CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 1

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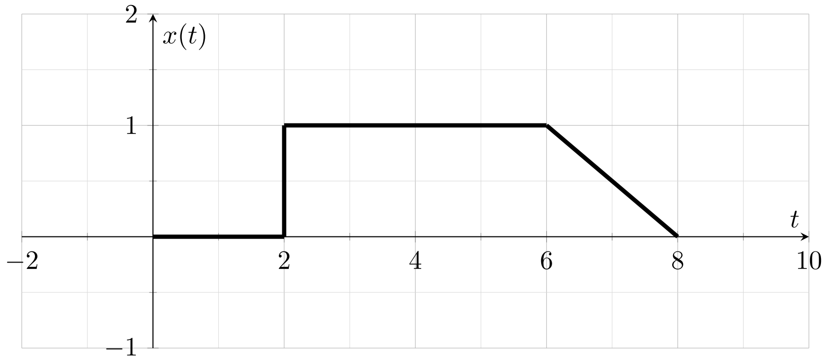
1. (a) $z = \frac{\sqrt{2}+\sqrt{2}j}{\sqrt{2}+\sqrt{3}j}$ Multiply by $\frac{\sqrt{2}+\sqrt{3}j}{\sqrt{2}+\sqrt{3}j}$, then $z = \frac{2\sqrt{2}+2\sqrt{6}+(2\sqrt{2}-2\sqrt{6})j}{16}$
   Then $Re(z) = \frac{\sqrt{2}+\sqrt{6}}{8}$ and $Im(z) = \frac{\sqrt{2}-\sqrt{6}}{8}$

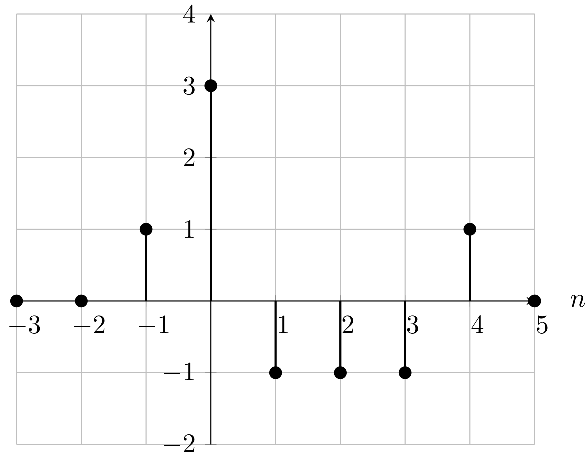
   (b) Magnitude is $\sqrt{(Re(z))^2 + (Im(z))^2}$.
   $(Re(z))^2 = (\frac{\sqrt{2}+\sqrt{6}}{8})^2 = \frac{2+6+2\sqrt{12}}{64} = \frac{8+4\sqrt{3}}{64}$
   $(Im(z))^2 = (\frac{\sqrt{2}-\sqrt{6}}{8})^2 = \frac{2+6-2\sqrt{12}}{64} = \frac{8-4\sqrt{3}}{64}$
   Magnitude$=\sqrt{(Re(z))^2 + (Im(z))^2} = \sqrt{\frac{16}{64}} = \frac{1}{2}$
   Phase$=arctan(\frac{Im(z)}{Re(z)}) = arctan(\frac{\frac{\sqrt{2}-\sqrt{6}}{8}}{\frac{\sqrt{2}+\sqrt{6}}{8}}) = -0.2618rad$

2. Here is the signal $x[\frac{1}{2}t - 2]$



3. (a) $x[n] = x[-3]\delta[n+3] + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + 2x[2]\delta[n-2] + x[3]\delta[n-3]$
   $x[n] = \delta[n+3] - \delta[n+2] - \delta[n+1] - \delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3]$

   (b) Here is the signal $x[2n+2] + x[1-n]$



   (c) $y[n] = \delta[n+1] + 3\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$

4. (a) Yes, it is periodic with period $N = 4$:

$$5\frac{\pi}{2}N = 2\pi m$$

$5\frac{\pi}{2}N$ must be a multiple of 2π for $N > 0$, and N should be an integer.

$$N = \frac{4}{5}m$$

The above equation must be hold to be periodic. Fundamental period is the smallest n value such that $x_2[n] = x_2[n + N]$ for integer value m. Therefore, for $m = 5$, fundamental period $N = 4$.

(b) No, it is not periodic, it is aperiodic:

$$5N = 2\pi m$$

$5N$ must be a multiple of 2π for $N > 0$, and N should be an integer.
The above equation must be hold to be periodic. Since there is no such N, and m which are integers to fit the equation above, there is no such N. Since π is not a real number, it is not periodic.

