CEng 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 2

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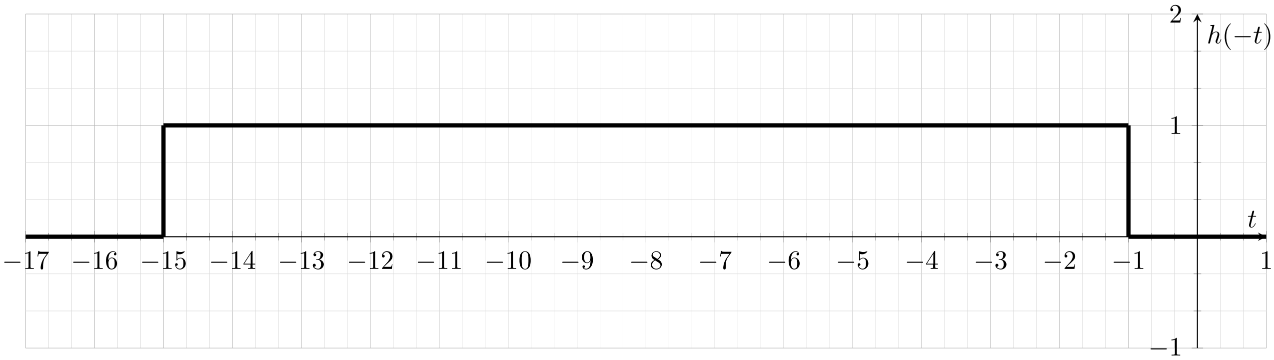
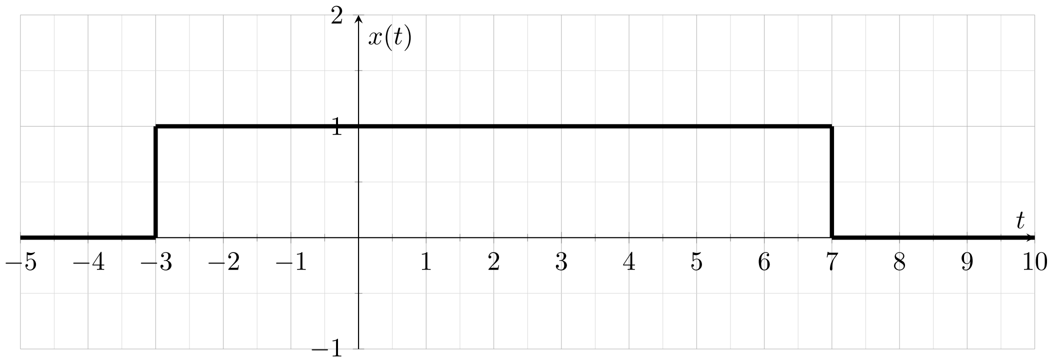
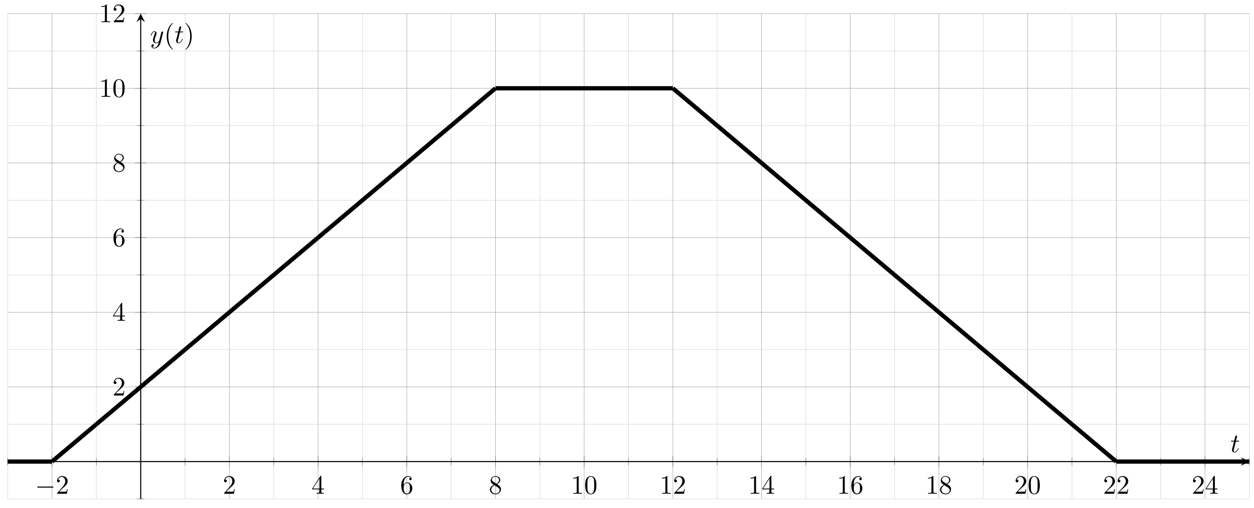
1. In this question flip, slide and multiply method is used on graphics.

