Manual of plotastrodata

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1 Read Data

The data class AstroData can take a fits file.

```
from plotastrodata.analysis_utils import AstroData
d = AstroData(fitsimage='file_name.fits')
```

AstroData can take more arguments, such as Tb and sigma. Tb=True means the data values will be converted from flux densities in the unit of Jy beam⁻¹ to brightness temperatures in the unit of K. sigma specifies how to measure the noise level of the data values: for example, the default option of 'hist' means to use the histgram of the data values. The argument of pvpa is the position angle of the PV cut direction in the unit of degree, which will be used to calculate the spatial resolution of the PV diagram.

The data class AstroFrame is necessary to form the AstroData instance in a useful format. AstroFrame can take quantities related to coordinate ranges.

vmin, vmax, and vsys are in the unit of km s⁻¹. rmax is in the unit of arcsec. Instead of rmax, other arguments (xmin, xmax, ymin, ymax, xoff, yoff) can be used to adjust the x and y ranges in more detail. When an argument of fitsimage is given, the central coordinates center is read from the given fits file; the rest frequency is also read from the fits file, which is used for the frequency-velocity conversion. Moreover, an argument of dist can specifies the distance used to change the unit of spatial coordinates from arcsec to au. An argument of swapxy is used to swap the x and y coordinates. An argument of pv must be True when the AstroData instance to be read is a position-velocity (PV) diagram, i.e., the first and second axes are the spatial and velocity coordinates. When quadrants=True, the first and third (or second and fourth) quadrants of the PV diagram will be averaged. xflip and yflip will be used to determine the plotting direction of the x and y axes, respectively; these have a meaning only when a figure is made from the data set.

Forming the AstroData instance needs the following command.

```
f.read(d)
```

After this command, the AstroData instance has useful attributes in the format of numpy array: a 1D array of d.x (as well as d.y and d.v), a 2D or 3D array of d.data, a 1D array of d.beam, and a float of d.sigma. d.x and d.y are the relative coordinates in the unit of arcsec from the

given center. Similarly, d.v is the relative coordinate in the unit of km s⁻¹ from the given vsys. d.beam is the beam components array([bmaj, bmin, bpa]), where bmaj and bmin are in the unit of arcsec, while bpa is in the unit of degree. When Tb=True above, the data values are converted to the brightness temperature and stored as d.data. d.sigma is the noise level measured in the way specified above in the same unit as d.data. AstroData can take two more arguments, restfreq and cfactor. restfreq is used to explicitly set the rest frequency in the unit of Hz, which is used to convert frequency to velocity and Jy beam⁻¹ to K. When this argument is zero (restfreq=0), the frequency axis is not converted to velocity. When the unit of the frequency/velocity axis in the input fits header is m/s, f.read(d) divides the coordinate by 1000. The argument cfactor specifies a constant factor that d.data is multiplied by. This will be useful when one wants to change the intensity unit from Jy beam⁻¹ to mJy beam⁻¹.

In addition to the input attributes, f.read(d) adds three attributes to the AstroData instance d: fitsimage_org, sigma_org, and fitsheader. fitsimage_org saves the input fitsimage, while fitsimage is updated to None after f.read(d). Similarly, sigma_org saves the input sigma, which may be a string, while sigma is updated to the calculated value. fitsheader is the fits header in the format of astropy.io.fits.open(fitsimage)[0].

When pv=True in AstroFrame, the read method changes the beam attribute of the AstroData instance. The first element will be the velocity resolution. The second element of beam will be $1/\sqrt{\cos^2(\text{bpa} - \text{pvpa})/\text{bmaj}^2 + \sin^2(\text{bpa} - \text{pvpa})/\text{bmin}^2}$. This is calculated from the intersection of the beam ellipse and the PV cut line. The third element will be 0; thus, the velocity resolution (first element) is regarded as the vertical length by default when this "beam" is plotted in a PV diagram. The original beam will be stored in beam_org.

2 Analyze Data

AstroData also has handy methods to analyze the 2D/3D data.