(c) Yes, it is periodic with period $T = \frac{\pi}{2}$:

$$4T = 2\pi$$

$4T$ must be a multiple of 2π.
Fundamental period is the smallest T value such that $x_3(t) = x_3(t + T)$. From the above equation, fundamental period is $\frac{\pi}{2}$.

5. We know that:

$$*** \int_{-\infty}^{t} \delta(t)\, dt = u(t)$$

Integrating both sides:

$$\int_{-\infty}^{t} \delta(at)\, dt = \int_{-\infty}^{t} \frac{1}{|a|}\delta(t)\, dt$$

Excluding the number $\frac{1}{|a|}$ from the integral since that does not depend on t.

$$\int_{-\infty}^{t} \delta(at)\, dt = \frac{1}{|a|}\int_{-\infty}^{t} \delta(t)\, dt$$

We can use ***:

$$\int_{-\infty}^{t} \delta(at)\, dt = \frac{1}{|a|}u(t)$$

Since the function $\delta(t)$ is equal to the $\delta(-t)$, we can say that sign of the t does not matter for the function $\delta(t)$. For both of the functions $\delta(-at)$, and $\delta(at)$, we can use $\delta(|a|t)$. Therefore, integrating the $\int_{-\infty}^{t} \delta(at)\, dt$ results in $\frac{u(t)}{|a|}$:

$$\frac{u(t)}{|a|} = \frac{1}{|a|}u(t)$$

Since the above equation holds, $\delta(at)$ is equal to the $\frac{1}{|a|}\delta(t)$.

6. (a) We can find the difference equation by considering $y_1[n]$ is the output of system S_1 and input of the S_2:

$$y[n] = 4x_1[n-2] + 2x_1[n-3]$$

(b) If the S_1 and S_2 is reversed, the system equation in part (a) would be the same. We can show that:

$$y_2[n] = y_1[n-2]$$

$$y_2[n] = y_1[n-2] = 4x_1[n-2] + 2x_1[n-3]$$

Since equation of the reversed version of the system is the same as part (a), it is commutative.

(c) From the superposition property:
For the input $ax_1[n]$:

$$y_{21}[n] = 4ax_1[n-2] + 2ax_1[n-3]$$

For the input $bx_2[n]$:

$$y_{22}[n] = 4bx_2[n-2] + 2bx_2[n-3]$$

For the input $ax_1[n] + bx_2[n]$:

$$y_{23}[n] = 4ax_1[n-2] + 2ax_1[n-3] + 4bx_2[n-2] + 2bx_2[n-3]$$

Since the sum of the $y_{21}[n]$, and $y_{22}[n]$ is equal to the $y_{23}[n]$, overall system S is linear from superposition property, and scaling property.

(d) Yes, the overall system S is time invariant. Since shifting parameter n causes identical shift in the output $y_2[n]$ as well.
$y[n] = H\{x_1[n]\}$ implies $y_2[n+a] = H\{x_1[n+a]\}$

7. (a)
```
from sympy import *
def signal_1(n,x,k=1):
  return simplify(n*x(n)*k)
x=Function('x')
x_1=Function('x_1')
x_2=Function('x_2')
n=Symbol('n')
k=Symbol('k')
m=Symbol('m')
o=x_1(n)*k
t=x_2(n)*m
y1=signal_1(n,x_1,k)#y1=k*n*x_1[n]
y2=signal_1(n,x_2,m)#y2=m*n*x_2[n]
y3=n*(o+t)#y3=n(k*x_1[n]+m*x_2[n])
result=simplify(y1+y2)
T1=simplify(y3)==simplify(result)#T1 checks superposition
y1=signal_1(n,x,k)#y1=k*n*x[n]
y2=k*signal_1(n,x)#y2=k*(n*x[n])
T2=simplify(y2)==simplify(y1)#T2 checks scaling
if T1 and T2:
  print("The given system is a Linear system")
else:
  print("The given system is a Non-Linear system")
```

(b)
```
from sympy import *
x=Function('x')
x_1=Function('x_1')
x_2=Function('x_2')
n=Symbol('n')
k=Symbol('k')
m=Symbol('m')
def signal_2(n,x,k=1):
  return simplify((k*x(n))**2)

y1=signal_2(n,x_1,k)#y1=(k*x_1[n])^2
y2=signal_2(n,x_2,m)#y2=(m*x_2[n])^2
o=x_1(n)*k
t=x_2(n)*m
y3=(o+t)**2#y3=(x_1(n)*k+x_2(n)*m)^2
y3 = simplify(expand(y3))
result=simplify(y1+y2)
```

```
T1=y3==result#T1 checks superposition
y1=signal_2(n,x,k)#y1=(k*x[n])^2
y2=k*signal_2(n,x)#y2=k*((x_1[n])^2)
T2=simplify(y2)==simplify(y1)#T2 checks scaling
if T1 and T2:
  print("The given system is a Linear system")
else:
  print("The given system is a Non-Linear system")
```