$$\text{pdf of histogram } (k) = \frac{\#\text{ of } k \text{ valued pixels} \to n}{\#\text{ of total pixels} \to M}$$

$$CDF = 255 \sum_{j=0}^{k} \frac{n}{M} \quad \left(\begin{array}{l} \text{new value of } k \text{ valued pixels} \\ \text{after histogram equalization applied} \end{array} \right)$$

→ from prob theory, if we have transformation $s = T(r)$, the probability density of the output variable s, denoted as $P_s(s)$ is related to input by

$$P_s(s) = P_r(r) \left| \frac{dr}{ds} \right|$$

$$\frac{ds}{dr} = \frac{d}{dr}\left[255 \int_0^r P_r(w)\,dw \right] = 255\, P_r(r)$$

$$\Rightarrow P_s(s) = \cancel{P_r(r)} \left| \frac{1}{255\, \cancel{P_r(r)}} \right| = \frac{1}{255}$$

→ since $P_s(s)$ is a constant, the probability of every value in the output is equal. This means the output histogram is perfectly flat (uniform).