The binning method rebins the 2D/3D data with a given width for each coordinate.

```
d.binning(width=[5, 4, 2])
```

This command takes an average over 5 channels in the velocity direction, 4 pixels in the y direction, and 2 pixels in the x direction. The number of channels and pixels are decreased accordingly after this method. If the length of width is 2, the two values are regarded as the widths for the y and x directions.

The centering method sets the coordinates so that the spatial center and the systemic velocity have the exact zero coordinates by interpolation.

```
d.centering(includexy=True, includev=False)
```

This command adjusts the x and y coordinates but does not adjust the velocity coordinate. This method will be useful when one wants to quickly get a radial profile along a line passing the center or a PV diagram (using the rotate method together).

The circularbeam method makes the beam shape circular by additional 2D Gaussian convolution. This method takes no argument.

```
d.circularbeam()
```

The new beam has the major and minor axes same as the old major axis. This method also updates the attribute of d.beam accordingly.

The deproject method deprojects the 2D/3D data with a given position angle (P.A.) and inclination angle.

```
d.deproject(pa=45, incl=45)
```

pa is the position angle from the north to the east in the unit of degree. incl is the inclination angle; incl=0 means the face-on configuration and thus no deprojection. This command replaces d.data and d.beam with the deprojected data and the deprojected beam, respectively.

The fit2d method performs MCMC fitting to the 2D image of d.data or d.data[chan], where chan is the channel number, by using emcee or ptemcee through the class of plotastrodata.fitting_utils.EmceeCorner.

model is a function with the shape of model(par, x, y), where par is the list of parameters. bounds is the 2D list of the parameter boundary in the shape of $[[p_{0,\min},p_{1,\min},...],[p_{0,\max},p_{1,\max},...]]$. kwargs_fit and kwargs_plotcorner are the arguments for the methods of EmceeCorner.fit and EmceeCorner.plotcorner, respectively. nsteps includes the number the burn-in steps. The output res is a dictionary consisting of popt, plow, pmid, phigh, model, and residual. The first four keys provide the best, lower percentile, 50 percentile, and higher percentile parameters. The last two keys provide the model and residual 2D images.

The histogram method returns the bins and the histogram in the bins of the attribute of d.data by using numpy.histogram().

```
hbin, hist = d.histogram(bins=10)
```

The arguments are the same as those for numpy.histogram(). The bins (hbin) have the same length as the histogram (hist), which is different from the original numpy.histogram.

The gaussfit2d method performs the 2D Gaussian fitting to the 2D image of d.data or d.data[chan], where chan is the channel number, by using scipy.optimize.curve_fit.

```
res = d.gaussfit2d(chan=12)
```

This command performs the fitting to the channel of 12 in d.data. For 2D data, the chan argument can be omitted. The output (res here) is a dictionary having keys of popt, pcov, model, and residual. popt and pcov are the optimized parameters (peak intensity, central x, central y, major FWHM, minor FWHM, and P.A.) and their covariance. The model and residual are 2D arrays with the same shape of the fitted 2D array.

The mask method puts numpy.nan on pixels that satisfies a given condition.

```
d.mask(dataformask=d2.data, includepix=[1e-3, 100], excludepix=[50, 200])
```

This command puts numpy.nan on pixels of d.data where d2.data is outside [1e-3, 100] or inside [50, 200], where d2 is an AstroData instance different from d. This method will be useful when one wants to put a mask on a moment 1 map using the values of a moment 0 map.

The profile method makes line profiles at given spatial positions.

```
v, f, g = d.profile(xlist=[-0.5, 0.2], ylist=[1, 0.8], ellipse=[0.3, 0.2, 45])
```

This command makes line profiles at (x,y)=(-0.5,1) and (0.2,0.8) in the unit of arcsec. The intensity of each profile is an average over a boxcar ellipse with the major axis of 0.3 arcsec, the minor axis of 0.2 arcsec, and the P.A. of 45 degrees. Instead of xlist and ylist, coordinate strings can be input using an argument of coords: coords=['00h00m00.0s 00d00m00.0s', '11h11m11.1s 11d11m11.1s']. When ellipse is omitted, the profile is made by picking up the values at the closest pixel to the given position. When the ellipse size is not so large compared to the pixel size, the integer argument ninterp may be useful; this makes the pixel size ninterp times finer by interpolartion. When the boolian argument flux is True, the output profile has the unit of Jy. Additionally, when the boolian argument gaussfit is True, the profiles will be fitted with a 1D Gaussian function. The output v above is d.v. The output f above is a list of 1D-array profiles, i.e., 2D array. The output g is a list of dictionaries; each dictionary has keys of best and error (square root of the diagonal components of the covarience).

