poker and game theory algorithm

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1 アルゴリズムの擬似コード

1.1 Chance sampling Counterfactual Regret Minimization

Algorithm 1 Chance sampling Counterfactual Regret Minimization [1]

Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : r_I[a] \leftarrow 0$
Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : s_I[a] \leftarrow 0$
ChanceSamplingCFR(h, i, t, π_1, π_2):
if h is terminal **then return** $u_i(h)$
else if h is a chance node **then**
    Sample a single outocme $a \sim \sigma_c(h, a)$
     return ChanceSamplingCFR(ha, i, t, π_1, π_2)
end if
Let I be the information set containing h
$\sigma^t(I) \leftarrow$ RegretMatching (r_I)
$v_\sigma \leftarrow 0$
$v_{I\rightarrow a}[a] \leftarrow 0$ for all $a \in A(I)$
for $a \in A(I)$ **do**
    if $P(h) = 1$ **then**
        $v_{I\rightarrow a}[a] \leftarrow$ ChanceSamplingCFR$(ha, i, t, \sigma^t(I, a) \cdot \pi_1, \pi_2)$
    else if $P(h) = 2$ **then**
        $v_{I\rightarrow a}[a] \leftarrow$ ChanceSamplingCFR$(ha, i, t, \pi_1, \sigma^t(I, a) \cdot \pi_2)$
    end if
    $v_\sigma \leftarrow v_\sigma + \sigma^t(I, a) \cdot v_{I\rightarrow a}[a]$
end for
if $P(h) = i$ **then**
    for $a \in A(I)$ **do**
        $r_I[a] \leftarrow r_I[a] + \pi_{-i} \cdot (v_{I\rightarrow a}[a] - v_\sigma)$
        $s_I[a] \leftarrow s_I[a] + \pi_i \cdot \sigma^t(I, a)$
    end for
end ifreturn v_σ

1.2 Vanilla Counterfactual Regret Minimization

Algorithm 2 Vanilla Counterfactual Regret Minimization [1]

Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : r_I[a] \leftarrow 0$
Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : s_I[a] \leftarrow 0$
VanillaCFR(h, i, t, π_1, π_2):
if h is terminal **then return** $u_i(h)$
else if h is a chance node **then**
    Sample a single outocme $a \sim \sigma_c(h, a)$
    return $\sum_{a \in A(h)} \sigma_c(h, a)$ VanillaCFR(ha, i, t, π_1, π_2)
end if
Let I be the information set containing h
$\sigma^t(I) \leftarrow$ RegretMatching (r_I)
$v_\sigma \leftarrow 0$
$v_{I \to a}[a] \leftarrow 0$ for all $a \in A(I)$
for $a \in A(I)$ **do**
    if $P(h) = 1$ **then**
        $v_{I \to a}[a] \leftarrow$ VanillaCFR$(ha, i, t, \sigma^t(I, a) \cdot \pi_1, \pi_2)$
    else if $P(h) = 2$ **then**
        $v_{I \to a}[a] \leftarrow$ VanillaCFR$(ha, i, t, \pi_1, \sigma^t(I, a) \cdot \pi_2)$
    end if
    $v_\sigma \leftarrow v_\sigma + \sigma^t(I, a) \cdot v_{I \to a}[a]$
end for
if $P(h) = i$ **then**
    for $a \in A(I)$ **do**
        $r_I[a] \leftarrow r_I[a] + \pi_{-i} \cdot (v_{I \to a}[a] - v_\sigma)$
        $s_I[a] \leftarrow s_I[a] + \pi_i \cdot \sigma^t(I, a)$
    end for
end ifreturn v_σ

1.3 External Sampling Monte Carlo Counterfactual Regret Minimization

Algorithm 3 External Sampling Monte Carlo Counterfactual Regret Minimization [2]

Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : r_I[a] \leftarrow 0$

Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : s_I[a] \leftarrow 0$

ExternalSamplingMCCFR(h, i):

if h is terminal **then return** $u_i(h)$

else if h is a chance node **then**

  Sample a single outocme $a \sim \sigma_c(h, a)$

   return ExternalSamplingMCCFR(ha, i)

end if

Let I be the information set containing h

$\sigma^t(I) \leftarrow$ RegretMatching

if $P(h) = i$ **then**

  Let u be an array indexed by actions and $u_\sigma \leftarrow 0$

  for $a \in A(I)$ **do**

    $u[a] \leftarrow$ ExternalSamplingMCCFR(ha, i)

