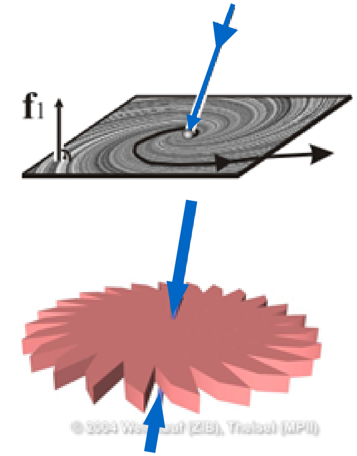
[Weinkauf & Theisel 05]

Vector field topology (3D)

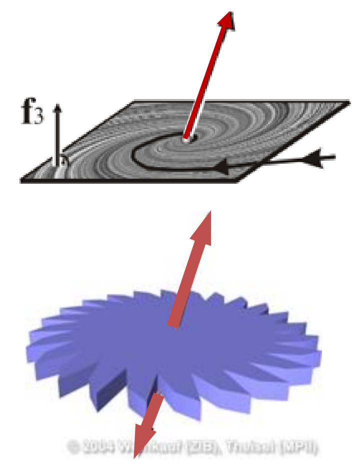
- Saddle connectors in 3D



[Weinkauf & Theisel 05]



Repelling spiral saddle

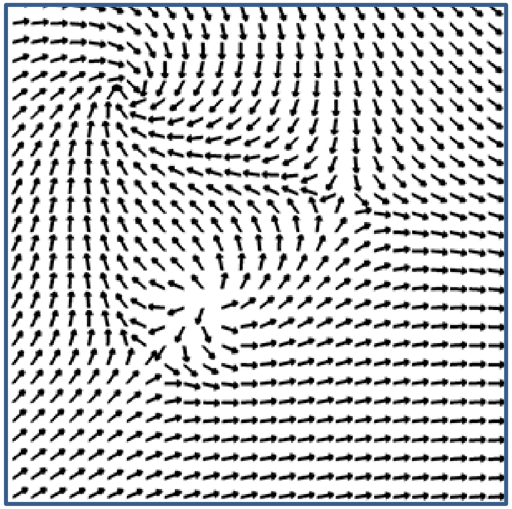


Attracting spiral saddle

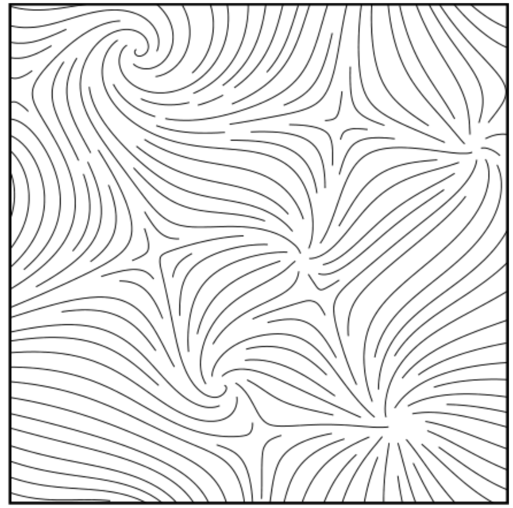
Vector field topology

- Summary
  - Draw only relevant stream lines (topological skeleton)
  - Partition domain into regions with similar flow features
  - Based on critical points
  - Good for 2D steady flows
  - Unsteady flows?
  - 3D?

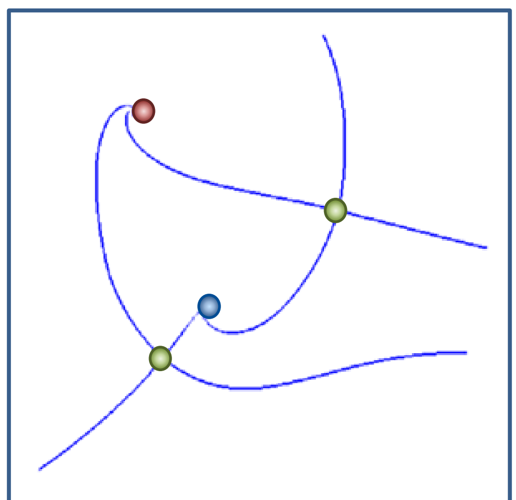
Flow visualization – Approaches



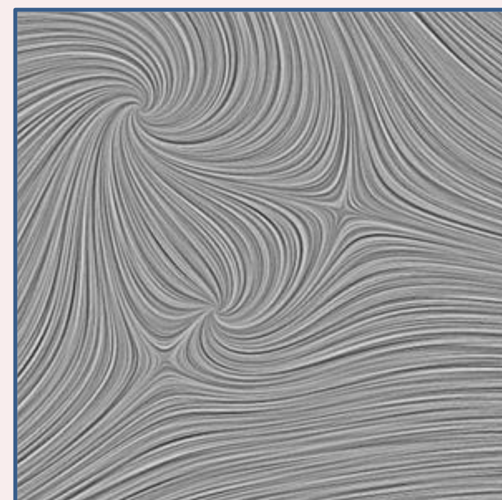
Direct flow visualization
(arrows, color coding, …)



Geometric flow vis.
(stream lines/surfaces, …)



Sparse (feature-based) vis.

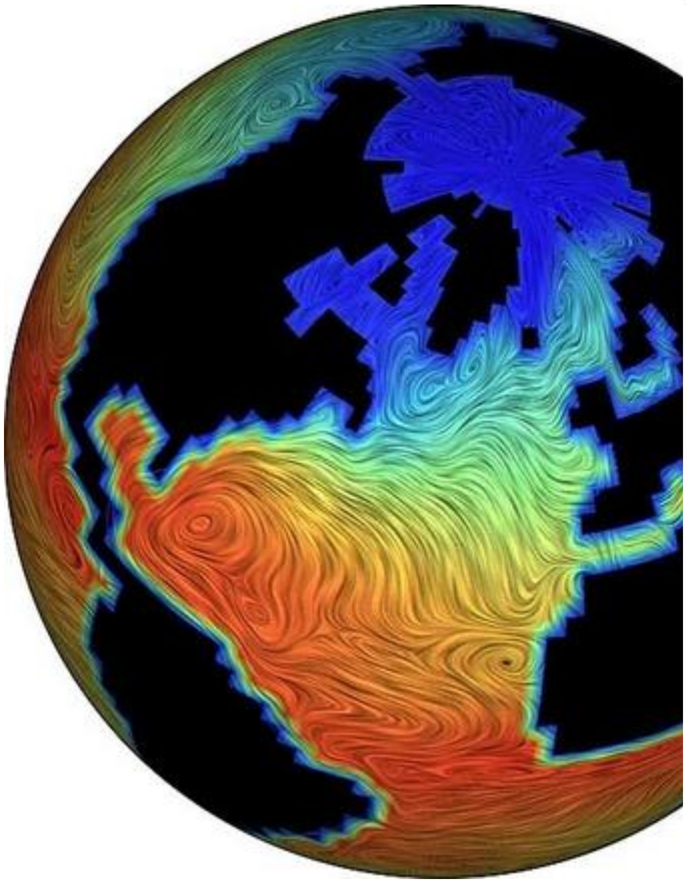
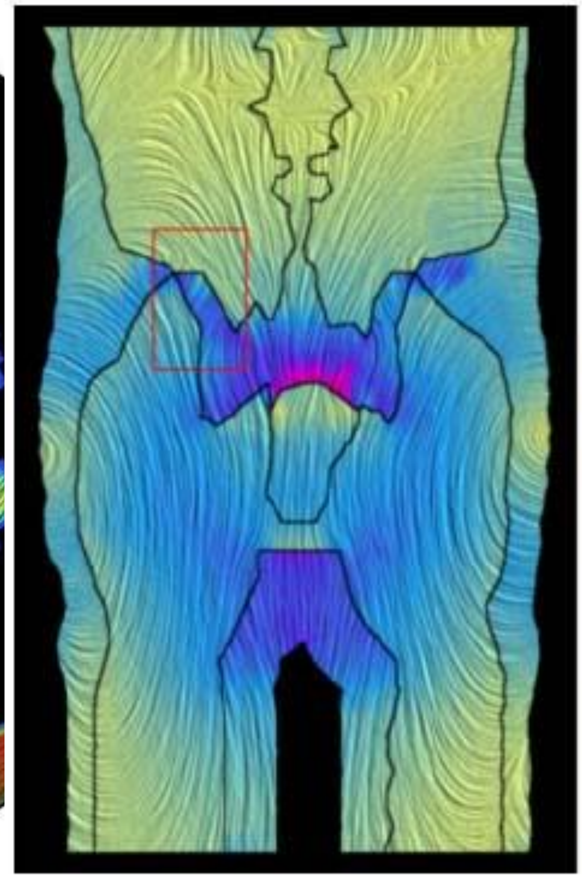


Dense (texture-based)

