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Abstract.

1. Introduction

Ortlama değerler arasındaki belirli eşitsizlikleri incelemek için Steffensen [11] i kanıtlamıştır.Aşağıdaki eşitsizlikler

Theorem 1.1. *f ve g'nin (a,b) üzerinde iki integrallenebilir fonksiyon olsun. f f azalıyor ve her $t\in(a,b)$, $0\le g(t)\le 1$.O zaman aşağıdaki eşitsizlik*

$$\int_{b-\lambda}^{b} f(t)dt \le \int_{a}^{b} f(t)g(t)dt \le \int_{a}^{a+\lambda} f(t)dt \tag{1.1}$$

geçerlidir, burada $\lambda=\int_a^b g(t)dt$

Steffensaen eşitsizliğinin (1.1) bazı küçğük genellemeleri,A'nın pozitif sabit olduğu $g(t)$ yerine $g(t)/A$ kullanılarak Hayashi(5) tarafında dikkate alınmıştır.[3, 5, , 8 − 11].

Son çalışmada [1],Alomari ve arkadaşları aşağıdaki sonucu kanıtladılar:

Theorem 1.2. *$f,g:[a,b]\to\mathbb{R}$ $0\le g(t)\le 1$ olacak şekilde integrallenebilir olsun. $\forall t\in[a,b]$ için $\int_a^b g(t)f(t)dt$ var . Eğer f is üzerinde kesin sürekli ise, [a,b]ile $f'\in L[a,b]$, $1\le p\le\infty$, sahibiz*

$$\left|\int_a^{a+\lambda} f(t)dt-\int_a^b f(t)g(t)dt\right|$$

$$\le\begin{cases}\frac{1}{2}[\lambda^2+(b-a-\lambda)^2]\|f'\|_{\infty,[a,b]} & \text{iff}' \in L_\infty[a,b];\\ \frac{\|f'\|_{p,[a,b]}}{(q+1)^{1/q}}[\lambda^{(q+1)/q}+(b-a-\lambda)^{(q+1)/q}] & \text{iff}' \in L_\infty[a,b],p>1;\\ \left[\int_{a+\lambda}^b g(t)dt\right]\|f'\|_1 & \text{iff}'\in L_1[a,b],\end{cases} \tag{1.2}$$

ve

$$\left|\int_a^b f(t)g(t)dt-\int_{b-\lambda}^b f(t)dt\right|$$

$$\le\begin{cases}\frac{1}{2}[\lambda^2+(b-a-\lambda)^2]\|f'\|_{\infty,[a,b]} & \text{iff}' \in L_\infty[a,b];\\ \frac{\|f'\|_{p,[a,b]}}{(q+1)^{1/q}}[\lambda^{(q+1)/q}+(b-a-\lambda)^{(q+1)/q}] & \text{iff}' \in L_\infty[a,b],p>1;\\ \left[\int_a^{b-\lambda} g(t)dt\right]\|f'\|_1 & \text{iff}'\in L_1[a,b],\end{cases} \tag{1.3}$$

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Bir $f:\mathbb{R}^+\to\mathbb{R}$ *fonksiyonu , burada* $\mathbb{R}^+=[0,\infty)$ *ikinci anlamda s-convex olduğu söylenir.Eğer*

$$f(\alpha x+\beta y)\le\alpha^s f(x)+\beta^s f(y)$$

$\forall x,y\in[0,\infty]$, $\alpha,\beta\ge 0$ *için* $\alpha+\beta=1$ *ve sabit ise* $s\in(0,1]$. *Bu s-convex fonskiyonları sınıfı genellikle* K_s^2 *gösterilir. ([6]görülür).* $s=1$ *için s-convex* $[0,\infty]$ *fonksiyonmların sıradan dışbükeyliğine indirgendiği görülür.*

Dragomir ve Fitzparrick, s-convex için geçerli olan Hadamard eşitsizliğinin bir varyantını kanıtladı.

$$2^{s-1}f\left(\frac{a+b}{2}\right)\le\frac{1}{b-a}\int_a^b f(x)dx\le\frac{f(a)+f(b)}{s+1} \tag{1.4}$$

2. The results

Mitrimovic ve arkadaşlarına bağlı olarak öne çıkan Lemma ile başlayalım.

