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Answer 1

a)
$$degree(a) = 3, degree(b) = 3, degree(c) = 3, degree(d) = 2, degree(e) = 3$$

Therefore, sum of the degrees of all nodes of G is 14.

b)The number of non-zero entries in the adjacency matrix representation of G is twice the number of edges.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Therefore, it is 14.

c)
$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Total number of entries is $7 \cdot 5 = 35$, and there are 14 non-zero entries. So, there are $35 - 14 = 21$ zero entries that can be seen in the table as well.

d)No, G doesn't have a complete graph with at least four vertices as a subgraph. It has complete graph with at most three vertices as a subgraph.

e)No, it is not because we can not color the graph with two colors. We can not group a,b,c or a,b,e or others. Therefore, it is not bipartite.

f)There are 7 edges. Each edge has 2 direction. Therefore, the total number is 2^7.

g)It is 4. There are more than one path that is longest. One of them is:

$$a \to c \to b \to e \to d$$

h)It is 1 because we can reach all vertexes from only one arbitrary node by traversing. Therefore, there is 1 connected component.

i)There is not any Euler circuit in G because the degrees of all vertexes are not even.

j)No, there is not, because it doesn't have 0 or 2 number of odd degree of vertexes. Since there are 4 odd degrees of vertexes, there is no Euler path in G.

k)Yes, G has a Hamilton Circuit:

$$a \to b \to c \to d \to e \to a$$

l)Yes, G has a Hamilton path:

$$a \to b \to c \to d \to e$$

Answer 2

1)All vertexes have the same degrees with the vertex they match.
2)Both graphs have the same number of edges and vertexes.
3)We can match up the vertices a and a',b and b', c and c',d and d', e and e', because:
a connects 2 vertices with degree 2, and a' provides this too.
b connects 2 vertices with degree 2, and b' provides this too.
c connects 2 vertices with degree 2, and c' provides this too.
d connects 2 vertices with degree 2, and d' provides this too.
e connects 2 vertices with degree 2, and e' provides this too.
Their set of edges can match up too. So, we can create a function from G to H in terms of edges sets and vertexes sets, which are bijective.
Since 1),2) and 3) are verified, G and H are isomorphic.

Answer 3

We have to create a set called visited that keeps track of vertices whose minimum distance from the first vertex is calculated. Initially, this set is empty. Current variable holds the current vertex. We have to create one vector called DP, which keeps the shortest distances that calculated, and the previous vertex(vertex before the vertex on the shortest path). Initialize all distances as infinite.

Assign the distance value as 0 for the first vertex so that it is picked first, and assign root the previous vertex of DP because it is the first vertex. Assign current variable to first vertex.

1)Assign a vertex, that is not in the visited set, and has a minimum distance to current vertex.
2)Add this vertex to visited set.
3)For all adjacent vertex v calculate the new distance which is the sum of distance value of vertex that we picked and weight of edge. If it is less than the distance value of v, then update the distance value of v.
4)Do this steps until all vertexes are visited.

At first, current vertex is s, and corresponding DP set. In the DP vector, all objects have two values, which are distance value, and the previous node. If the distance of a vertex is not calculated, the previous vertex value is set to null.
current = s, visited = {s}
$DP = \{((s = 0), root), ((w = 3), s), ((u = 4), s), ((v = 5), s), ((x = \infty), null), ((y = \infty), null), ((z = \infty), null), ((t = \infty), null)\}$

current = w, visited = {s,w}
$DP = \{((s = 0), root), ((w = 3), s), ((u = 4), s), ((v = 5), s), ((x = 11), w), ((y = \infty), null), ((z = 15), w), ((t = \infty), null)\}$

current = u, visited = {s,w,u}
$DP = \{((s = 0), root), ((w = 3), s), ((u = 4), s), ((v = 5), s), ((x = 11), w), ((y = 15), u), ((z = 15), w), ((t = \infty), null)\}$

current = v, visited = {s,w,u,v}
$DP = \{((s = 0), root), ((w = 3), s), ((u = 4), s), ((v = 5), s), ((x = 7), v), ((y = 11), v), ((z = 15), w), ((t = \infty), null)\}$

current = x, visited = {s,w,u,v,x}
$DP = \{((s = 0), root), ((w = 3), s), ((u = 4), s), ((v = 5), s), ((x = 7), v), ((y = 8), x), ((z = 13), x), ((t = \infty), null)\}$

current = y, visited = {s,w,u,v,x,y}
$DP = \{((s = 0), root), ((w = 3), s), ((u = 4), s), ((v = 5), s), ((x = 7), v), ((y = 8), x), ((z = 12), y), ((t = 17), y)\}$

current = z, visited = {s,w,u,v,x,y,z}
$DP = \{((s = 0), root), ((w = 3), s), ((u = 4), s), ((v = 5), s), ((x = 7), v), ((y = 8), x), ((z = 12), y), ((t = 15), z)\}$

We should stop here because all vertexes are visited, there is no vertex that is not visited. Distance of the shortest path is 15. We can retrace the path from the last vertex until we reach the first vertex(this path is reversed). So, it is:

$$t - z - y - x - v - s$$

Then the correct path is:

$$s \to v \to x \to y \to z \to t$$

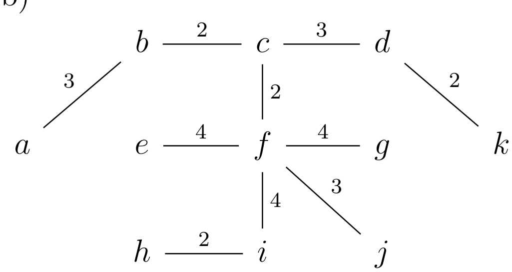
Answer 4

Prim's Algorithm
1)Select any vertex from the graph, and mark it as visited.
2)When selecting a new vertex, the smallest vertex among the neighbours of all visited vertexes and which does not create a cycle at the same time is marked as visited.
3)Repeat 2 until visiting all vertexes.

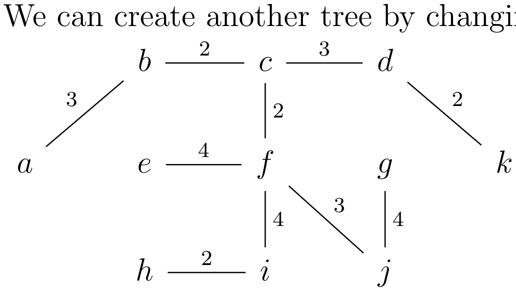
a) Select vertex a as the first vertex. Then the order of edges added is:

$$\{a, b\}, \{b, c\}, \{c, f\}, \{c, d\}, \{d, k\}, \{f, j\}, \{f, i\}, \{i, h\}, \{f, g\}, \{f, e\}$$

b)



c)
We can create another tree by changing the edges f-g to g-j.



Since there are more than one minimum spanning tree it is not unique.