Thermal and Electrical Waves Experiment

Second Year Laboratory 2025 - 2026

Blackett Laboratory, Physics Department

Imperial College London

Overview of the experiment

Welcome to the 2nd year lab 'Waves' experiment! Over the next 8 sessions you will investigate and learn about the **propagation of waves through different media** and how the propagation of such **waves depends on frequency**.

The experiment is divided into 3 parts:

- In part 1 (**"Thermal Waves"**) you will investigate the propagation of **temperature oscillations through a solid** on a timescale of seconds. These are the waves that Fourier studied when he developed his famous harmonic analysis technique. Thus, an important aspect of the data analysis is the familiarisation you will gain with the important and generally applicable Fourier techniques. In addition, you will gain useful experience of data analysis as you correlate your temperature wave data. This should take approximately 3 sessions.

- In part 2 (**"Electrical Waves"**) you will investigate the propagation of **voltage oscillations along a lumped transmission line** on timescales of microseconds. You will gain hands on practical experience of measuring parameters of the transmission line such as characteristic impedance, phase and group velocities, and the effects of impedance matching and mismatching. This should take approximately 3 sessions.

- In part 3 ("**Extensions**") you are invited to investigate in more detail any aspect of the Thermal or Electrical Waves experiments. This is a chance to show creativity and explore a theme which interests you, and to prepare results for your report. The final 2 sessions are allocated for this part.

Despite the very different timescales involved for thermal and electrical waves, you will find a lot of similarities between the physics describing their behaviour and the mathematical tools we use to understand them.

How you will be assessed

As with the other cycles, you will be expected to ask for your lab book to be signed off in weeks 2, 3, and 4 of the cycle. Also, you must write your reflective log by the end of week 5 of the cycle.

In this experiment, you will submit slides for a 10-minute presentation, with about 5-minute Q&A, followed by feedback. The presentation will be completed in pairs in front of two demonstrators. The deadline for submission of your slides is the same as other cycles: **4 pm** on the Tuesday (Mon/Thu groups) or Wednesday (Tues/Fri groups) of week 5 of the cycle. The presentations will happen in person on Thursday (Mon/Thu groups) or Friday (Tues/Fri groups) of week 5 of the cycle. We expect equal contributions from the pair to both the presentation and Q&A.

The marking scheme for the presentation is the same as for the lab reports in the other experiments but has more weighting for the "concise transmission of information" component. More information about the presentation can be found in the introductory presentation to the Waves experiment.

Some additional remarks

This script contains all the basic information you will need for the whole experiment. It also contains **additional information in the "Section 4 – Appendix**" at the end which can be useful if you want more depth on physics behind the experiment.

The experiment has been planned as 8 x 3-hour teaching sessions, however you will need to spend time outside the lab hours to prepare for each session, perform further data analysis (which should be recorded in your lab book), etc.

What is expected of you as a student?

- **That you will go through the script <u>before each lab session</u>**. Each lab session has defined goals (linked to tasks throughout the script), thus spending time reading the script for the first time during the sessions is not a very efficient use of your time.

- **No one expects you to know all the physics and maths presented in the script**. This is a teaching lab and thus we expect that you will learn new things!

- That you will make annotations, write down questions, look up things that you do not fully understand (or that you have never heard of). Where? **In your lab book**.

- That you will talk to your demonstrators. However **always try thinking about a possible answer first**.

What is expected of your demonstrators?

- They will **teach and guide you throughout each session**. They will be inquisitive of your work throughout the experiment. Your demonstrators **are not 'invigilators', they are here to teach you**.

- They will help answering your questions however, most of the times, **they will guide you towards being able to the answer the question yourselves**. They may not know the answer to your question (there may not be one correct answer!). They will work together with you to help figure it out.

Part 1: Thermal Waves experiment

1.1. Introduction and aims of the experiment

This experiment is an investigation of the propagation, by thermal conduction, of a periodic temperature “wave” through a solid.

The aims of this experiment include familiarisation with:

- Quantitative ideas of thermal conduction
- The treatment of temperature as a wave
- Experience in data analysis

The overall objective of this experiment is to obtain a best estimate of the thermal diffusivity of brass and its associated uncertainty.

1.2. Experiment overview

The thermal diffusivity of a material D depends on its thermal conductivity, specific heat and density. We will assume that the thermal diffusivity is constant over the duration of the experiment.

To measure D, a regular temperature fluctuation is established at one point in a solid and the variation of temperature is measured at another point. The solid in this case is a cylinder of metal. You have access to three types: aluminium, brass, and steel. This script will assume that you are using brass, so please start with this, but feel free to try other materials as well.

A regular temperature wave is launched into the bottom face of the cylinder by using a Peltier heat pump to add or remove energy. Peltier devices, also known as thermoelectric heat pumps, can move heat from one side to another according to the electrical current flowing through them. You can control the voltage applied to the pump to add or remove heat from the cylinder.

This injection / removal of heat changes the temperature of the surface of the cylinder, and these temperature changes will propagate through the cylinder as waves. In this experiment, we will see how these waves propagate and investigate effects such as dispersion and attenuation, and how these manifest in arbitrary-shape temperature waves.

The cylinder has equally spaced temperature sensors along its length. These are not directly in the centre of the cylinder, but are all placed at the same radius. These are *thermistors,* whose resistance varies with temperature in a predictable way. To work out the temperature of each thermistor, the driver applies a small voltage to each thermistor, measures how much current flows through it, and calculates its resistance. See the appendix for more details.

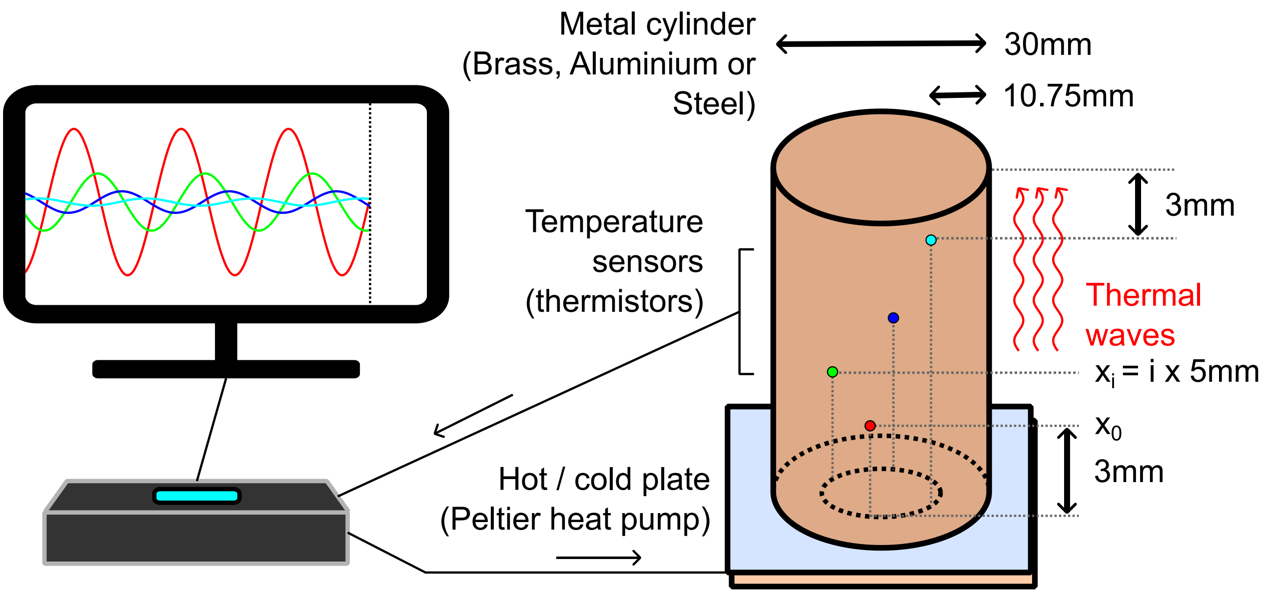


Fig. 1.1: Overview of the Thermal Waves experimental setup.

A metal cylinder placed on a hot / cold plate (a thermoelectric cooler, or Peltier heat pump). The heat pump is controlled by the computer and can heat / cool the bottom surface of the metal cylinder. Thermistor temperature sensors are buried inside the cylinder, and their temperature is logged by the computer. Thermal waves propagate along the cylinder, experiencing a frequency-dependent attenuation as they travel.

1.3. Experimental details

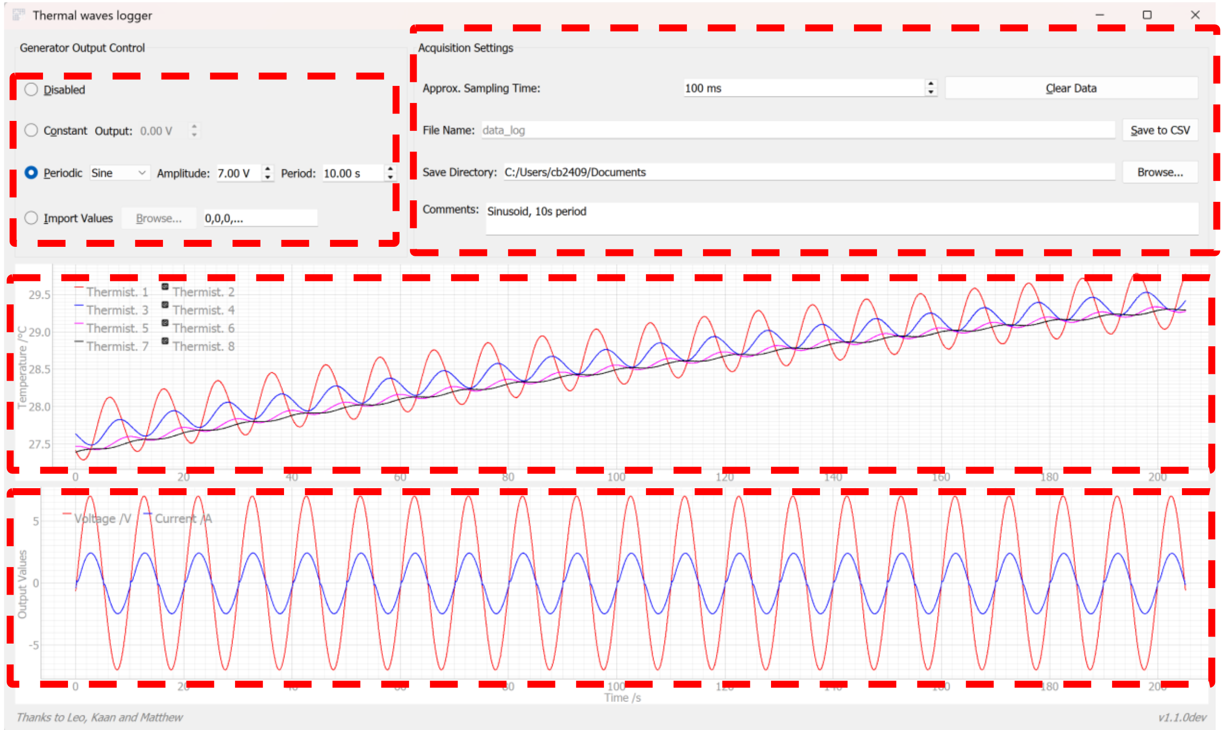
Fig. 1.1 shows a sketch of the experimental setup. You should have:

- A driver unit, connected to your computer with a USB cable
- A power supply for the driver unit
- Three thermal modules with brass, aluminium and steel cylinders
- Two cables to connect the temperature probes to the driver unit

The driver is controlled by a USB connection to your computer which has the “Thermal waves” software pre-installed. Let’s try it out.

1. Control of input temperature wave

You can send common waveforms or define your own.



2. Save data to .csv files

All the data shown will be saved: use the "Clear Data" button to start a new dataset.
Make sure to write a useful comment to identify your dataset.

3. Temperature logs

Temperature recorded from the 8 thermistors. You can hide channels by clicking the legend, but all channels will still be stored in the output files.

4. Output logs

The voltage and current applied to the heat pump. Note that the driver applies the requested voltage, but the resultant current varies slightly with temperature.

Fig. 1.2: Thermal waves computer interface

The interface to control the thermal waves experiment. (1) You can control the voltage applied to the Peltier heat pump. You will only need the pre-programmed waveforms to complete this script; however, you can also provide arbitrary functions if you wish. These should be saved to a text file as a list of voltages separated by commas – each point will be output for 100ms and the whole list will be repeated indefinitely. (2) Data can be saved to .csv files which you can read with Excel or Python. All data on the screen will be saved: to start a new dataset, clear the screen. (3 and 4) View of the temperature data from the 8 thermistors and the voltage / current outputs to the heat pump. Note that the data shown above has strong transient effects: this dataset would be hard to analyse! After enough time, the temperatures will reach a steady state.

Task 1.1:

a) Connect the **brass** thermal module to your driver and make sure the driver is powered on and connected to the computer.
b) Launch the "Thermal waves" program (see fig. 1.2) from the start menu of your computer.
c) Familiarise yourself with the experiment by sending some sine waves through the brass cylinder. Do they behave as you expected? What frequencies of wave work well? How large temperature variations can you cause on the first thermistor?

1.3.1. Note on transients

When you drive e.g. a sinusoidal temperature wave you will notice that there is a slow change in the average temperature of the cylinder which is written on top of your sinusoidal signal. These temperature transients can partially obscure the temperature wave you want to measure, as shown in Fig. 1.3.

One way of avoiding these transients is by allowing your wave to settle for at least 10 cycles before taking your data. *Be careful when collecting your datasets – a strong transient could lead you to large uncertainties in your thermal diffusivity values!*

For some insight into why these transients occur, you could research Peltier heat pumps – and think through what will happen when there is a difference between the "heating" vs "cooling" efficiency

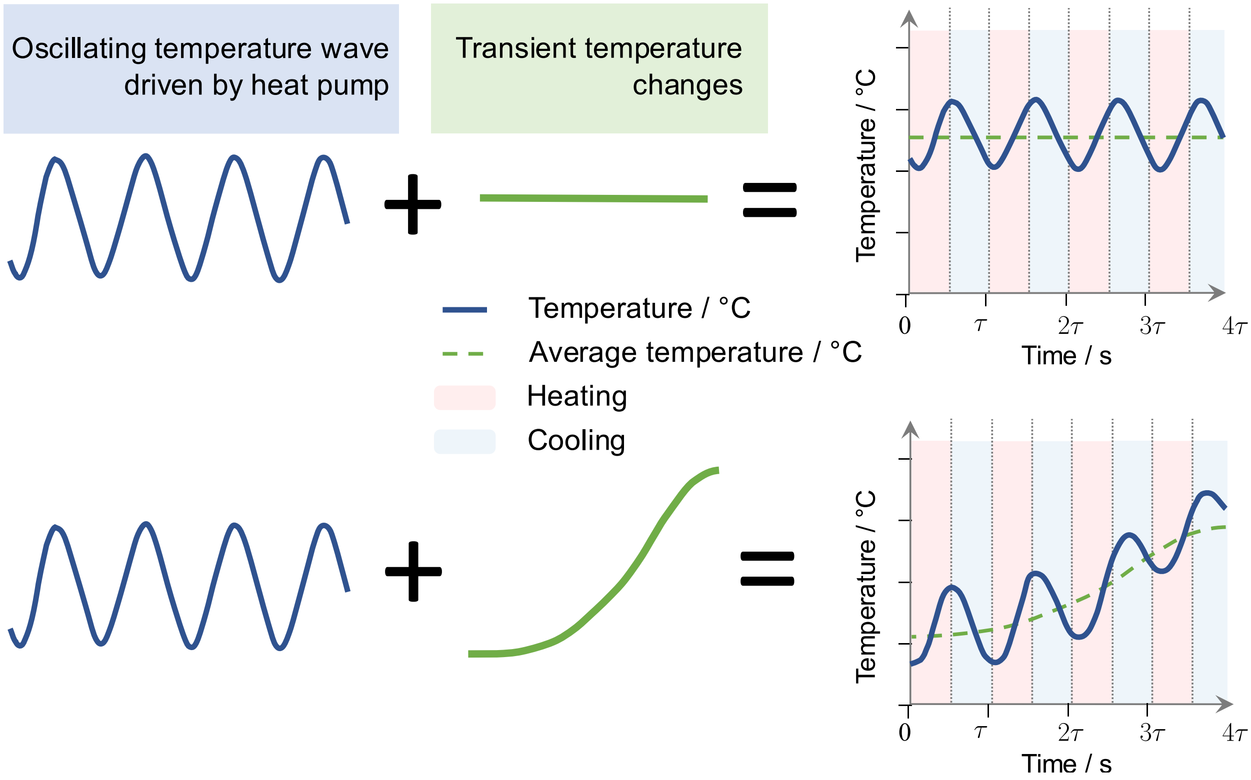


Fig. 1.3: Measured wave as a combination of transient temperature settling and driven oscillation. *(a) Data if the steady-state of the heat pump + cylinder system has been reached (b) Data if cylinder was initially at room temperature. In case (b), it takes a few cycles for the temperatures along the cylinder to reach steady state. Depending on how you analyse your data, this may distort your results unless avoided.*

1.4. Plane slab model

The flow of heat in a medium is described by the heat equation, which for a plane slab (i.e. in 1-dimension) is given by:

$$\frac{\partial T(x,t)}{\partial t} = D\frac{\partial^2 T(x,t)}{\partial x^2}, \tag{1.1}$$

where $T(x,t)$ is the temperature in the medium as a function of space and time and D is the thermal diffusivity of the medium.

Eq. (1.1) is a **diffusion equation**, which is a **linear differential equation** like Schrodinger's equation and the wave equation in the Electrical Wave experiment.

A solution for $T(x,t)$ decays in space and oscillates in time with angular frequency ω:

$$T(x,t) = T_0 + Ce^{-\sqrt{\omega/2D}\,x}\sin\left(\sqrt{\omega/2D}\,x - \omega t + \phi_0\right), \tag{1.2}$$

Where T_0, C and ϕ_0 are constants dependent on initial and boundary conditions.

For a derivation of this equation and its solution, please refer to the Appendix.

Task 1.2:

a) From Eq. (1.1), what are the units of thermal diffusivity?
b) Look up examples of typical materials and their thermal diffusivities. Remember to record your sources of information.
c) Prove that Eq. (1.2) is a solution to the heat equation

In the plane slab model, the cylinder is treated as a 1-dimensional slab of thickness $x_i = i\,\Delta d$ with the x-axis along the axis of the cylinder. Waves propagate along the x-axis, and the thermistor sensors are placed every $\Delta\mathrm{d} = 5\mathrm{mm}$.

We define $x = 0$ at the first thermistor.

Task 1.3: Consider a sinusoidal temperature of angular frequency ω starting from $x = 0$ and propagating along the cylinder.

a) Evaluate $T(x_i, t)$ at the first and second thermistors ($i=0,1$). Which one has greater amplitude? Is the thermal wave at x_1 leading or lagging the one at x_0?

b) The amplitude transmission factor is defined as
$$\gamma_i = \frac{Amplitude|_{x_i}}{Amplitude|_{x_0}},$$
and the phase lag is defined as
$$\Delta\phi_i = Phase|_{x_i} - Phase|_{x_0}.$$
Show that in the case considered here $\gamma_i = e^{-\sqrt{\omega/2D}\, i\Delta d}$ and $\Delta\phi_i = \sqrt{\omega/2D}\, i\Delta d$. Comment on their angular frequency dependence.

c) Hence show that the thermal diffusivity is given by:
$$D = \frac{\omega(i\Delta d)^2}{2\,\ln(\gamma_i)^2} = \frac{\omega(i\Delta d)^2}{2\,\Delta\phi_i^2}$$
This suggests that if either the attenuation or phase lag are measured for a known value of i and ω, then D can be found. This will be the principle behind your analysis.

