A Decomposition-based Multi-modal Multi-objective Evolutionary Algorithm with Problem Transformation into Two-objective Subproblems

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ABSTRACT

In some real-world multi-objective optimization problems, Pareto optimal solutions with different design parameter values are mapped to the same point with the same objective function values. Such problems are called multi-modal multi-objective optimization problems (MMOPs). For MMOPs, multi-modal multi-objective evolutionary algorithms (MMOEAs) have been developed for approximating both the Pareto front (PF) and the Pareto sets (PSs). However, most MMOEAs use population convergence in the objective space as the primary evaluation criterion. They do not necessarily have a high PS approximation ability. To better approximate both PF and PSs, we propose a decomposition-based MMOEA where an MMOP is transformed into a number of two-objective subproblems. One objective of each subproblem is a scalarizing function defined by a weight vector for the original MMOP, while the other is defined by a decision space diversity. Experimental results show a high approximation ability of the proposed method for both PF and PSs.

CCS CONCEPTS

• **Computing methodologies → Search methodologies**.

KEYWORDS

Multi-modal multi-objective evolutionary algorithm, MOEA/D

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1 INTRODUCTION

Multi-objective optimization problems (MOPs) frequently appear in the real world [11, 13]. The set of optimal solutions in the decision space is called the Pareto set (PS). The image of the PS in the objective space is called the Pareto front (PF). In some applications, there also exist a particular type of MOPs called multi-modal multi-objective optimization problems (MMOPs) [6, 12]. In MMOPs, some Pareto optimal solutions with the same objective function values have different design parameter values. To offer more alternatives for the decision maker of MMOPs, it is important to find the Pareto optimal solutions on all the PSs.

In general, multiobjective evolutionary algorithms (MOEAs) cannot effectively solve MMOPs since they do not consider the diversity of solutions in the decision space, but only the convergence and the diversity in the objective space. To address this issue, multi-modal multi-objective evolutionary algorithms (MMEAs) have been proposed such as DN-NSGA-II [6], DNEA [8], MMODE [15], MMODE_CSCD [4] for MMOPs.

MMEAs need a specialized ability to approximate both the PF and the PSs better to find the Pareto optimal solutions on all the PSs. However, most MMEAs use population convergence in the objective space as the primary evaluation criterion [7]. As a result, they do not necessarily have a high approximation ability of the PSs in the decision space.

To better approximate both the PF and PSs, we propose a decomposition based MMEA, called multi-modal multi-objective to two-objective (MM2T) in this paper. MM2T first transforms an MMOP into a number of single-objective subproblems using uniformly distributed weight vectors like MOEA/D [17]. Then, each subproblem is transformed into a new two-objective problem defined by the minimization of a scalizing function based on a weight vector for the original MMOP and the maximization of a decision space diversity measure. We also propose a differential evolution (DE) [10] based crossover operator where parent selection is performed considering both the PF and PSs approximation.

The rest of this paper is organized as follows. Section 2 explains the general background. Section 3 describes the proposed MM2T. Then, Section 4 compares MM2T with representative MMEAs on several MMOPs. Finally, Section 5 concludes this paper.

2 GENERAL BACKGROUND

2.1 Multiobjective Optimization Problems

An M-objective minimization problem is defined as:

$$\begin{aligned} &\text{Minimize} \quad \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_M(\boldsymbol{x})), \\ &\text{subject to} \quad \boldsymbol{x} \in \boldsymbol{X}, \end{aligned} \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{x})$ is an M-dimensional objective vector, $f_i(\boldsymbol{x})$ is the ith objective function, $\boldsymbol{x}$ is a D-dimensional decision vector, and $\boldsymbol{X}$ is the feasible region in the decision space. When the trade-off relations between objectives exist in MOPs, there is no single optimal solution, rather a set of Pareto optimal solutions.

2.2 Multi-modal Multi-objective Optimization Problems

In some MOPs especially engineering optimization problems like a conceptual design optimization problem of hybrid rocket engine [3] and distillation plant layout [9], there exist multiple different Pareto optimal solutions correspond to one point on the PF. Those solutions are useful as alternatives for the decision maker who selects solutions based on the objective function values. This type of problems are called MMOPs. Each point on the PF corresponds to two different solutions on the PS like:

$$\mathbf{f}(\boldsymbol{x}_1) = \mathbf{f}(\boldsymbol{x}_2) \quad \text{subject to } \boldsymbol{x}_1 \neq \boldsymbol{x}_2. \tag{2}$$

Most of MOEAs have no diversity maintenance mechanism in the decision space. Thus, once the population is biased toward a certain PS, new offspring solutions tend to be generated around the PS. As a result, solutions near or on the single PS are obtained although they approximate the PF.

