Homework 1

1) a) $\lim_{n\to\infty} \frac{\log_2(n^2+1)}{n} \xrightarrow{\text{l'hopital's rule}} \lim_{n\to\infty} \frac{2n}{n^2+1} \xrightarrow{\text{l'hopital's Rule}} \lim_{n\to\infty} \frac{2}{2n} = 0$

so $\log_2(n^2+1) \in O(n)$: True.

b) $\lim_{n\to\infty} \frac{\sqrt{n^2+n}}{n} = \lim_{n\to\infty} \sqrt{\frac{n^2+n}{n^2}} = \lim_{n\to\infty} \sqrt{1+\frac{1}{n}} = 1$

so $\sqrt{n^2+n} \in \Omega(n)$. True.

c) $\lim_{n\to\infty} \frac{n^{n-1}}{n^n} = \lim_{n\to\infty} \frac{\cancel{n.n.n.\cdots\cdot n}}{\cancel{n.n.n.\cdots}\, n.n} = \lim_{n\to\infty} \frac{1}{n} = 0$

so $n^{n-1} \notin \Omega(n^n)$. Statement is false.

d) if $f(n) \in O(g(n))$ then we can say $O(f(n)) \subset O(g(n))$

$\lim_{n\to\infty} \frac{(2^n+n^3)}{4^n} = \lim_{n\to\infty} \left(\frac{1}{2}\right)^n + \frac{n^3}{4^n} = \underbrace{\lim_{n\to\infty}\left(\frac{1}{2}\right)^n}_{0} + \underbrace{\lim_{n\to\infty}\frac{n^3}{4^n}}_{0} = 0+0=0$

so $2^n+n^3 \in O(4^n)$ and $O(2^n+n^3) \subset O(4^n)$. True.

e) $\lim_{n\to\infty} \frac{2\log_3 \sqrt[3]{n}}{3\log_2 n^2} = \lim_{n\to\infty} \frac{\frac{2}{3}\log_3 n}{6\cdot\log_2 n} \xrightarrow{\text{l'hopital's rule}} \lim_{n\to\infty} \frac{\frac{1}{n\ln(3)}}{\frac{1}{n\ln(2)}} = \lim_{n\to\infty} \frac{\ln 2}{\ln 3} = \frac{\ln 2}{\ln 3}$

(we can ignore the constants)

Result is smaller than 0. So statement is false.

f) $\lim_{n\to\infty} \frac{\log_2\sqrt{n}}{(\log_2 n)^2} = \lim_{n\to\infty} \frac{\frac{1}{2}\cdot\cancel{\log_2 n}}{\log_2 n\cdot\cancel{\log_2 n}} = \lim_{n\to\infty} \frac{1}{2\cdot\log_2 n} = 0$

$\log_2\sqrt{n} \in O(\log_2 n)^2$
$(\log_2 n)^2 \notin O(\log_2\sqrt{n})$) Statement is false.

2) $10^n > 2^n > n^3 = 8^{\log n} > n^2 \log n > n^2 > \sqrt{n} > \log n$

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$\Rightarrow 10^n - 2^n$

Let a^x, b^x if $a^x > b^x$, $a > b$.

so $\underline{10^n > 2^n}$

$\Rightarrow 2^n - n^3$

$\lim_{n\to\infty} \frac{n^3}{2^n} = 0$ (from growth function definition $(x^a < a^x)$) so $\underline{2^n > n^3}$

$\Rightarrow n^3 - 8^{\log n}$

$8^{\log n} = n^{\log 8} = n^3$ so $\underline{n^3 = 8^{\log n}}$

$\Rightarrow 8^{\log n} - n^2 \log n$

$8^{\log n} = n^3$

$\lim_{n\to\infty} \frac{n^2 \log n}{n^3} = \lim_{n\to\infty} \frac{\log n}{n} \xrightarrow{\text{l'hopital rule}} \lim_{n\to\infty} \frac{1}{n} = 0$

so $\underline{8^{\log n} > n^2 \log n}$

$\Rightarrow n^2 \log n - n^2$

$\lim_{n\to\infty} \frac{n^2}{n^2 \log n} = \lim_{n\to\infty} \frac{1}{\log n} = 0$

$\underline{n^2 \log n > n^2}$

$\Rightarrow n^2 - \sqrt{n}$

$n^2 - n^{1/2}$

Let $x^a - x^b$ $x^a > x^b$ iff $a > b$

so $\underline{n^2 > \sqrt{n}}$

$\Rightarrow \sqrt{n} - \log n$

$\lim_{n\to\infty} \frac{\log n}{\sqrt{n}} \Rightarrow$ assume $n = x^2$

$\lim \frac{2\log x}{x} = \lim \frac{2}{\ln(x)} = 0$

so $\underline{\sqrt{n} > \log n}$

3) a) In this function the basic operation is comparison operators. Since there is no nested loops or recursive calls in the function complexity only depends on loop. Loops iterates only size of Array times. So complexity is $O(n) = \Omega(n) = \Theta(n)$

b) In this function, if we examine the pattern of this loop, whenever even number assignes to i, count is increased 1 in the next iteration. After that loops iterates one more time and calculates $i = (i-1)*i$ statement and generates even i and repeates same pattern. So we assume this eas every else statement it increases count. The growth of i is depends on else statement.