1.5. First measurement of thermal diffusivity

We are now ready to make a measurement of thermal diffusivity by considering either the attenuation or the phase lag of sinusoidal waves across the thermistors.

In physics, it is usually best to solve a simpler problem before tackling a more complex one to test your method and build your understanding. We will therefore extract an estimate of the diffusivity by considering just two thermistors: i=0 and i=3. We will do this at a single frequency, $f = 1 / 10\text{s}$. Later we will scale this up to consider all the thermistors at various frequencies.

✔ **Task 1.4:** a) Drive a sine wave with $\tau = 10$ s and the maximum amplitude of 7 V. Show just thermistors 0 and 3.

b) From the software, without saving a .csv dataset, read out estimates of the phase lag and attenuation. Use these to calculate two estimates of diffusivity. You may want to wait until the temperature transients are sufficiently small.

c) Research typical diffusivities of brass. Do your estimates agree with each other and with your research?

1.6. Dependence on distance

Our model predicts that the amplitude of the temperature wave will decay **exponentially** with distance, and that the phase lag will increase **linearly**. By considering multiple thermistors we can test both these relations and extract a better estimate for diffusivity. We could do this manually, but processing our data with a computer will allow us to handle more and longer datasets, reducing the effect of statistical noise and allowing us to explore the physics of our experiment with higher resolution.

This script will assume that you use python, but feel free to use your language of choice, including Excel.

You can load a dataset using this python function:

```python
def load_dataset(path):
    import pandas as pd
    import re

    data = pd.read_csv(path, header=3)

    timestamp = data.iloc[:, 0].to_numpy()
    output_voltage = data.iloc[:, 1].to_numpy()
    output_current = data.iloc[:, 2].to_numpy()
    thermistor_temperatures = data.iloc[:, 3:].to_numpy()

    comments = re.search(r"Comments: (.*)$", open(path).read(), re.MULTILINE)[1]
```

```
    return timestamp, output_voltage, output_current, thermistor_temperatures,
comments


timestamp, output_voltage, output_current, thermistor_temperatures, comments = (
    load_dataset(r"path/to/your/data.csv")
)
```

Task 1.5: a) Write a python script to extract the phase and amplitude of a single thermistor's readings. Use this to extract the phase difference and attenuation for the i=0 and i=3 thermistors. Does it agree with your rough estimate from Task 1.4?

b) Repeat this for all the thermistors and plot the attenuation factor against distance.

c) Find a theoretical form for this function using the plane wave model and fit it to your data to extract D.

d) Do the same for the phase lag $\Delta\phi_{\mathrm{i}}$.

For this task and for many more, you will need to fit a model to your data. There are many ways to do this and you can use whichever you prefer. But, if you'd like a hint, you could try the "curve_fit" function from the python package SciPy:

https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html#scipy.optimize.curve_fit

1.7. Dependence on frequency

Our model predicts that both the attenuation and the phase lag should have a strong dependence on frequency. We should test this. Also, as we move to different frequencies, we might find deviations from our predictions as the assumptions contained in our model break down.

When testing a physical theory, it is important to find its limits – every theory we have so far stops working at some point, and it is at this boundary where the cutting edge of research exists.

By testing different frequencies we can get a handle on the **systematic uncertainty** of our measurement, at least the part of it that comes from breakdown in our model. These are typically the hardest uncertainties to characterise, and require physical insight into the experiment.

The other form of uncertainty is statistical, and we can estimate this too. One common method to do this is to repeat datasets in identical conditions and use the variance in their output to estimate the underlying noise in the measurement. We can do that with our data and then **propagate those uncertainties** into uncertainties on the parameters of our model, i.e. the thermal diffusivity.

✔ **Task 1.6:** a) Send sine wave of different periods, e.g. 15s, 20s, 30s, 60s. Repeat each dataset a few times and use this to estimate the noise level on your extracted phase and amplitudes. Don't forget to let the temperatures stabilize!

b) Plot the attenuation and phase lags for each frequency and extract estimates of D for each frequency with each method. How well does the model work at all these frequencies?

1.8. [OPTIONAL] Fourier analysis

If you have reached this point before session 4, have a try at this section. Otherwise, move on to the electrical section and you can come back later if you want to.

In our thermal waves datasets, we can create waves with non-sinusoidal shapes. These contain higher harmonics beyond the fundamental frequency and the total waves is the sum (or superposition) of all these harmonics. Because the heat diffusion equation is linear, each harmonic propagates independently so the total solution can be found by solving each harmonic separately.

This is a very powerful and general way of dealing with linear differential equations, which are a powerful way of dealing with the world! You will use this technique throughout your physics career.

1.8.1. The Fourier series

A periodic function in time $T(t)$ with period τ can be expressed as a Fourier series, i.e. an infinite sum of sine and cosine functions:

$$T(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi n}{\tau}\, t\right) + \, b_n \sin\left(\frac{2\pi n}{\tau}\, t\right) \right] . \tag{1.3}$$

where a_n and b_n are constant coefficients (amplitudes) given by the expressions:

$$a_n = \frac{2}{\tau} \int_0^{\tau} T(t) \cos\left(\frac{2\pi n}{\tau} t\right)\, dt \tag{1.4}$$

$$b_n = \frac{2}{\tau} \int_0^{\tau} T(t) \sin\left(\frac{2\pi n}{\tau} t\right)\, dt \tag{1.5}$$

Alternatively, Eq. (1.3) can be re-written in ‘amplitude-phase’ form as:

$$T(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \beta_n \sin\left(\frac{2\pi n}{\tau}\, t \, - \, \Delta\phi_n\right) . \tag{1.6}$$

where β_n and $\Delta\phi_n$ are the amplitude and phase lag respectively:

$$\beta_n = \sqrt{{a_n}^2 + {b_n}^2} \tag{1.7}$$

$$\Delta\phi_n = \, -\arctan\,(a_n/b_n) \tag{1.8}$$

Each component of these series labelled by *n* represents a ‘harmonic’ mode, a sinusoidal function with angular frequency $\omega_n = \frac{2\pi n}{\tau}$.

In the expressions above, a_n and b_n are real coefficients, however it is often convenient to use the complex convention instead to write

$$T(t) = \sum_{n=0}^{\infty} k_n \, e^{i\omega_n t}$$

Where k_n are now complex coefficients. The "*complex convention"* is that we implicitly take the **real** part of this expression when calculating T.

If you feel comfortable with this, try using it here instead of real coefficients.

Task 1.7: Fourier analysis [Optional task]

a) Use a square wave with period 60s to create a non-sinusoidal wave on the first thermistor. Note that the temperature wave looks triangular – why?

b) Use python to extract the a_n and b_n coefficients from each thermistor's dataset. You will need to use equations 1.2 and 1.3, and to numerically integrate your data. You will need to trim your datasets so they are an integer number of periods long.

c) Plot some of the datasets with your Fourier approximations overlaid on top. Do they agree? Did you use enough harmonics?

d) Use these coefficients to calculate the phase lags and attenuation for each harmonic. Plot these on the same axes as the phase lags and attenuations you measured using sinusoidal waves. Do they agree?

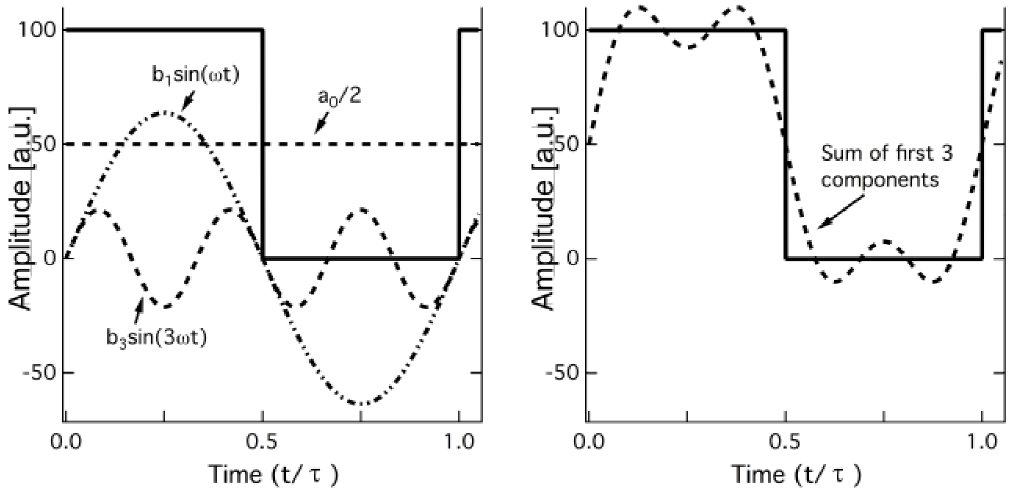


Fig. 1.4: An example of Fourier analysis. *A square wave (bold) is broken down into* $n=3$ *Fourier components (harmonics). As* $n\rightarrow\infty$*, the Fourier approximation tends closer to the true waveform. This calculation could have been done analytically, however for datasets acquired by an experiment we must use numerical integration.*

Visit the ImpVis (Imperial Visualisations) website and check out the 'Fourier series' visualisation to see a similar plot to Fig. 1.4 and much more:

https://impvis.co.uk/launch/fourier-series/

Part 2: Electrical Waves experiment

2.1. Introduction and aims of the experiment

You will study the propagation of electrical signals, specifically **square pulses** and **sine waves**, in **a dispersive medium**. Think about the most common medium to transport such signals, a cable, a continuous pair of conductors arranged in a cylindrical symmetry. We will **model such a cable as a 'lumped transmission line'**, a series of discrete inductors and capacitors.

