Complex system – Vicsek model analysis 109202502 林于軒

Introduction

Vicsek model is a mathematical model used to describe the collective behavior of active matters. It consists in point-like self-propelled particles that evolve at constant speed and align their velocity with their neighbors' one in presence of noise. Such a model shows collective motion at high density of particles or low noise on the alignment.

The mathematical way to describe the model by it assumes that flocking is due to the combination of any kind of self-propulsion and of effective alignment. The discrete time evolution of one particle is set by two equations:

At each time step Δt, each agent aligns with its neighbors within a given distance r with a noise $\eta(t)$:

$$\theta_i(t+\Delta t) = \langle \theta_j \rangle_{|r_i - r_j| < r} + \eta_i(t)$$

The particle then moves at constant speed v in new direction:

$$r_i(t+\Delta t) = r_i(t) + v\Delta t \begin{pmatrix} \cos\theta_i(t) \\ \sin\theta_i(t) \end{pmatrix}$$

This model shows a phase transition from a disordered motion to large-scale ordered motion. At large noise or low density, particles are on average not aligned, and they can be described as a disordered gas. At low noise and large density, particles are globally aligned and move in the same direction (collective motion).

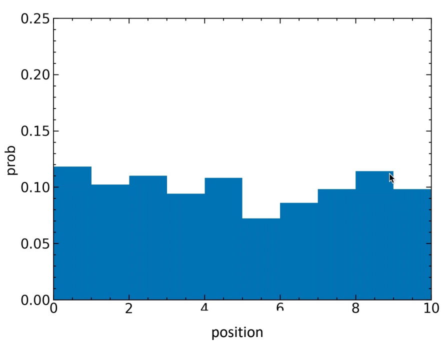
Data collection

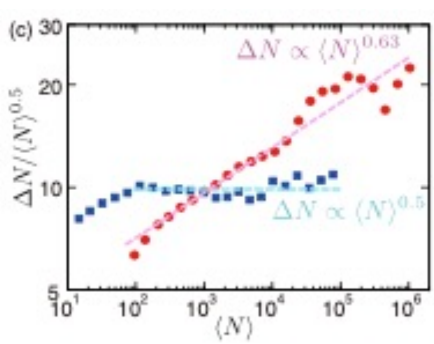
The data was collected by python simulation, with simulation code from: Philip Mocz (2021). Create Your Own Active Matter Simulation (With Python)
https://github.com/pmocz/activematter-python

The parameters that we are going to change is:

amplitude of noise(eta) and density of the system($\rho = \frac{N}{L^2}$)

We collected the particles position and angle by time. Thus, we can plot the figure of probability density distribution of the particle

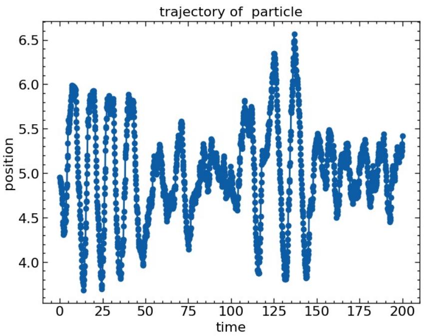


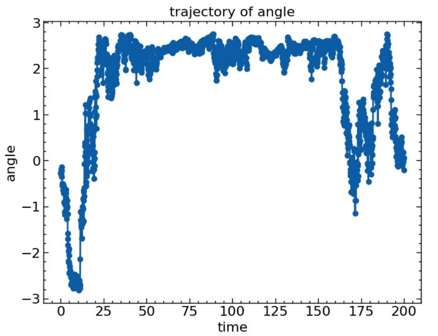


The PDF of the left plot(video, see in ppt page 9)shows the time evolution of particles, which it starts with particles are evenly disperse, and once it shows the nematic order, the probability at a certain position will become high and it moves with the system's velocity.