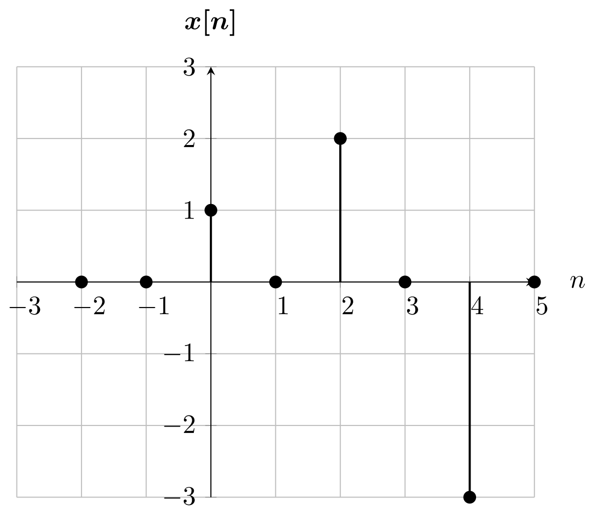


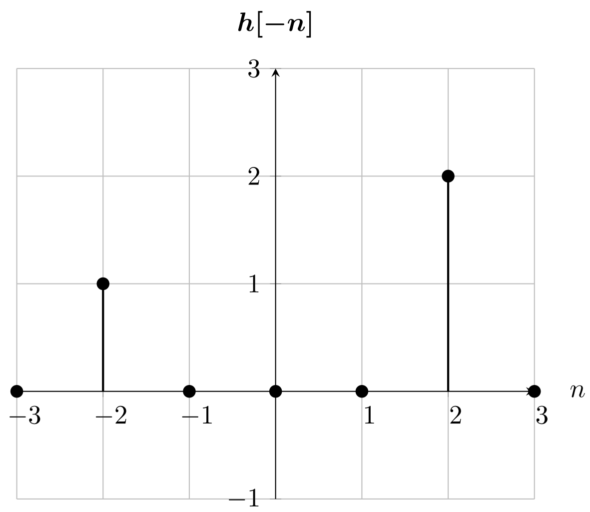


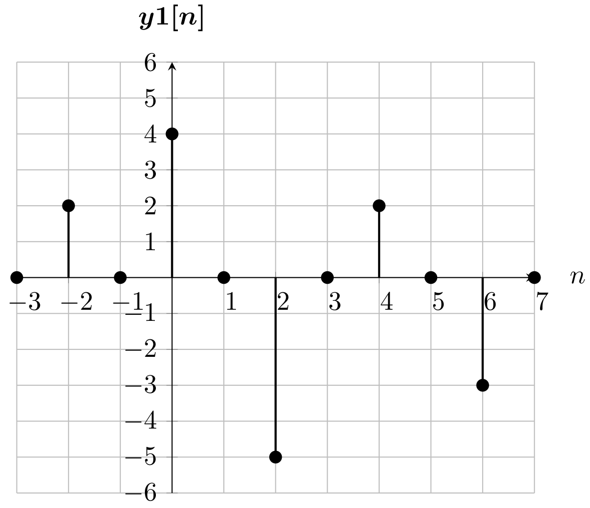
Output is:



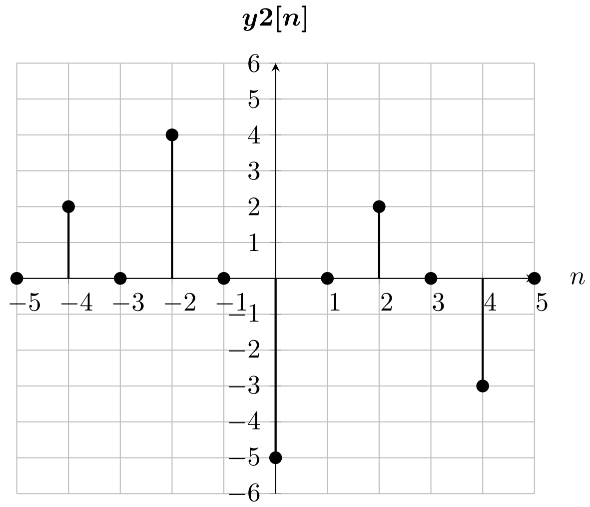
2. (a) Again we use flip, slide and multiply method.



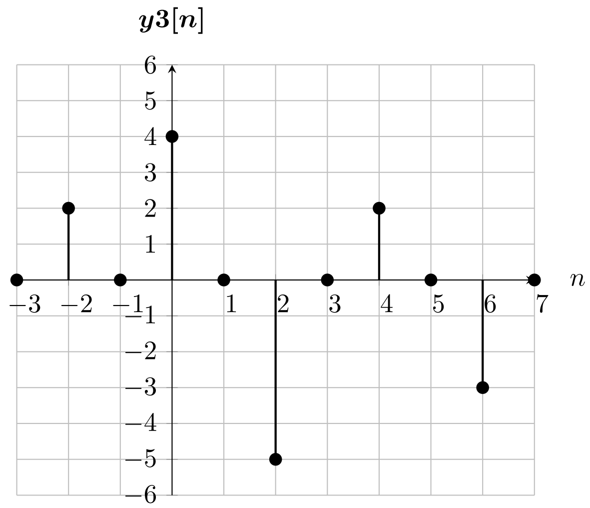




(b) x[n+2] just shifts the graph on a in -x direction by 2.



(c) h[n-2] just shifts the graph on b in +x direction by 2. y3[n] is the same with y1[n].



3. (a) $h[n] = \frac{1}{5}\delta[n-1] + \delta[n]$
   (b) $y[n] = \frac{1}{5}\delta[n-3] + \delta[n-2]$
   (c) Yes it is Bibo stable. It satisfies Jensen's inequality.

$$c = x[n], d = x[n-1]$$

then

$$y[n] = \frac{1}{5}d + c$$

Since every bounded input produces a bounded output, this system is BIBO stable.

(d) Since $y[n]$ depends on past values of n ($x[n-1]$), it has memory.

(e) It is not invertible because addition is not invertible. There can be multiple inputs that mapped to same y[n] value.

4. (a) Differential equation which represents the system is:

$$y''(t) - 2y'(t) + y(t) = 2x'(t)$$

(b) Homogenous equation:

$$y_h(t) = ae^{bt}$$

Put the homogenous part to the differential equation that found in part a:

$$b^2ae^{bt} - 2bae^{bt} + ae^{bt} = 0$$

$$ae^{bt}(b^2 - 2b + 1) = 0$$

For 'b' to satisfy this equation, it needs to b 1 since a cannot be 0. Since there are two same roots for the b, we find that $y_h(t) = c_1e^t + c_2te^t$.
For the particular solution, $y_p(t) = ax(t)$. Since $x(t)$ is zero, particular solution comes zero.

$$y(t) = y_h(t) + y_p(t) = c_1e^t + c_2te^t + 0 = c_1e^t + c_2te^t$$

Finally, we need to find the coefficient of the $y_h(t)$ which are c_1, and c_2. Since the system is initially at rest, $y(0) = 0$:

$$c_1e^0 + c_20e^0$$

$$c1 = 0$$

$$y'(t) = c_2e^t + c_2t$$

From the initially rest condition:

$$y'(0) = c_2 = 0$$

Since c_1, and c_2 is zero, $y_h(t)$ is also zero. Since the both $y_h(t)$, and $y_p(t)$ is zero, $y(t)$ is also zero.

