Disclaimer: *Below you find some example questions, which should help you prepare for the exam. However, note that the actual questions at the exam can be very different and can cover all material presented in the lecture!*

Vector Field Visualization (part I)

a) Calculate the Jacobian matrix $\mathbf{J}$ of the 3D vector field $\boldsymbol{v}(x,y,z) = (y^2 - 1,\ x - y,\ xz)^T$.

$$J = \nabla v(xyz) = \begin{pmatrix} \frac{\partial V_x}{\partial x}, & \frac{\partial V_x}{\partial y}, & \frac{\partial V_x}{\partial z} \\ \frac{\partial V_y}{\partial x}, & \frac{\partial V_y}{\partial y}, & \frac{\partial V_y}{\partial z} \\ \frac{\partial V_z}{\partial x}, & \frac{\partial V_z}{\partial y}, & \frac{\partial V_z}{\partial z} \end{pmatrix} = \begin{pmatrix} 0 & 2y & 0 \\ 1 & -1 & 0 \\ z & 0 & x \end{pmatrix}$$

b) Calculate the Jacobian matrix, divergence and curl of the 3D vector field $v(x,y,z) = (\cos(xy), \sin(x), z)^T$.

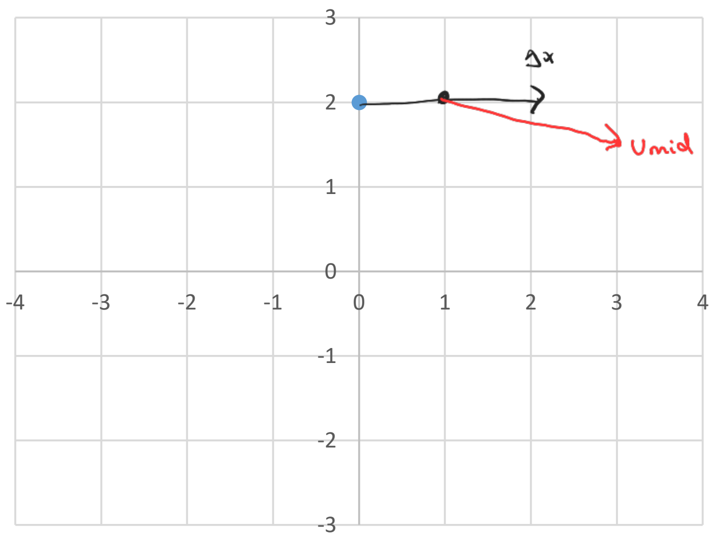
$$J = \begin{pmatrix} -y\sin(xy) & -x\sin(x) & 0 \\ \cos(x) & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\operatorname{div} v(x,y,z) = \nabla \cdot v(x,y,z) = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$= -y\sin(y) + 0 + 1$$

$$\operatorname{curl} v(x,y,z) = \nabla \times v(x,y,z) = \begin{pmatrix} \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \\ \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \\ \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 - 0 \\ 0 - 0 \\ \cos x + x\sin(xy) \end{pmatrix}$$

c) Given a 2D vector field $\boldsymbol{v}(x,y) = (y, -\frac{x}{2})^T$. Compute the next point $\boldsymbol{x}_1$ of a stream line with seed point $\boldsymbol{x}_0 = (0,2)^T$ using the *midpoint integration* method (also known as Runge-Kutta of 2nd order) with a step size $\Delta t = 1$. Draw the resulting point and the used vector(s) in the illustration below (don't forget to annotate them). Specify exactly how the point and vectors are calculated.



Euler Step

$\Delta x = \Delta t \cdot v(x,t)$

$\Delta x = 1 \cdot \binom{2}{0}$

Mid Point

$V_{mid} = v(1,2) = \binom{2}{-0{,}5}$

$X_1 = X_0 + \Delta t \cdot V_{mid}$

$x_1 = \binom{0}{2} + 1 \cdot \binom{2}{-0{,}5} = \binom{2}{1{,}5}$

d) In Figure 6, a vector field at four different times is shown. The grid vertex marked by a dot is selected as initial position for particles that are seeded into the flow. Assume that a particle can only move diagonally, vertically, or horizontally (depending on the vector at the particles current position), and that it always moves from the current grid point to the next grid point in the respective direction.