Once it is in a nematic order, the number fluctuation will be like the right plot, in specific region, the number of particle will be greater than $\langle N\rangle$

(right figure from : Daiki Nishiguchi & Ken H. Nagai & Hugues Chaté & Masaki Sano(2017). Long-range nematic order and anomalous fluctuations in suspensions of swimming filamentous bacteria)



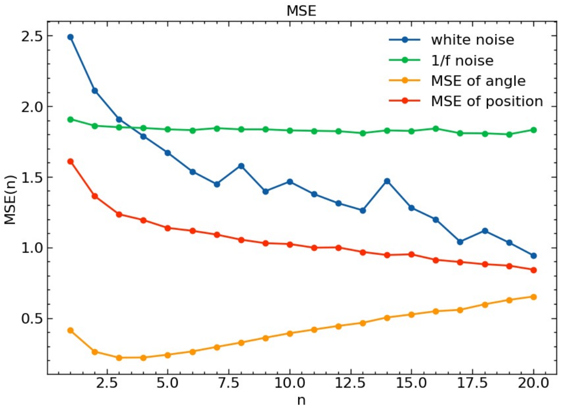


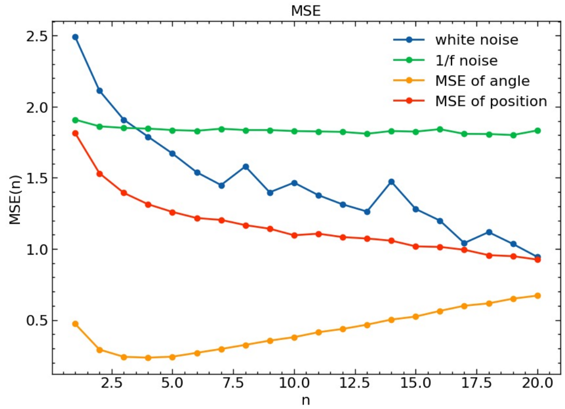
The two plot shows the collected data of particle position and angle variation by time.

Data analysis

1. Multiscale entropy

By using MSE analysis, I was able to have the information of the complexity or irregularity of my different position and angle's time series data.





In the MSE analysis, we could see that the position looks more like a white noise, which is a non-stationary time series with long-range dependencies, the complexity of the data decreases at longer time scales. While the angle looks more like a 1/f noise, which is a stationary time series, the complexity of the data is similar at all time scales.

The result we got from MSE was not surprise by looking at the trajectory of the above plot, on the plot of positions' trajectory, we can see a periodic sine-wave like shape, which indicates that the position of this system was followed in a certain period, so that the information it brought in for long time scale wasn't that much.

For Shuffled MSE, shuffling the data for shuffled MSE disrupts the underlying structure, resulting in a different MSE analysis pattern. It randomizes the temporal order, breaking inherent patterns and correlations. Shuffled data may show a more uniform spectral density and reduced complexity, indicating a lack of long-range dependencies.

Thus, we assume that if the original data follows white noise, with a power spectral density decreasing with frequency, shuffling destroys its long-range dependencies. Shuffled data is unlikely to retain white noise characteristics. However, if the original data exhibits 1/f noise, shuffling maintains its properties in MSE analysis. 1/f noise has equal power across frequencies and lacks discernible patterns. Shuffling only rearranges values without introducing systematic changes or correlations, preserving the statistical properties. Thus, shuffled 1/f noise data is likely to display characteristics of white noise in the MSE analysis.

While in our data it doesn't follows the assumption, it is still white noise for position and 1/f noise for angle after we shuffled the data.

