Middle East Technical University Department of Computer Engineering

CENG 424

Logic For Computer Science

Fall 2024-2025

Assignment 5

Regulations

1. The homework is due by **23:55 on January 12th, 2025**. Late submission is not allowed.
2. Submissions will be via ODTUClass, do not send your homework via e-mail.
3. You typeset your homework in any typesetting tool (LaTex, Word, etc.) Please do **not submit handwritten answers**. Furthermore, you must upload the homework as **a single pdf file**. Other formats will not be considered for grading. **make sure that the page layouts are properly organized and your answers are clear and readable.**
4. Write your names and surnames to the first page.
5. Name your submission files as $< yourName\_yourSurname\_424HW5 >$.pdf.
6. Send an e-mail to **garipler@metu.edu.tr** if you need to get in contact.
7. **This is an individual homework, which means you have to answer the questions on your own. Any contrary case including but not limited to getting help from automated tools, sharing your answers with each other, extensive collaboration etc. will be considered as cheating and university regulations about cheating will be applied.**
8. **You will be invited to a short Q&A session on your solutions on a date to be determined later (expectedly around the end of the finals period).**

Preliminaries

Specifically for this assignment, let us define a new class of finite state machines species by slightly extending the definition of nondeterministic finite state automata.

Definition. A **nondeterministic weighted finite state machine (NWFSM)** is a nondeterministic finite automaton

1. without final states
2. extended with a function w from the of transitions δ to the set of real numbers $\mathbb{R}$ such that w map each transition to a real number.

We call the value of $w(t, k)$ $(t \in \delta, k \in \mathbb{R})$ the **weight of the transition** t.

Formally, a Weighted Nondeterministic Finite Automaton (WNFA) is defined as a tuple $N = (Q, \Sigma, \delta, s, w)$ where:

- Q is a finite set of states.
- Σ is a finite alphabet of input symbols.
- $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation, where each element $(q, a, q') \in \delta$ represents a transition from state q to state q' on input symbol a.
- $s \in Q$ is the initial state.
- $w : \delta \to \mathbb{R}$ is a weight function that assigns a real number to each transition, i.e., $w((q, a, q')) = k$, where k is the weight of the transition (q, a, q').

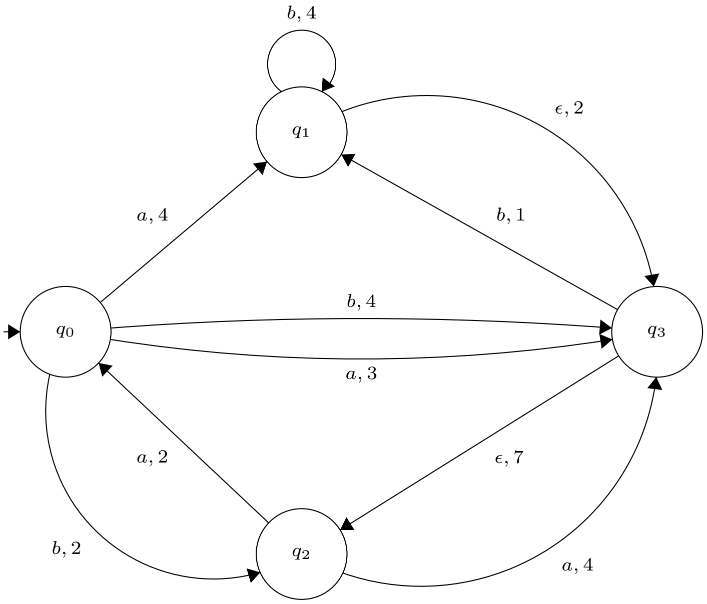

Figure 1: NWFSM N_1

Figure 1 illustrates the NWFSM N_1. Formally, N_1 is defined as

$N_1 = (Q_1, \Sigma_1, \delta_1, s_1, w_1)$ where

- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{a, b\}$
- $\delta = \{(q_0, a, q_1), (q_0, a, q_3), (q_0, b, q_2), (q_0, b, q_3), (q_1, \epsilon, q_3), (q_1, b, q_1), (q_2, a, q_0), (q_2, a, q_3), (q_3, \epsilon, q_2), (q_3, b, q_1)\}$
- $s = q_0$
- $w = \{(q_0, a, q_1) \rightarrow 4, (q_0, a, q_3) \rightarrow 3, (q_0, b, q_2) \rightarrow 2, (q_0, b, q_3) \rightarrow 4, (q_1, \epsilon, q_3) \rightarrow 2,$
  $(q_1, b, q_1) \rightarrow 4, (q_2, a, q_0) \rightarrow 2, (q_2, a, q_3) \rightarrow 4, (q_3, \epsilon, q_2) \rightarrow 7, (q_3, b, q_1) \rightarrow 1\}$

Furthermore, we define **the total weight of a path** as the sum of the weights of the transitions constituting that path.

To illustrate, on of the paths the string *abb* traces on N_1 is $(q_0, a, q_3)(q_3, b, q_1)(q_1, b, q_1)$. The total weight of that path is 8, which is calculated as $w((q_0, a, q_3)) + w((q_3, b, q_1)) + w((q_1, b, q_1)) = 3 + 1 + 4 = 8$

Problem

The **highest weighted path** a given string s traces on a given NWFSM N is defined as the path having the highest total weight among the paths s can trace on N. To illustrate, the highest weighted path the string *abb* traces on N_1 is $(q_0, a, q_1)(q_1, b, q_1)(q_1, b, q_1)$.

You are asked to devise an algorithm that outputs the Relational Logic formulation of the problem of finding the highest weighted path a given string traces on a given NWFSM.

Question 1

Write down the pseudocode of the algorithm you devised for the given problem. Note that the algorithm should not solve the problem but represent in the Relational Logic.

Question 2

Using your algorithm, find the highest weighted path for the string *abb* on the NWFSM N_1 given at Figure1. Show each step your algorithm performs on those inputs.

Question 3

There are some classes/types of tools that can be used to find the solution of the problem using its Relational Logic representation generated by your algorithm. Choose one those tool classes and briefly explain (with at most 180 words)

1. What kind of changes you need to make to the output format of your algorithm in order to be able use the that type of tools?
2. How do that type of tools work, how can they solve the problem(s) by just taking its(/their) Relational-Logic-like representation?