Lemma 2.1. $f,g:[a,b]\subset\mathbb{R}^+\to\mathbb{R}$ *olcak şekilde integrallenebilir olsun* $0\le q(t)\le 1$ *,tüm* $t\in[a,b]$ *öyleki* $\int_a^b q(t)f'(t)dt$ *var*

$$\int_a^{a+\lambda} f(t)dt-\int_a^b f(t)g(t)dt$$

$$=-\int_a^{a+\lambda}\left(\int_a^x(1-g(t))dt\right)f'(x)dx-\int_{a+\lambda}^b\left(\int_x^b g(t)dt\right)f'(x)dx \tag{2.1}$$

ve

$$\left[\int_a^b f(t)g(t)dt-\int_{b-\lambda}^b f(t)dt\right]$$

$$=\int_a^{b-\lambda}\left(\int_a^x g(t)dt\right)f'(x)dx-\int_{b-\lambda}^b\left(\int_x^b(1-g(x))dx\right)f'(x)dx \tag{2.2}$$

where $\lambda:=\int_a^b g(t)dt.$

Proof. Using Lemma (**) and since $|f'|$ is P-function we have

$$-\int_a^{a+\lambda}\left(\int_a^x(1-g(t))dt\right)f'(x)dx-\int_{a+\lambda}^b\left(\int_a^x 1-g(x)dx\right)f'(x)dx$$

$$\le-\left(\int_a^{a+\lambda}(1-g(t))dt\right)f(a+\lambda)-\int_a^{a+\lambda}f(x)d\left(\int_a^x(1-g(t))dt\right)$$

$$+\left(\int_{a+\lambda}^a g(t)dt\right)f(a+\lambda)-\int_{a\lambda}^a f(x)d\left(\int_a^x(1-g(t))dt\right)$$

$$\le-\left(\int_a^{a+\lambda}(1-g(t))dt\right)f(a+\lambda)-\int_a^{a+\lambda}f(x)(1-g(x))dx$$

$$+\left(\int_{a+\lambda}^a g(t)dt\right)f(a+\lambda)-\int_{a+\lambda}^b f(x)g(x)dx$$

$$=-\lambda f(a+\lambda)f(a+\lambda)\int_a^{a+\lambda}g(t)dt+\int_a^{a+\lambda}f(x)dx$$

$$
\begin{aligned}
& - \int_{a+\lambda}^{b} f(x)g(x)dx + f(a+\lambda)\int_{a+\lambda}^{b} g(t)dt - \int_{a+\lambda}^{b} f(x)g(x)dx \\
= \; & -\lambda f(a+\lambda) f(a+\lambda)\int_{a}^{a+\lambda} g(t)dt + f(a+\lambda)\int_{a+\lambda}^{b} g(t)dt \\
& + \int_{a}^{a+\lambda} f(x)dx - \int_{a}^{a+\lambda} f(x)g(x)dx \\
= \; & \int_{a}^{a+\lambda} f(x)dx - \int_{a}^{a+\lambda} f(x)g(x)dx
\end{aligned}
$$

whice gives the desired representation (2.1). The identity (2.2) can be also proved in a similar way, we shall omit the details.

2.1 Inequalities involving s-convexity , inequalities for absolutely continuous functions whose first derivatives are (s-concave) are given: In the following

Theorem 2.1. *$f, g : [a, b] \subset \mathbb{R}^+ \to \mathbb{R}$ olcak şekilde integrallenebilir olsun $0 \le q(t) \le 1$,tüm $t \in [a, b]$ öyleki $\int_a^b q(t)f'(t)dt$ var.if f is absolutely continuous on [a,b] such tyhat $|f'|$ is s-convex on [a,b], for some fixed $s \in (0, 1]$ then we have*

$$
\begin{aligned}
& \left| \int_{a}^{a+\lambda} f(t)dt - \int_{a}^{b} f(t)g(t)dt \right| \\
\le \; & \frac{1}{(s+1)(s+2)}[\lambda^2 |f'(a)| + (b-a-\lambda)^2 |f'(b)|] \\
+ \; & \frac{1}{s+2}[\lambda^2 + (b-a-\lambda)^2] |f'(a+\lambda)|
\end{aligned}
\tag{2.3}
$$