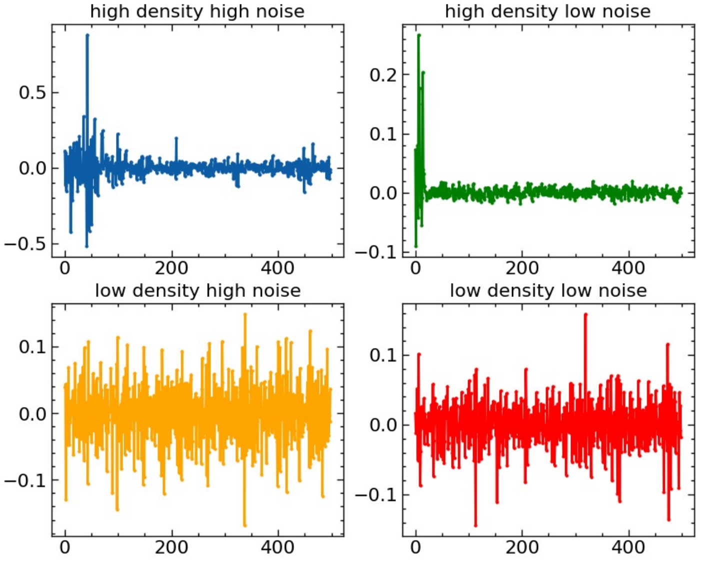
2. Detrended fluctuation analysis(DFA):

By doing the DFA analysis, we may find some characteristic of the data:

Identification of Long-Range Correlations: DFA analysis can detect the presence of long-range correlations in a time series. The DFA exponent (α) can help quantify the strength and nature of these correlations. Since α close to 0.5 indicates uncorrelated or random data, while values greater than 0.5 indicate positive correlations (persistent behavior), and values less than 0.5 indicate negative correlations (anti-persistent behavior), we can fit the slope to find the value α and find out the correlation of the data

Characterization of System Dynamics: DFA analysis can provide information about the self-similarity and fractal properties of a time series. It helps in understanding the organization and complexity of the system generating the data.

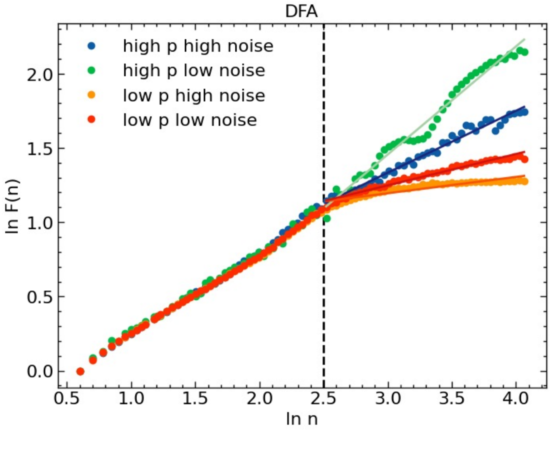
The DFA analysis was doing by the steps of angle for four different phases:



By comparing the four plot of the steps, we can found that:

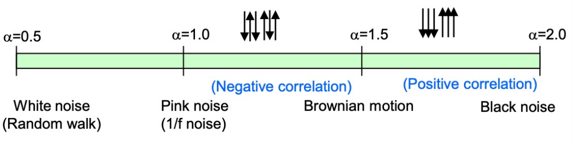
- It takes longer time to arrange into nematic order for high noise in high density system, and also cause a greater fluctuation
- A higher noise in low density region will cause a weaker long-range dependence and random or un-correlated behavior at longer time scale

Taking the different phases undergo detrended fluctuation analysis, we got the result



Slope of each line

	ln n < 2.5	ln n > 2.5
High density high noise	0.572 ± 0.005	0.417 ± 0.008
High density low noise	0.548 ± 0.007	0.727 ± 0.016
Low density high noise	0.544 ± 0.004	0.109 ± 0.007
Low density low noise	0.550 ± 0.004	0.211 ± 0.006



In DFA analysis, we got the result that before log n < 2.5, whether the noise and density difference, they all got a similar slope, which is about 0.5, the log n < 2.5 region is the region of about the particle goes around for one cycle, so it is determined by systems initial condition(box size, initial velocity). In four systems, they have same initial condition of box size and velocity, thus causing the same slope.

After log n > 2.5, we can divide it roughly into two areas, for high density system, α is approximately greater than 0.5, either high noise or low noise, they will shows a nematic order, which we can found positive correlation of the particles' angle through DFA analysis, and in particle the high density low noise system, which is in a smectic phase, shows a slope of 0.7 which indicated it's strong correlation of different particles' angle.