2.3 MOEA/D

MOEA/D [17] is one of the most representative decomposition-based EMOAs. MOEA/D decomposes an M-objective MOP into N single-objective subproblems using a set of uniformly distributed weight vectors $\boldsymbol{w} = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_N\}$ in the objective space. Each weight vector $\boldsymbol{w}_i = (w_{i,1}, w_{i,2}, \ldots, w_{i,M})$ is defined as follows:

$$\sum_{k=1}^{M} w_{i,k} = 1, \quad w_{i,k} \in \left\{\frac{0}{H}, \frac{1}{H}, \ldots, \frac{H}{H}\right\}, \tag{3}$$

where H is a user-specified positive integer. Each subproblem optimizes a scalarzing function such as the Tchebycheff function [17]. The Tchebycheff function is given as follows:

$$g(\boldsymbol{w}_i, \boldsymbol{x}) = \max_{1 \le j \le M} \{w_{i,j} |f_j(\boldsymbol{x}) - z_j^*|\}, \tag{4}$$

where $\boldsymbol{z}^* = (z_1^*, z_2^*, \ldots, z_M^*)$ is the estimated ideal point which is the best value of each objective during the optimization.

3 PROPOSED ALGORITHM

In this paper, we propose MM2T, which can highly approximate both PF and PSs. Unlike other MMEAs, MM2T searches for solutions considering convergence to the PF and diversity in the decision space. In addition, we propose a random choice of two parent selection schemes in a DE crossover operator with a 0.5 probability. One parent selection scheme focuses on convergence to the PF, while the other focuses on diversity in the decision space.

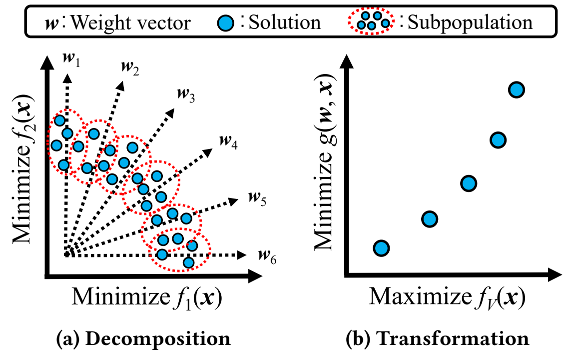

Figure 1: Problem decomposition and transformation of MM2T.

Like MOEA/D, MM2T uses weight vectors to decompose an MMOP into subproblems, but there are two differences. First, MM2T maintains a subpopulation of multiple solutions for each subproblem shown in Fig. 1a, while MOEA/D maintains one solution for each subproblem. Second, each subproblem in MM2T is a two-objective optimization problem, while that in MOEA/D is a single-objective optimization problem. As shown in Fig. 1b, each subproblem is transformed into a two-objective problem composed of the minimization of the scalarizing function value $g(\boldsymbol{w}, \boldsymbol{x})$ with the original weight vector and the maximization of the diversity measure $f_V(\boldsymbol{x})$ in the decision space. The framework of MM2T is shown in Algorithm 1. The next subsections explain each part of MM2T.

3.1 Population Initialization

For each subproblem, MM2T maintains S solutions as a subpopulation, i.e., if the number of weight vectors for the problem decomposition is N, the entire population has $S \times N$ solutions. MM2T randomly generates N solutions during population initialization, then evaluates the N solutions and selects S solutions from the N solutions for each subpopulation. That is, some solutions are shared among different subpopulations. In other words, only N solutions are evaluated in initialization.

3.2 Subpopulation Update

At first, the generation of each subpopulation is performed using the initial population of N solutions. During optimization, the update of each subpopulation is performed using $N + S$ solutions, which are the sum of all N offspring generated from each subpopulation and the current subpopulation S.

