Student Information

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Answer 1

a)

$F(t_A) = t_A/100$
$F(t_B) = t_B/100$
Since T_A and T_B are independent we can multiply the $F(t_A)$ and $F(t_B)$ to obtain $F(t_A, t_B)$.
$F(t_A, t_B) = t_A/100 \cdot t_B/100 = \frac{t_A \cdot t_B}{10000}$

To obtain $f(t_A, t_B)$, we can derive the joint cdf $F(t_A, t_B)$.
$f(t_A, t_B) = \frac{\partial^2 F(t_A, t_B)}{\partial t_A \partial t_B} = \frac{1}{10000}$

b)

Since $P(t_A)$ and $P(t_B)$ are independent, we can multiply them to find $P(t_A < 30 \cap 40 < t_B < 60)$
$P(t_A < 30) = \frac{30}{100}$
$P(40 < t_B < 60) = \frac{20}{100}$
$P((t_A < 30) \cap (40 < t_B < 60)) = P(t_A < 30) \cdot P(40 < t_B < 60) = \frac{600}{10000} = 0.06$

c)

We need to multiply the area under the graph of the equation $t_A = t_B + 10$ by 1/10000, as the highest value that t_A can take is $t_B + 10$, and then we have to add remaining part which is $\frac{10 \cdot 100}{10000}$.
$\int_0^{90} \int_0^{t_B+10} \frac{1}{10000} dt_A dt_B + 10 \cdot 100 = \frac{4950}{10000} + \frac{1000}{10000} = \frac{5950}{10000} = 0.595.$

d)

The area between the functions $t_A = t_B + 20$, $t_B = t_A + 20$, $t_B = 100$, and $t_A = 100$ gives us the desired situation. We should then multiply this area by the density function. I will calculate the area in between by subtracting the area inside the square that is not valid from it. This will make the calculation easier, and I will obtain the valid area by subtracting the area inside the square that is not valid from the total area of the square. The invalid area is the sum of top-left and bottom-right which is:
$80 * 80/2 + 80 * 80/2 = 80 * 80 = 6400$(This is invalid area)
We have to subtract 6400 from 10000 to obtain result.

$$\frac{10000 - 6400}{10000} = \frac{3600}{10000} = 0.36$$

Therefore, the answer is 0.36.

Answer 2

a)

$\mu = 0.65$
$var = 0.4(0-0.6)^2 + 0.6(1-0.6)^2 = 0.144 + 0.096 = 0.24$
$\sigma = \sqrt{var} = \sqrt{0.24} = 0.4899$
From the central limit theorem:
$P(x >= \frac{65}{100} \cdot 150) = P(Z >= \frac{\frac{65}{100} \cdot 150 - \frac{60}{100} \cdot 150}{0.4899 \cdot \sqrt{150}}) = 1 - P(Z < \frac{\frac{65}{100} \cdot 150 - \frac{60}{100} \cdot 150}{0.4899 \cdot \sqrt{150}}) = 1 - P(Z < 1.25)$
$P(Z < 1.25) = 0.8944$
$1 - P(Z < 1.25) = 1 - 0.8944 = 0.1056$
Therefore, the answer is 0.1056.

b)

$\mu = 0.1$
$var = 0.9(0-0.1)^2 + 0.1(1-0.1)^2 = 0.009 + 0.081 = 0.09$
$\sigma = \sqrt{var} = \sqrt{0.09} = 0.3$
From the central limit theorem:
$P(x < \frac{15}{100} \cdot 150) = P(Z < \frac{\frac{15}{100} \cdot 150 - \frac{10}{100} \cdot 150}{0.3 \cdot \sqrt{150}}) = P(Z < \frac{\frac{15}{100} \cdot 150 - \frac{10}{100} \cdot 150}{0.3 \cdot \sqrt{150}}) = P(Z < 2.04)$
$P(Z < 2.04) = 0.9793$
Therefore, the answer is 0.9793.