    $u_\sigma \leftarrow u_\sigma + \sigma(I, a) \cdot u[a]$

  end for

  for $a \in A(I)$ **do**

    $\tilde{r}(I, a) \leftarrow u[a] - u_\sigma$

    $r_I[a] \leftarrow r_I[a] + \tilde{r}(I, a)$

  end forreturn u_σ

else

  Sample a single outocme $a \sim \sigma(I)$

  $u \leftarrow$ ExternalSamplingMCCFR(ha, i)

  for $a \in A(I)$ **do**

    $s_I[a] \leftarrow s_I[a] + \sigma^t(I, a)$

  end forreturn u

end if

1.4 Outcome Sampling Monte Carlo Counterfactual Regret Minimization

Algorithm 4 Outcome Sampling Monte Carlo Counterfactual Regret Minimization [2]

Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : r_I[a] \leftarrow 0$
Initialize: $\forall I \in \mathcal{I}, \forall a \in A(I) : s_I[a] \leftarrow 0$
Initialize: $\forall I \in \mathcal{I} : c_I \leftarrow 0$
OutcomeSamplingMCCFR$(h, i, t, \pi_i, \pi_{-i}, s)$:
if h is terminal **then return** $u_i(h)/s, 1$
else if h is a chance node **then**
    Sample a single outcome $a \sim \sigma_c(h, a)$
    return OutcomeSamplingMCCFR$(ha, i, t, \pi_i, \pi_{-i}, s)$
end if
Let I be the information set containing h
$\sigma^t(I) \leftarrow$ RegretMatching
Let $\sigma'(I)$ be a sampling distibution at I
if $P(h) = i$ **then**
    $\sigma'(I) \leftarrow \epsilon\cdot$ UnIf(I) $+(1-\epsilon)\sigma(I)$
else
    $\sigma'(I) \leftarrow \sigma(I)$
end if
Sample an action a' with probability $\sigma'(I, a)$
if $P(h) = i$ **then**
    $(u, \pi_{tail}) \leftarrow$ OutcomeSamplingMCCFR$(ha', i, t, \pi_i \cdot \sigma(I, a), \pi_{-i}, s \cdot \sigma'(I, a))$
    for $a \in A(I)$ **do**
        $W \leftarrow u \cdot \pi_{-i}$
        compute $\tilde{r}(I, a)$
        $r_I[a] \leftarrow r_I[a] + \tilde{r}(I, a)$
    end for
else
    $(u, \pi_{tail}) \leftarrow$ OutcomeSamplingMCCFR$(ha', i, t, \pi_i, \pi_{-i} \cdot \sigma(I, a), s \cdot \sigma'(I, a))$
    for $a \in A(I)$ **do**
        $s_I[a] \leftarrow s_I[a] + (t - c_I) \cdot \pi_{-i} \cdot \sigma(I, a)$
    end for
    $c_I \leftarrow t$
end ifreturn $(u, \pi_{tail} \cdot \sigma(I, a))$

1.5 General Fictitious Self-Play

Algorithm 5 General Fictitious Self-Play [3]

function $\textsc{FictitiousSelfPlay}(\Gamma,\ n\ ,\ m)$
    Initialize: completely mixed π_1
    $\beta_2 \leftarrow \pi_1$
    $j \leftarrow 2$
    while within cumpuational budget **do**
        $\eta_j \leftarrow \textsc{Mixingparameter}(j)$
        $\mathcal{D} \leftarrow \textsc{GenerateData}(\pi_{j-1}, \beta, n, m, \eta_j)$
        for each player $i \in \mathcal{N}$ **do**
            $\mathcal{M}^i_{RL} \leftarrow \textsc{updaterlmemory}(\mathcal{M}^i_{RL}, \mathcal{D}^i)$
            $\mathcal{M}^i_{SL} \leftarrow \textsc{updateslmemory}(\mathcal{M}^i_{SL}, \mathcal{D}^i)$
            $\beta^i_{j+1} \leftarrow \textsc{reinforcementlearning}(\mathcal{M}^i_{RL})$
            $\pi^i_j \leftarrow \textsc{supervisedlearning}(\mathcal{M}^i_{SL})$
        end for
        $j \leftarrow j + 1$
    end while
    return π_{j-1}
end function

function $\textsc{GenerateData}(\pi, \beta, n, m, \eta)$
    $\sigma \leftarrow (1-\eta)\pi + \eta\beta$
    $\mathcal{D} \leftarrow n$ episode $\{t_k\}_{1\leq k\leq n}$ sampled from strategy profile σ
    for each player $i \in \mathcal{N}$ **do**
        $\mathcal{D}^i \leftarrow m$ episode $\{t^i_k\}_{1\leq k\leq n}$ sampled from strategy profile (β^i, σ^{-i})
        $\mathcal{D}^i \leftarrow \mathcal{D}^i \cup \mathcal{D}$
    end for
    return $\{D_k\}_{1\leq k\leq N}$
end function