Let Q be the combined solution set used to update the subpopulations. First, we calculate the scalarizing function value $g(\boldsymbol{w}, \boldsymbol{x})$ of solution $\boldsymbol{x}$ in Q and the diversity measure $f_V(\boldsymbol{x})$ in the decision space. Then, we rank $\boldsymbol{x}$ using the Pareto ranking [1] and the crowding distance [1] in the two-objective space of $g(\boldsymbol{w}, \boldsymbol{x})$ and $f_V(\boldsymbol{x})$. For the diversity measure $f_V(\boldsymbol{x})$ in the decision space, we use the following equation (5) used in DNEA [8].

$$f_V(\boldsymbol{x}) = -\sum_{\boldsymbol{y} \in Q} \max\left\{0, 1 - \frac{d_{var}(\boldsymbol{x}, \boldsymbol{y})}{\sigma_{var}}\right\}, \tag{5}$$

ALGORITHM 1: MM2T

Input:
the number of subpopulations: N,
the subpopulation size: S,
weight vectors: $\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_N$.
Output:
subpoulations: $P = \{P_1, P_2, \ldots, P_N\}$.

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Initialization:
Randomly generate N solutions and set them to R.
Initialize the ideal point z* = (z*_1, z*_2, ..., z*_M).
for k = 1 to N do
  | Update subpopulation P_k with R.
end
while the stopping criterion is not met do
  Set R = ∅.
  for k = 1 to N do
    Select parent solutions from P_k.
    Apply differential evolution and mutation operator
      to generate a new solution y according to Eq. 8.
    Calculate the objective values of y.
    for t = 1 to M do
      if z*_t > f_t(y) then
        | z*_t = f_t(y)
      end
    end
    R = R ∪ {y}
  end
  for k = 1 to N do
    | Update subpopulation P_k with R.
  end
end
return P.
```

where $\boldsymbol{y}$ is Q, $d_{var}(\boldsymbol{x}, \boldsymbol{y})$ is the Euclidean distance between solutions $\boldsymbol{x}$ and $\boldsymbol{y}$ in the decision space, and σ_{var} is the niche radius. σ_{var} is formulated for solution $\boldsymbol{x}$ using the set $B_{\boldsymbol{x}}$ of the top n_{best} solutions $\boldsymbol{y}$ for which $d_{var}(\boldsymbol{x}, \boldsymbol{y})$ is small, as shown in (6) below.

$$\sigma_{var} = \sum_{\boldsymbol{x} \in Q} \frac{\sum_{\boldsymbol{y} \in B_{\boldsymbol{x}}} d_{var}(\boldsymbol{x}, \boldsymbol{y})}{n_{best} \times |Q|}, \tag{6}$$

where n_{best} is defined by (7):

$$n_{best} = \min\{3, \max(|Q| - 1, 0)\}. \tag{7}$$

The $f_V(\boldsymbol{x})$ value becomes larger if solution $\boldsymbol{x}$ is farther away in the decision space compared to the other solutions in the same subpopulation. Thus, a solution with a large $f_V(\boldsymbol{x})$ value is regarded as a diverse solution in the decision space.

3.3 Offspring Generation

In MM2T, one offspring is generated in each subpopulation at every generation. In other words, the number of offspring solutions is the same as the number of weight vectors, N. DE is used as a crossover operator. The i-th decision variable o_i of the offspring $\boldsymbol{o}$ is specified by three parent solutions $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, $\boldsymbol{p}_3$ as follows:

$$o_i = \begin{cases} p_{1,i} + F(p_{2,i} - p_{3,i}) & \text{if} \quad rand < CR, \\ p_{1,i} & \text{otherwise}, \end{cases} \tag{8}$$

where $F \in [0, 1]$ is a scaling factor, CR is a crossover probability, $rand$ is a uniform random number of $[0, 1)$.

To better approximate both PF and PSs, $\boldsymbol{p}_1$ is randomly selected by one of the following two schemes with a 0.5 probability.

PF approximation-oriented selection
$\boldsymbol{p}_1$ is selected by a binary tournament based on $g(\boldsymbol{w}, \boldsymbol{x})$.
PS approximation-oriented selection
$\boldsymbol{p}_1$ is selected by a binary tournament based on $f_V(\boldsymbol{x})$.

$\boldsymbol{p}_2$ and $\boldsymbol{p}_3$ are selected in a common manner regardless of the used $\boldsymbol{p}_1$ selection scheme. $\boldsymbol{p}_2$ is selected by a binary tournament based on $g(\boldsymbol{w}, \boldsymbol{x})$. The nearest neighbor solution of $\boldsymbol{p}_2$ is selected as $\boldsymbol{p}_3$. The Euclidean distance in the decision space is used for comparison for $\boldsymbol{p}_3$ selection. Parent solutions $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, $\boldsymbol{p}_3$ are determined within the corresponding subpopulation.