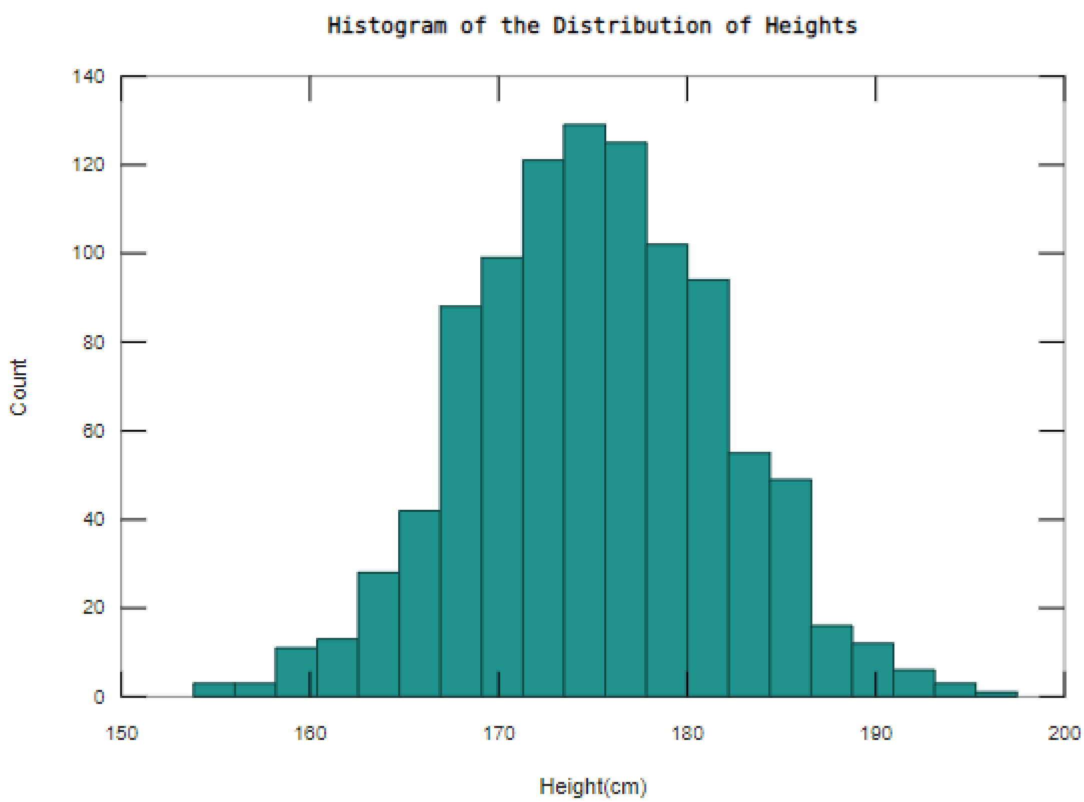
Answer 3

Let x is the height of an adult. Using the normal distribution:
$P(170 < x < 180) = P(\frac{170-175}{7} < Z < \frac{180-170}{7}) = P(Z < \frac{180-175}{7}) - P(Z < \frac{170-175}{7})$
$= P(Z < 0.714) - P(Z < -0.714)$
$P(Z < 0.714) - P(Z < -0.714) = 0.7624 - 0.2376 = 0.5248$
Therefore, the probability that a randomly selected adult will have a height between 170 cm and 180 cm is 0.5248.

Answer 4

a)

```
heights = normrnd(175, 7, [1, 1000]);
hist(heights, 20)
xlabel('Height(cm)')
ylabel('Count')
title('Histogram of the Distribution of Heights ')
```