The aims of this experiment include familiarisation with:

- A lumped transmission line circuit and its main parameters, both theoretically and experimentally.
- The reflection of waves at interfaces between media of different impedances.
- The concept of dispersion, phase and group velocity.

These phenomena, studied here in the context of electrical signals in a transmission line, have wider application in many fields of physics, for instance the propagation of information in fibre optics, antennas, etc.

2.2. Theory

You will work with a lumped transmission line circuit consisting of 40 sections as illustrated roughly in Fig. 2.1. "Lumped" means that the *L* and *C* occur as individual inductors and capacitors, in contrast to a "continuous" line, such as a coaxial cable where the *L* and *C* are distributed along the inner and outer conductors.

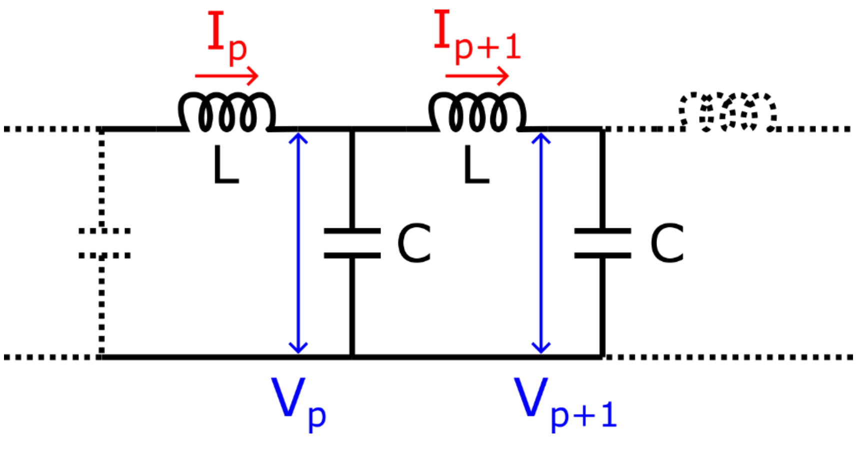


Fig. 2.1: *Circuit diagram of a single section of the lumped transmission line circuit.*

Under certain conditions, we find that the voltage *V(x,t)* is described by the wave equation:

$$\frac{\partial^2 V(x,t)}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V(x,t)}{\partial x^2} \tag{2.1}$$

where *x* is a spatial coordinate along the sections of the transmission line.

As this is a wave equation, we can distinguish the term *1 / LC* as being related to the speed of propagation of the wave, the phase velocity, measured in sections/sec:

$$V_{phase} \quad = \quad 1/\sqrt{LC} \tag{2.2}$$

A solution for *V(x,t)* is oscillatory in space and time with a frequency ω and wavenumber *k*:

$$V(x,t) = V_0 \exp\big(i(kx - \omega t)\big) \tag{2.3}$$

where V_0 is a constant dependent on the initial conditions. Here we use the complex convention, so you must take the real part of V to find its value in the time domain.

> *For a detailed derivation of the Wave equation in a lumped transmission line, please refer to the Appendix. What assumptions underly this derivation? Can you repeat the derivation without making those assumptions? How does this affect the circuit's behaviour?*

2.2.1. Characteristic impedance and cut-off frequency

To derive the characteristic impedance of the line Z_0, we assume an infinitely long version of the circuit in Fig. 2.1 with inductors and capacitors having impedances Z_L and Z_C respectively. Thus, it is possible to derive the following expression (as shown in the 'Feynman Lectures', Vol. 2, 22-6):

$$Z_0 \quad = \quad \sqrt{Z_L Z_C (1/(1 + Z_L/4Z_C))} \tag{2.4}$$

and replacing $Z_L = i\omega L$ and $Z_C = 1 / i\omega C$ this results in:

$$Z_0(\omega) \quad = \quad \sqrt{(L/C)(1/(1 - \omega^2 LC/4))} \tag{2.5}$$

In the limit of $\omega \ll \omega_c = \sqrt{4/LC}$, the characteristic impedance is constant:

$$Z_0 \quad \approx \quad \sqrt{L/C} \tag{2.6}$$

where ω_c is known as the 'cut-off frequency'.

2.2.2. Reflections at changes of impedance

If a signal propagating in a medium A of impedance Z_a meets an interface with a medium B where the impedance changes to Z_b the signal will be only partially transmitted from A to B with the rest being reflected back into A.

If the signal is a voltage incident from A with amplitude V_i then a signal with amplitude V_r is reflected from the interface back into A. The voltage reflection coefficient R can be found to be

$$R = \frac{V_r}{V_i} = \frac{(Z_b - Z_a)}{(Z_b + Z_a)} \quad . \tag{2.7}$$

R is plotted as a function of Z_b / Z_a in Fig. 2.2 and some key values are tabulated below.

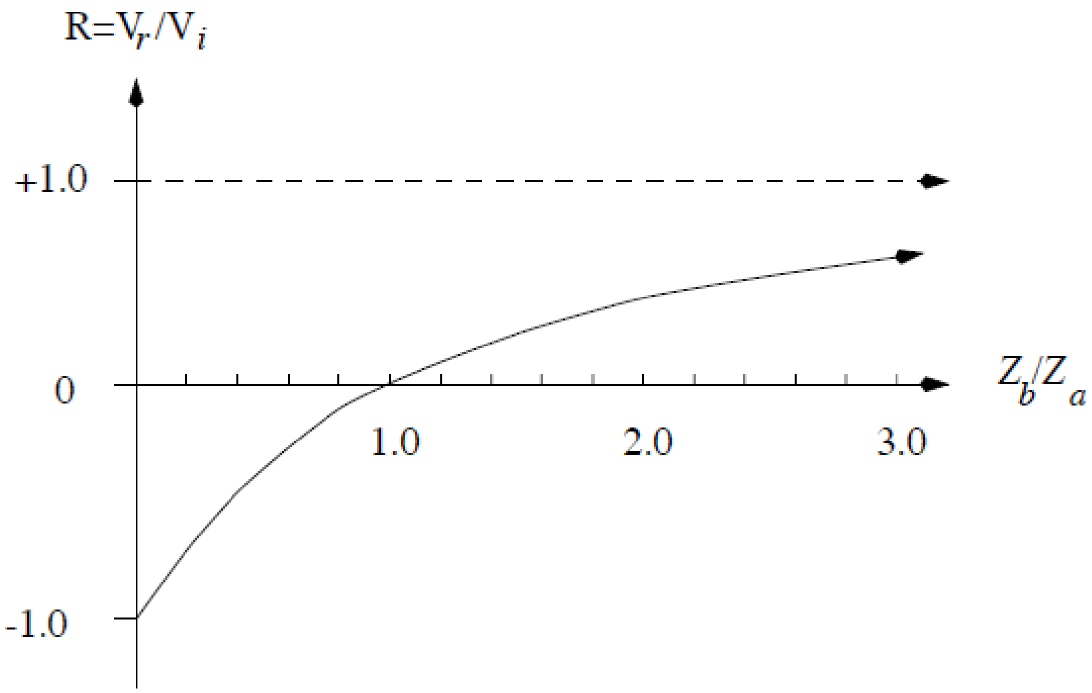


Fig. 2.2. *The voltage reflection coefficient V_r / V_i as a function of Z_b / Z_a.*

So if	$Z_a = Z_b$	$R = 0$	i.e. there is no reflection
if	$Z_b = 0$	$R = -1$	i.e. complete reflection with inversion
if	$Z_b = \infty$	$R = +1$	i.e. complete reflection without inversion

Task 2.1: Transmission line theory

a) Demonstrate that Eq. (2.3) is a solution to the wave equation and, in doing so, derive the dispersion relation $\omega(k)$.

b) Research about what are transmission lines and some known examples/applications. **Remember to keep track of your sources**.

c) Look up typical values of L, C, Z_0 and V_{phase} for a radiofrequency cable, for instance the coaxial BNC cables in your experimental kit. These are typically 'RG-59/U' or similar.

2.3. Experimental setup

The equipment you will need for this part of the experiment is composed of:

- 1 x lumped transmission line circuit made of 40 sections
    - Note that each circuit is labelled and may have slight differences from others – Use the same one throughout the experiment!
- 1 x oscilloscope: Rohde & Schwarz model RTB2004, 100 MHz bandwidth, with built-in signal generator (and MANY other features, e.g. touch screen)
- 2 x BNC probes
- A digital multimeter
- Coaxial BNC cables

Details of the 40-section lumped transmission line circuit are shown in Fig. 2.3. Note that, in the actual circuit, the 40 sections are arranged in a “C” shape starting from the BNC input on the top-right part of the circuit and ending with the switch and variable resistor on the bottom-right. There are 40 inductors and 43 capacitors, each with the same nominal value of L = 330 μH and C = 15 nF.