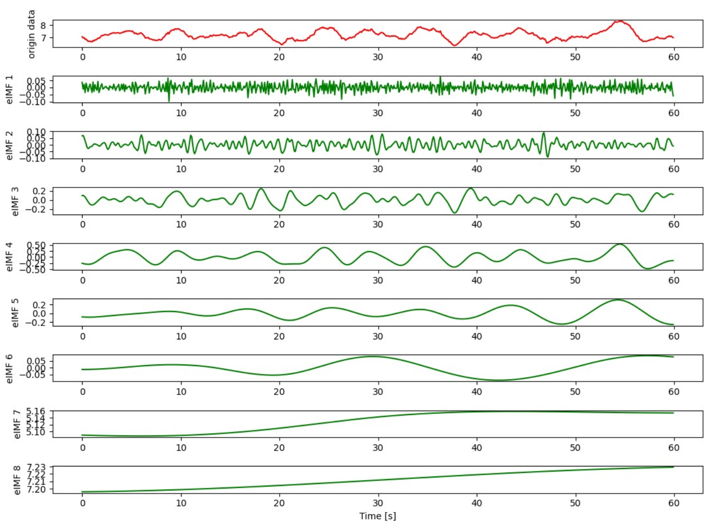
Another region is α smaller than 0.5, which distinguish the region of lower density, in low density region, whether low or high noise the system will show a slope smaller than 0.3, shows a uncorrelated feature in long range.

From DFA analysis, we could easily distinguish the phase of high and low density system.

3. Empirical Mode Decomposition:

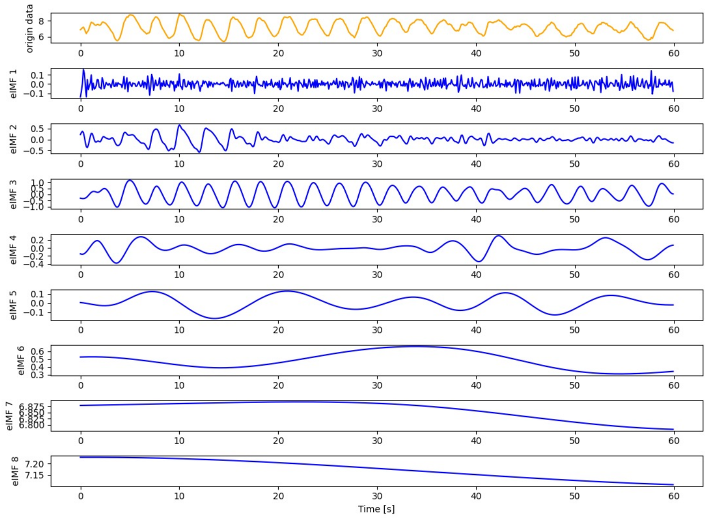
We take the two high density system with same fixed parameter: N=1000 L = 10 R = 0.5 dt =0.1 undergo EMD analysis:

High noise



In high noise system, we have velocity $v_0 = 2$ and noise $\eta = 3$, which means that the noise is 1.5 times greater than its velocity

low noise



In low noise system, we have velocity $v_0 = 4$ and noise $\eta = 1$, which means that the velocity is 4 times greater than its noise

In most of the high frequency modes, we found that for high noise system, there’s no clear periodic function in high frequency region. But we could found multiple groups of different frequency IMFs occurs in low frequency region, which means that the particle has more turn arounds or random walks.

While in low noise system, we could found a sinusoidal function in IMF3 which its period $\mathrm{T}{\sim}2.5sec$. Due to the initial conditions, it takes time $T{\sim}2$sec for a particle to go directly through the box. We could conclude that for a low noise system, the particles most go straight through the box, not much random walks.

