Middle East Technical University
Computer Engineering Department

# CENG 382

Spring 2023 - Homework 2 & 3

**Regulations** :

- **Submission**: We provide a latex template for your solutions. Use that template and create a hw2.pdf file that includes your solutions for the questions below in this homework.
- **Deadline**: 23:55, 31 May, 2024 (Fridays) (last day of classes).
- **Late Submission**: Not allowed.
- Submit `hw2.pdf` to the odtuclass page of the course.
- Don't submit a PDF file that is non-readable or converted from image/screenshot.
- Please justify your answers.

**Questions** :

1. Consider the following continuous time system:

$$\dot{x_1} = 2x_2^2 - x_1 - 1$$
$$\dot{x_2} = -2x_1x_2 - x_2$$

Show that fixed point $\widetilde{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ is stable. Use the Lyapunov function $V(x_1, x_2) = (x_1+1)^2 + x_2^2$.

2. Consider the following discrete time system:

$$x_1(k+1) = \frac{1}{3}x_1 + \frac{1}{3}x_4$$
$$x_2(k+1) = \frac{1}{3}x_2 + \frac{1}{3}x_3$$
$$x_3(k+1) = \frac{1}{3}x_1 - \frac{1}{3}x_4$$
$$x_4(k+1) = \frac{1}{3}x_2 - \frac{1}{3}x_3$$

Show that the fixed point $\widetilde{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is stable. Use the Lyapunov function $V(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2 + x_3^2 + x_4^2$.

3. Consider the continuous time system:

$$\dot{x_1} = -4x_2$$
$$\dot{x_2} = Ax_2 + 4x_1 - 3x_2^3$$

Explain the behavior of this system for different values of "A" using Poincare-Bendixson theorem. You can use the Lyapunov function $V(x_1(k), x_2(k)) = x_1^2 + x_2^2$.

4. Consider the following discrete-time system:

$$x(k+1) = -3x(k)^2 + 4$$

   (a) Find the fixed points of the system and show whether they are stable or not.
   (b) Find the periodic points of prime period 1 and 2.
   (c) Find whether the system tends to the periodic points with prime period 2 or not.

5. Consider the system given below;

$$x(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(k)$$

   (a) Show that this system is controllable. Justify your answer.
   (b) (10pts) Find the finite sequence of inputs which drives the system from initial state $x(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$

6. Determine whether the system below is observable or not. Justify your answer.

$$x(k+1) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x(k) \qquad y(k) = \begin{bmatrix} 1 & \frac{1}{2} & 1 \end{bmatrix} x(k)$$

7. Find the fixed points the 1D systems given below and explain whether they are stable or not by using linearization. If linearization test fails, analyze the system dynamics around the fixed points in order to determine stability.

   (a) $3x(k+1) = x^2(k) + 3x(k) - 4$

   (b) $\dot{x} = \frac{1}{3}x^3 - 3x^2 + 9x - 9$

8. Considering the following system, find the fixed points and explain whether they is stable or not by using linearization.

$$\dot{x}_1 = 2x_1^2 + x_1x_2 - 6$$
$$\dot{x}_2 = x_1 + x_2$$

9. Consider the following system.

$$x(k+1) = \begin{bmatrix} 0 & 0 & 1 \\ -2 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} x(k) \qquad y(k) = \begin{bmatrix} 0 & -2 & -4 \end{bmatrix} x(k)$$

   (a) Determine whether this system is observable by using the definition observability. That is to say, you should not calculate the observability matrix here, rather try to show you are able to obtain the initial state by following the output sequence.

   (b) Now, calculate the observability matrix and show that the result you found in part (a) is accurate. Show each step to receive full grade.