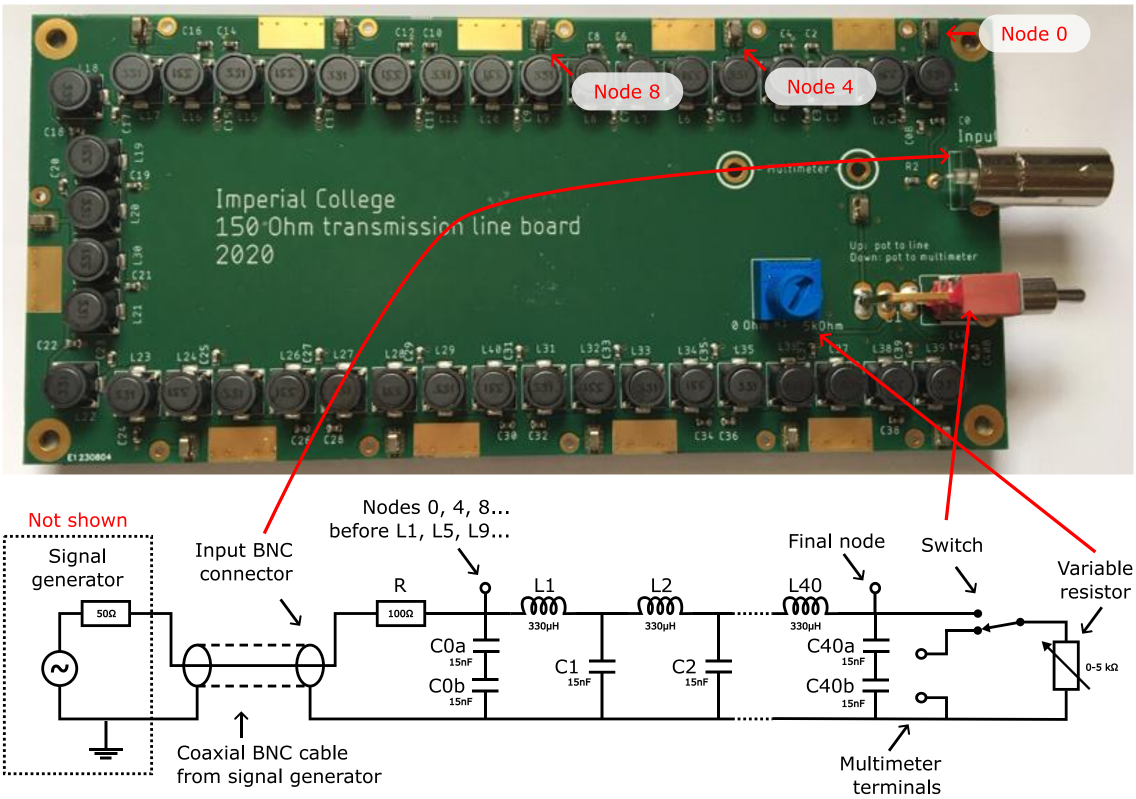


Fig. 2.3: Details of the lumped transmission line circuit. *(top) Photo of the circuit (bottom) Equivalent circuit. Each capacitor and inductor in the circuit have the same value of L and C.*

2.4. Measurement 1: Square pulses

2.4.1. Initial investigations

Your first task is to produce a series of square pulses and send them through the transmission line to investigate the behaviour. Note that square pulses are not exactly the same as square waves (like those in the Thermal Waves experiment). Although square pulses are still periodic, they will be set up such that only a single pulse propagates through the transmission line circuit.

Your oscilloscope is a Rohde & Schwarz model RTB2004 with a built-in signal generator (Fig. 2.4). The output signal comes from the 'Aux Out' BNC connector on the bottom left of the oscilloscope. There are 4 input BNC channels ('Ch1' to 'Ch4) on the bottom right.

About the oscilloscope: *Besides the actual transmission line circuit, your oscilloscope is the most important piece of equipment in this experiment. It will allow you to send and measure signals in the line with high accuracy. Think of the oscilloscope as a multimeter combined with a computer. Do not be discouraged by the daunting number of buttons and options displayed either in the panel or on screen. You might find that many times a specific option can be found in several different ways!*

Fig. 2.4: *Front view of the Rohde & Schwarz RTB2004 oscilloscope. The labels show (1) 'Aux Out' connector from the built-in signal generator, (2)-(3) input BNC channels 'Ch1' and 'Ch2', (4) 'Universal rotary knob', useful to select values when it is blue, (5) 'Gen' button to quickly access to the signal generator options, (6) Use the 'Autoset' and 'Preset' buttons at your own risk! (7) 'Apps' quick access button. Note that knobs such as (4) have push functions which can be quite useful to set up your signals quickly.*

A recap of oscilloscopes: Oscilloscopes are multi-purpose devices used to measure voltages and plot how they change over time. They are ubiquitous in research physics laboratories because they allow you to accurately plot and analyse almost any quantity that you can measure.

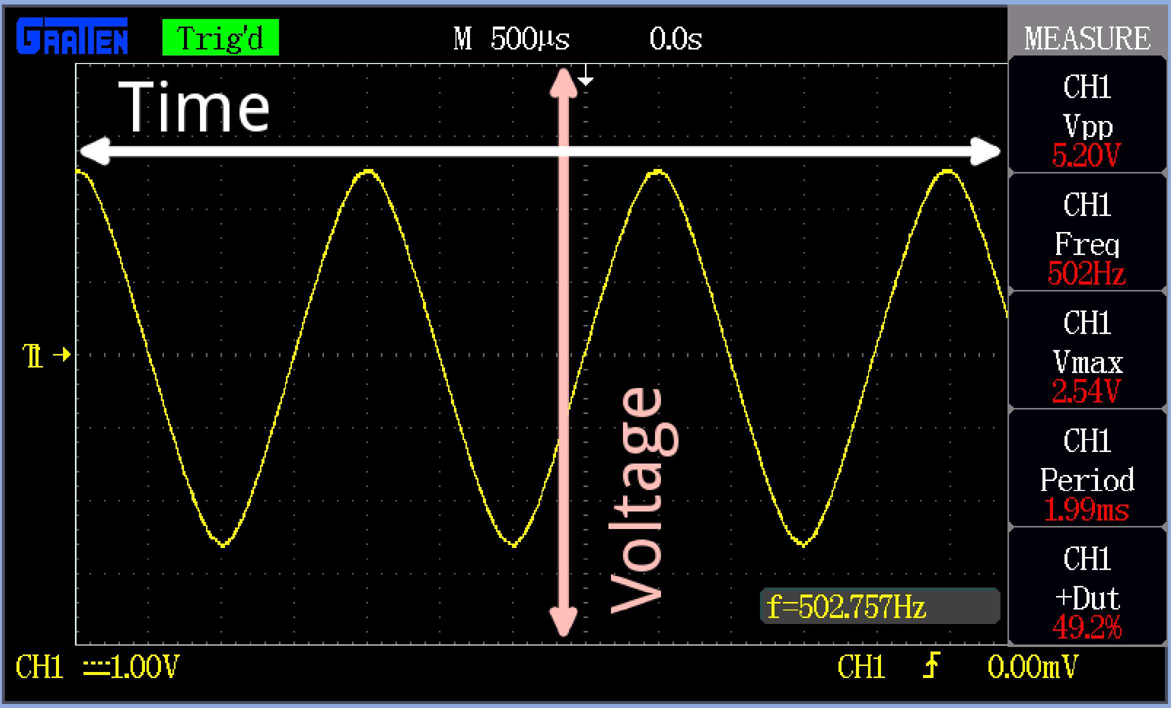


Figure reproduced from https://learn.sparkfun.com/tutorials/how-to-use-an-oscilloscope

Normally, an oscilloscope plots a measured voltage on the Y axis against time on the X axis. You control the scale of both the X and the Y axes, allowing you to zoom into interesting feature of the signal you are measuring. You also control when an oscilloscope starts measuring by setting its trigger level. For a recap of these concepts, see https://learn.sparkfun.com/tutorials/how-to-use-an-oscilloscope/all.

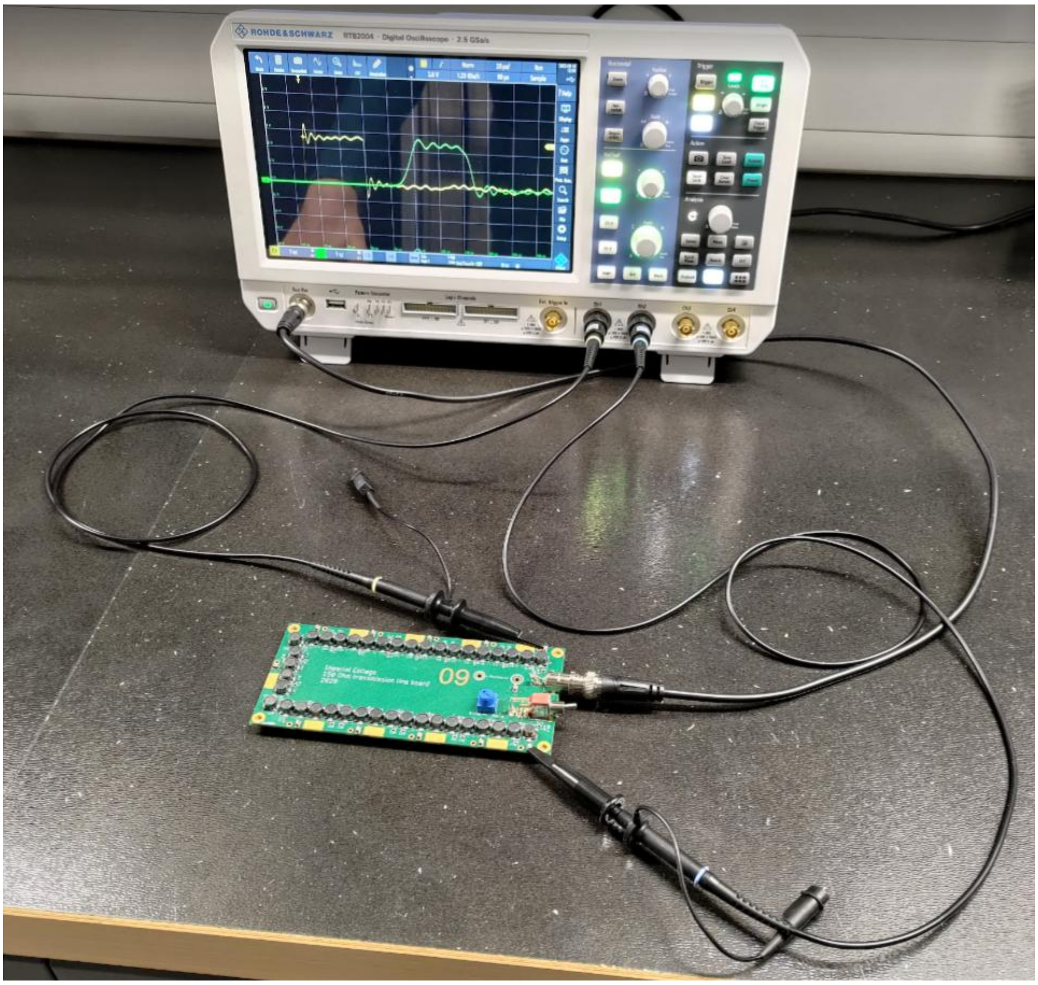


Fig. 2.5: *View of the connections needed between the oscilloscope and transmission line, and the signals you will measure. Note that the probes can conveniently hook into the hoops on the circuit board: here they are shown hooked into nodes 0 and 40.*

Task 2.2: Setting up a square pulse using the oscilloscope's 'Function Generator'

a) Use the "Pulse" function of the Function Generator (see (5) in Fig. 2.4) to produce a 50 μs pulse ranging from 0 V to 5 V and repeating at 2 kHz. Ensure that the "Load" option of the signal generator is set to "High-Z".