1.6 Batch Fictitious Self-Play

Algorithm 6 Batch Fictitious Self-Play [4]

Initialize: completely mixed average strategy profile π_1

Initialize: replay memories $\mathcal{M}_{RL}, \mathcal{M}_{SL}$

Initialize: the constant numbers of episode that are sampled simulataneously, n, and alternatingly, m

for $k = 1, ..., K-1$ **do**

    SAMPLESIMULTANEOUSEXPERIENCE $(n, \pi_k, \mathcal{M}_{RL})$

    Each player i updates their approximate best response strategy

    $\beta^i_{k+1} \leftarrow$ REINFORCEMENTLEARNING$(\mathcal{M}^i_{RL})$

    SAMPLEALTERNATINGEXPERIENCE $(m, \pi_k, \beta_{k+1}, \mathcal{M}_{RL}, \mathcal{M}_{SL})$

    Each player i updates their average strategy

    $\pi^i_{k+1} \leftarrow$ SUPERVISEDLEARNING$(\mathcal{M}^i_{SL})$

end for

return π_k

function SAMPLESIMULTANEOUSEXPERIENCE$(n, \pi_k, \mathcal{M}_{RL})$

    Sample n episodes from strategy profile π

    For each player i store their experienced transitions, $(u^i_t, a_t, r_{t+1}, u^i_{t+1})$ in their reinforcement learning memory $\mathcal{M}^i_{RL}$

end function

function SAMPLEALTERNATINGEXPERIENCE$(m, \pi_k, \beta_{k+1}, \mathcal{M}_{RL}, \mathcal{M}_{SL})$

    for each player $i \in \mathcal{N}$ **do**

        Sample m episodes from strategy profile (β^i, π^{-i})

        Store player i's experienced transitions, $(u^i_t, a_t, r_{t+1}, u^i_{t+1})$ in their reinforcement learning memory $\mathcal{M}^i_{RL}$

        Store player i's experienced own behaviour, (u^i_t, a_t) or $(u^i_t, \beta^i(u^i_t))$ in their supervised learning memory $\mathcal{M}^i_{SL}$

    end for

end function

1.7 Table-lookup Batch FSP with Q-learning and counting model

Algorithm 7 Table-lookup Batch FSP with Q-learning and counting model [4]

Initialize Q-learning parameters, e.g. learning stepsize
Initialize all agents' table-lookup action-values, $\{Q^i\}_{1 \in N}$
Initialize all agents' table-lookup counting models, $\{N^i\}_{1 \in N}$
Initialize and run Algorithm 6 (Batch FSP) with REINFORCEMENTLEARNING and SUPERVISEDLEARNING methods below

function REINFORCEMENTLEARNING($\mathcal{M}^i_{RL}$)
  Restore previous iteration's Q^i-values
  Update (decay) learning stepsize and Boltzmann temperature
  updated Q^i-values with Q-learning from $\mathcal{M}^i_{RL}$
   return Boltzmann[Q^i]
end function

function SUPERVISEDLEARNING($\mathcal{M}^i_{SL}$)
  Restore previous iteration's counting model N^i, and average strategy, π^i
  for each $(u_t, \rho_t))$ in $\mathcal{M}^i_{SL}$ **do**
    $\forall a \in \mathcal{A}(u_t) : N^i(u_t, a) \leftarrow N^i(u_t, a) + \rho_t(a)$
    $\forall a \in \mathcal{A}(u_t) : \pi^i(u_t, a) \leftarrow N^i(u_t, a)/N^i(u_t)$
  end for
  Empty $\mathcal{M}^i_{SL}$
   return π^i
end function

1.8 Neural Fictitious Self-Play

Algorithm 8 Neural Fictitious Self-Play [5]