The rotate method rotates the attribute d.data by the given angle in the unit of degree by interpolation.

```
d.rotate(pa=45)
```

When a pixel refers to a position outside the original image, this pixel has numpy.nan after this method.

The slice method makes a radial profile for a 2D image or a PV diagram for a 3D image along a given direction and length by interpolation.

```
r, f = d.slice(length=3, pa=45, dx=0.2)
```

This commands makes a radial profile (or PV diagram) along a cut with a length of 3 arcsec at P.A.=45 degrees with a separation of 0.2 arcsec. The output r and f are both 1D arrays of the positional offsets and the intensity at the positions. When the separation dx is omitted, the absolute value of the x pixel size is adopted.

The todict method returns the attributes as a dictionary.

```
a = d.todict()
```

This output includes keys of data, x, y, v, fitsimage, beam, Tb, restfreq, cfactor, sigma, and center. This dictionary can be input to methods of PlotAstroData as **a.

The writetofits method exports the instance as a fits file.

```
d.writetofits(fitsimage='new_file_name.fits')
```

The output fits file reuses the header components of the fits file used to make the AstroData instance ('file_name.fits' above); some header components are automatically updated after the above-mentioned methods for analysis, such as CDELT1.

3 Plot Data

The class PlotAstroData can take an AstroData instance through the method of d.todict().

```
from plotastrodata.plot_utils import PlotAstroData

p = PlotAstroData(rmax=3.0)
p.add_color(**d.todict())
p.add_scalebar(length=50 / 140, label='50 au')
p.set_axis()
p.savefig('figure_name.png')
```

These commands make a color map using the AstroData instance d. PlotAstroData can take the same arguments as AstroFrame to define the plotting ranges; particularly rmax (also, vmin and vmax for a cube or a PV diagram) is necessary. The method p.add_color() can take a fits file directly instead of the AstroData instance, as p.add_color(fitsimage='file_name.fits'). This method can actually take the same arguments as AstroData to specify the data to be plotted as a color map; **d.todict() does this indirectly. The command p.add_scalebar() can be omitted if the map does not need to show a scale bar. The command p.set_axis() (or p.set_axis_radec()) is necessary even without any argument.

The class PlotAstroData can take more arguments. veldigit specifies the number of digits of the velocity label on the channel maps. channelnumber specifies which channel is used to make a 2D image; when this is None (default), channel maps are made instead of a 2D image. nrows and ncols specify the grid shape of the channel maps. fontsize specifies the basic font size in the figure. nancolor specifies which color is used when the value is nan. dpi specifies the resolution of a raster image file. figsize is the same as the argument for Figure.figure of matplotlib. fig and ax can be used to input external Figure and Axes objects. More detailed usage can be found in the example.py file and https://plotastrodata.readthedocs.io/en/latest/#. The following is the explanation of each method of PlotAstroData.

3.1 Maps

The add_color method plots the given data in the format of a color map.

The stretch argument can be 'linear', 'log', 'asinh', or 'power'. The Arcsin hyperbolic stretch can be adjusted by another argument of stretchscale as asinh(data / stretchscale). The power-law stretch can be adjusted by another argument of stretchpower as ((data / vmin)^(1 - stretchpower) - 1) / (1 - stretchpower) / ln(10), where vmin is a given minimum value. The arguments starting with 'cb' adjust the colorbar. In addition, two arguments, xskip and yskip, can be used to skip spatial pixels in this method as well as add_contour, add_segment, and add_rgb. Similarly, show_beam, beamcolor, and beampos can be used in these four methods to adjust beam color and position, as well as switch whether to show the beam.