Texture-based flow visualization

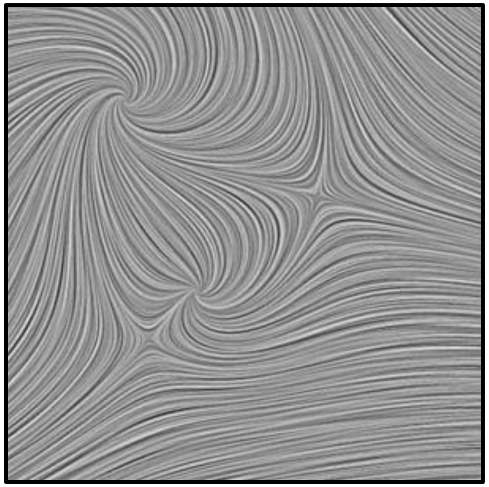
SIEMENS
Ingenuity for life

- Global method to visualize vector fields

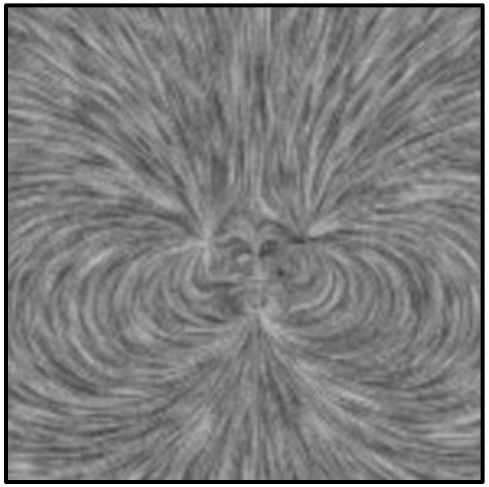



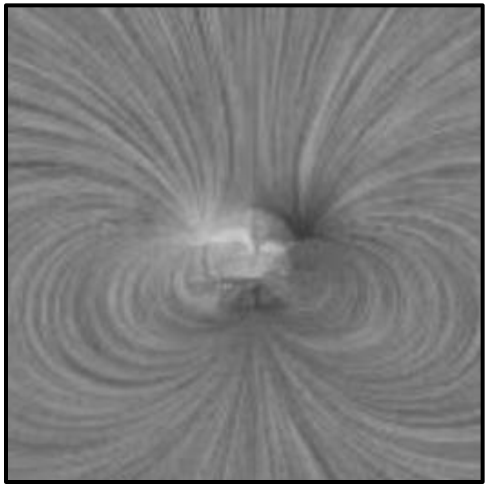

Why texture-based flow vis.

- Dense sampling
  - Better coverage of information
  - Critical point detection and classification
  - (Partially) solved problem of seeding



- Flexibility in visual representation
  - Good controllability of visual style
  - From line-like (crisp) to fuzzy

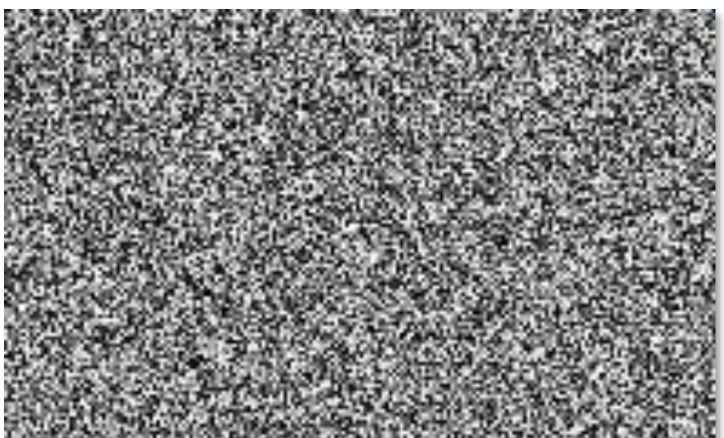




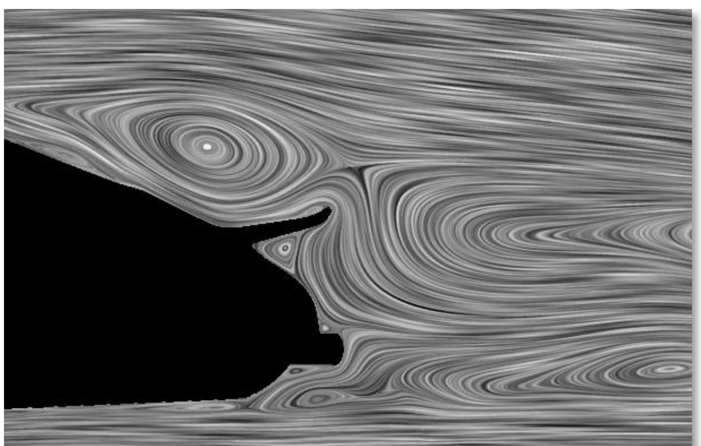
Line Integral Convolution

SIEMENS
Ingenuity for life

- Line Integral Convolution (LIC)
  - Global visualization technique (not only one particle path)
  - Start with a random texture (white noise)
  - Smear out the texture along trajectories of vector field
  - Results in low correlation between neighboring lines but high correlation along them (in flow direction)



White noise (no correlation)

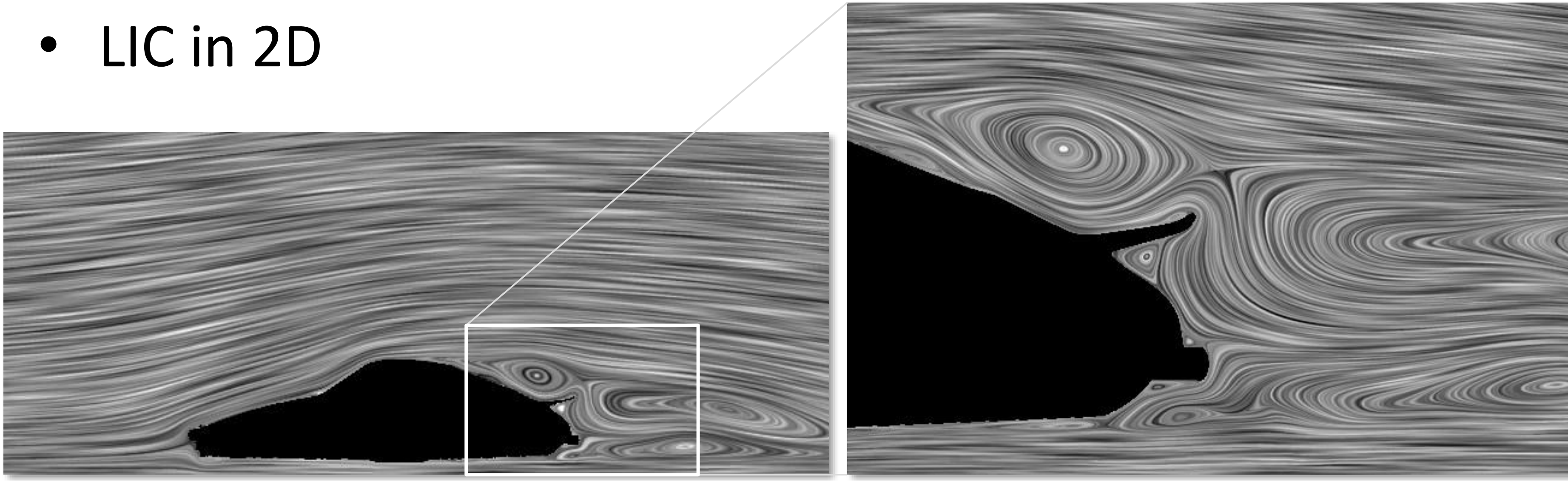


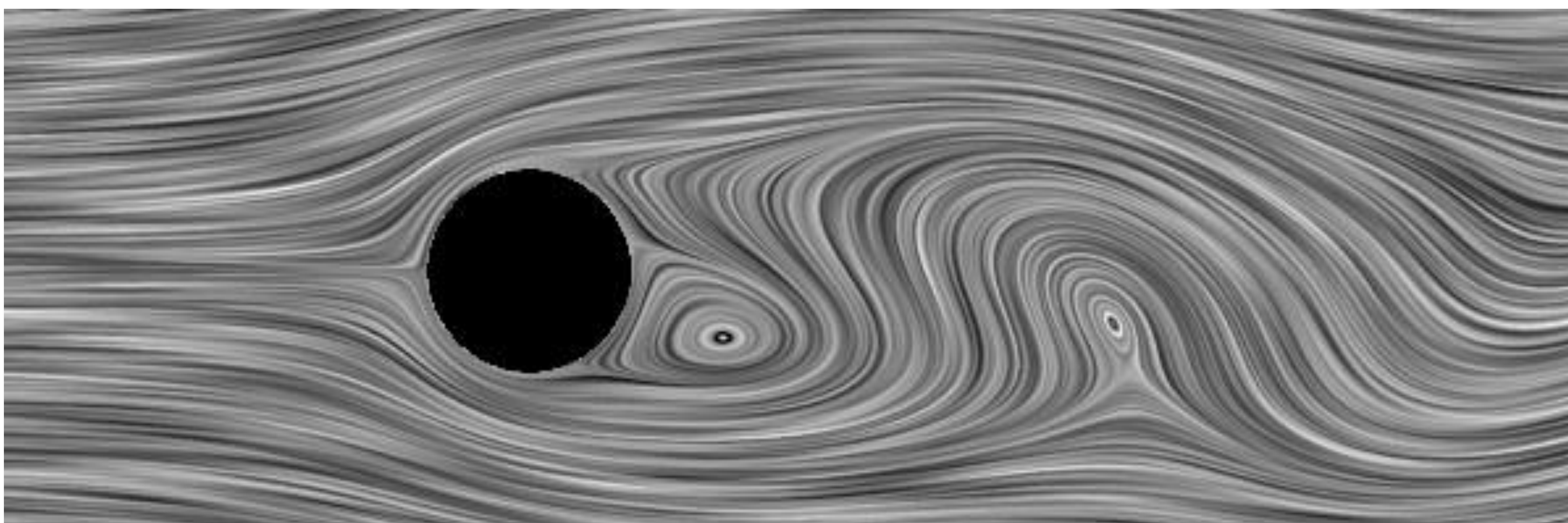
Texture values along trajectories are correlated (visually coherent)