Initialize game Γ and excute an agent via RUNAGENT for each player in the game

function RUNAGENT(Γ)
  Initialize replay memories $\mathcal{M}_{RL}$ (circular buffer) and $\mathcal{M}_{SL}$ (reservoir)
  Initialize average-policy network $\Pi(s, a|\theta^{\Pi})$ with random parameters θ^{Π}
  Initialize action-value network $Q(s, a|\theta^{Q})$ with random parameters θ^{Q}
  Initialize target network parameters $\theta^{Q'} \leftarrow \theta^{Q}$
  Initialize anticipatory parameter η
  for each episode **do**
    Set policy $\sigma \leftarrow \begin{cases} \epsilon - greedy(Q), \text{with probability } \eta \\ \Pi, \text{with probability } 1 - \eta \end{cases}$
    for $t = 1, ..., T$ **do**
      Observe information state s_t and sample action a_t from σ
      Execute action a_t in game and observe reward r_{t+1} and next information state s_{t+1}
      Store transitions $(s_t, a_t, r_{t+1}, s_{t+1})$ in reinforcement learning memory M_{RL}
      if agent follows best response policy $\sigma = \epsilon - greedy(Q)$ **then**
        Store behavior tuple (s_t, a_t) in supervised learning memory M_{SL}
      end if
      Periodically update θ_{π} with stochastic gradient descent on loss
        $\mathcal{L}(\theta^{\pi}) = \mathbb{E}_{(s,a)\sim M_{RL}}[-log\Pi(s, a|\theta^{\Pi})]$
      Periodically update θ_Q with stochastic gradient descent on loss
        $\mathcal{L}(\theta^{Q}) = \mathbb{E}_{(s,a,r,s_{prime})\sim M_{RL}}[(r + max_{a'}Q(s', a'|\theta^{Q'}) - Q(s, a|\theta^{Q}))^2]$
      Periodically update target network parameters $\theta^{Q'} \leftarrow \theta^{Q}$
    end for
  end for
end function

1.9 Neural Fictitious Self-Play with PPO

Algorithm 9 Neural Fictitious Self-Play with PPO [5]

Initialize game Γ and excute function PPO

function PPO(Γ)
  Initialize number of parallel processing N
  Initialize number of players NP
  Initialize replay memories $\mathcal{M}_{RL}$ and $\mathcal{M}_{SL}$ (reservoir)
  Initialize average-policy network $\Pi(s, a|\theta^{\Pi})$ with random parameters θ^{Π}
  Initialize action-value network $\beta(s, a|\theta^{\beta})$ with random parameters θ^{β}
  Initialize $\theta^{\beta}_{old} \leftarrow \theta^{\beta}$
  for each iteration **do**
    for actor $= 1, 2, ..., N$ **do**
      target-player = (actor -1) % NP
      Set policy $\sigma \leftarrow \begin{cases} \beta_{\theta_{old}}\ strategy & \text{if } P(s) = \text{target-player for s in } \sigma \\ \Pi\ strategy & \text{if } P(s) \text{ != target-player for s in } \sigma \end{cases}$
      make environment including chance, terminal and un-target-player node
      $t = 0$
      done = False (whether one episode finish)
      observe s_0 from environment
      while not done **do**
        Observe information state s_t and sample action a_t from σ
        Execute action a_t in game and observe state s_{t+1}, reward r_{t+1} and done
        Store transitions $(s_t, a_t, pd_t, r_{t+1})$ in reinforcement learning memory M_{RL}
        Store behavior tuple (s_t, a_t) in supervised learning memory M_{SL}
        $t+ = 1$
      end while
      Compute adavantage estimates $\hat{A}_1, ..., \hat{A}_t$
    end for
    Optimize surrogate L_{RL} wrt θ^{β} in M_{RL}
    $\beta_{\theta_{old}} \leftarrow \beta_{\theta}$
    Optimize L_{SL} wrt θ^{π} with cross entropy in M_{SL}
    $\Pi_{\theta_{old}} \leftarrow \Pi_{\theta}$
    empty M_{RL}
  end for
end function

1.10 Soft Actor-Critic

Algorithm 10 Soft Actor-Critic [6]

Initialize game Γ and excute function SAC

function SAC(Γ)
  Initialize replay memories $\mathcal{M}_{RL}$ and $\mathcal{M}_{SL}$ (reservoir)
  Initialize $\theta^{\pi}, \theta^{Q}, \phi, \alpha, \eta$
  for each iteration **do**
    Set policy $\sigma \leftarrow \begin{cases} BR, \text{with probability } \eta \\ \Pi, \text{with probability } 1-\eta \end{cases}$
    for $t = 1, ..., T$ **do**
      Observe information state s_t and sample action a_t from σ
      Execute action a_t in game and observe reward r_{t+1} and next information state s_{t+1}
      Store transitions $(s_t, a_t, r_{t+1}, s_{t+1})$ in reinforcement learning memory M_{RL}
      if agent follows best response policy $\sigma = BR$ **then**
        Store behavior tuple (s_t, a_t) in supervised learning memory M_{SL}
      end if
      Periodically update θ_{π} with stochastic gradient descent on loss
        $\mathcal{L}(\theta^{\pi}) = \mathbb{E}_{(s,a)\sim M_{RL}}[-log\Pi(s, a|\theta^{\Pi})]$
      Periodically update θ^{Q}, ϕ, α with stochastic gradient descent on loss
        $\mathcal{L}(\theta^{Q}), \mathcal{L}(\phi), \mathcal{L}(\alpha)$
    end for
  end for
end function

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