The add_contour method plots the given data in the format of a contour map.

```
p.add_contour(**d.todict(), levels=[-3, 3, 6, 9])
```

The levels argument spedifies the contour levels in the unit of d.sigma.

The add_rgb method plots the given data in the format of three-color maps. The input three data sets are mixed as Red. Green, and Blue.

```
p.add_rgb(**d.todict(), stretch=['linear', 'log', 'linear'])
```

For this method, the input is a combination of three data sets, and thus d.data here must be a 1D list (not a numpy array) of three 2D/3D numpy arrays. In the same way as add_color, stretchscale and stretchpower are available in this method.

The add_segment method plots the given data in the format of a segment map, as is often used for polarization maps.

The input format of the data is different from that for add_color and add_contour. One of the following pairs must be given: (ampfits, angfits), (Qfits, Ufits), (amp, ang), or (stQ, stU). The latter two pairs are supposed to be in the numpy.array format. The ampfactor argument can be used to adjust the segment length. When angonly=True, the segment length is set to be uniform. The rotation can be used to rotate the segments in the unit of degree. The cutoff argument specifies the intensity threshold to calculate the amplitude and angle from the Stokes Q and U intensities.

3.2 Patches, markers, text, lines, arrows, and scale bar

The class PlotAstroData also has methods for plotting regions (patches), markers, text, lines, and arrows. The following is the explanation of each method for this purpose.

The add_region method overlays an ellipse or a rectangular. the given data in the format of a segment map, as is often used for polarization maps.

The argument patch may be 'ellipse' or 'rectangle'. The patch position can be specified by a 2D list like [0.1, 0.2] or a coordinate string like '01h23m45.6s 01d23m45.6s'. The list means a fractional position from left to right and bottom to top: e.g., [0, 0], [0.5, 0.5], and [1, 1] are bottom left, center, and top right, respectively. majlist, minlist, and palist specifies the patch size in the unit of arcsec and the orientation in the unit of degree. Other arguments for matplotlib.patches.Ellipse or matplotlib.patches.Rectangle can be used in this method. Another method add_beam uses this method and is executed internally in add_color, add_contour, add_segment, add_rgb.

The add_marker method overlays markers.

```
p.add_marker(poslist=[[0.5, 0.5], [0.1, 0.2]])
```

Except for the position list poslist (same as in add_region), the marker properties can be specified by the arguments for Axes.plot of matplotlib.

The add_text method overlays text.

Except for the position list poslist (same as in add_region) and the list of text slist, the text properties can be specified by the arguments for Axes.text of matplotlib.

The add_line method overlays lines.

The position list poslist (same as in add_region) specifies the start position of the line. anglelist and rlist specify the position angle in the unit of degree and the length in the unit of arcsec of the line, respectively. Other arguments for Axes.plot of matplotlib can be used in this method.

The add_arrow method overlays arrows.

poslist, anglelist, and rlist are the same in add_line. Other arguments for Axes.quiver of matplotlib can be used in this method.

The add_scalebar method overlays a scale bar.

```
p.add_scalebar(length=0.5,
label='70 au',
barpos=[0.8, 0.1])
```

The bar length length is in the unit of arcsec. The bar label label is located above the bar. barpos is a fractional position from left to right and from bottom to top (same as poslist). In addition, the arguments of color, fontsize, linewidth, and bbox can be used in this method. bbox is a dictionary argument for Axes.text of matplotlib.

3.3 Setting axes and saving figures

The method set_axis (or set_axis_radec) is necessary even without any argument. This method provides axes of relative coordinates and can also adjust things related to the axes.

```
p.set_axis(title='figure title')
```

The title argument can be either string or dictionary; the latter is a set of arguments for Axes.set_title or Figure.suptitle of matplotlib. This method can also take more arguments related to Axes.set_* of matplotlib: xscale, yscale, xlim, ylim, xlabel, ylabel, xticks, yticks, xticklabels, and yticklabels are in the same format as in Axes.set_* of matplotlib. xticksminor and yticksminor are either list or integer; the latter means how many times more ticks are needed than the major ticks. Two more dictionary arguments, grid and aspect, available as for Axes.grid and Axes.set_aspect of matplotlib.