[Cabral & Leedom 93]

Line Integral Convolution

- LIC in 2D

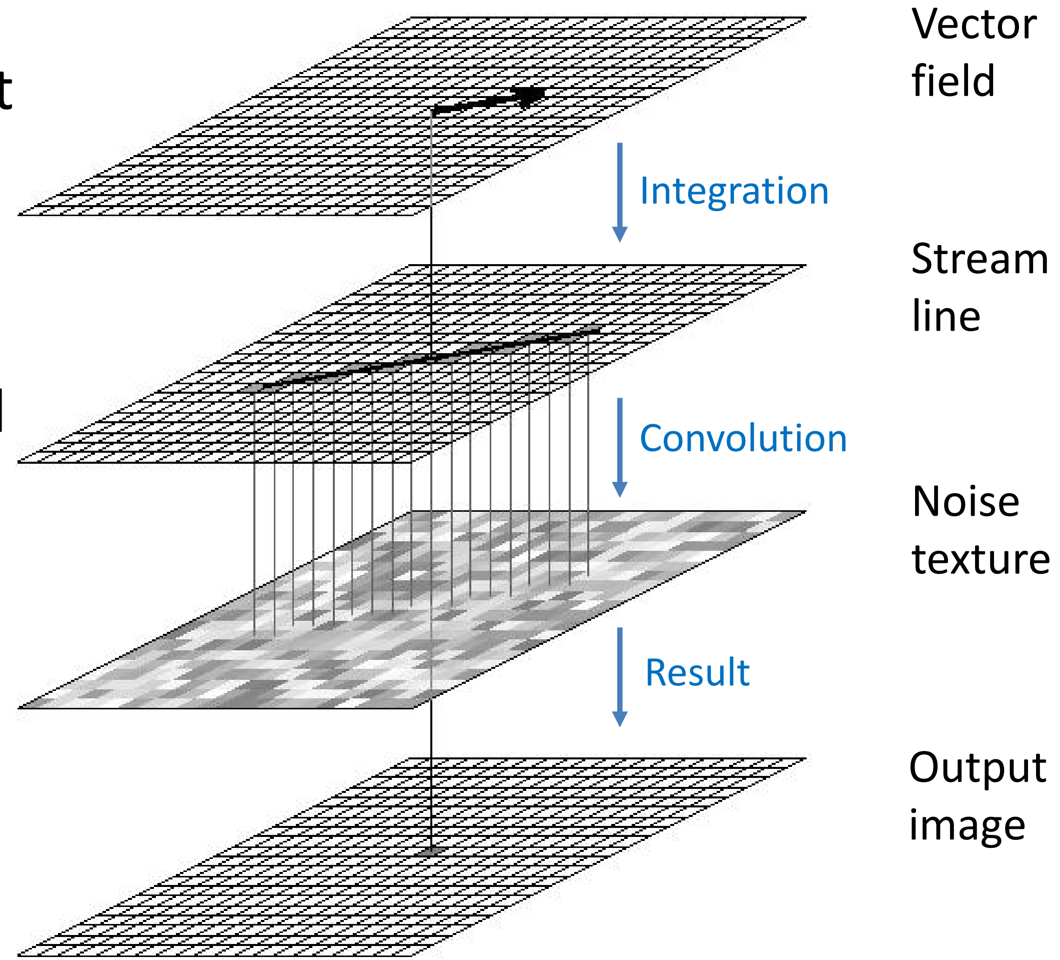




[Cabral & Leedom 93]

Line Integral Convolution

- Algorithm for 2D LIC
  - Look at stream line that passes through a pixel
  - Smear out - convolve - noise texture in direction of vector field (along stream line)



[Cabral & Leedom 93]

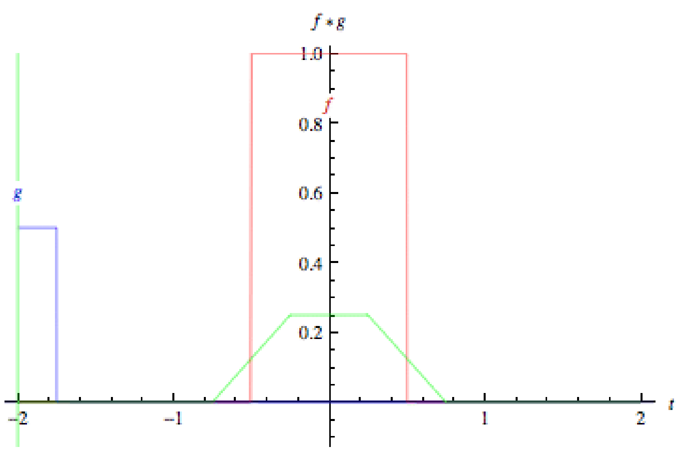
Filtering by convolution

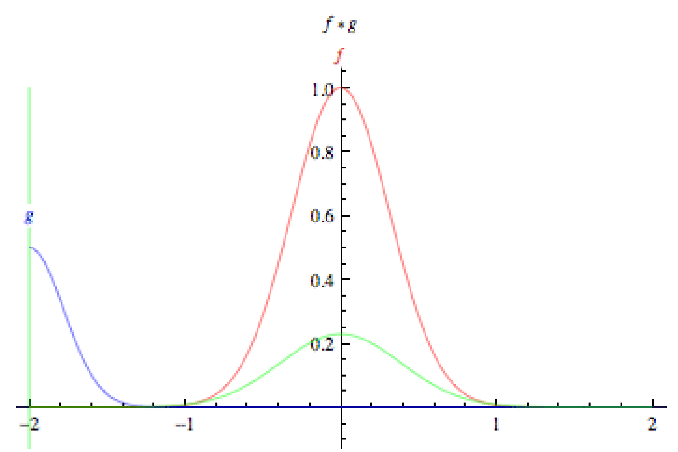
SIEMENS
Ingenuity for life

- Sliding a function $g(x)$ along a function $f(x)$

$$s(x') = [f * g](x') = \int_{-\infty}^{\infty} f(x)g(x' - x)dx$$

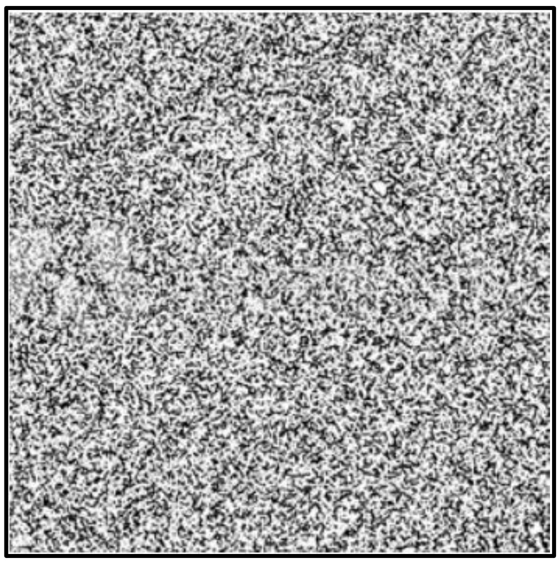
  - Function f is averaged with a weight function g
    - $(x' - x)$ centers g around x'



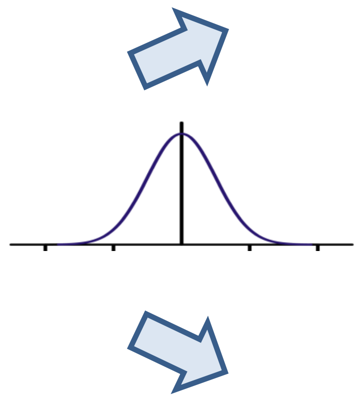


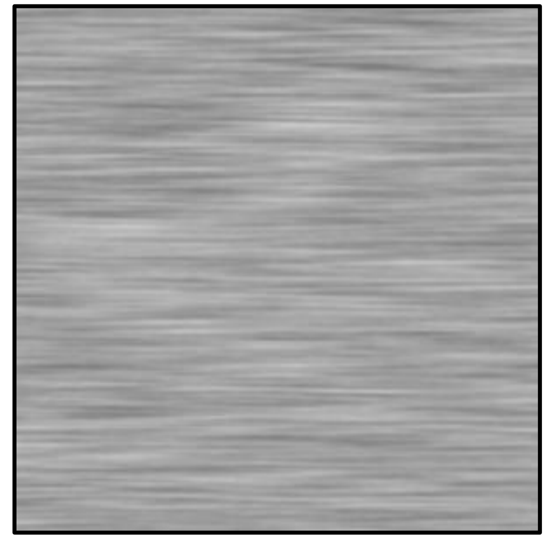
http://mathworld.wolfram.com/Convolution.html

Filtering by convolution

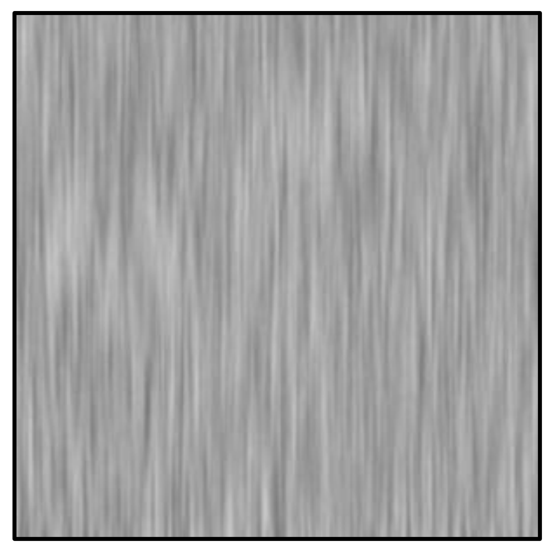


White noise (no correlation)