$$y(t) = 0$$

(c) From the part (b), $y_h(t)$:

$$y_h(t) = c_1e^t + c_2te^t$$

$$y_p(t) = (At + B)u(t)$$

Put the $y_p(t)$ to the equation:

$$-2A + At + B = 4$$

$$A = 0$$

$$B = 4$$

$$y_p(t) = 4u(t)$$

Then the $y(t) = y_h(t) + y_p(t) = c_1e^t + c_2te^t + 4$.
Since the initial rest condition holds:

$$y(0) = c_1 + 4 = 0$$

$$c_1 = -4$$

$$y'(0) = -4 + c_2 + 0 = 0$$

$$c_2 = 4$$

Therefore, final equation is:

$$y(t) = (-4e^t + 4te^t + 4)u(t)$$

5. (a) We can solve it by using recursive method:

$$y[0] = \frac{1}{5}y[-1] + 2x[-2] = 0$$

Since the system is initially at rest, both $y[-1]$, and $x[-2]$ are 0. Therefore $y[0] = 0$.

$$y[1] = \frac{1}{5}y[0] + 2x[-1] = 0$$

$$y[2] = \frac{1}{5}y[1] + 2x[0] = 2x[0]$$

$$y[3] = \frac{1}{5}y[2] + 2x[1] = \frac{2}{5}x[0] + 2x[1]$$

$$y[4] = \frac{1}{5}y[3] + 2x[2] = \frac{2}{5^2}x[0] + \frac{2}{5}x[1] + 2x[2]$$

So the overall equation is:

$$y[n] = (\sum_{k=0}^{n-2} \frac{2x[k]}{5^{n-2-k}})u[n-2]$$

Impulse response of this system is:

$$h[n] = (\sum_{k=0}^{n-2} \frac{2\delta[k]}{5^{n-2-k}})u[n-2] = (\frac{2}{5^{n-2}})u[n-2]$$

(b) Transfer function of the system is:

$$(*)H[e^{\lambda}] = \frac{\sum_{k=0}^{M} b_k e^{-\lambda k}}{\sum_{k=0}^{N} a_k e^{-\lambda k}}$$

We can write the whole equation like that:

$$(**)\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

We can find the coefficients:

$$a_1 = -\frac{1}{5}, a_0 = 1, b_2 = 2$$

Since (*), and (**), transfer function is:

$$H[e^{\lambda}] = \frac{2e^{-2\lambda}}{-0.2e^{-\lambda} + 1}$$

(c) Here is the block diagram representation of this system using the adders and unit delay operators:

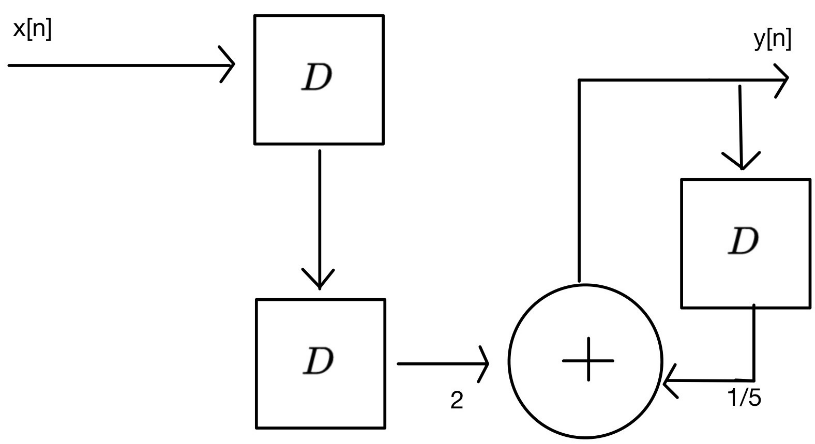


Figure 1: Block Diagram Representation of The System

6. (a) Here is the block diagram representation of this system using integrators and adders:

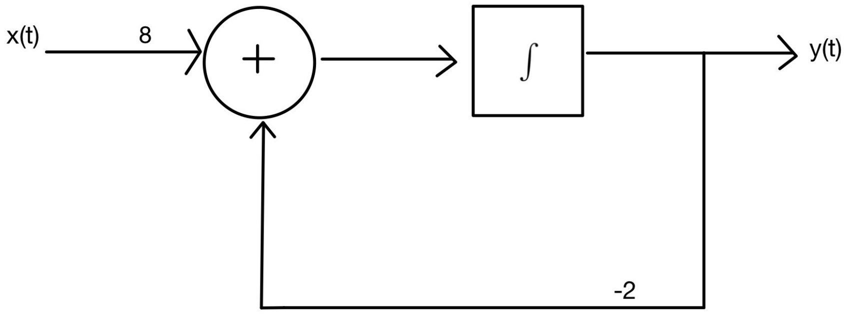


Figure 2: Block Diagram Representation of The System

(b) Here is the block diagram representation of this system using differentiators and adders:

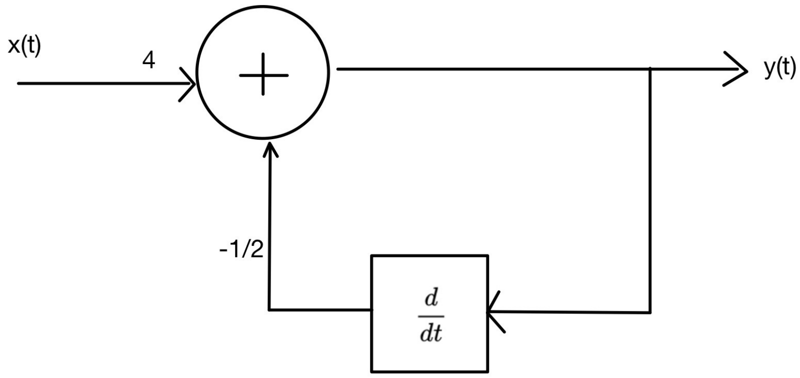
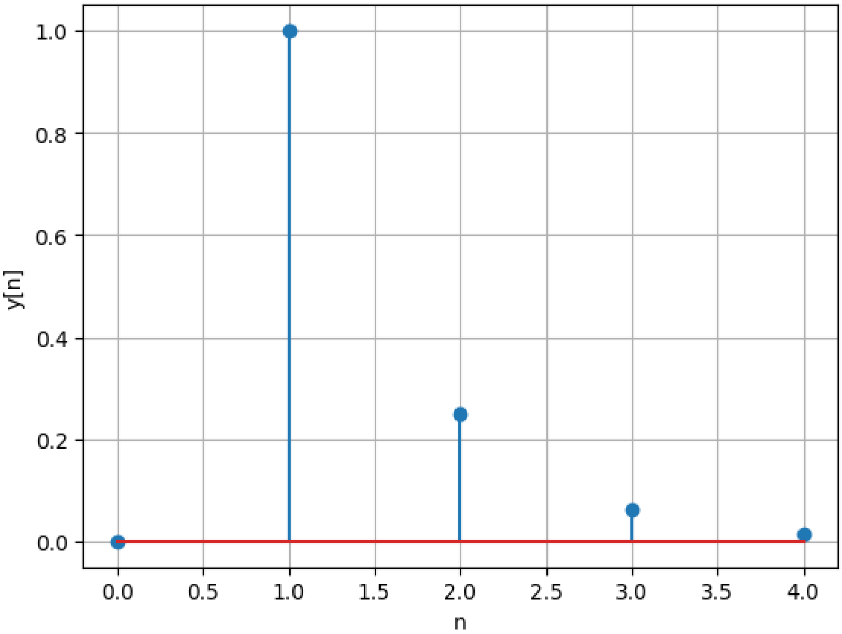


Figure 3: Block Diagram Representation of The System

7.
```
import matplotlib.pyplot as plt
def y_en(n):
    if n <= 0:
        return 0 + (n-1==0)
    else:
        return 0.25 * y_en(n-1) + (n-1==0)

y = [y_en(i) for i in range(5)]

plt.stem(range(5), y, use_line_collection=True)
plt.xlabel('n')
plt.ylabel('y[n]')
plt.title('Output of the Difference Equation')
plt.grid(True)
plt.show()
```



1.0
0.8
0.6
0.4
0.2
0.0
y[n]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
n