The method set_axis_radec provides axes in the right ascension and declination coordinates.

```
ylabel='Dec. (ICRS)',
nticksminor=2,
grid=None)
```

title, xlabel, ylabel, and grid are the same as in set_axis. nticksminor specifies how many times more ticks are needed than the major ticks.

The method savefig saves the figure as an image file and/or shows it on the screen.

In addition to the two arguments, the arguments for Figure.savefig of matplotlib can be used for this method.

The method get_figax returns the Figure and Axes objects.

```
fig, ax = p.get_figax()
```

3.4 Other plotting functions

The function plot3d is a function independent of the PlotAstroData class. This function provides an HTML file to show a 3D figure that can be interactively rotated in a web browser.

```
plot3d(**d.todict(),
    levels=[3, 6, 9],
    cmap='Jet',
    alpha=0.08,
    xlabel='R.A. (arcsec)',
    ylabel='Dec. (arcsec)',
    vlabel='Velocity (km/s)',
    eye_p=0,
    eye_i=180,
    outname='filename',
    show=True)
```

The data to be plotted can be an instance of AstroData. levels specifies which surfaces are plotted. cmap is an argument (named colorscale) used in the 'mesh3d' type data in plotly.graph_objs. alpha is also used through the 'mesh3d' type data in plotly.graph_objs. eye_p and eye_i specify the default viewing direction. The output has the extension of .html with the file name of outname. When show=True, a web browser will be excecuted automatically to show the 3D figure.

The plot_utils module has more functions to make other types of figures. plotprofile provides a figure of line profiles and plotslice provides a 1D slice of a 2D image. These two functions use AstroData.profile and AstroData.slice, respectively. Hence, it may be more flexible to make figures in one's preferred way using the outputs of these method than using plotprofile and plotslice.

4 Relative and Absolute Coordinates

The relation between relative and absolute coordinates is not trivial on the plane of the sky. For example, the point 30 degrees (= 2h) east of '00h00m00s 60d00m00s' has coordinates of '03h16m25.6s 48d35m25s', rather than '2h00m00s 60d00m00s'. The module coord_utils provides useful functions for the coordinate transformation.

```
from plotastrodata.coord_utils import xy2coord
from plotastrodata.coord_utils import coord2xy
s = xy2coord(xy=[[30, 90], [0, 0]], coordorg='00h00m00s 60d00m00s')
print(s)
#['03h16m25.58528421s +48d35m25.36040662s', '06h00m00s +00d00m00s']
a = coord2xy(coords=s, coordorg='00h00m00s 60d00m00s')
print(a)
#[[30, 90], [0, 0]]
```

This example calculates 30 degrees east (and 0 degrees north) and 90 degrees east (and 0 degrees north) of '00h00m00s 60d00m00s' and puts back the resulting coordinates to the relative coordinates in the unit of degree ([offset to the east, offset to the north]).

5 Rotation

Comparison of a physical model and an observational result often requires 3D rotation between the model coordinates and the observational coordinates. The los_utils module provides a useful coordinate transformation. It starts with a parabolic streamer, as an example, on a coordinate set, $(x_{\text{sys}}, y_{\text{sys}}, z_{\text{sys}})$, named system coordinates, as shown in Figure 1(a). The direction of such a streamer is defined with polar and azimuthal angles (θ_0 and ϕ_0 , respectively) in a 3D coordinate, $(x_{\text{phy}}, y_{\text{phy}}, z_{\text{phy}})$, named physical coordinates. Figures 1(b) and 1(c) show the relation between $(x_{\text{sys}}, y_{\text{sys}}, z_{\text{sys}})$ and $(x_{\text{phy}}, y_{\text{phy}}, z_{\text{phy}})$, through an intermediate coordinate set, $(x_{\text{azi}}, y_{\text{azi}}, z_{\text{azi}})$, named azimuthal coordinates.