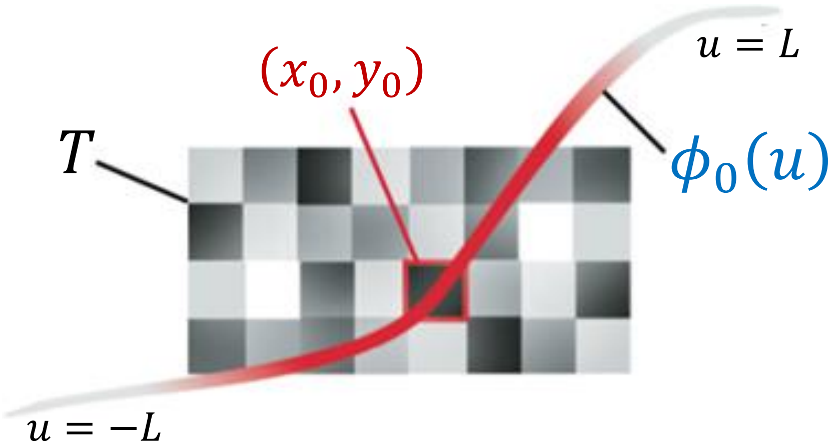
Horizontal Gaussian blur



Vertical Gaussian blur

Line Integral Convolution

- Algorithm for 2D LIC
  - Let $\phi_0(u)$ be the stream line containing the point (x_0, y_0)
  - $T(x, y)$ is the randomly generated noise texture



[Cabral & Leedom 93]

Line Integral Convolution

- Algorithm for 2D LIC
  - Let $\phi_0(u)$ be the stream line containing the point (x_0, y_0)
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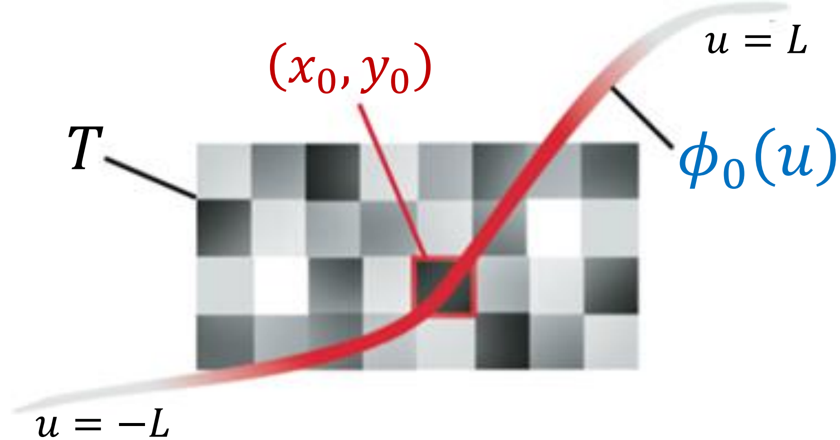

  - Compute the intensity at (x_0, y_0) as

$$I(x_0, y_0) = \int_{-L}^{L} k(u) \cdot T\big(\phi_0(u)\big)\, du$$

convolution with a kernel $k(u)$

[Cabral & Leedom 93]

Line Integral Convolution

- Algorithm for 2D LIC
  - Let $\phi_0(u)$ be the stream line containing the point (x_0, y_0)
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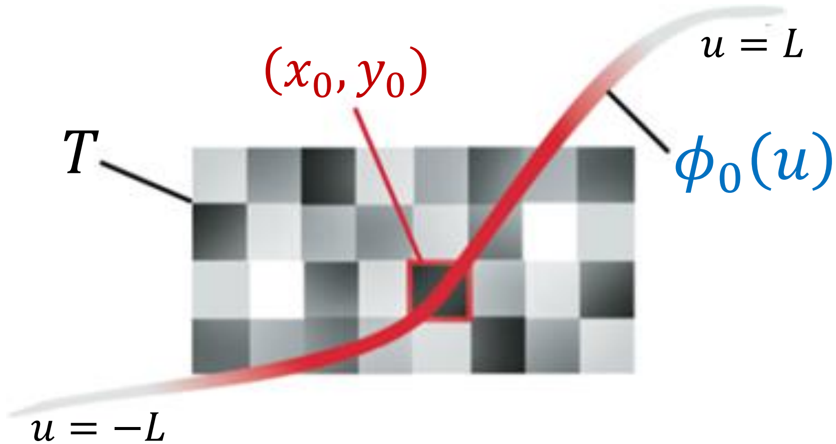
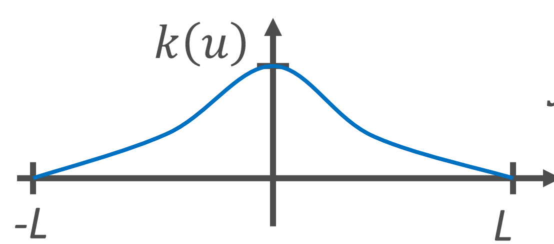


  - Compute the intensity at (x_0, y_0) as

$$I(x_0, y_0) = \int_{-L}^{L} k(u) \cdot T(\phi_0(u))\, du$$

convolution with a kernel $k(u)$

Smoothing filter kernel
- Finite support [-*L*, *L*]
- Normalized, usually symmetric
- E.g., Gaussian or box filter



$$\int_{-L}^{L} k(u)\, du = 1$$

[Cabral & Leedom 93]

Line Integral Convolution

- Algorithm for 2D LIC
  - Let $\phi_0(u)$ be the stream line containing the point (x_0, y_0)
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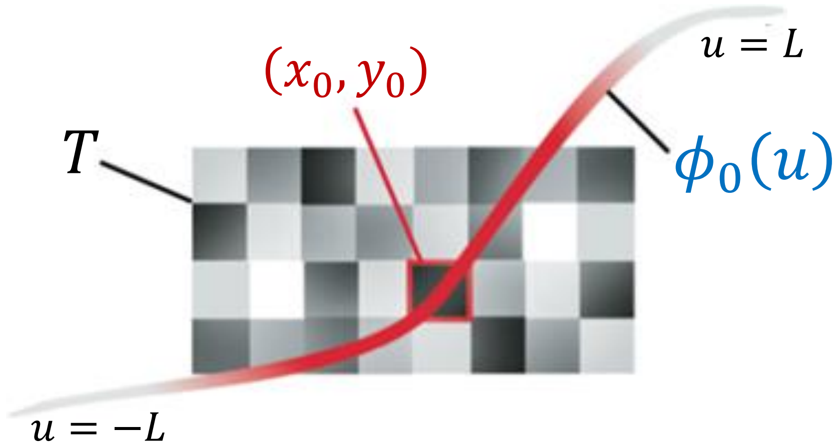

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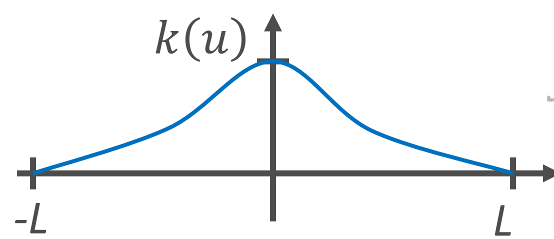
- Smoothing filter kernel
  - Finite support [-L, L]
  - Normalized, usually symmetric
  - E.g., Gaussian or box filter



$$\int_{-L}^{L} k(u)\, du = 1$$

[Cabral & Leedom 93]

