Homework 13 (2021)

JAIST — School of Information Science — I232E Information Theory

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1. *Arimoto-Blahut algorithm for channel capacity* Implement Arimoto-Blahut algorithm for computing the capacity of a DMC in any programming language you like. The Information Theory Lecture Notes contains a partial example in Matlab — note that this implementation does not check for convergence. Use your implementation to compute the capacity C and capacity-achieving input distribution p_X^* of the following channel:

$$p_{\mathsf{Y}|\mathsf{X}}(y|x)y|x = \frac{1}{45}\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 2 & 8 & 1 & 3 & 6 & 4 & 7 & 9 \\ 5 & 8 & 2 & 9 & 7 & 4 & 6 & 3 & 1 \end{bmatrix}.$$

2. *Arimoto-Blahut algorithm for rate-distortion computation* Modify your source code of the previous problem to implement the Arimoto-Blahut algorithm for computing the $R(D)$ curve for a source. Your implementation should have three input arguments: the source distribution $p_{\mathsf{X}}(x)x$, the distortion metric as a $|\mathcal{X}|$-by-$|\widehat{\mathcal{X}}|$ matrix d, and a parameter λ. The output should be $R(D)$ and D. If you are using Matlab, use a function declaration like:

```
function [RD,D] = rateDistortionComputation(p,d,lambda)
```

(a) Use your implementation to find the $R(D)$ curve for the binary source with $p_{\mathsf{X}}(x)x = [0.2, 0.8]$ and Hamming distortion metric. By changing λ, obtain pairs $R(D), D$ sweep an $R(D)$ curve. On the same graph, plot $R(D) = h(p) - h(D)$ to validate your algorithm.

(b) Use your implementation to plot the $R(D)$ curve for a ternary source with $p_{\mathsf{X}}(x)x = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$ and distortion metric in matrix form:

$$d(x, \widehat{x}) = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 0 & 1 \\ 4 & 1 & 0 \end{bmatrix}.$$

What is the value of $R(0)$? How can you find $R(0)$ exactly? For what minimum value of D^* do you obtain $R(D^*) = 0$? What scheme can achieve $R(D^*) = 0$ for this value of D?