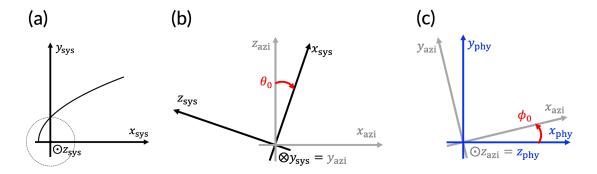


Figure 1: (a) The streamer system. (b) The system and azimuthal coordinates. (c) The azimuthal and physical coordinates.

When a model in the physical coordinates is observed, the reltaion beteen the physical and observational coordinates $(x_{\text{obs}}, y_{\text{obs}}, z_{\text{obs}})$ requires inclination and position (or orientation) angles. Although this relation also requires another azimuthal angle, the ϕ_0 devined above will work for the azimuthal rotation. Figures 2(a) and 2(b) show the relation between $(x_{\text{phy}}, y_{\text{phy}}, z_{\text{phy}})$ and $(x_{\text{obs}}, y_{\text{obs}}, z_{\text{obs}})$, through an intermediate coordinate set, $(x_{\text{ori}}, y_{\text{ori}}, z_{\text{ori}})$, named oriented coordinates. In this definition, when the zero position angle (P.A.= 0) means a configuration where the z_{phy} direction is projected to the y_{obs} direction. In other words, this P.A. is not that of the disk major axis but that of the blueshifted outflow in a typical protostellar system.

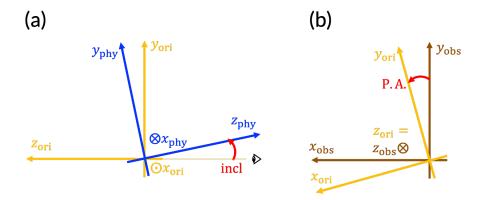


Figure 2: (a) The physical and oriented coordinates. (b) The oriented and observational coordinates.

Figure 3 summarizes the relation between the system and observational coordinates with the four angles (θ_0 , ϕ_0 , incl., and P.A.) through the physical coordinates. The relation can be expressed

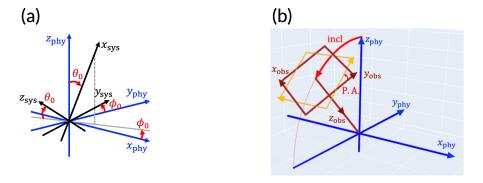


Figure 3: (a) The system and physical coordinates. (b) The physical and observational coordinates.

by the rotational matrices $R_x(\theta)$, $R_y(\theta)$, and $R_z(\theta)$ as follows:

$$\boldsymbol{x}_{\mathrm{sys}} = R_y(\frac{\pi}{2} - \theta_0)\boldsymbol{x}_{\mathrm{azi}},$$
 (1)

$$\boldsymbol{x}_{\text{ori}} = R_z(-\phi_0)\boldsymbol{x}_{\text{phy}},\tag{2}$$

$$\mathbf{x}_{\text{sys}} = R_y (\frac{\pi}{2} - \theta_0) \mathbf{x}_{\text{azi}}, \tag{1}$$

$$\mathbf{x}_{\text{ori}} = R_z (-\phi_0) \mathbf{x}_{\text{phy}}, \tag{2}$$

$$\mathbf{x}_{\text{phy}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} R_x (-\text{incl.}) \mathbf{x}_{\text{ori}}, \tag{3}$$

$$\boldsymbol{x}_{\text{ori}} = R_z(\text{P.A.})\boldsymbol{x}_{\text{obs}}.$$
 (4)