ve

$$
\begin{aligned}
& \left| \int_{a}^{b} f(t)g(t)dt - \int_{b-\lambda}^{b} f(t)dt \right| \\
\le \; & \frac{1}{(s+1)(s+2)}[\lambda^2 |f'(b)| + (b-a-\lambda)^2 |f'(a)|] \\
+ \; & \frac{1}{s+2}[\lambda^2 + (b-a-\lambda)^2] |f'(b-\lambda)|
\end{aligned}
$$

where $\lambda := \int_a^b g(t)dt.$

□

Proof. Utilizing the triangle inequality on (2.1), and since $|f'|$ is s-convex, we have

$$
\begin{aligned}
& \left| \int_{a}^{a+\lambda} f(t)dt - \int_{a}^{b} f(t)g(t)dt \right| \\
\le \; & \left| \int_{a}^{a+\lambda} \left(\int_{a}^{x} (1-g(t))dt \right) f'(x)dx \right| - \left| \int_{a+\lambda}^{b} \left(\int_{a}^{x} g(t)dt \right) f'(x)dx \right| \\
\le \; & \int_{a}^{a+\lambda} \left| \int_{a}^{x} (1-g(t))dt \right| |f'(x)|dx - \int_{a+\lambda}^{b} \left| \int_{a}^{x} g(t)dt \right| |f'(x)|dx \\
\le \; & \int_{a}^{a+\lambda} \left| \int_{a}^{x} (1-g(t))dt \right| \left(\frac{(x-a)^s}{\lambda^s} |f'(a+\lambda)| + \frac{(a+\lambda-x)^s}{\lambda^s} |f'(a)| \right) dx
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{a+\lambda}^{b}\left|\int_{x}^{b} g(t)dt\right|\left(\frac{(x-a-\lambda)^s}{(b-a-\lambda)^s}|f'(b)|+\frac{(b-x)^s}{(b-a-\lambda)^s}|f'(a+\lambda)|\right)dx\\
\leq\ & \frac{|f'(a+\lambda)|}{\lambda^s}\int_a^{a+\lambda}\left(\int_a^x|1-g(t)|dt\right)(x-a)^s dx\\
&+\frac{|f'(a)|}{\lambda^s}\int_a^{a+\lambda}\left(\int_a^x|1-g(t)|dt\right)(a+\lambda-x)^s dx\\
&+\frac{|f'(b)|}{(b-a-\lambda)^s}\int_{a+\lambda}^{b}\left(\int_x^b|g(t)|dt\right)(x-a-\lambda)^s dx\\
&+\frac{|f'(a+\lambda)|}{(b-a-\lambda)^s}\int_{a+\lambda}^{b}\left(\int_x^b|g(t)|dt\right)(b-x)^s dx\\
\leq\ & \frac{|f'(a+\lambda)|}{\lambda^s}\int_a^{a+\lambda}(x-a)^{s+1}dx+\frac{|f'(a+\lambda)|}{\lambda^s}\int_a^{a+\lambda}(x-a)(a+\lambda-x)^s dx\\
&+\frac{|f'(b)|}{(b-a-\lambda)^s}\int_{a+\lambda}^{b}(b-x)(x-a-\lambda)^s dx+\frac{|f'(a+\lambda)|}{(b-a-\lambda)^s}\int_{a+\lambda}^{b}(b-x)^{s+1}dx\\
=\ & \frac{1}{(s+1)(s+2)}\left[\lambda^2|f'(a)|+(b-a-\lambda)^2|f'(b)|\right]\\
&+\frac{1}{s+2}\left[\lambda^2+(b-a-\lambda)^2\right]|f'(a+\lambda)|
\end{aligned}
$$

whice proves the first ineequality in (2.3). In similar way and using (2.2) we may deduce the desired inequality (2.4), and we shall omit details.