Line Integral Convolution

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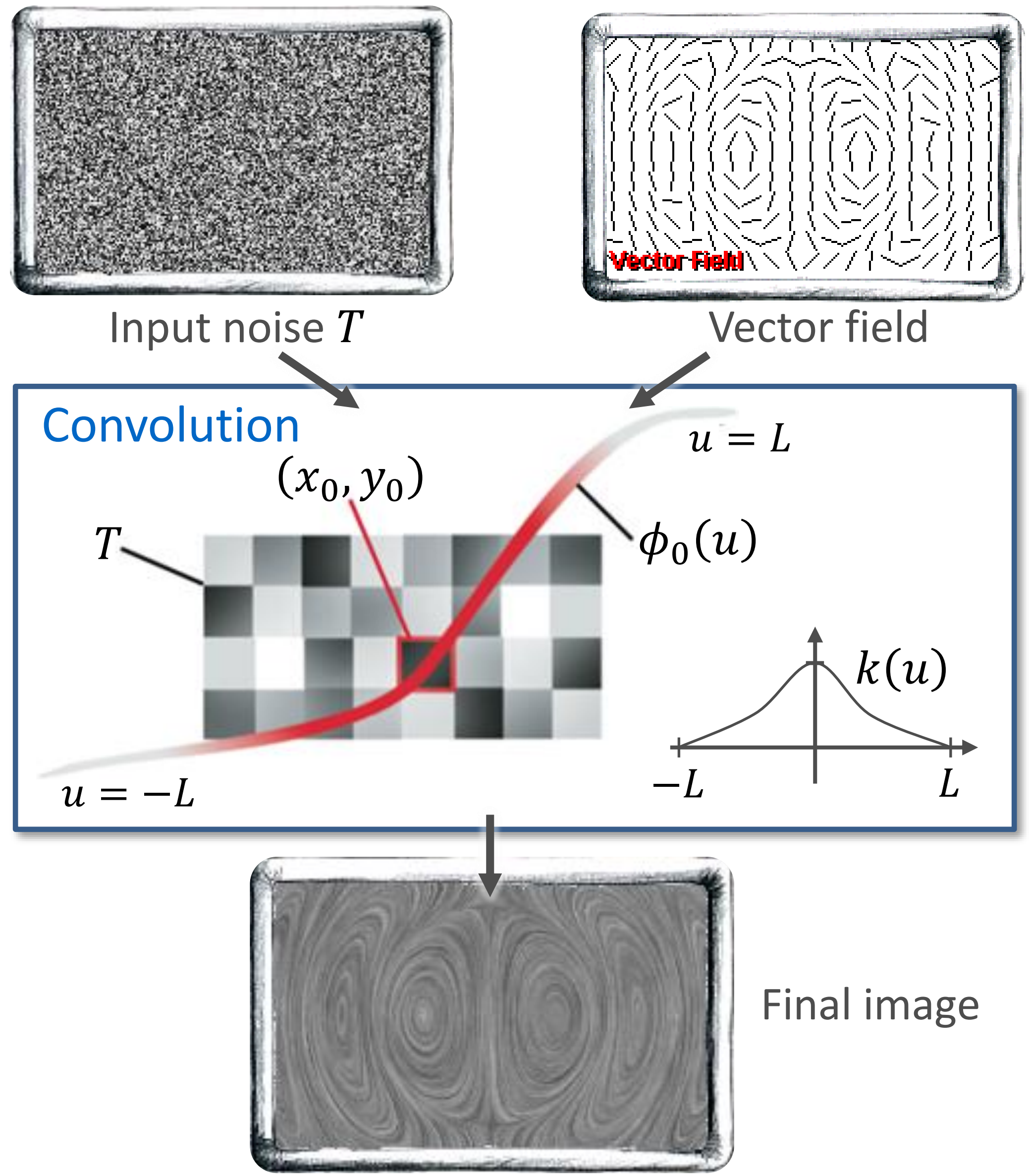
$$I(x_0, y_0) = \int_{-L}^{L} k(u) \cdot T\big(\phi_0(u)\big)\, du$$

- LIC is a convolution of
  - a noise texture $T(x, y)$
  - and a smoothing filter $k(u)$

- Noise texture values are picked up along the stream line $\phi_0(u)$ through $T\big(\phi_0(u)\big)$



Line Integral Convolution

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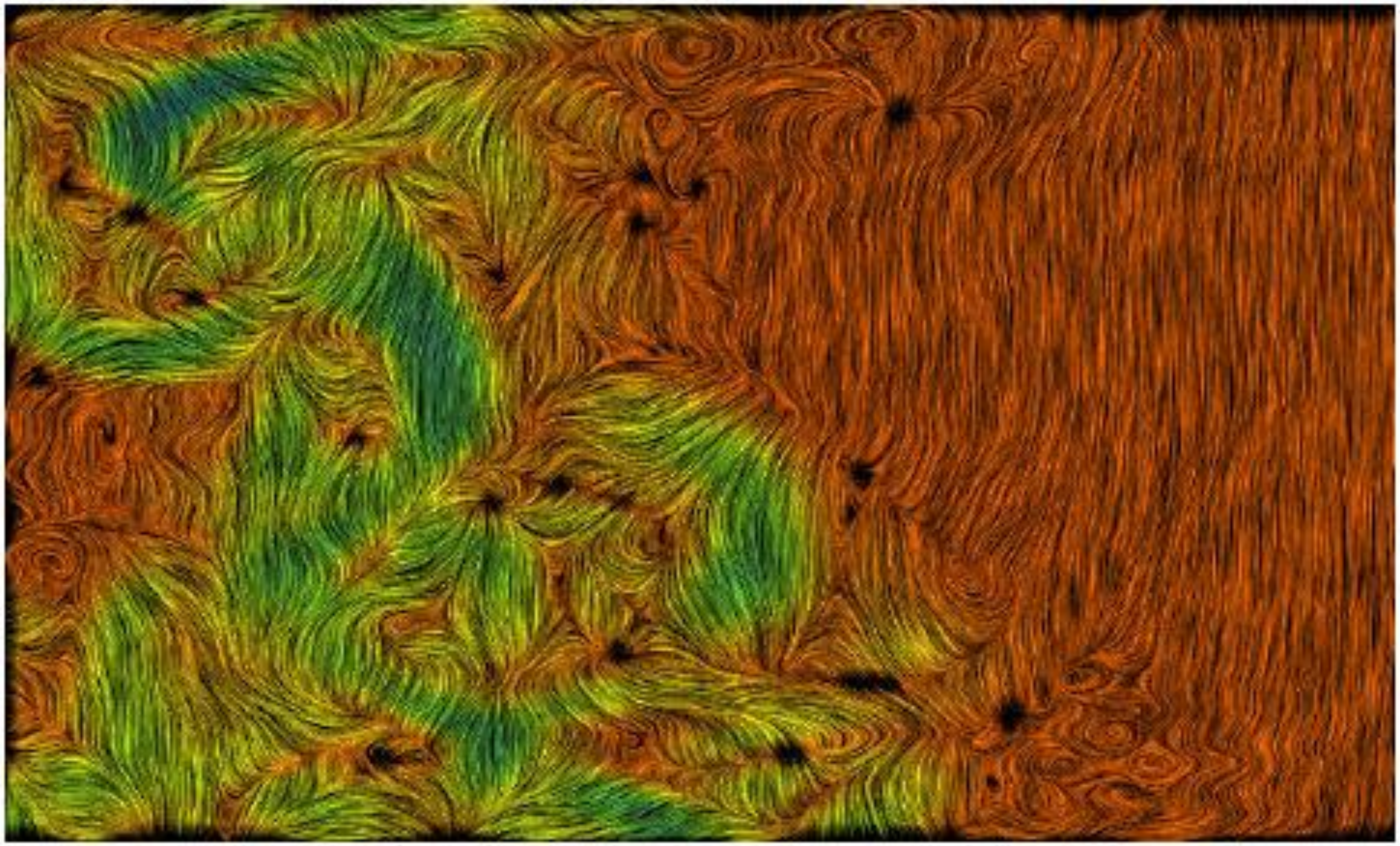
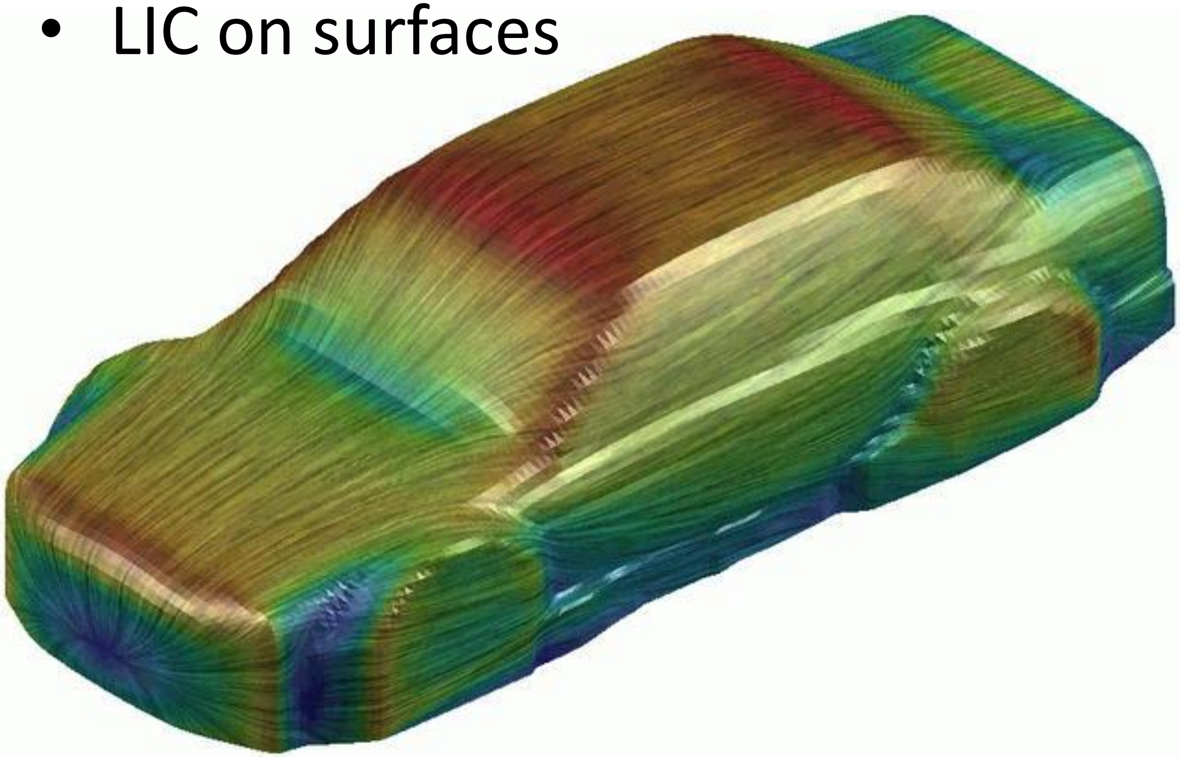


