Homework 9 (2021)

JAIST — School of Information Science — I232E Information Theory

Instructor: Brian M. Kurkoski and Lei Liu TA: Shunqi Huang and Ning Liu

1. *Input-constrained capacity* Consider an additive white Gaussian noise (AWGN) channel :

$$Y = X + Z,$$

where $Z \sim \mathcal{N}(0, \sigma^2)$. Define SNR $= -10 \log_{10} \sigma^2$ dB. Plot the capacity curves with respect to SNR $\in [-10, 20]$ dB for the following cases:

   (a) *Gaussian:* For $X \in \mathcal{N}(0, 1)$.

   (b) *BPSK:* For $X \in \{-1, +1\}$ with uniform distribution.

   (c) *4PAM:* For $X \in \left\{ \frac{-3}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, \frac{+1}{\sqrt{5}}, \frac{+3}{\sqrt{5}} \right\}$ with uniform distribution.

   (d) *8PAM:* For $X \in \left\{ \frac{-7}{\sqrt{21}}, \frac{-5}{\sqrt{21}}, \frac{-3}{\sqrt{21}}, \frac{-1}{\sqrt{21}}, \frac{+1}{\sqrt{21}}, \frac{+3}{\sqrt{21}}, \frac{+5}{\sqrt{21}}, \frac{+7}{\sqrt{21}} \right\}$ with uniform distribution.

   Note: You can use any programming language you like. The matlab source code of binary-input AWGN channel is provied in Section 9.6.

2. Consider an additive white Gaussian noise (AWGN) channel :

$$Y = X + Z,$$

where $Z \sim \mathcal{N}(0, \sigma^2)$. Define SNR $= -10 \log_{10} \sigma^2$ dB. We assume that $X \in \{-b, -a, a, b\}$ with $p_{-a} = p_a$ and $p_{-b} = p_b$, and the power of X is normalized, i.e., $\mathrm{E}\{X^2\} = 1$.

   (a) Given σ^2, find the capacity C of the above AWGN channel. Plot the capacity curve with respect to SNR $\in [-10, 20]$ dB.

   (b) Find the optimal $\{a, b\}$ and $\{p_a, p_b\}$ that achieve the capcity in (a). Plot respectively the curves of $\{a, b, p_a, p_b\}$ with respect to SNR $\in [-10, 20]$ dB.

   Note: You can use any method you like to slove this problem.