Since $i = i^2 - i$ statement increases i exponentially the complexity is $O(\log(\log n))$.

4) a) $\int_0^n g(x)\,dx \leq f(x) \leq \int_1^{n+1} g(x)\,dx$

$\int x^2 \log x\,dx \Rightarrow \frac{x^3 \log x}{3} - \int \frac{x^2}{3}\,dx = \frac{x^3 \log x}{3} - \frac{x^3}{9} = \frac{x^3(3\log x - 1)}{9} \Rightarrow$ lets put this in a statement

$uv - \int v\,du$

$u = \log x \quad v' = x^2$

$u' = \frac{1}{x} \quad v = \frac{x^3}{3}$

$$\left.\frac{x^3(3\log x - 1)}{9}\right|_0^n \leq f(x) \leq \left.\frac{x^3(3\log x - 1)}{9}\right|_1^{n+1}$$

$$\frac{n^3(3\log n - 1)}{9} - \underbrace{\frac{0(3\log 0 - 1)}{9}}_{\text{tanımsız.}} \leq f(x) \leq \frac{(n+1)^3(3\log(n+1) - 1)}{9} + \frac{1}{9}$$

Alternatif çözüm
diğer sayfada.

$$\frac{n^3(3\ln-1)}{9} - \underbrace{\frac{0\cdot(3\log 0-1)}{9}}_{\log 0 \text{ is undefined.}} \leq f(x) \leq \frac{(n+1)^3(3\log(n+1)-1)}{9} + \frac{1}{9}$$

for this part, Let's try this;

$$1+\int_1^n x^2\log x\,dx = 1+\left(\frac{x^3(\log x-1)}{9}\Big|_1^n\right) = 1+\frac{n^3(\log n-1)}{9}+\frac{1}{9}$$

So statement becomes;

$$\frac{n^3(\log n-1)}{9}+\frac{10}{9} \leq f(x) \leq \frac{(n+1)^3(3\log(n+1)-1)}{9}+\frac{1}{9}$$

So $\underline{f(x) \in \Theta(n^3\log n)}$

b) $$\int_0^n g(x)\,dx \leq f(x) \leq \int_1^{n+1} g(x)\,dx$$

$$\int_0^n x^3\,dx \leq f(x) \leq \int_1^{n+1} x^3\,dx$$

$$\frac{x^4}{4}\Big|_0^n \leq f(x) \leq \frac{x^4}{4}\Big|_1^{n+1} \Rightarrow \frac{n^4}{4} \leq f(x) \leq \frac{(n+1)^4}{4}$$ So $\underline{f(x) \in \Theta(n^4)}$

c) $$\int_0^n \frac{1}{2\sqrt{x}}\,dx \leq f(x) \leq \int_1^{n+1} \frac{1}{2\sqrt{x}}\,dx$$

$$\sqrt{x}\Big|_0^n \leq f(x) \leq \sqrt{x}\Big|_1^{n+1}$$

$$\sqrt{n} \leq f(x) \leq \sqrt{n+1}-1$$ So $\underline{f(x) \in \Theta(\sqrt{x})}$

d) $\int_0^n \frac{1}{x} dx \leq f(x) \leq \int_1^{n+1} \frac{1}{x} dx$

$\ln x \Big|_0^n \leq f(x) \leq \ln(x) \Big|_1^{n+1}$

$\underbrace{\underset{\text{tanımsız}}{\ln 0 - \ln n}}_{\text{alternatif yol}} \leq f(x) \leq \ln(n+1)$

:)

$1 + \int_1^n \frac{1}{x} dx \leq f(x)$

$1 + \ln x \Big|_1^n \leq f(x)$

$\ln(n) + 1 \leq f(x) \leq \ln(n+1)$ so $f(x) \in \Theta(\ln(n))$

5) Best case: first element is searching element.

$L[1] = x$ $B(n) \in \Theta(1)$

Worst case: Element does not in the lists or $L[n] = x$ (last element)

$W(n) \in \Theta(n)$ which n is length of list.

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