

1.1

$$\neg(p \vee r) \wedge (\neg p \rightarrow (q \vee r)) \wedge \neg q = 1 \quad (a)$$

| a

$$\neg(p \vee r) = 1 \quad (b)$$

$$(\neg p \rightarrow (q \vee r)) = 1 \quad (c)$$

$$\neg q = 1$$

| b

$$\neg p = 1$$

$$\neg r = 1$$

c

$$\neg p = 0$$

closed

c

$$(q \vee r) = 1$$

(d)

d

$$q = 1$$

closed

d

$$r = 1$$

closed

Hence, it is not satisfiable.

$$1.2. \quad \neg((P \rightarrow q) \rightarrow \neg((P \rightarrow t) \wedge (P \wedge t \rightarrow q))) = 1 \quad (a)$$

$$\begin{array}{c} | \\ a \\ ((P \rightarrow q) \rightarrow \neg((P \rightarrow t) \wedge (P \wedge t \rightarrow q))) = 0 \quad (b) \end{array}$$

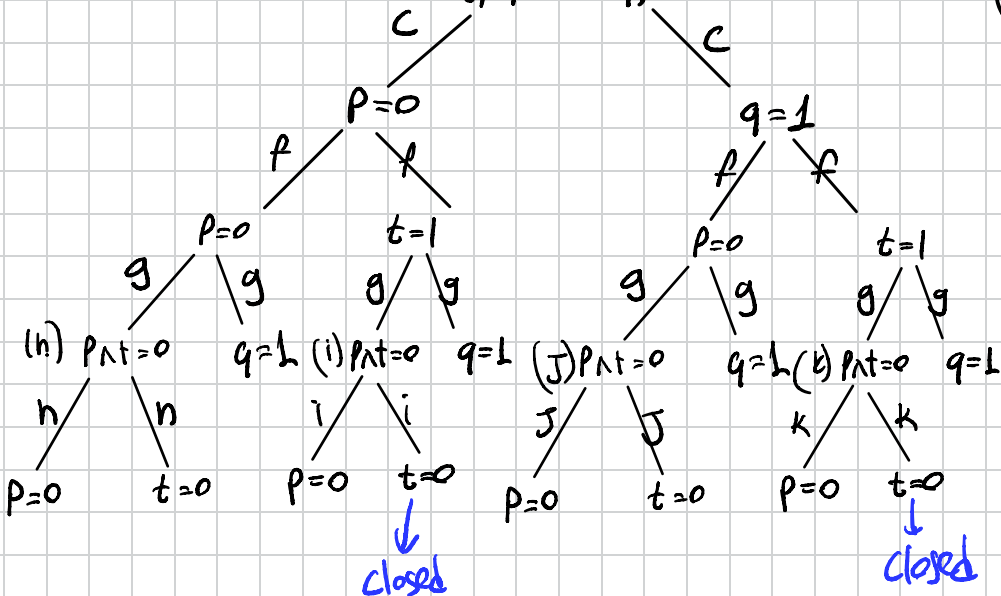
$$\begin{array}{c} | \\ b \\ (P \rightarrow q) = 1 \quad (c) \end{array}$$

$$\neg((P \rightarrow t) \wedge (P \wedge t \rightarrow q)) = 0 \quad (d)$$

$$\begin{array}{c} | \\ d \\ ((P \rightarrow t) \wedge (P \wedge t \rightarrow q)) = 1 \end{array}$$