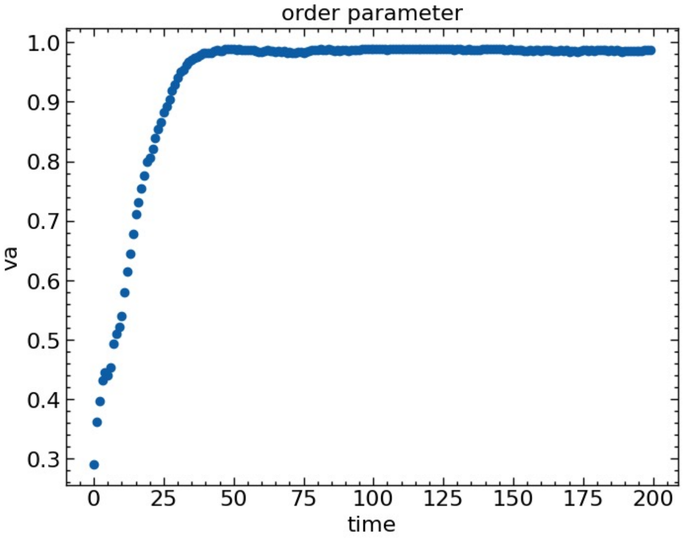
4. Correlation analysis

To well-defined the phase of different noise and density, we will do the correlation analysis.

The order parameter is a measurement of the average velocity direction of all particles in the system, which reflects the degree of alignment and collective behavior of the particles.

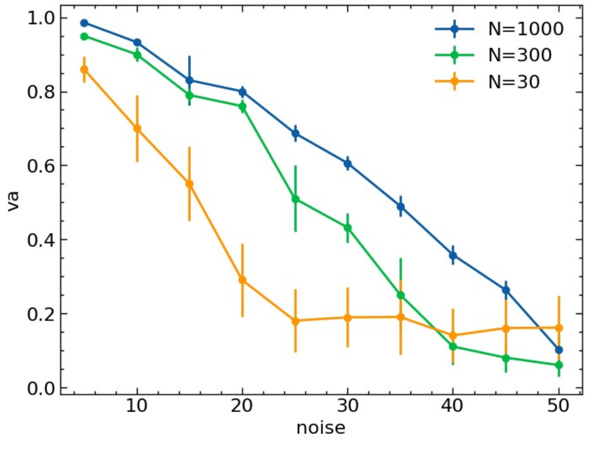
The order parameter is defined as $v_a = \frac{1}{Nv_0} | \sum_i v_i |$ which the velocity $v_j(t) = v_0 e^{-i\theta t}$.

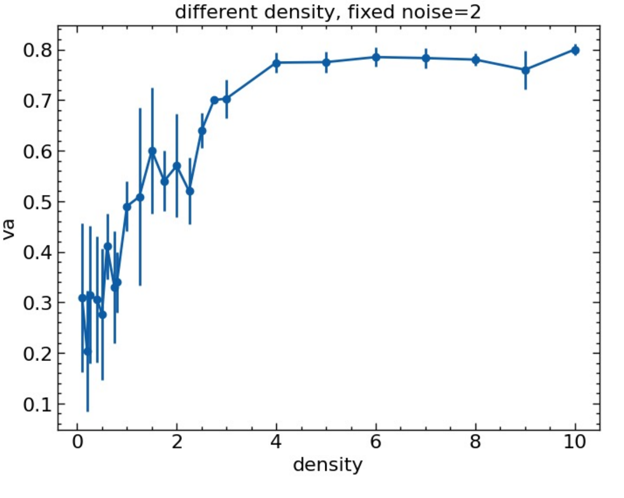
Thus, if the order parameter $v_a = 0$ that means the system is a random motion system, while $v_a = 1$ then the system is in perfect alignment.



The picture shows a high density low noise system’s order parameter over time, it starts at a random position with low correlation and gradually become perfect alignment.

The figure below plot from each different parameters(noise, density)’s system, for their time evolution plot of order parameter, we calculate va by taking the average after it is stabilize from time (only calculated the mean va of time 100-200)





The two plot show the comparison of different number of particle(indicates different density) and different amplitude of noise with the order parameter. The left plot shows the noise scan from 0.5-5 (velocity v=1), with three different density. We can see that the order parameter drops fast in the line N=30, which indicate that low density systems are less resistant to higher noise. And after a certain noise, we could found that whether high or low density system were all showing a uncorrelated behavior.