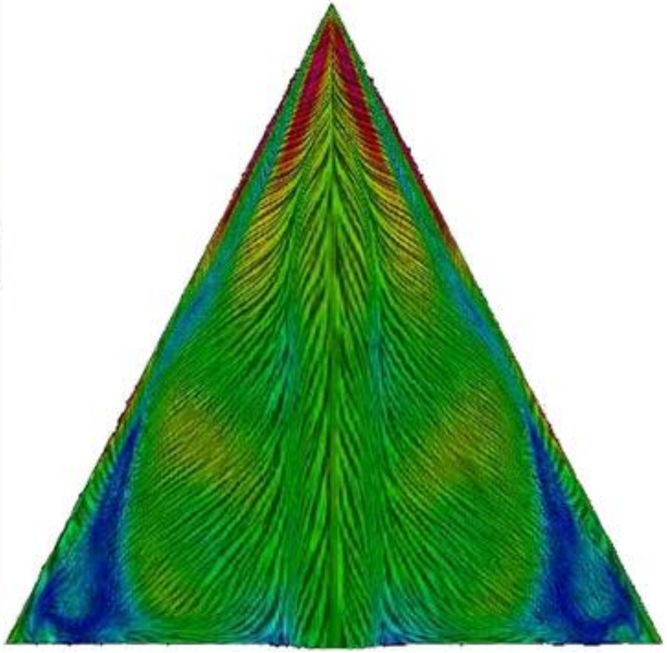
LIC and color coding of velocity magnitude

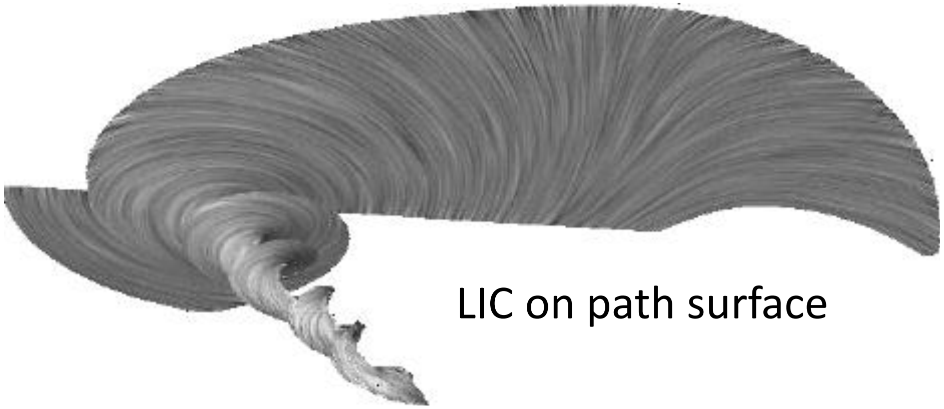
[Cabral & Leedom 93]

Line Integral Convolution

- LIC on surfaces







LIC on path surface

Data sources – artificial flows

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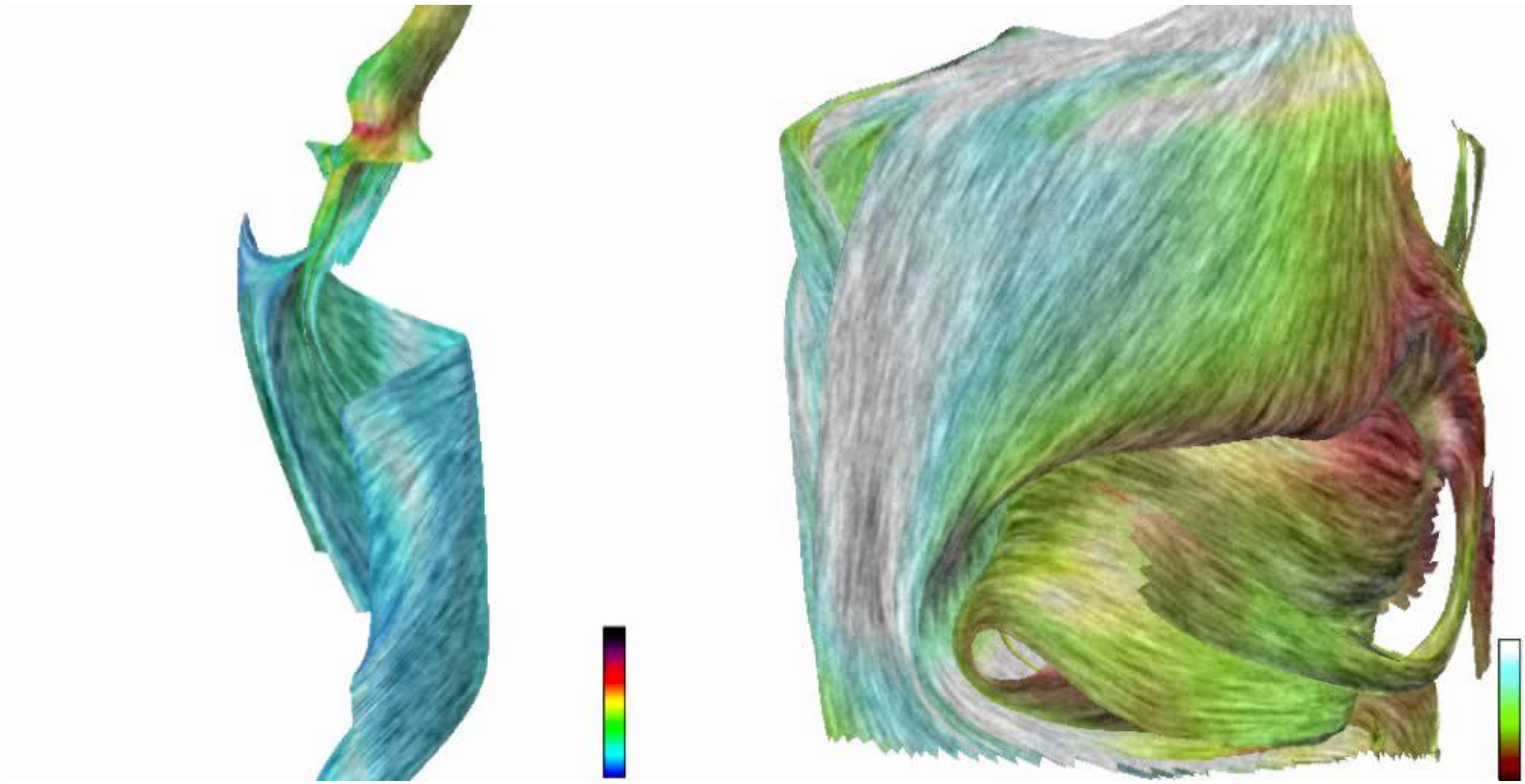
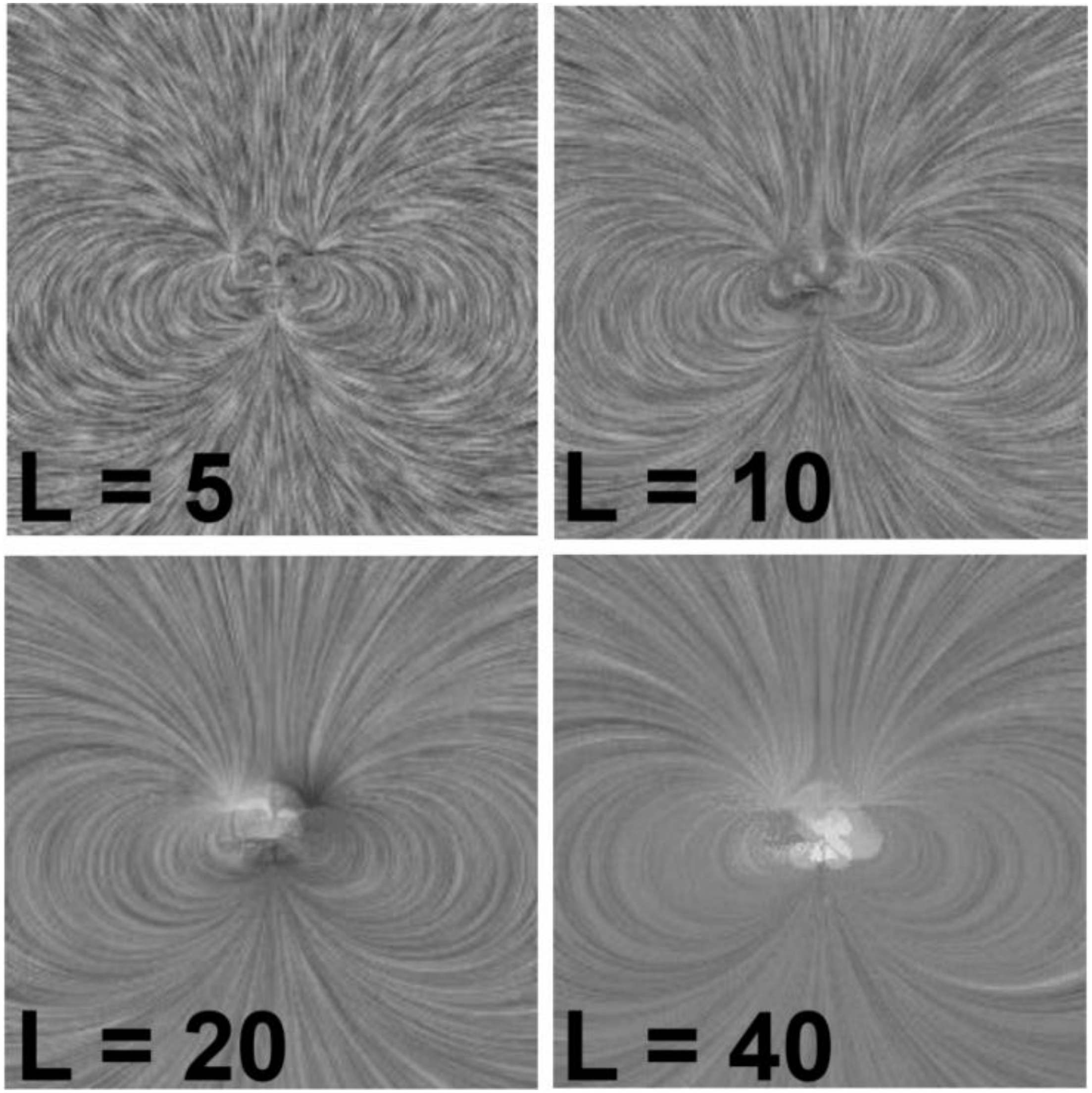
- Visualization of surface flows