$$\begin{array}{c} | \\ e \\ (P \rightarrow t) = 1 \quad (f) \end{array}$$

$$(P \wedge t \rightarrow q) = 1 \quad (g)$$



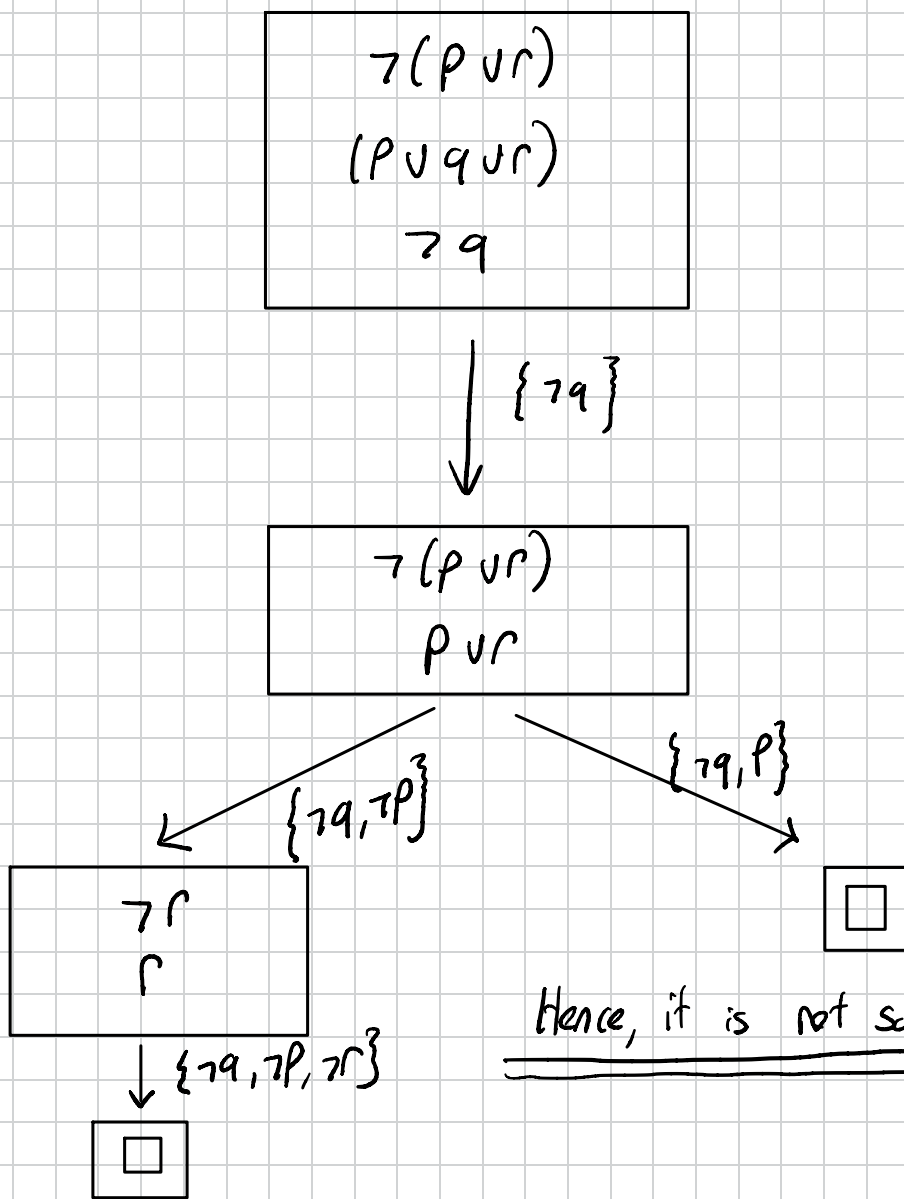
Hence, it is satisfiable.

2.1.

$$\neg(p \vee r) \wedge (\neg p \rightarrow (q \vee r)) \wedge \neg q$$

$$\neg(p \vee r) \wedge (\neg p \vee (q \vee r)) \wedge \neg q$$

$$\neg(p \vee r) \wedge (p \vee q \vee r) \wedge \neg q$$



2.2

$$\neg(((P \wedge q \wedge \neg w) \rightarrow r) \rightarrow \neg(((P \wedge r) \rightarrow (q \vee w)) \wedge P \wedge (q \rightarrow \neg(r \wedge w)))) \wedge (q \vee \neg r \wedge w)$$

Diagram showing the decomposition of the formula into sub-propositions:

- P_2 is the entire formula.
- P_3 is $(P \wedge r)$.
- P_4 is $(q \vee w)$.
- P_5 is $\neg(P_3 \rightarrow P_4)$.
- P_6 is $P \wedge (q \rightarrow \neg(r \wedge w))$.
- P_7 is $(q \vee \neg r \wedge w)$.

$$\neg(P_2 \rightarrow \neg(P_5 \wedge P_6)) \wedge P_7$$

$$\neg(\neg P_2 \vee \neg(P_5 \wedge P_6)) \wedge P_7$$

$$P_2 \wedge P_5 \wedge P_6 \wedge P_7$$

$$P_2 = P_1 \rightarrow r$$

$$\neg P_1 \vee r$$

$$\neg(P \wedge q \wedge \neg w) \vee r$$

$$\neg P \vee \neg q \vee w \vee r$$

$$P_5 = \neg P_3 \vee P_4$$

$$\neg(P \wedge r) \vee (q \vee w)$$

$$\neg P \vee \neg r \vee q \vee w$$

$$P_6 = q \rightarrow \neg(r \wedge w)$$

$$\neg q \vee \neg r \vee \neg w$$

General formula:

$$(\neg P \vee \neg q \vee w \vee r) \wedge (\neg P \vee \neg r \vee q \vee w) \wedge P \wedge (\neg q \vee \neg r \vee \neg w) \wedge (q \vee \neg r \vee w)$$

$$\begin{array}{l} \neg P \vee \neg q \vee w \vee r \\ \neg P \vee \neg r \vee q \vee w \\ P \\ \neg q \vee \neg r \vee \neg w \\ q \vee \neg r \vee w \end{array}$$

$\downarrow \{P\}$

$$\begin{array}{l} \neg q \vee w \vee r \\ \neg r \vee q \vee w \\ \neg r \vee \neg w \end{array}$$

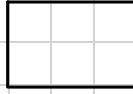
$\downarrow \{P, q\}$

$$\begin{array}{l} w \vee r \\ \neg r \vee \neg w \end{array}$$

$\downarrow \{P, q, r\}$

$$\neg w$$

$\downarrow \{p, q, r, zw\}$



Hence, it is satisfiable with the model

$\{p \leftarrow 1, q \leftarrow 1, r \leftarrow 1, w \leftarrow 0\}$