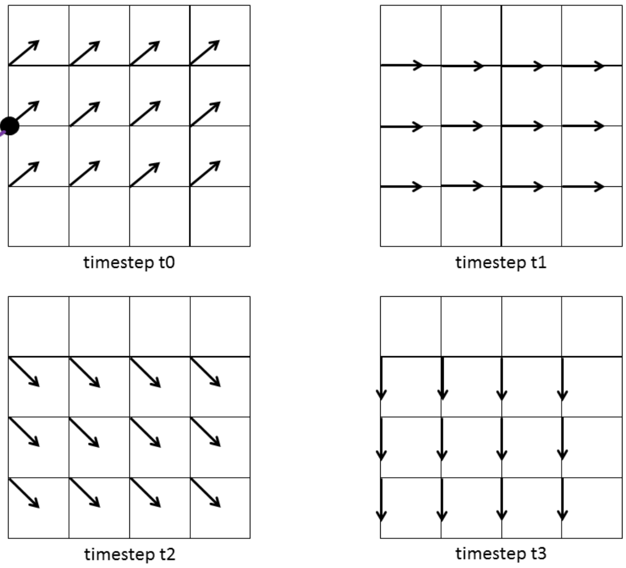
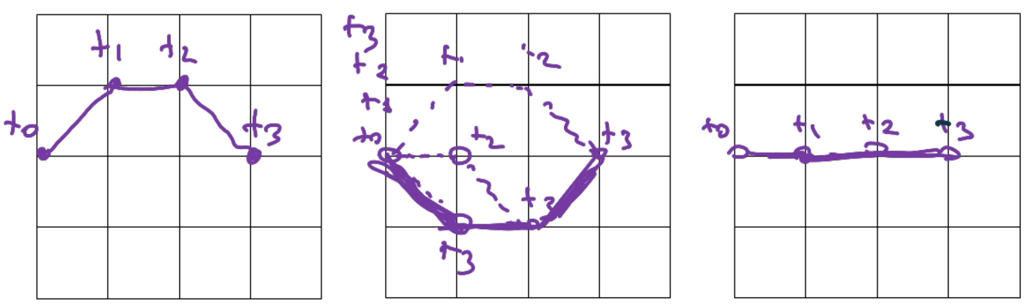


Figure 6

Illustrate in the grids given below a) the pathline of a particle released into the flow at the marked position at time t0, b) the streakline of particles released into the flow at the marked position at time t0, and c) the streamline of a particle released into the flow at the marked position in the fixed vector field at time t1.

Note: Only draw 3 edge 4 node for this example since we have 4 timestamps, so we need 4 nodes



Pathline | Streakline | Streamline in field at timestep t1

Only follow 1 particle

follow all particles have same seed point.

Seed point

e) Which characteristic line approaches do you know for *unsteady/time-varying* flow?
What happens if you apply them to steady flows? → We get a streamlines. → Change

Unsteady time-varying Flows → Path lines, Streaklines

steady → Streamlines → Vector field is not change

f) Answer whether the following statements are true or false.

The Jacobian matrix at a point in a constant 3D vector field has non-zero elements on the main diagonal.	F, if there is no change it would be 0
If the Jacobian matrix at every point in a 3D vector field is the identity matrix, then the vector field is divergence free. divergence → Sum of diagnol	F
The divergence at every point in a 3D vector field is a scalar value.	T
Streamlines in a steady 3D vector field never cross.	T
Path lines in a time-varying 2D vector field never cross.	F

Always ← Scalar

→ Identity

g) Give two examples for direct flow visualization.

Direct Flow

- Color Coding - Glyphs -
- Arrow Plots

h) What challenges does arrow-based direct flow visualization have?

- Ambiguity when representing length of the vectors 2D in 3D
- Inherent occlusions effect
- Disordered, hard to interpret if magnitude of velocity is high and changes quickly

i) Give two examples for geometric (integration-based) flow visualization.
How do these techniques relate to direct flow visualization?

Streamlines, Streak lines

j) What is Runge-Kutta integration (2nd/4th order)?
What's the advantage over Euler integration?

It is a integration method. It is ODE solver and can be used for compute chracteritic lines. It uses second or more order, and faster then Euler

k) Characteristic lines are tangential to the flow. What does that mean?

Each dangent on the line shows direction of the flow.