[Laramee 2006]

Line Integral Convolution

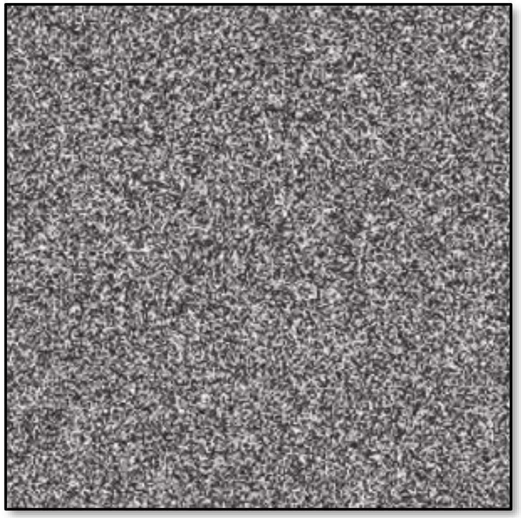
- Influence of filter length



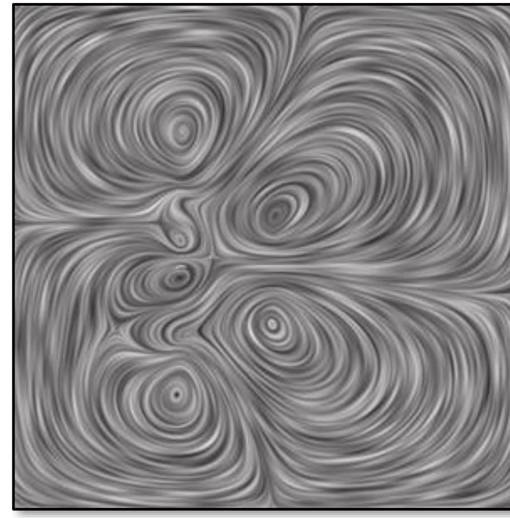
[Schumann & Müller 00]

Line Integral Convolution

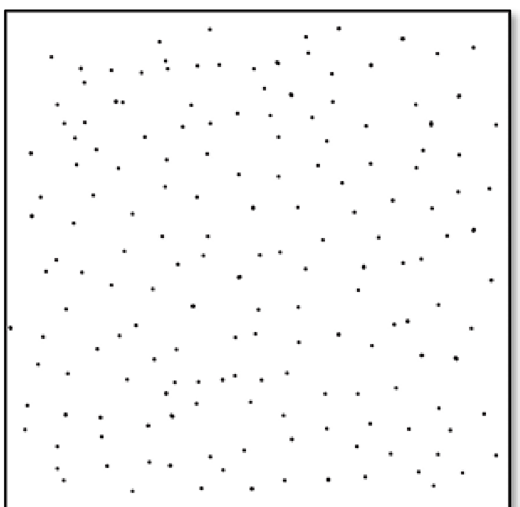
- Influence of input noise



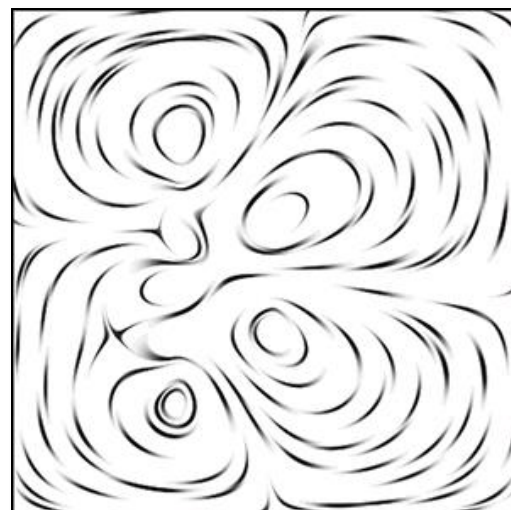
White noise texture



LIC result for white noise



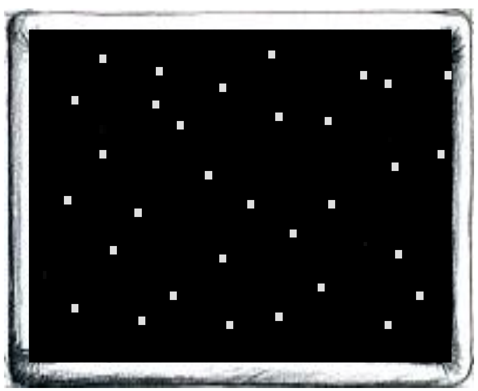
Sparse noise texture

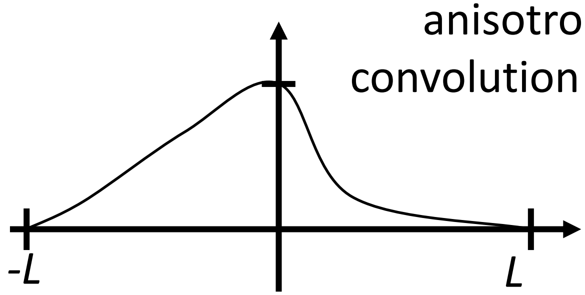


LIC result for sparse noise

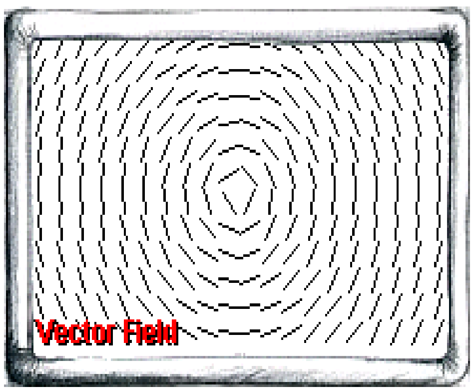
Line Integral Convolution

- Oriented LIC (OLIC)
  - Visualizes orientation (in addition to direction)
  - Uses a sparse texture; i.e. smearing of individual drops
  - Asymmetric convolution kernel





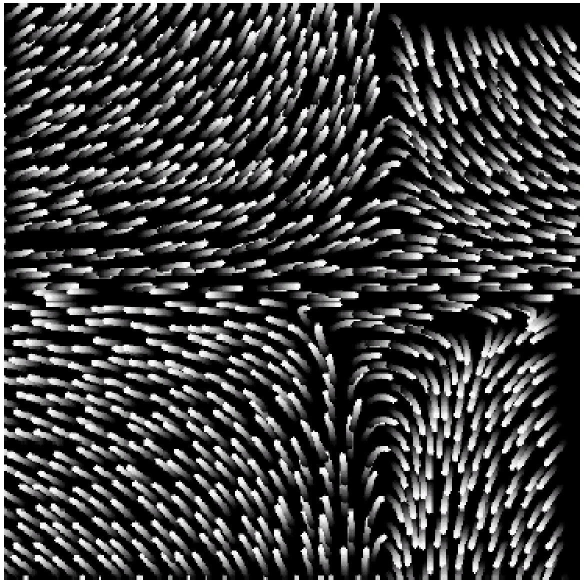
anisotropic
convolution kernel



[Wegenkittel & Gröller 97]

Line Integral Convolution

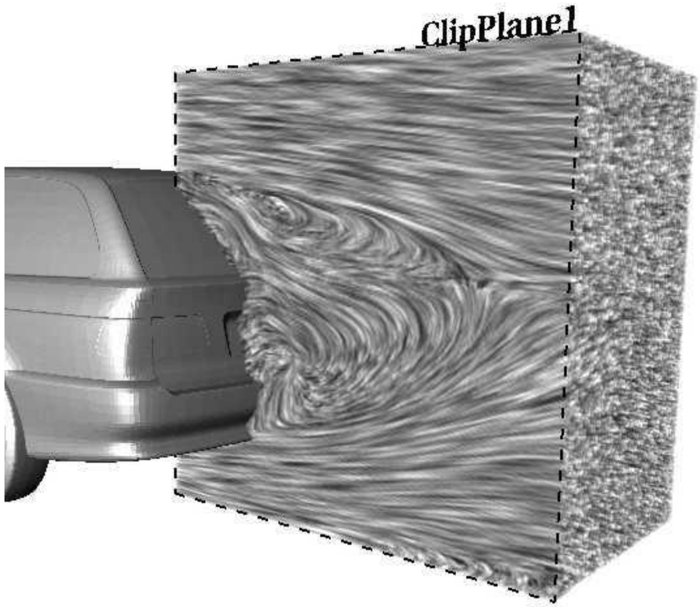
- Oriented LIC (OLIC)