4 COMPUTATIONAL EXPERIMENTS

4.1 Experimental Settings

We compare the proposed MM2T with four representative MMEAs (i.e., DN-NSGA-II [6], DNEA [8], MMODE [15], MMODE_CSCD [4]) through computational experiments using 16 MMOPs defined in [16] (i.e., MMF1-9, MMF1_e, MMF1_z, MMF14, MMF14_a, SYM-PART simple, SYM-PART rotated, Omni_test).

As performance metrics, we use inverted generational distance (IGD) [14] and its decision space version, IGDX [18]. IGD evaluates the approximation to the PF in the objective space, while IGDX evaluates the approximation to the PSs in the decision space. Both measures are calculated from all the solutions generated during optimization. The reference points for IGD and IGDX calculation are the same as those provided in CEC2019 competition [5].

Table 1 lists the parameter settings. The parameters of the existing methods are based on the proposed papers for each method.

4.2 Experimental Results

We first performed the Friedman test [2] at the significance level of 0.05 over all the algorithms for all the MMOPs. The Friedman test was used to test the null hypothesis that all algorithms perform equally. The p values for IGD and IGDX were 1.707e-04 and 1.280e-02, respectively. Then, we performed the Nemenyi post-hoc analysis [2] for all pairwise comparisons based on the ranks of results over all the algorithms for all the MMOPs. The null hypothesis was also rejected at the significance level of 0.05.

Figures 2 and 3 show the critical difference diagrams for IGD and IGDX, respectively. In the critical difference diagram, an algorithm with better search performance has lower average ranks, i.e., on the right side of the diagram. In theory, the difference in the search performance among algorithms within a critical distance (i.e., a red line) is not statistically significant. From these diagrams, we can see that MM2T has the best average ranks for both IGD and IGDX. We can generally say that MM2T is superior to MMODE and MMODE_CSDC on the objective space, while it is superior to DNEA and DN-NSGA-II on the decision space.

Table 1: PARAMETER SETTINGS

Common settings of the used MMEAs	
Population size N	$100 \times D$
The number of evaluations	$5,000 \times D$
The number of runs	21
MM2T	
Subpopulation size S	40
Scalarizing function	Tchebycheff function[17]
Crossover	DE (CR: 1.0, F: 20)
Mutation	PM (Probability: 0.5, μ_m: 20)
DN-NSGA-II[6]	
CF	$N/2$
Crossover	SBX (Probability: 1.0, μ_c: 20)
Mutation	PM (Probability: 0.5, μ_m: 20)
DNEA[8]	
Crossover	SBX (Probability: 1.0, μ_c: 20)
Mutation	PM (Probability: $1/D$, μ_m: 20)
MMODE[15]	
Q	$N/2$
Crossover	DE (CR: 1.0, F: 20)
Mutation	PM (Probability: $1/D$, μ_m: 20)
MMODE_CSCD[4]	
n	10
Crossover	DE (CR: 0.8, F: 20)

D: The number of decision variables.

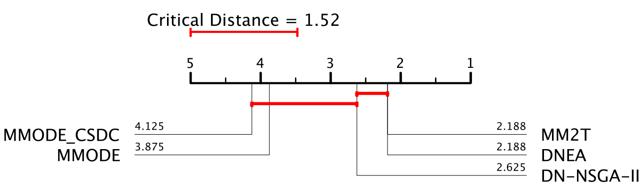

Figure 2: Critical difference diagram for IGD based on the Nemenyi test using the average ranks.

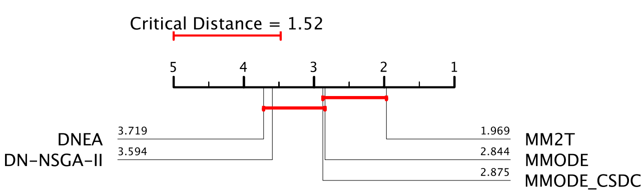

Figure 3: Critical difference diagram for IGDX based on the Nemenyi test using the average ranks.

5 CONCLUSION

In this paper, we proposed MM2T to achieve high approximation performance to both PF and PSs. MM2T decomposes an MMOP into a number of single-objective subproblems like MOEA/D and then transforms each subproblem to a two-objective problem that simultaneously considers convergence in the objective space and diversity in the decision space. In addition, two types of parent selection schemes that emphasize PF or PS approximation are probabilisitically used when generating offspring solutions. The experimental results show that MM2T has high approximation performance to both PF and PS compared to the existing MMEAs on average.

Our experiments were limited to small problems with up to three objectives and three decision variables. As future work, we will clarify the characteristics of MM2T through experiments using a variety of MMOPs with more objectives and variables.

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