The transformation between $x_{
m obs}$ and $x_{
m sys}$ can be done through the functions obs2sys and sys2obs in los_utils.

```
from plotastrodata.los_utils import obs2sys
from plotastrodata.los_utils import sys2obs
import numpy as np

xobs, yobs, zobs = sys2obs(xsys=1/np.sqrt(2), ysys=1/np.sqrt(2), zsys=0,
pa=0, incl=30, phi0=0, theta0=90, polar=False)
print(xobs, yobs, zobs)
#-0.707107 0.612372 0.353553
#-1/sqrt(2) cos(30deg)/sqrt(2) sin(30deg)/sqrt(2)
xsys, ysys, zsys = obs2sys(xobs=xobs, yobs=yobs, zobs=zobs, pa=0, incl=30,
phi0=0, theta0=90, polar=False)
print(xsys, ysys, zsys)
#0.7071077 0.707107 0.0000000
#1/sqrt(2) 1/sqrt(2) 0
```

The input coordinate (e.g., xsys or xobs) may be a numpy array. When polar=True, xsys, ysys, and zsys are the radius, polar angle, and azimuthal angle, respectively; the radius is in the unit of arcsec, and the polar and azimuthal angles are in the unit of radian. xobs, yobs, and zobs are the Cartesian coordinates regardless of the polar argument. In particular, the combination of polar=True and $\theta_0 = 90$ is useful when a model is defined in the physical coordinates in the polar format.

In addition to the functions for the spatial coordinate transformation, los_utils has another function, polarvel2losvel, to transform polar velocities (v_r, v_θ, v_ϕ) to line-of-sight velocity (i.e., $v_{z,obs}$).

```
from plotastrodata.los_utils import polarvel2losvel
import numpy as np

v = polarvel2losvel(v_r=-1/np.sqrt(2), v_theta=0, v_phi=1/np.sqrt(2),
theta=np.pi/2, phi=np.pi/2, incl=30, phi0=0, theta0=90)
print(v)
#-0.353553
#-1/sqrt(2)/2
```

theta and phi may be numpy arrays and in the unit of radian. incl, phi0, and theta0 are the same as in obs2sys and sys2obs. This velocity transformation is useful for calculating the line-of-sight velocity when the 3D velocity field of a model is written in the polar format as a function of the polar coordinates. This velocity transformation can be derived from the following ideas. First, a vector is expressed as a matrix product of a basis matrix and a component column vector.

$$E_{1}x_{1} := \begin{pmatrix} e_{x,1} & e_{y,1} & e_{z,1} \end{pmatrix} \begin{pmatrix} x_{1} \\ y_{1} \\ z_{1} \end{pmatrix} = \begin{pmatrix} e_{x,2} & e_{y,2} & e_{z,2} \end{pmatrix} \begin{pmatrix} x_{2} \\ y_{2} \\ z_{2} \end{pmatrix} =: E_{2}x_{2}$$

$$(5)$$

$$x_{1} = E_{1}^{-1}E_{2}x_{2}$$

$$(6)$$

This provides the change-of-basis matrix. This matrix can also change the basis for velocity vectors.

$$v_1 = E_1^{-1} E_2 v_2 \tag{7}$$

Hence, we already know the relation between v_{ori} and v_{sys} .

$$\mathbf{v}_{\text{ori}} = R_x(\text{incl.}) \begin{pmatrix} -1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1 \end{pmatrix} R_z(\phi_9) R_y(\theta_0 - \frac{\pi}{2}) \mathbf{v}_{\text{sys}}$$
 (8)

The polar velocities can be expressed with the Cartesian velocities as follows:

$$\begin{pmatrix} v_r \\ v_\theta \\ v_\phi \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} =: E_r^{-1} \boldsymbol{v}. \tag{9}$$

Then, the polar velocities are linked to v_{sys} .

$$\mathbf{v}_{\text{sys}} = E_r \begin{pmatrix} v_{r,\text{sys}} \\ v_{\theta,\text{sys}} \\ v_{\phi,\text{sys}} \end{pmatrix} = (E_r^{-1})^T \begin{pmatrix} v_{r,\text{sys}} \\ v_{\theta,\text{sys}} \\ v_{\phi,\text{sys}} \end{pmatrix}$$
(10)

Finally, the line-of-sight velocity is calculated as the z component of $v_{z,ori}$ because the rotation about P.A. does not change the line-of-sight velocity.

