

## Important Information

1. This lab is due by the deadline specified in CatCourses.
2. Your solution must be submitted electronically through CatCourses.
3. Labs MUST be solved individually.
4. Your solution must be coded in Matlab and strictly follow the given requirements. Failure to comply with the desired structure for the input/output files leads to an immediate 0.

## Extended Kalman filter

In this lab assignment you will use the extended Kalman filter (EKF) to estimate the pose (state) of a robot moving in a known environment. The state is  $\mathbf{x} = [x \ y \ \theta]^T$  and corresponds to the position  $(x, y)$  and heading  $\theta$  of the robot. The input vector is bidimensional,  $\mathbf{u} = [u_v \ u_\theta]^T$ . The state transition equation is the following:

$$\mathbf{x}(t + \Delta t) = g(\mathbf{x}(t), \mathbf{u}(t + \Delta t)) + \varepsilon$$

where  $g(\mathbf{x})$  is defined as follows (simplified motion model):

$$\begin{aligned}x(t + \Delta t) &= x(t) + u_v(t + \Delta t)\Delta t \cos \theta \\y(t + \Delta t) &= y(t) + u_v(t + \Delta t)\Delta t \sin \theta \\\theta(t + \Delta t) &= \theta(t) + u_\theta \Delta t\end{aligned}$$

and  $\varepsilon \sim N(0, R)$ , where  $R$  is the following matrix:

$$R = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}.$$

The robot moves between 3 markers at known locations and is equipped with a sensor returning the distance between the robot and the markers, i.e. the sensor model is the following:

$$\mathbf{z}(t + \Delta t) = h(\mathbf{x}(t + \Delta t)) + \delta.$$

The  $i$ -th ( $1 \leq i \leq 3$ ) component of the vector  $\mathbf{z}(t + \Delta t)$  is then

$$z_i(t + \Delta t) = \sqrt{(x(t + \Delta t) - x_i)^2 + (y(t + \Delta t) - y_i)^2}$$

where  $x_i, y_i$  are the known coordinates of the  $i$ -th landmark and  $\delta \sim N(0, Q)$  where

$$Q = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}.$$

Assume  $\mathbf{x}(0) \sim N(\mu_0 = [0 \ 0 \ 0]^T, \Sigma_0)$

$$\Sigma_0 = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}.$$

and let  $\Delta t = 0.5$ . Markers are located at locations  $(5; 5), (4; 7), (-3; 2)$ . Assume that  $t_0 = 0$ .

1. using the inputs available in `inputs.txt` and the sensor readings stored in the file `sensor_readings.txt` provide a final estimate for  $\mathbf{x}$ . Note that the files have equal length, i.e. you have one input and one sensor reading per iteration. The first column in `inputs.txt` gives  $u_v$  and the second gives  $u_\theta$ , whereas every row in `sensor_readings.txt` gives the noisy measurements from the three landmarks (in the same order as they are listed above). Note that the sensor readings and the input have the same frequency, i.e., every 0.5s. Your code should print the estimated mean and covariance matrix at each time step.
2. use the same input file `inputs.txt` and the sensor readings from `sporadic_sensor_readings.txt` solve the same problem. However in this case sensor readings come only from time to time. In this case the file has four columns and the first one is the time when the sensor reading was collected (the other three are as above). Your code should print the estimated mean and covariance matrix at each time step.
3. write a script that visualizes the trajectory of the robot during the estimation process, and also displays the uncertainty ellipsoid for the  $x, y$  components of the state while the estimation evolves.

Properly document your matlab files and include a text file explaining how to run them.