Disclaimer: *Below you find some example questions, which should help you prepare for the exam. However, note that the actual questions at the exam can be very different and can cover all material presented in the lecture!*

Vector Field Visualization

a) A critical point in a 2D vector field $\boldsymbol{v}(x,y)\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a point (x,y) where $\boldsymbol{v}(x,y) = (0,0)^T$. How many critical points does the vector field $\boldsymbol{v}(x,y) = (y^2 - 1,\ x - y)^T$ have and where are these points located?

$y^2 - 1 = 0$ $\quad y = \pm 1$ $\quad x - y = 0$ $\quad x = y$ $\quad$ 2 critical points $\begin{pmatrix}1\\1\end{pmatrix}$ $\begin{pmatrix}-1\\-1\end{pmatrix}$

b) Classify each critical point from Assignment a by computing the eigenvalues of the respective Jacobian matrix $\mathbf{J}$. **Hint:** The eigenvalues of $\mathbf{J}$ are values of λ that satisfy the equation $\det(\mathbf{J} - \lambda \cdot \mathbf{I}) = 0$, where $\mathbf{I}$ is the identity matrix.

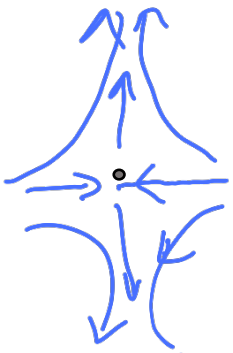
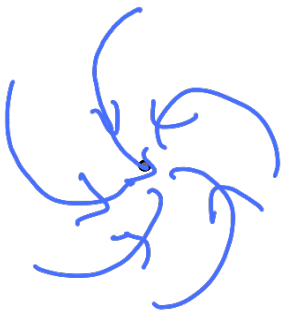
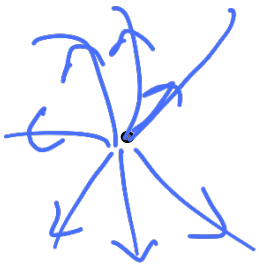
$J = \begin{pmatrix} 0 & 2y \\ 1 & -1 \end{pmatrix}$ $\quad J_{y=1} = \begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix}$ $\quad |J - \lambda I|$

$-\lambda(-1-\lambda) - 2 = 0$ $\quad \lambda^2 + \lambda - 2 = 0$ $\quad \lambda = 2,\ \lambda = -1$

$J_{y=-1} = 0 - 2 = -2 \neq -1$

c) Below are three examples of critical points together with the eigenvalues λ_1, λ_2 of the respective Jacobian matrices. Classify each critical point according to the eigenvalues and sketch the typical flow around it.

$\lambda^2 + \lambda + 2 = 0$ ✗

		
$\lambda_1 = 2 > 0$ $\lambda_2 = -1 < 0$	$\lambda_1 = -1 + i$ $\lambda_2 = -1 - i$	$\lambda_1 = 1$ $\lambda_2 = 3$
Type: Saddle point	Type: Sink	Type: Source

d) Answer whether the following statements are true or false.

LIC is a local method for visualizing a vector field.	F
The larger the extent of the convolution kernel used in LIC, the lower is the correlation between adjacent intensity values along a streamline.	F
LIC images show high correlation between the intensity values at adjacent streamlines.	F
LIC is restricted to 2D vector fields.	F
The convolution kernel used in LIC must be symmetric.	F