Hint: You will need to use the duty cycle setting to achieve this.

b) Use a BNC coaxial cable to deliver this pulse to the "Input" of the transmission line and use a probe on channel 1 of the oscilloscope to monitor the pulse at the first test-point of the circuit. Adjust the oscilloscope to clearly visualise the pulse.

You can save oscilloscope screenshots and data traces to the USB flash drive provided by inserting it in the front USB slot.

- *To save a screenshot (.PNG by default), press the*  *button*
- *To save data traces, press* 

and go to "Waveforms" to save voltage as a function of time as, e.g. .CSV or .TXT

Remember: Do not take the USB stick home with you!

Alternative method to save data

You can also copy the data onto your computer through the USB cable plugged into your oscilloscope. To do this, open File Explorer and navigate to:

This PC → Rohde&Schwarz RTB2004 → Live Data → Channel → Display Data

In this directory, you will find files called "CHx.CSV" which contain the live data for each active channel on your oscilloscope. You may copy these files into OneDrive using File Explorer, or by opening them in Excel and using "Save as".

Whichever method you use, remember to name your data files clearly, so that you can keep track of which data they contain

Task 2.3: Sending square pulses through the transmission line

a) Have a careful look at your transmission line circuit and make sure you can distinguish the main components following the schematic diagram in Fig. 2.3.

b) Check that the switch at the end of the line is 'down' (i.e. open circuit, variable resistor disconnected from the circuit). Plug a 2nd probe into CH2 and hook the probe into a series of different points along the line. Keep a record of your measurements and explain the time delays and behaviour of the pulses observed.

   What could be causing the pulse to lose its ideal square shape?

2.4.2. Measure the characteristic impedance of the line

Now move the switch 'up' to connect the variable resistor (potentiometer, 0 to 5 kΩ) at the end of the circuit and look at the behaviour of the square pulse as you vary the resistance.

Task 2.4: Pulse reflection and characteristic impedance Z_0 of the line

a) Connect the potentiometer to the circuit by flicking the switch at the end of the line to the 'up' position. Vary the potentiometer and look at the pulses along the line at a fixed position, e.g. section 20 half-way along the circuit. Explain the change of amplitude and polarities observed and the relation with the value of the variable resistor by referring to the content in Section 2.2.2 ('Reflections at changes of impedance').

b) Vary the potentiometer until the reflected pulse vanishes. Measure the value of resistance in the potentiometer (to do so, make sure you disconnect the potentiometer from the circuit by leaving the switch 'down').

   This is the characteristic impedance Z_0 of the transmission line. Explain.

c) Estimate the uncertainty δZ_0 of your measurement.

d) Compare you calculated value of Z_0 to the value given by Eq. (2.6) using the manufacturer's values for the inductors and capacitors of $L = 330 \pm 20\%$ µH and $C = 0.015 \pm 10\%$ µF. Are these two values compatible?

2.4.3. Measure the speed of propagation of pulses

Task 2.5: Speed of propagation of pulses

During this measurement, ensure that the line is terminated to avoid reflection.

a) Measure the propagation speed of the pulse through the transmission line in units of 'sections/second'. (You may find the Cursor feature useful here). By taking velocity measurements over various intervals along the transmission line, estimate the uncertainty of your measurement.

b) Now calculate the phase velocity of the transmission line from Eq. (2.2) and its uncertainty using the manufacturer's values for the inductors and capacitors. In this case your values of L and C should be in units of [H/section] and [F/section].

2.5. Measurement 2: Sine waves

You will now investigate the frequency response of the transmission line by sending sine waves with variable frequency down the line. This contrasts with the square pulse with, approximately a single 'low' frequency. If the transmission line is dispersive, the speed of the waves going through the line will change as a function of frequency.

Your aim in this part is to measure the dispersion relation of the transmission line, i.e. a graph of ω vs *k*, where ω is the angular frequency and *k* is the wavenumber.

2.5.1. Speed of propagation of sine waves with different frequencies

In general, the phase speed of a wave is defined as:

$$V_{phase} = \lambda f \tag{2.8}$$

where λ is the wavelength, in units of sections in this case, and f the frequency in units of Hz (for electromagnetic waves in vacuum, $V_{phase} = c$).

To measure the phase speed for a given frequency, we will look at the relative phase of a sinusoidal wave at two fixed points along the line, and in doing so estimate the wavenumber in terms of the 'distance' between those two points. This is shown schematically in Fig. 3.6.

Setup

Firstly, adjust the variable resistor so there are no reflections of signals from the end of the line back towards the pulse generator, i.e. 'match' the end of the line with its characteristic impedance so it seems to be infinitely long.

As with the square pulses, connect 'Aux Out' to the input of the transmission line using a coaxial BNC cable. On the 'Function generator', chose 'Sine' with a start frequency of '20 Hz', a peak-to-peak amplitude of '4 V' and an offset of '0 V' to obtain a sine wave that oscillates between -2 V and 2 V.

Now connect your first probe (CH1) at the start of the circuit and your second probe (CH2) at the end of your circuit on section 40.

To look at the phase difference of the sine wave at the input and output of the line, we could look at the two signals superimposed visually on the oscilloscope display. However, it is much more accurate to display phase differences by using the 'XY' mode on the oscilloscope, also known as 'Lissajous figures'. Press the 'Apps' button ((7) in Fig. 3.4) and chose the 'XY' view. You should see something similar to Fig. 3.7.

> *When working in 'XY mode', make sure you adjust the values of vertical position, magnitude, time scale and number of samples as you vary the frequency, as these will influence how the Lissajous figures are displayed.*

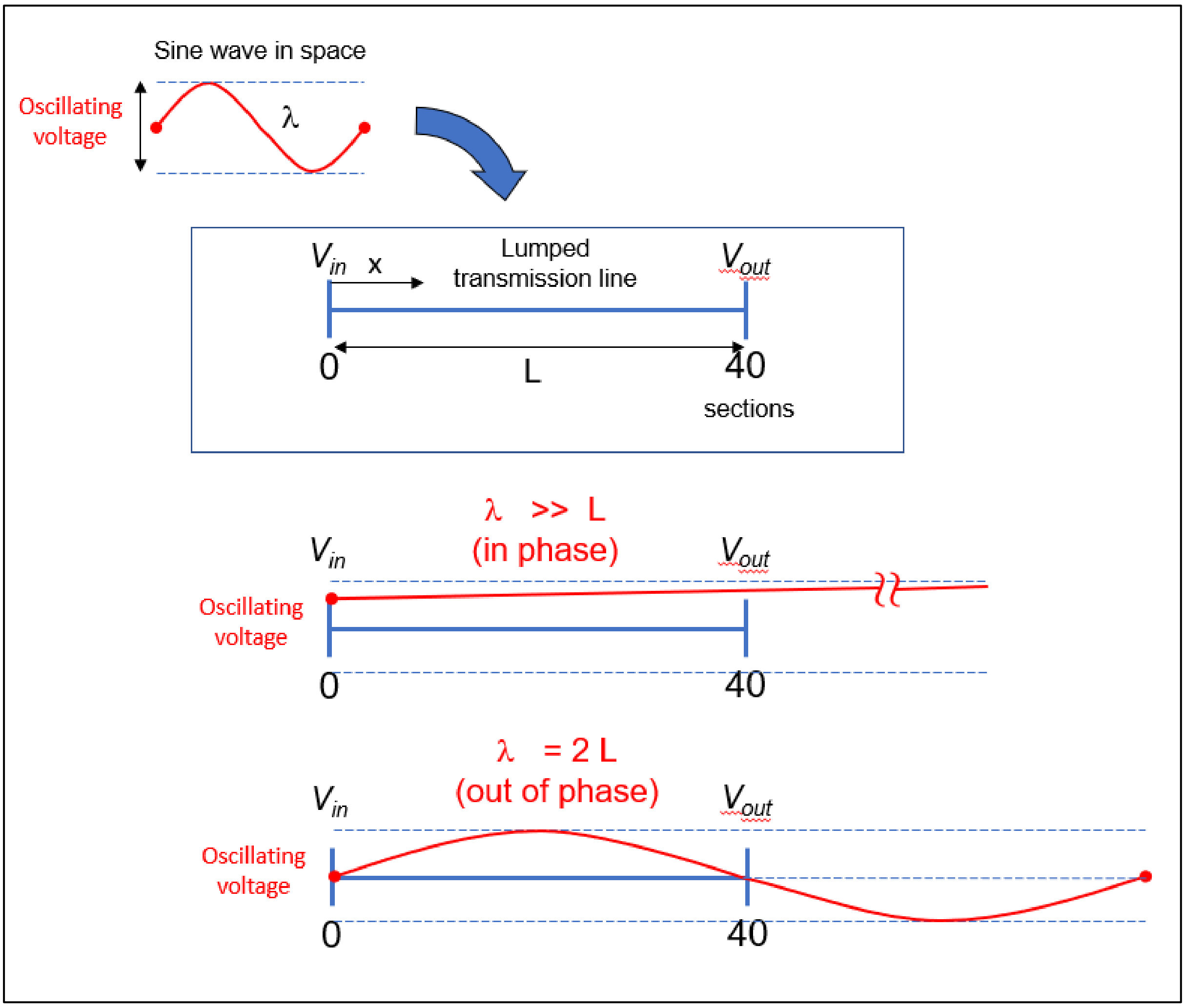


Fig. 2.6: *Schematic diagram showing a wave with a variable wavelength λ sent through a transmission line with length L = 40 sections.*