□

3. SONUÇ

In (2.3) if one chooses $s = 1$ then

$$
\begin{aligned}
&\left|\int_a^{a+\lambda} f(x)dx-\int_a^b f(x)g(x)dx\right|\\
\leq\ & \frac{1}{6}\lambda^2|f'(a)|+\frac{1}{3}\left[\lambda^2+(b-a-\lambda)^2\right]|f'(a+\lambda)|+\frac{1}{6}(b-a-\lambda)^2|f'(b)| \qquad (3.1)
\end{aligned}
$$

also, in (2.4)if $s = 1$, then

$$
\begin{aligned}
&\left|\int_a^b f(t)g(t)dt-\int_{b-\lambda}^b f(t)dt\right|\\
\leq\ & \frac{1}{6}\lambda^2|f'(b)|+\frac{1}{3}\left[\lambda^2+(b-a-\lambda)^2\right]|f'(b-\lambda)|+\frac{1}{6}(b-a-\lambda)^2|f'(a)| \qquad (3.2)
\end{aligned}
$$

4. AÇIKLAMA

In the inequalities 2.3 and 2.4, choose $\lambda = 0$, then we have

$$
\begin{aligned}
&\left|\int_a^b f(t)g(t)dt\right|\\
\leq\ & \frac{(b-a)^2}{(s+1)(s+2)}\min\left\{(s+1)|f'(a)|+|f'(b)|,\ |f'(a)|+(s+1)|f'(b)|\right\} \qquad (4.1)
\end{aligned}
$$

Theorem 4.1. *$f, g : [a, b] \subset \mathbb{R}^+ \to \mathbb{R}$ olcak şekilde integrallenebilir olsun $0 \le q(t) \le 1$,tüm $t \in [a, b]$ öyleki $\int_a^b q(t)f'(t)dt$ var.if f is absolutely continuous on [a,b] such tyhat $|f'|$ is s-convex on [a,b], for some fixed $s \in (0, 1]$ then we have*

$$
\begin{aligned}
&\left| \int_a^{a+\lambda} f(t)dt - \int_a^b f(t)g(t)dt \right| \\
\le\ & \frac{1}{s+1}\left(\int_{a+\lambda}^b g(t)dt \right) [\lambda|f'(a)| + (b-a)|f'(a+\lambda)| + (b-a-\lambda)|f'(b)|] \\
\le\ & \frac{(b-a-\lambda)}{s+1}[\lambda|f'(a)| + (b-a)|f'(a+\lambda)| + (b-a-\lambda)|f'(b)|] \qquad (4.2)
\end{aligned}
$$

ve

$$
\begin{aligned}
&\left| \int_a^b f(t)g(t)dt - \int_{b-\lambda}^b f(t)dt \right| \\
\le\ & \frac{1}{s+1}\left(\int_{b-\lambda}^b g(t)dt \right) [(b-a-\lambda)|f'(a)| + (b-a)|f'(b-\lambda)| + \lambda|f'(b)|] \\
\le\ & \frac{\lambda}{s+1}[(b-a-\lambda)|f'(a)| + (b-a)|f'(b-\lambda)| + \lambda|f'(b)|] \qquad (4.3)
\end{aligned}
$$

where $\lambda := \int_a^b g(t)dt$.

Proof. From Lemma 1, we may write

$$
\begin{aligned}
&\left| \int_a^{a+\lambda} f(t)dt - \int_a^b f(t)g(t)dx \right| \\
\le\ & \sup_{x\in[a,a+\lambda]} \left(\int_0^x (1-g(t))dt \right) \int_{a+\lambda}^b |f'(x)|dx \\
&+ \sup_{x\in[a,a+\lambda]} \left[\int_x^b (g(t))dt \right] \int_{a+\lambda}^b |f'(x)|dx
\end{aligned}
$$

Since |f'| is convex on [a,b]. then by (1.4) we have

$$\int_a^{a+\lambda} |f'(x)|dx \le \lambda + \frac{|f'(a)|, |f'(a+\lambda|}{s+1}$$

ve

$$\int_a^{a+\lambda} |f'(x)|dx \le (b-a-\lambda) + \frac{|f'(a+\lambda)|, |f'(b|}{s+1}$$

bu nedenle elimizde

$$
\begin{aligned}
&\left| \int_a^{a+\lambda} f(t)dt - \int_a^b f(t)g(t)dt \right| \\
\le\ & \lambda\, \frac{|f'(a)|, |f'(a+\lambda|}{s+1} \int_a^{a+\lambda} (1-g(t))dt
\end{aligned}
$$

