Student Information

Full Name : Yusuf Sami Lök
Id Number : 2521748

Answer 1

Write out the first few equations, and then multiply the n^{th} equation by x^n.
(n=2) $\quad a_2x = 3a_1x + 4a_0x$
(n=2) $\quad a_3x^2 = 3a_2x^2 + 4a_1x^2$
(n=2) $\quad a_4x^3 = 3a_3x^3 + 4a_2x^3$
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. . $\quad$ . . . .
$a_2x + a_3x^2 + a_4x^3 + ... = \Sigma_{n=2}^{\infty} a_n x^{n-1}$
$3a_1x + 3a_2x^2 + 3a_3x^3 + ... = 3\Sigma_{n=1}^{\infty} a_n x^n$
$4a_0x + 4a_1x^2 + 4a_2x^3 + ... = 4\Sigma_{n=0}^{\infty} a_n x^{n+1}$
Once we add first equations all up, we obtain the equation:
$\Sigma_{n=2}^{\infty} a_n x^{n-1} = 3\Sigma_{n=1}^{\infty} a_n x^n + 4\Sigma_{n=0}^{\infty} a_n x^{n+1}$
Set the $f(x) = \Sigma_{n=0}^{\infty} a_n x^n$. We can write the equation in terms of $f(x)$.
$\dfrac{f(x) - a_0 - a_1x}{x} = 3f(x) - 3a_0 + 4f(x)x$ and we know $a_0 = a_1 = 1$.
$f(x) = \dfrac{2x-1}{4x^2+3x-1} = \dfrac{A}{4x-1} + \dfrac{B}{x+1}$
Solving the system $\rightarrow A = \dfrac{-2}{5}, B = \dfrac{3}{5}$
$f(x) = \dfrac{2}{5}\dfrac{1}{(1-4x)} + \dfrac{3}{5}\dfrac{1}{(1-(-x))}$
Hence, the solution is

$$a_n = \frac{2}{5} \cdot (4^n) + \frac{3}{5} \cdot (-1)^n$$

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Answer 2

a)

We can write the sequence as the summation of two generating functions:
$2 + 2x^2 + 2x^3 + ...$
$3x + 9x^2 + 27x^+ .....$
$2(1 + x^2 + x^3 + ...) = 2\Sigma_{n=0}^{\infty} x^n = 2 \cdot \dfrac{1}{1-x}$
$(3x + 3^2x^2 + 3^3x^3 + ...) = \Sigma_{n=1}^{\infty} 3^n x^n = \dfrac{1}{1-3x} - 1$

We can find the closed form of generating function in this way:
$f(x) = \frac{2}{1-x} + \frac{3x}{1-3x}$
Hence, the solution is:

$$f(x) = \frac{-3x^2 - 3x + 2}{3x^2 - 4x + 1}$$

b)

We should use the partial fraction version of the function $G(x)$:
$G(x) = \frac{A}{2x-1} + \frac{B}{x-1}$
Once we solve the system $\rightarrow A = -5$ and $B = -2$.
$G(x) = \frac{5}{1-2x} + \frac{2}{1-x}$
$\frac{5}{1-2x} = 5\Sigma_{n=0}^{\infty} 2^n x^n =< 5, 10, 20, 40, ... >$
$\frac{2}{1-x} = 2\Sigma_{n=0}^{\infty} x^n =< 2, 2, 2, 2, ... >$
Then we get the summation of the two sequences:
Hence the answer is:

$$G(x) = 5\Sigma_{n=0}^{\infty} 2^n x^n + 2\Sigma_{n=0}^{\infty} x^n$$

$$a_n = 2 + 5 \cdot 2^n$$

The corresponding sequence is:

$$< 7, 12, 22, 42, 82, ... >$$

Answer 3

a)

Check whether R is reflexive(aRa) or not:

$$a^2 + a^2 = n^2$$

(1)

$$a^2 + n^2 = a^2$$

(2)

If at least one of the two are correct, the right triangle is possible.
From (1), we have $n = a\sqrt{2}$, n and a are integers. Since R is defined on as Z, $a\sqrt{2}$ can not be any integer. Hence, the first equation (1) is not possible. Since n can not be zero (because it is

the edge of a triangle), the equation (2) is not possible too. Because R is not reflexive, R is not a equivalence relation.

b)

Since $2x_1 + y_1 = 2x_1 + y_1$,$(x_1, y_1)R(x_1, y_1)$. Hence, R is reflexive. (1)
If $(x_1, y_1)R(x_2, y_2)$, then $2x_1 + y_1 = 2x_2 + y_2$. So we can write this as $2x_2 + y_2 = 2x_1 + y_1$, which shows that $(x_2, y_2)R(x_1, y_1)$. Hence R is symmetric. (2)
If $(x_1, y_1)R(x_2, y_2)$ and $(x_2, y_2)R(x_3, y_3)$ then we can write the equation:

$$2x_1 + y_1 = 2x_2 + y_2 = 2x_3 + y_3$$

So $2x_1 + y_1 = 2x_3 + y_3$. Which shows that $(x_1, y_1)R(x_3, y_3)$. Thus, R is transitive. (3)
Because of (1),(2) and (3), R is equivalence relation.
Assume (x,y) is the equivalence class form of (1,-2). So:

$$2x + y = 2 - 2 = 0,$$

$$y = -2x$$

Say x is any real number n, then the equivalence class is:

$$(n, -2n)$$

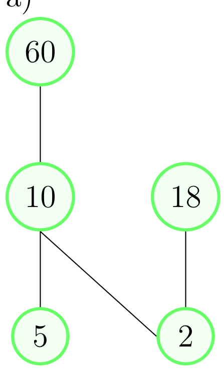
for all $n \in R$. It represents the line

$$y = -2x$$

in the cartesian coordinate system.

Answer 4

a)



b)
Rows and columns are 2,5,10,18,60 respectively.

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c)
All pairs:

$$(x, y) = \{(10, 2), (10, 5), (18, 2)(60, 2), (60, 5), (60, 10)\}$$

Matrix representation of R_s:
Rows and columns are 2,5,10,18,60 respectively.

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

d)
If we can change one element in A and add another to create a total ordering that includes all elements, it is not possible. There are three situations that cause problems, which are 2,5 and 5,18 and 10,18, so we have to change at least two of them because these 3 conditions do not have a common number. Hence, it is not possible.

If we can remove two elements and add one, We can remove the 2 troublesome numbers from the set and replace it with a number that can divide all the numbers, so it is possible. For example, we can remove 5 and 18 from the set, and add 1 to it. In this way, we can construct a total ordering that includes all elements.

$$\{1, 2, 10, 60\}$$

Hence, it is possible.