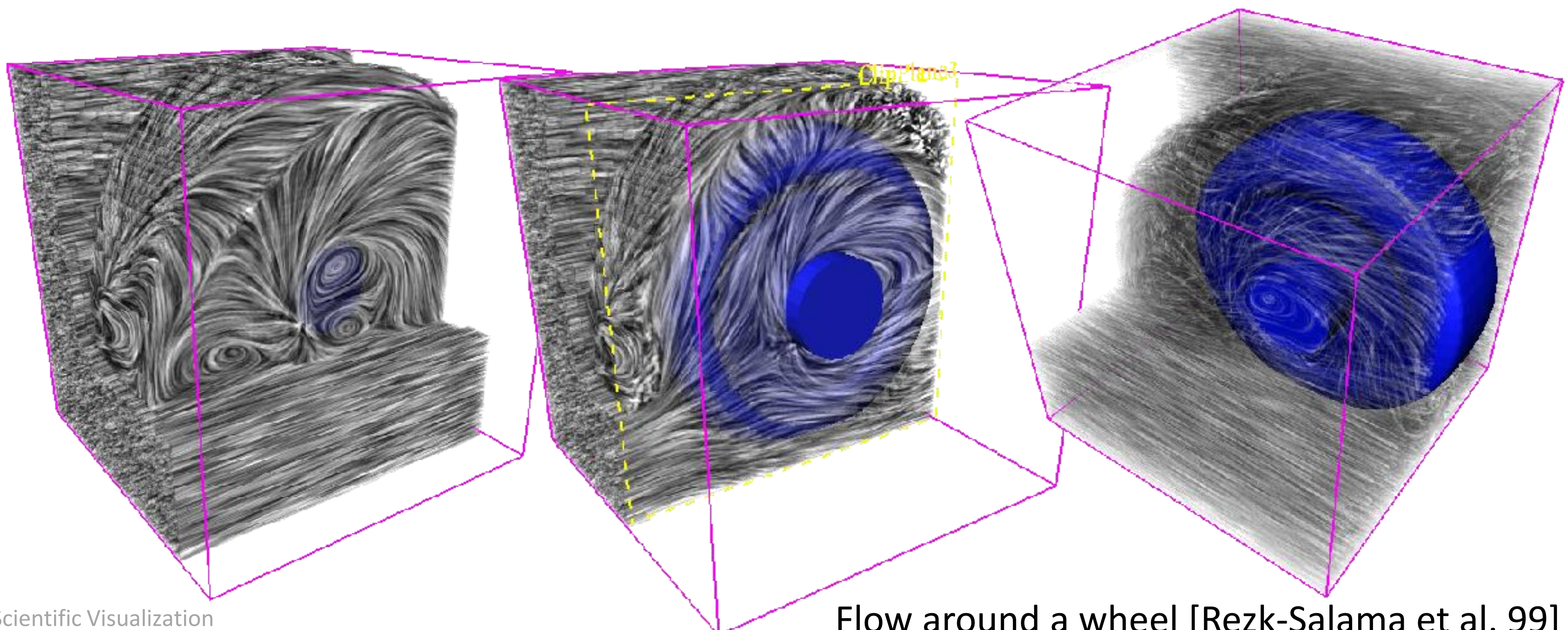


[Wegenkittel & Gröller 97]

Line Integral Convolution

- 3D LIC
  - Only good if non-relevant data is discarded

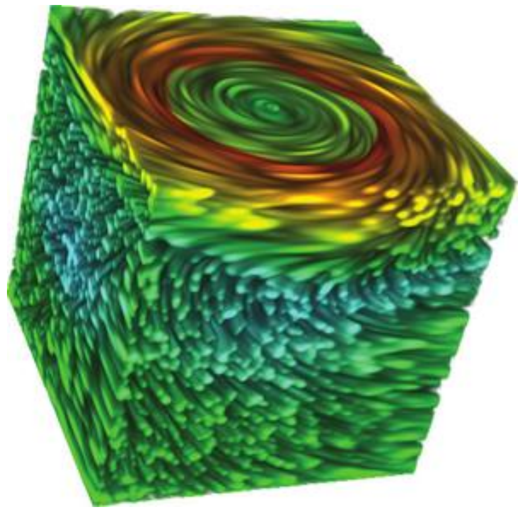




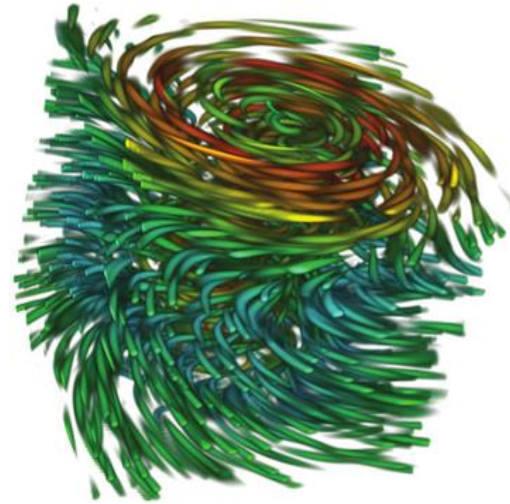
Flow around a wheel [Rezk-Salama et al. 99]

Line Integral Convolution

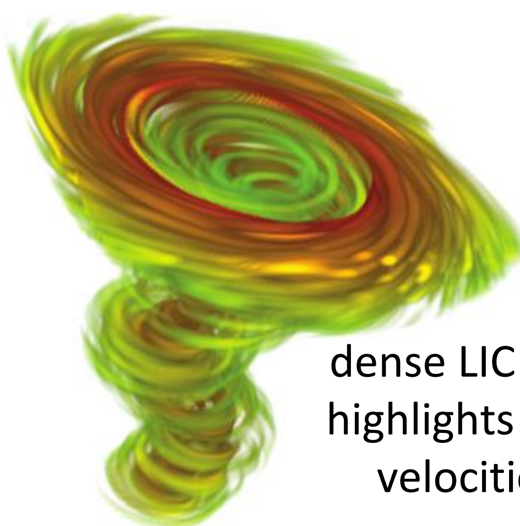
- 3D LIC



dense LIC with
white noise input



sparse LIC with
spares noise



dense LIC that
highlights high
velocities

sparse LIC that
highlights high
velocities

Line Integral Convolution

- Summary
  - Dense representation of flow fields
  - Convolution along characteristic lines
    → correlation along these lines
  - For 2D and (3D flows)

References

- Jobard & Lefer 97: Creating evenly-spaced streamlines of arbitrary density, In *Visualization in Scientific Computing*, 1997.
- Spencer et al. 09: Evenly-spaced streamlines for surfaces: An image-based approach, *Computer Graphics Forum*, 28(6), 2009.
- Cabral & Leedom 93: Imaging vector fields using line integral convolution. In *Proc. ACM SIGGRAPH*, 1993.
- Wegenkittel & M. Gröller 97: Fast oriented line integral convolution for vector field visualization via the Internet. In *Proc. IEEE Visualization*, 1997.

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Contact information

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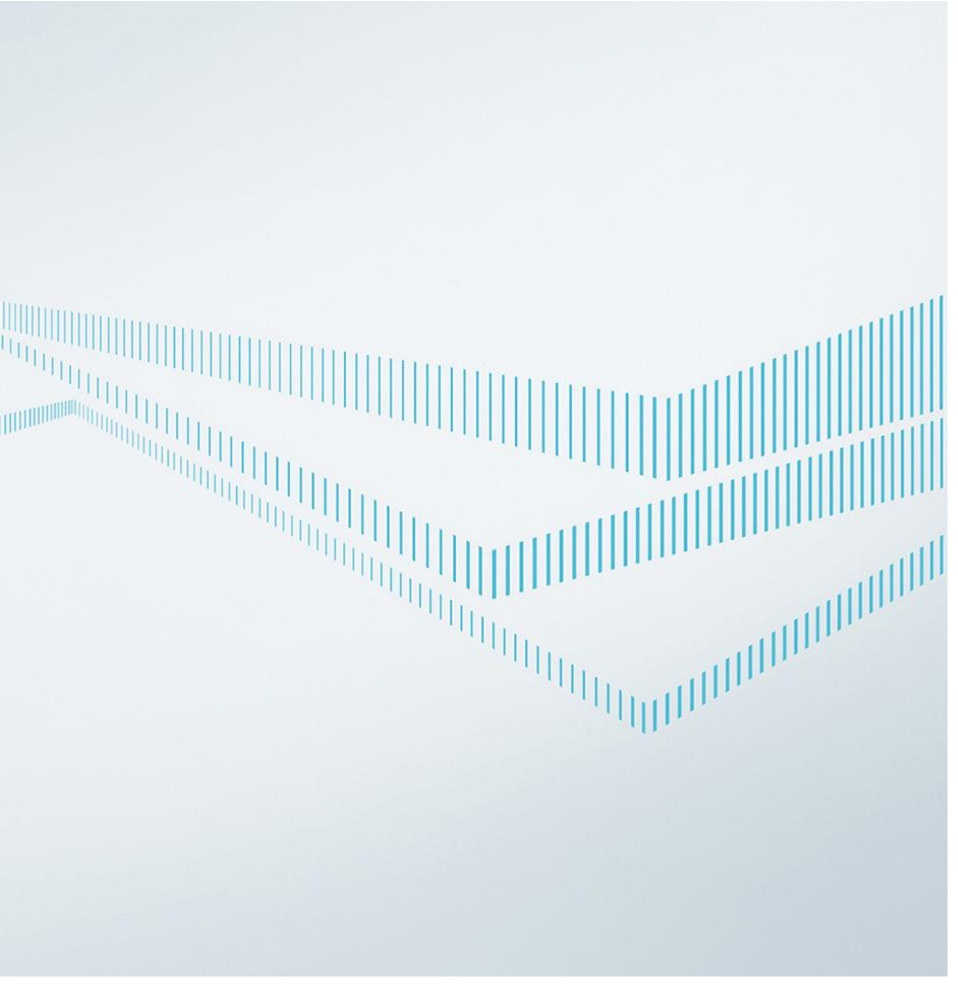

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