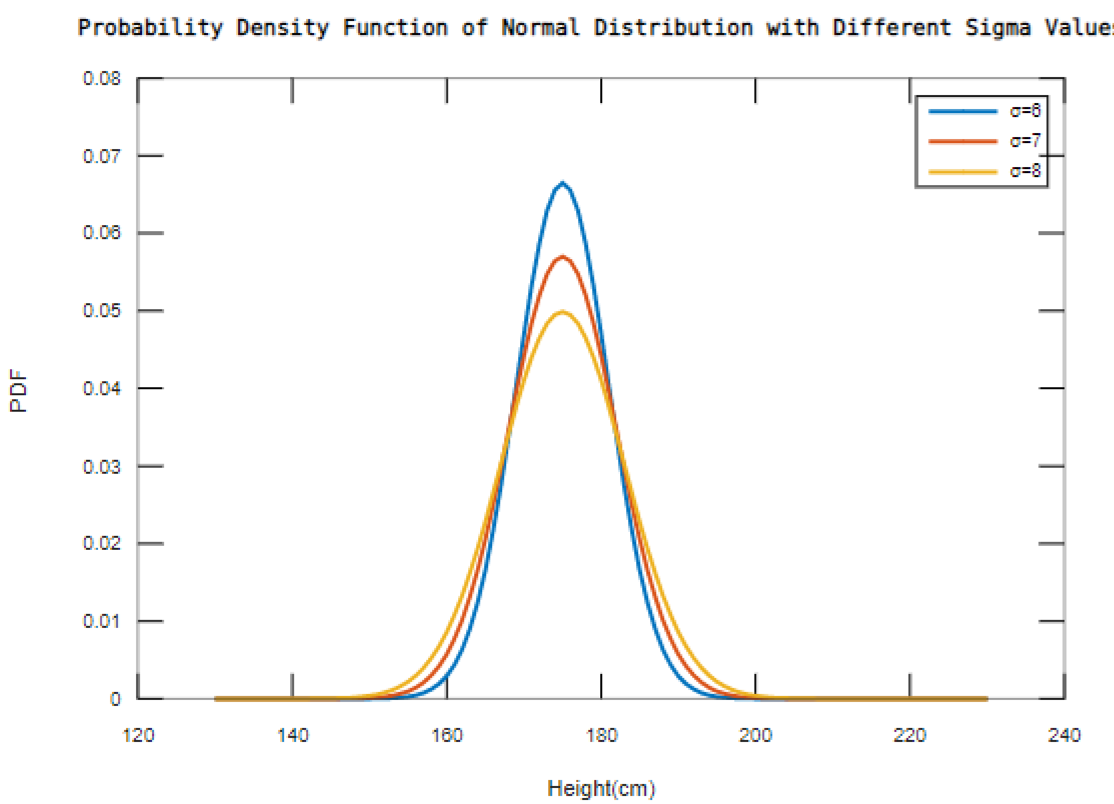


As seen here, more values have been formed around 175 cm in the table, and the frequency of occurrence decreases as we move away from 175. Of course, a graph exactly matching the theoretical distribution cannot be observed in this way, but we can approach the real value but cannot reach it.

b)

```
range = 130:230;
y1 = normpdf(range, 175, 6);
y2 = normpdf(range, 175, 7);
y3 = normpdf(range, 175, 8);

plot(range, y1,'LineWidth',2, 'DisplayName', '\sigma=6');
hold on;
plot(range, y2,'LineWidth',2, 'DisplayName', '\sigma=7');
plot(range, y3,'LineWidth',2, 'DisplayName', '\sigma=8');
xlabel('Height(cm)');
ylabel('PDF');
title('Probability Density Function of Normal Distribution with Different Sig
legend('show');
```



As seen in the table, the smaller the sigma value, the closer it is to the mean value. The reason for this can actually be explained by using standard deviation as a deviation rate from the mean value. In other words, as the standard deviation increases, it deviates more from the mean value.

c)

```
p45 = 0;
p50 = 0;
p55 = 0;

first = 150 * 45 / 100;
second = 150 * 50 /100;
third = 150 * 55 / 100;


for c = 1:1000
    a = normrnd(175,7,[1,150]);
    i=1;
    n=0;
    for s = 1:150
        if(a(i) < 180 && a(i) > 170)
            n++;
        end
        i++;
    end
    if(n >= third)
        p45++;p50++;p55++;
    elseif(n >= second)
        p50++;p45++;
    elseif(n >= first)
        p45++;
    end
end
probabilityOfAtLeast45 = p45 / 1000
probabilityOfAtLeast50 = p50 / 1000
probabilityOfAtLeast55 = p55 / 1000
```

```
probabilityOfAtLeast45 = 0.9790
probabilityOfAtLeast50 = 0.7680
probabilityOfAtLeast55 = 0.2790
```

As expected, the value of at least 45 is greater than at least 50, which is greater than at least 55. However, these values can vary in each loop, and as the number of loops increases, it approaches the value we calculated theoretically. This is an approximation with 1000 loops, and it is not possible to reach the real values with a small number of loops.