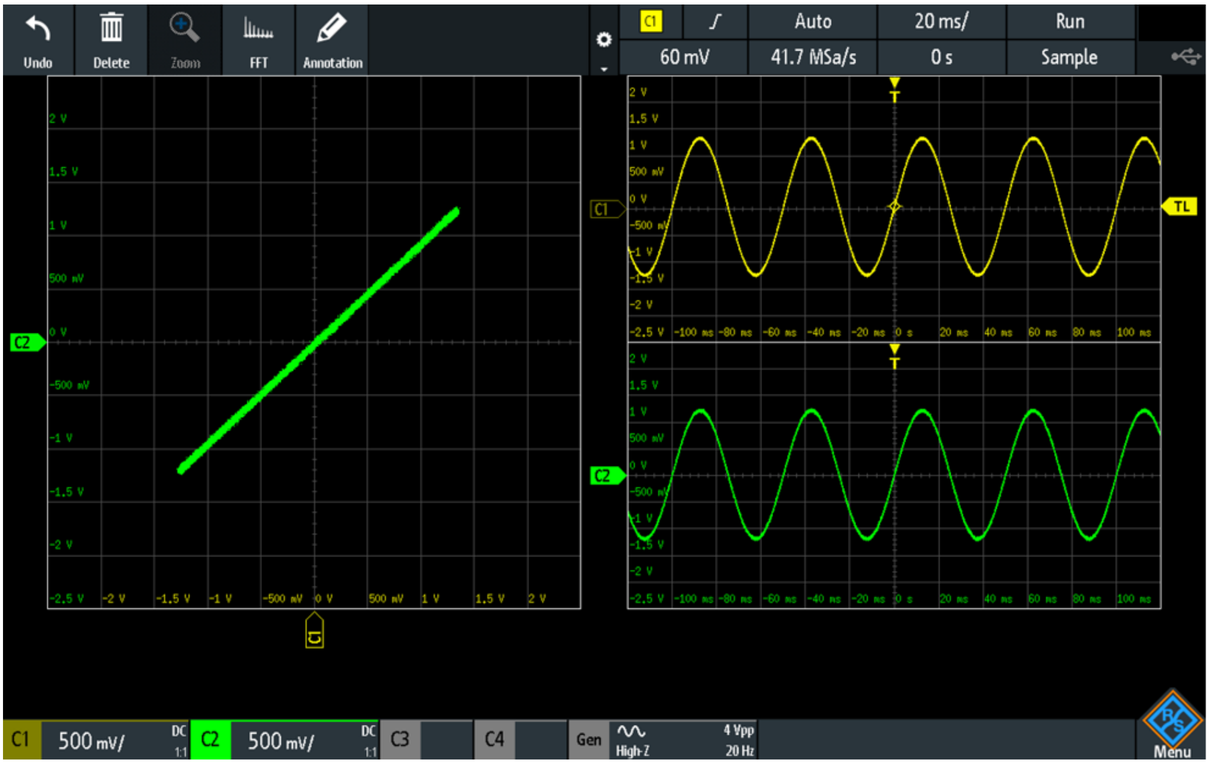


Fig. 2.7: *Oscilloscope screenshot showing the XY mode for sine waves at the input and output of the transmission line.*

Note that this image is for visual purposes only as you probably can't read the small numbers and labels. This is an example of what you SHOULD NOT DO in your presentation.

Task 2.6: Sine waves - Measuring phase differences in the line

a) Explain, with the aid of figures, the shape of the Lissajous figures as you increase the frequency.

b) Using the sketch in Fig. 2.6, find a general expression for the wavelength λ_n, as a function of the length of the line and whether the wave is in or out of phase. From this, calculate an expression for the wavenumber k_n.
Hint: From the first 2 examples in the figure, work out the next 'in/out' of phase case and carry on from there.

c) Take the measurements below (and remember to record uncertainties!):

- Measure all the frequencies at which the wave is in and out of phase by scanning the frequency range starting from ~20 Hz, where the signals are nearly in phase, to the maximum frequency at which signals will propagate down the line.
- As you scan in frequency, measure the voltage amplitudes at the input and output of the line and calculate their 'amplitude ratio', i.e. V_{out} / V_{in} as a function of frequency.

You should now have a table with frequency, whether the wave is in/out phase, V_{in}, V_{out} and amplitude ratio. Now:

Task 2.7: Sine waves - Frequency response of the transmission line

a) From your measurements, plot the dispersion relation of the line ω vs k. Explain the main features of this plot

b) Work out an expression for the group velocity $V_{group} = \Delta\omega / \Delta k$.

c) Plot V_{phase} and V_{group} vs frequency in the same graph. How is V_{phase} related to your measurements with square pulses? How is V_{phase} and V_{group} related to the dispersion relation?

d) Plot amplitude ratio vs frequency and measure the cut off frequency of your line (with its uncertainty). Compare your estimate with the theoretical value derived from Eq. (2.5).

You can also look at the frequency response of the lumped transmission line by performing Fast Fourier Transform (FFT) tool in your oscilloscope. Try having a go at this even if it is purely treated as a black box!

Part 3: Extensions

For the last two sessions, the goal is to investigate in detail any part of the Thermal or Electrical experiments you found interesting. This is a good opportunity to plan what will go in your presentation – pick a theme, identify a research question, and demonstrate to us that you understand it well! Don't forget to characterise measurement errors and be sure to link your results to the underlying physics.

A few ideas you could explore are given below. However, creativity is encouraged (but not for the sake of it: you should aim to explore interesting physics, not just to do something unusual).

Better ways to take the electrical wave data

The Lissajous plot is only one method to measure the amplitude and phase of sine waves transmitted through the circuit – other methods are possible! E.g., you could send any periodic signal through the transmission line, and analyse the amplitude ratios and phase differences of the Fourier coefficients at the beginning and the end of the line. Tips:

- Be sure to use input waves which excite a sufficiently broad range of frequency components up to the cutoff frequency
- Be careful when assigning the Fourier component phase lags (2π's!), and discard harmonics which are too weakly excited by the input wave.

Better lumped transmission line theory

It turns out that the wave equation model presented in equation 3.1 (and derived in Appendix 5.2) wasn't very accurate for describing the amplitude transmission or dispersion relation. Can you find a more accurate model?

If statistically significant discrepancies remain between your model and the data, what could account for these discrepancies? Can you test your theories by taking different datasets?

Pulse distortion and data transmission

Transmission lines are often used to transmit data, usually in digital formats where the goal is to send as much data as possible in a given amount of time. For this application, dispersion of pulses is a problem! You could investigate how our transmission line model would affect data transfer by answering questions like:

a) Why is the pulse getting distorted? Does the pulse distortion match your expectations based on the frequency-domain data you gathered in Part 3?

b) Decrease the duty cycle from 10% to 1%. What happens to the pulse as it propagates down the line? Comment on why this could be important in practical transmission lines. What would happen if you sent data down the line? (Try, if you like).

Thermal waves in different materials

You characterised the thermal properties of brass in Task 1. How do steel and aluminium behave?

Thermal waves modelling

For low-frequency thermal waves, the data does not perfectly match the model in Eqn. (1.2). Looking at the Appendix section 4.1, is there a more complete model which could match the data better?

Fourier transform

We have suggested using Fourier analysis often during this script, however a more common technique in modern technology is the Fourier transform. This allows you to express any signal as an integral of harmonics, not just cyclical ones.

Use this to explore non-repeating signals in either the thermal or electrical experiments.

Part 4: Appendix: Derivations and additional information

4.1. Thermal Waves – Additional information

4.1.1. Heat equation in 1-dimension - Plane wave model

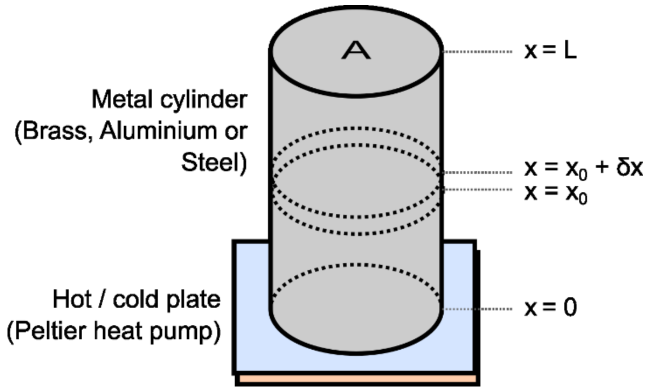


Fig. 4.1: Geometry of the plane-wave thermal model of the metal cylinder. *Heat propagation is modelled to occur along only one dimension, x. At $x = 0$, the Peltier heat pump is modelled to inject a sinusoidal heat wave. At $x = L$, the air is modelled as non-conductive, i.e. no heat is transferred beyond the $x = L$ boundary.*

In the geometry shown in Fig. 4.1, the temperature gradient is assumed to exist only in the x direction, so heat conduction occurs in one dimension only. The rate of heat conduction per unit area is given by Fourier's Law[1] as the product of the thermal conductivity κ and the temperature gradient $\partial T(x,t)/\partial x$ in the direction of conduction. Consider the slab volume element in Fig. 4.1: if Q is the heat contained in the slab, the net rate of heat flowing into the volume element at $x = x_0$ is

$$\frac{\partial Q}{\partial t} = -A \cdot \kappa \left(\left.\frac{\partial T}{\partial x}\right|_{x=x_0} - \left.\frac{\partial T}{\partial x}\right|_{x=x_0+\delta x} \right) \tag{4.1}$$

where $\mathrm{T} = \mathrm{T}(\mathrm{x}, \mathrm{t})$ represents the temperature. Using the chain rule and the mass of the slab $\mathrm{m} = \rho \cdot A \cdot \delta x$, where ρ is the material's density and A is cross-sectional area of the cylinder, we can rewrite

$$\frac{\partial Q}{\partial t} = m \left(\frac{1}{m} \frac{\partial Q}{\partial T} \right) \frac{\partial T(x,t)}{\partial t} = m\, c_V \frac{\partial T(x,t)}{\partial t} , \tag{4.2}$$

where c_V is the constant volume heat capacity of the material. Substituting our new expression for $\partial Q / \partial t$ into Eq. (4.1), re-arranging the terms and defining diffusivity $\mathrm{D} = \frac{\kappa}{\rho \cdot c_V}$ gives

$$\left.\frac{\partial T(x,t)}{\partial t}\right|_{x_0} = \frac{\kappa}{\rho \cdot c_V} \left(\frac{\left.\frac{\partial T(x,t)}{\partial x}\right|_{x_0+\delta x} - \left.\frac{\partial T(x,t)}{\partial x}\right|_{x_0}}{\delta x} \right) \xrightarrow[\delta x \to 0]{} D \left.\frac{\partial^2 T(x,t)}{\partial x^2}\right|_{x_0} , \tag{4.3}$$

the 1D diffusion equation we have cited in the main text as Eq. (1.1).

