Middle East Technical University
Computer Engineering Department

CENG 382

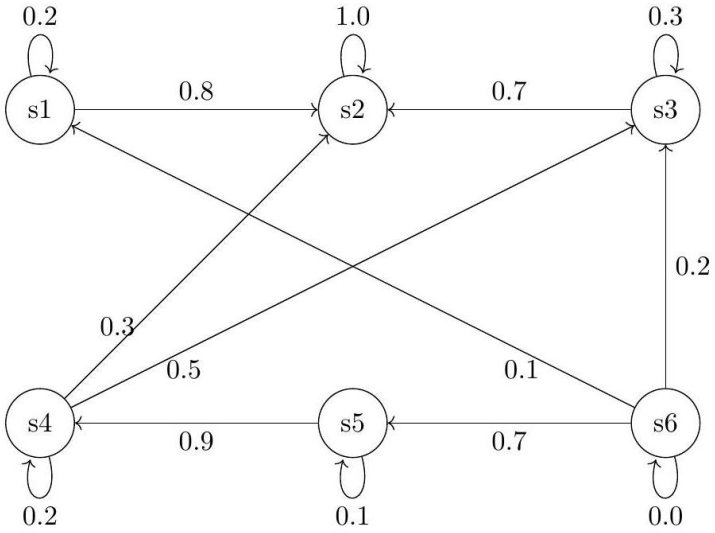
Spring 2023 - Homework 1

Regulations :

- **Submission**: We provide a latex template for your solutions. Use that template and create a hw1.pdf file that includes your solutions to the questions below in this homework.
- **Deadline**: 23:55, 25 March, 2024 (Monday) (few days before the midterm).
- **Late Submission**: Not allowed.
- Submit `hw1.pdf` to the odtuclass page of the course.
- Don't submit a PDF file that is non-readable or converted from image/screenshot.
- Please justify your answers.

Questions :

1. (12 pts) Represent the following higher order equations as a system of first order equations.
   (a) (6 pts) $y''(t) - 5y'(t) + 6y(t) = 0$
   (b) (6 pts) $y(k+3) = y(k+1) + y(k)$
2. (9 pts) Find an exact formula for $x(k)$, where $x(k+1) = ax(k) + b, x(0) = x_0$, and a, b, and x_0 have the following values:
   (a) (3 pts) $a = 1, b = 0, x_0 = 3$.
   (b) (3 pts) $a = 0.5, b = -1, x_0 = 0$.
   (c) (3 pts) $a = -1, b = 1, x_0 = 7$.
3. (6 pts) For each of the discrete time systems in Q2, determine whether or not $|x(k)| \to \infty$. Determine if the system has a fixed point and whether or not the system is approaching that fixed point.
4. (9 pts) Find the exact value of $x(t)$, where $x' = ax + b, x(0) = x_0$, and a, b, and x_0 have the following values:
   (a) (3 pts) $a = 1, b = 0, x_0 = 1$.
   (b) (3 pts) $a = 0, b = 1, x_0 = 0$.
   (c) (3 pts) $a = -1, b = 2, x_0 = 3$.
5. (6 pts) For each of the continuous time systems in Q4, determine whether or not $|x(t)| \to \infty$. Determine if the system has a fixed point and whether or not the system is approaching that fixed point.
6. (9 pts) Find state transition matrix $\Phi(k, l)$ for system $x(k+1) = \begin{bmatrix} \frac{k+2}{k+1} & 0 \\ 0 & 1/2 \end{bmatrix} x(k)$. Comment on the behavior of the system as $k \to \infty$.
7. * **Extra question, this will not be graded**: Consider system $x(k+1) = \begin{bmatrix} \frac{k+2}{k+1} & 1 \\ 0 & 1/2 \end{bmatrix} x(k)$. Comment on the behavior of the system as $k \to \infty$. You do not have to compute an exact formula for $\Phi(k, l)$. However, its structure will be helpful to predict $x(k)$ as $k \to \infty$.
8. (9 pts) Let $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, and $x_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Consider the discrete time dynamical system $x(k+1) = Ax(k)$.
   (a) (6 pts) Find an exact formula for x(k).
   (b) (3 pts) Comment on the behavior of $x(k)$ as $k \to \infty$.
9. (9 pts) Let $A = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and $x_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Consider the continuous time dynamical system $x'(t) = Ax(t) + b$.
   (a) (6 pts) Find an exact formula for x(t).
   (b) (3 pts) Comment on the behavior of $x(t)$ as $t \to \infty$.
10. (12 pts) Consider the system shown below:
$$x(k+1) = Ax(k)$$
$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{16} \\ -1 & 0 \end{bmatrix}$$
    (a) (6 pts) Show that the matrix A is not diagonalizable. Justify your answer.
    (b) (6 pts) If we try to compute A^k in order to solve the system, what happens as $k \to \infty$? Justify your answer.
11. (9 pts) Consider the state diagram representing of a Markov Chain shown in below:



(a) (3 pts) Find the matrix representing the transition probabilities of the state diagram above.

(b) (6 pts) How this markov chain behaves in the long term? Justify your answer.