$$+(b-a-\lambda)\frac{|f'(a+\lambda)|,|f'(b)|}{s+1}\int_{a+\lambda}^{b} g(t)dt$$

$$\le max\left\{\int_a^{a+\lambda}(1-g(t))dt, \int_{a+\lambda}^{a} g(t)dt\right\}\left(\lambda\frac{|f'(a)|,|f'(a+\lambda)|}{s+1} +(b-a-\lambda)\frac{|f'(a+\lambda)|,|f'(b)|}{s+1}\right)$$

$$\le \frac{1}{s+1}\left(\int_{a+\lambda}^{b} g(t)dt\right)[\lambda|f'(a)|+(b-a)|f'(a+\lambda)|+(b-a-\lambda)|f'(b)|]$$

whice proves the first inequality in (2.8). The second inequality in (2.8) follows directly, since $0 \le g(t) \le 1$ for all $t \in [a,b]$, then

$$0 \le \int_{a+\lambda}^{b} g(t)dt \le (b-a-\lambda)$$

The inequalities in (2.9) may be proved in the same way using the idenity (2.2), we shall omit the detais. □

5. Inequalities involving s-concavity

Theorem 5.1. *$f, g : [a,b] \subset \mathbb{R}^+ \to \mathbb{R}$ olcak şekilde integrallenebilir olsun $0 \le q(t) \le 1$,tüm $t \in [a,b]$ öyleki $\int_a^b q(t)f'(t)dt$ var.if f is absolutely continuous on [a,b] such tyhat $|f'|$ is s-convex on [a,b], for some fixed $s \in (0,1]$ then we have*

$$\left|\int_a^{a+\lambda} f(t)dt - \int_a^b f(t)g(t)dt\right| \le 2^{s-1}\left(\int_{a+\lambda}^{b} g(t)dt\right)\left(\lambda\left|f'\left(a+\frac{\lambda}{2}\right)\right| + (b-a-\lambda)\left|f'\left(\frac{a+b+\lambda}{2}\right)\right|\right) \le 2^{s-1}(b-a-\lambda)\left(\lambda\left|f'\left(a+\frac{\lambda}{2}\right)\right| + (b-a-\lambda)\left|f'\left(\frac{a+b+\lambda}{2}\right)\right|\right) \tag{5.1}$$

ve

$$\left|\int_a^b f(t)g(t)dt - \int_{b-\lambda}^{b} f(t)dt\right| \le 2^{s-1}\left(\int_{b-\lambda}^{b} g(t)dt\right)\left((b-a-\lambda)\left|f'\left(\frac{a+b-\lambda}{2}\right)\right| + \lambda\left|f'\left(b-\frac{\lambda}{2}\right)\right|\right) \le \lambda 2^{s-1}\left((b-a-\lambda)\left|f'\left(\frac{a+b-\lambda}{2}\right)\right| + \lambda\left|f'\left(b-\frac{\lambda}{2}\right)\right|\right) \tag{5.2}$$

where $\lambda := \int_a^b g(t)dt.$

Proof. Utulizing the triangle inequality on (2.1), and since $|f'|$ is s-concave on [a,b] then by $[a,b]$ then by (1.4)we may state

$$\left|\int_a^{a+\lambda} f(t)dt - \int_a^b f(t)g(t)dt\right|$$

$$\leq \sup_{x\in[a,a+\lambda]}\left(\int_a^x (1-g(t))dt\right)\int_a^{a+\lambda}|f'(x)|dx+\sup_{x\in[a+\lambda,a]}\left(\int_x^b g(t)dt\right)\int_a^{a+\lambda}|f'(x)|dx$$

$$\leq 2^{s-1}\lambda\left|f'\left(a+\frac{\lambda}{2}\right)\right|\int_a^{a+\lambda}(1-g(t))dt+2^{s-1}(b-a-\lambda)\left|f'\left(\frac{a+b+\lambda}{2}\right)\right|\int_{a+\lambda}^{a}g(t)dt$$

$$=2^{s-1}\left(\int_{a+\lambda}^{b}g(t)dt\right)\left(\lambda\left|f'\left(a+\frac{\lambda}{2}\right)\right|+(b-a-\lambda)\left|f'\left(\frac{a+b+\lambda}{2}\right)\right|\right) \tag{5.3}$$

whice proves the first inequality in (2.10). The second inequality in (2.10) follows directly, since $0\leq g(t)\leq 1$ for all $t\in[a,b]$, then

$$0\leq\int_{a+\lambda}^{b}g(t)dt\leq(b-a-\lambda)$$

The inequalities in (2.11) may be proved in the same way using the idenity (2.2), we shall omit the detais.

