

The case $|P| = 4$.

- (1) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = 1$.
- (2) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(2a + 3b) = s$.
 - (i) $p = 2$, $q \neq 3, 5$,
 - (ii) $p = 2$, $q = 3$, $r \neq 5$,
 - (iii) $p = 2$, $q = 3$, $r = 5$,
 - (iv) $p = 2$, $q = 5$, $r \neq 3$,
 - (v) $p = 2$, $q = 5$, $r = 3$.
- (3) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(a + 3b) = s$, $N_P(2a + b) = 1$.
 - (i) $p = 2$, $r \neq 3, 5$,
 - (ii) $p = 2$, $r = 3$, $q \neq 5$,
 - (iii) $p = 2$, $r = 3$, $q = 5$,
 - (iv) $p = 2$, $r = 5$, $q \neq 3$,
 - (v) $p = 2$, $r = 5$, $q = 3$.
- (4) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(a + 3b) = s$, $N_P(2a + b) = s$.
 - (i) $p = 2$, $q, r \neq 3$,
 - (ii) $p = 2$, $q = 3$,
 - (iii) $p = 2$, $r = 3$.
- (5) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(a + 2b) = ps$, $N_P(2a + b) = 1$.
- (6) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(2a + b) = s$, $N_P(a + 3b) = 1$.
 - (i) $p = 2$, $q \neq 3, 5$,
 - (ii) $p = 2$, $q = 3$, $r \neq 5$,
 - (iii) $p = 2$, $q = 3$, $r = 5$,
 - (iv) $p = 2$, $q = 5$, $r \neq 3$,
 - (v) $p = 2$, $q = 5$, $r = 3$.
- (7) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(2a + b) = s$, $N_P(a + 2b) = ps$.
 - (i) $p = 2$, $q \neq 5$,
 - (ii) $p = 2$, $q = 5$.
- (8) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(2a + b) = 1$, $N_P(a + 2b) = 1$, $(p, q \neq 2)$.
- (9) $P = \{p, q, r, s\}$, $N_P(a) = p$, $N_P(b) = q$, $N_P(a + b) = r$, $N_P(a + 2b) = s$, $N_P(2a + b) = 1$.
 - (i) $p \neq 3$, $r \neq 2$,

$$(ii) \ p = 3, \ q \neq 5, \ r = 2,$$

$$(iii) \ p = 3, \ q = 5, \ r = 2,$$

$$(iv) \ p = 5, \ q \neq 3, \ r = 2,$$

$$(v) \ p = 5, \ q = 3, \ r = 2,$$

$$(vi) \ p \neq 3, 5, \ r = 2.$$

$$(10) \ P = \{p, q, r, s\}, \ N_P(a) = p, \ N_P(b) = q, \ N_P(a + b) = r, \ N_P(a + 2b) = s, \ N_P(2a + b) = s.$$

$$(i) \ r \neq 2,$$

$$(ii) \ p, q \neq 5, \ r = 2,$$

$$(iii) \ p = 5, \ r = 2.$$

$$(11) \ P = \{p, q, r, s\}, \ N_P(a) = p, \ N_P(b) = q, \ N_P(a + b) = rs, \quad (p = 2).$$

$$(12) \ P = \{p, q, r, s\}, \ N_P(a) = p, \ N_P(b) = q, \ N_P(a + b) = rs, \quad (p, q \neq 2).$$

$$(13) \ P = \{p, q, r, s\}, \ N_P(a) = pq, \ N_P(b) = r, \ N_P(a + b) = s, \quad (p = 2).$$

$$(14) \ P = \{p, q, r, s\}, \ N_P(a) = pq, \ N_P(b) = r, \ N_P(a + b) = s, \quad (r = 2).$$

$$(15) \ P = \{p, q, r, s\}, \ N_P(a) = pq, \ N_P(b) = r, \ N_P(a + b) = s, \quad (p, q, r \neq 2).$$

$$(16) \ P = \{p, q, r, s\}, \ N_P(a) = pq, \ N_P(b) = r, \ N_P(a + b) = 1.$$

$$(17) \ P = \{p, q, r, s\}, \ N_P(a) = pq, \ N_P(b) = rs.$$

$$(18) \ P = \{p, q, r, s\}, \ N_P(a) = pqr, \ N_P(b) = s.$$