$$v_{\rm los} = v_{z,\rm obs} = v_{z,\rm ori} \tag{11}$$

6 Fourier Transform

The Fourier transform frequently appears in the context of (radio) astronomy. Although it can be easily done by a computer, the numerical way of calculating the Fourier transform is different from the mathematical way. The main difference is the phase center. The fft_utils module provides an output closer to the mathematical way. This module is a wrapper of numpy.fft. The following shows a comparison between the plotastrodata.fftcentering module and the original numpy.fft module.

```
# Example of numpy.fft.fft
import numpy as np

x = np.linspace(-99.5, 99.5, 200)
f = np.where(np.abs(x) < 10, 1, 0)

u = np.fft.fftshift(np.fft.fftfreq(len(x), d=x[1]-x[0]))
F = np.fft.fftshift(np.fft.fft(f))</pre>
```

```
# Example of plotastrodata.fft_utils
from plotastrodata.fft_utils import fftcentering
import numpy as np

x = np.linspace(-99.5, 99.5, 200)
f = np.where(np.abs(x) < 10, 1, 0)

F, u = fftcentering(f=f, x=x, xcenter=0)</pre>
```

Figure 4 shows the input and output functions obtained by the example scripts above using numpy.fft.fft and plotastrodata.fft_utils.fftcentering. One will expect that the phase center is at x = 0 or the center of x, and then the expected Fourier transform of Figure 4(a) will be Figure 4(c). The phase center is indeed specified in fftcentering by the argument of xcenter=0. In comparison, Figure 4(b) can be produced by xcenter=x[0].

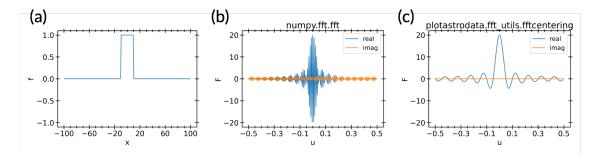


Figure 4: (a) The input function for the Fourier transform. (b) The real and imaginary parts obtained by numpy.fft.fft. (c) The real and imaginary parts obtained by from plotastrodata.fft_utils.fftcentering.

The inverse Fourier transform can be done by ifftcentering as follows.

```
from plotastrodata.fft_utils import fftcentering
from plotastrodata.fft_utils import ifftcentering
import numpy as np

x = np.linspace(-99.5, 99.5, 200)
f = np.where(np.abs(x) < 10, 1, 0)

F, u = fftcentering(f=f, x=x, xcenter=0)
ynew, xnew = ifftcentering(F=F, u=u, xcenter=0, x0=x[0], outreal=True)
print(np.max(np.abs(xnew - x)) < 1e-10, np.max(np.abs(ynew - y)) < 1e-10)
#True, True</pre>
```

The last two outputs are the same as the two inputs within the numerical error, which is consistent with iFT[FT[f]] = f. ifftcentering also requires the phase center value and additionally the starting value of the iFTed coordinate (xnew in the above example). This is because the Fourier transform is independent of absorbute values of the input coordinate. The argument of outreal=True means that the output ynew has only real values.

It is also possible for fft_utils to shift the phase of given FTed function and coordinate, by fft_utils.shiftphase.

```
from plotastrodata.fft_utils import shiftphase
import numpy as np

x = np.linspace(-99.5, 99.5, 200)
f = np.where(np.abs(x) < 10, 1, 0)

u = np.fft.fftshift(np.fft.fftfreq(len(x), d=x[1]-x[0]]))
F = np.fft.fftshift(np.fft.fft(f))</pre>
```

The output of shiftphase is the same as the first output (F in the example) obtained by fftcentering. The offset argument of xoff is determined by "new center (0) – old center (x[0])".

The functions introduced above (fftcentering, ifftcentering, and shiftphase) have counterparts for 2D arrays: fftcentering2, ifftcentering2, and shiftphase2. fftcentering2 requires an argument of y as well as x; these two arguments can be either 1D or 2D arrays (after numpy.meshgrid). The center is also expressed by two arguments xcenter and ycenter. Similarly ifftcentering takes arguments of u, v, xcenter, ycenter, x0, and y0, and shiftphase takes arguments of u, v, xoff, and yoff.