Another result in incorporated in the following theorem:

Theorem 5.2. *$f,g:[a,b]\subset\mathbb{R}^+\to\mathbb{R}$ olcak şekilde integrallenebilir olsun $0\leq q(t)\leq 1$,tüm $t\in[a,b]$ öyleki $\int_a^b q(t)f'(t)dt$ var.if f is absolutely continuous on [a,b] such tyhat $|f'|$ is s-convex on [a,b], for some fixed $s\in(0,1]$ then we have*

$$\left|\int_a^{a+\lambda}f(t)dt-\int_a^b f(t)g(t)dt\right|\leq\frac{2^{s-1/q}}{(p+1)^{1/p}}\left(\lambda^2\left|f'\left(a+\frac{\lambda}{2}\right)\right|+(b-a-\lambda)^2\left|f'\left(\frac{a+b+\lambda}{2}\right)\right|\right) \tag{5.4}$$

ve

$$\left|\int_a^b f(t)g(t)dt-\int_{b-\lambda}^b f(t)dt\right|\leq\frac{2^{s-1/q}}{(p+1)^{1/p}}\left((b-a-\lambda)^2\left|f'\left(b-\frac{\lambda}{2}\right)\right|+\lambda^2\left|f'\left(\frac{a+b-\lambda}{2}\right)\right|\right) \tag{5.5}$$

where $\lambda:=\int_a^b g(t)dt$.

Proof. From Lemma 1 and using the Hölder ineequality for $q>1$, and $p=\dfrac{q}{q-1}$, we obtain

$$\left|\int_a^{a+\lambda}f(t)dt-\int_a^b f(t)g(t)dt\right|$$

$$\leq\int_a^{a+\lambda}\left|\int_a^x(1-g(t))dt\right||f'(x)|dx+\int_{a+\lambda}^{a}\left|\int_x^b g(t)dt\right||f'(x)|dx$$

$$\leq\left(\int_a^{a+\lambda}\left|\int_a^x(1-g(t))dt\right|^p dx\right)^{1/p}+\left(\int_{a+\lambda}^{b}|f'(x)|^q dx\right)^{1/q}:=M \tag{5.6}$$

where p is the conjugate of q.

By the inequality (1.4), we have

□

$$\int_a^{a+\lambda} |f'(x)|^q dx \le 2^{s-1}\lambda \left| f'\left(a+\frac{1}{2}\lambda\right) \right|^q$$

ve

$$\int_{a+\lambda}^{b} |f'(x)|^q dx \le 2^{s-1}(b-a-\lambda) \left| f'\left(\frac{a+b+\lambda}{2}\right) \right|^q$$

whice gives by (2.14)

$$\begin{aligned} M &\le 2^{(s-1)/q}\lambda^{1/q} \left| f'\left(a+\frac{\lambda}{2}\right) \right| \left(\int_a^{a+\lambda} (x-a)^p dx \right)^{1/p} \\ &+ 2^{(s-1)/q}(b-a-\lambda)^{1/q} \left| f'\left(\frac{a+b+\lambda}{2}\right) \right| \left(\int_{a+\lambda}^{a} (b-x)^p dx \right)^{1/p} \\ &= \frac{2^{(s-1)/q}}{(p+1)^{1/p}} \left(\lambda^{1+\frac{1}{p}+\frac{1}{q}} \left| f'\left(a+\frac{\lambda}{2}\right) \right| + (b-a-\lambda)^{1+\frac{1}{p}+\frac{1}{q}} \left| f'\left(\frac{a+b+\lambda}{2}\right) \right| \right) \\ &= \frac{2^{(s-1)/q}}{(p+1)^{1/p}} \left(\lambda^2 \left| f'\left(a+\frac{\lambda}{2}\right) \right| + (b-a-\lambda)^2 \left| f'\left(\frac{a+b+\lambda}{2}\right) \right| \right) \end{aligned}$$

giving the inequality (2.12)

The inequality (2.3) may be proved in the same way using the identity (2.2), we sahll omit the deails.

Remark 2 The interested reader may obtain several inequalities for log-convex, quasi novex, r-convex and h- convex fınctions by replacing the condition $|f'|$

□

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