---

[1] For more about Fourier's Law, see e.g. https://en.wikipedia.org/wiki/Heat_flux and its references

4.1.2. Solution for 1D diffusion equation

Using a trial solution of the form $T(x,t) = X(x)\theta(t)$ we can write a solution to (4.3):

$$T(x,t) = \left(C_+ e^{\beta x} + C_- e^{-\beta x}\right) e^{-i\omega t}, \tag{4.4}$$

where C_+ and C_- are complex amplitudes and β is a complex propegation coefficient. Plugging this into Eq. (4.3), we can derive the constraint

$$\beta = \sqrt{\frac{\omega}{2D}}(i-1), \tag{4.5}$$

where the exponentially decaying solution (positive β) represents waves propagating upwards through the metal cylinder, and negative β represents downwards.

Note that we have used the complex convention to simplify the maths: physically, the temperature of the cylinder is the real part, $\mathrm{Re}\big(T(x,t)\big)$.

4.1.3. Boundary conditions

At $x = 0$, we assume that the Peltier heat pump injects heat sinusoidally with complex amplitude $\dot{Q}_0$:

$$\frac{\partial Q}{\partial t} = \dot{Q}_0 e^{-i\omega t} = -A\kappa \left.\frac{\partial T}{\partial x}\right|_{x=0} \quad , \tag{4.6}$$

At $x = L$, we assume zero heat loss from the top of the cylinder:

$$\frac{\partial Q}{\partial t} = 0 = -A\kappa \left.\frac{\partial T}{\partial x}\right|_{x=L} \quad , \tag{4.7}$$

From Eq. (4.4), we can calculate:

$$\frac{\partial T}{\partial x} = \beta \left(C_+ e^{\beta x} - C_- e^{-\beta x}\right) e^{-i\omega t} \ , \tag{4.8}$$

4.1.4. Upwards only approximation

In the high-frequency, heavily-damped limit $2\sqrt{\frac{\omega}{2D}}L \gg 1$, this solution simplifies to only an upward-propagating wave, resulting in the simplified solution:

$$\mathrm{T}(x,\ t) \approx Re\left[C_+ e^{\sqrt{\frac{\omega}{2D}}(i-1)x - i\omega t}\right] = C\ e^{-\sqrt{\frac{\omega}{2D}}x} \sin\left(\sqrt{\frac{\omega}{2D}}x - \omega t + \phi_0\right) \tag{4.9}$$

This is Eq. (1.2) in the core script, with an additional constant offset term T_0.

4.2. Electrical Waves – Additional information

4.2.1. Wave equation in 1-dimension

The waves propagating in a lumped transmission line are voltage / current waves. In the limit that the wavelength (measured in number of sections) is much larger than unity, we can derive the wave equation from some simple assumptions.

Some basic current-voltage relations for inductors and capacitors:

Capacitor: The voltage dropped across a capacitor is the charge stored divided by the capacitance $V = Q/C$. The current through the capacitor is, of course, the time derivative of the charge $I = dQ/dt$. Consequently, $I = C \cdot dV/dt$.

Inductor: The voltage dropped across an inductor is the product of its inductance and the rate of change of current flowing through it $V = L \cdot dI/dt$.

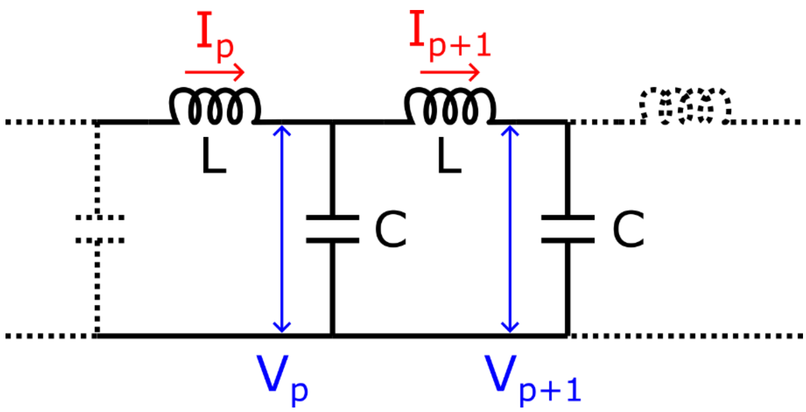


Fig. 4.2: *Sections of lumped transmission line*

Let's consider a section of the lumped transmission line as shown in Fig. 4.2. Using the designation of current and voltage in Fig. 4.2, we have the following relations:

- The current flowing through the capacitor at node p+1:
$$I_p - I_{p+1} = C \cdot \partial V_p/\partial t$$
- The voltage dropped across the inductor between nodes p and node p+1:
$$V_p - V_{p+1} = L \cdot \partial I_{p+1}/\partial t$$
- The voltage dropped across the inductor between node p-1 and node p:
$$V_{p-1} - V_p = L \cdot \frac{\partial I_p}{\partial t}$$

Taking the difference between the last two expressions:

$$V_{p+1} + V_{p-1} - 2V_p = L\frac{\partial}{\partial t}\left(I_p - I_{p+1}\right) . \qquad (4.11)$$

Substituting the 1st expression of difference in current between adjacent nodes into Eq. 4.11 we obtain:

$$V_{p+1} + V_{p-1} - 2V_p = L\frac{\partial}{\partial t}\left(C\frac{\partial V_p}{\partial t}\right) = LC\frac{\partial^2 V_p}{\partial t^2} . \qquad (4.12)$$

The LHS of the expression can be re-cast as differentials as function of node number:

$$V_{p+1} + V_{p-1} - 2V_p = \frac{\left(\frac{V_{p+1} - V_p}{\Delta p}\right) - \left(\frac{V_p - V_{p-1}}{\Delta p}\right)}{\Delta p} \approx \frac{\partial^2 V_p}{\partial p^2}\,, \qquad (4.13)$$

where Δp = 1. This leads to the wave equation:

$$\frac{\partial^2 V}{\partial p^2} = LC \cdot \frac{\partial^2 V}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 V}{\partial t^2}\,, \qquad (4.14)$$

where v is the phase speed in sections per unit time.

Part 5: Health and safety: Risk assessment

Imperial College London

RISK ASSESSMENT AND STANDARD OPERATING PROCEDURE

1. PERSON(S) CARRYING OUT THIS ASSESSMENT – This assessment has been carried out by the head of experiment.	
Name (Head of Experiment)	Charles Baynham and Richard Hobson
Date	01 October 2025

2. PROJECT DETAILS.						
Project Name	Thermal and Electrical Waves				Experiment Code	W
Brief Description of Project Outline	Wave propagation experiment for Year 2 physics undergraduates					
Location (*)	Campus	South Ken	Building	Blackett	Room	B407

(*) Note face to face teaching location in Blackett is optional for students who can come into campus. However, for students working remotely, the same hazards should be applicable.

3. HAZARD SUMMARY – Think carefully about all aspects of the experiment and what the work could entail. Write down any potential hazards you can think of under each section – this will aid you in the next section. If a hazard does not apply then leave blank.			
Manual Handling		Electrical	X
Mechanical		Hazardous Substances	
Lasers		Noise	
Extreme Temperature	X	Pressure/Steam	
Trip Hazards	X	Working At Height	
Falling Objects		Accessibility	X
Other			

4. CONTROLS – List the multiple procedures which may be carried out during the experiment along with the controls/ precautions that you will use to minimise any risks. Remember to take into consideration who may be harmed and how – other people such as students, support staff, cleaners etc will be walking past the experimental setup even when you aren't around.	
Brief description of the procedure and the associated hazards	**Controls to reduce the risk as much as possible**
Accessibility	All bags, coats, jumpers, etc. to be placed away from aisles, doors and walkways to have clear evacuation paths.
Electrical High currents in Peltier heater	No adjustment of electrical, mechanical or other parts of the experiments. In doubt, please inform a demonstrator or the Head of Experiment.
High Temperatures	Temperature range of Peltier is between -20 to +80 degrees. Device heating and cooling is enclosed in plastic so risk of accidental contact with skin is lowered.
Trip hazards	No cables are to be left on the floor near the equipment.

5. EMERGENCY ACTIONS – What to do in case of an emergency, for example, chemical spillages, pressure build up in a system, overheating in a system etc. Think ahead about what should be done in the worst case scenario.
All present in room Blackett B407 must be aware of the available escape routes and follow instructions in the event of an evacuation. The nearest fire escape route if following the green sign at the back of the lab, towards Huxley.