For fixing noise and doing density scan plot, we could found that it shows a exponential-like behavior, the order parameter growth once the density of the system becomes higher, and once it pass a certain density, the system becomes a nematic order system, even the density becomes higher the order parameter was fixed at a certain value, the value depends on the amplitude of the noise.

5. Phase transition

To find the phase transition point, the Binder cumulant is a dimensionless quantity that characterizes the behavior of an order parameter. It is used as a tool to analyze the behavior of a physical system and identify the phase transition point

The Binder cumulant is defined as $G = 1 - \frac{\langle \phi^4 \rangle}{3\langle \phi^2 \rangle^2}$, where ϕ is the order parameter.

As we approach the critical point, the order parameter distribution becomes broader and shows greater fluctuations. At the critical point, the distribution becomes scale-invariant.

1.Below critical point:

In the disordered phase, away from the critical point, the fluctuations are significant, resulting in a broad order parameter distribution.

The fourth moment $\langle \phi^4 \rangle$ is typically much larger than the square of the second moment $\langle \phi^2 \rangle^2$.

As a result, $G \sim 0$.

2. Above the critical point:

In the ordered phase, away from the critical point, the order parameter distribution narrows due to reduced fluctuations.

The fourth moment $\langle \phi^4 \rangle$ is still larger than the square of the second moment $\langle \phi^2 \rangle^2$ but approaches it as the system size increases.

Consequently, G approaches a value less than 1 but greater than 0.

3. At the critical point:

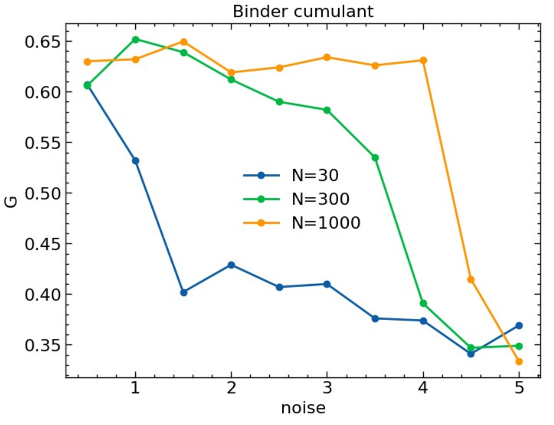
At the critical point, the order parameter distribution becomes scale-invariant, and the fluctuations become critical.

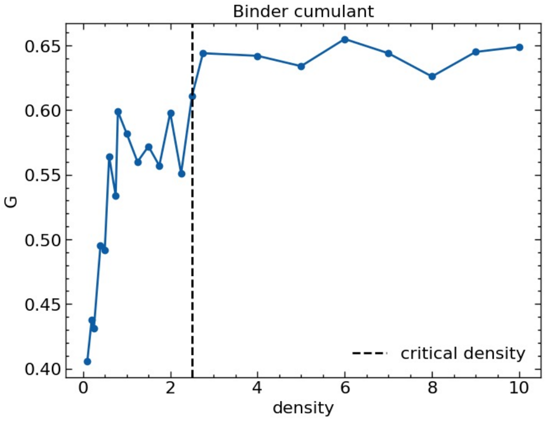
The fourth moment $\langle \phi^4 \rangle$ scales with the square of the second moment $\langle \phi^2 \rangle^2$ as the system size increases.

Therefore, at the critical point, $\langle \phi^4 \rangle / \langle \phi^2 \rangle^2$ approaches a constant value.

This results in G taking on a universal value that does not depend on the system size or microscopic details.

By using the equation and substitute the value of order parameter we obtain from the previous figure, we got:





By the plot, we can found the phase transition point critical point of a system of box size L=10.

The critical noise of different Ns:

	Critical noise
N = 30	4-4.5
N =300	3.5-4
N = 1000	1.0-1.5

The phase transition point of density with fixed noise $\eta = 2$ occurs at $\rho \sim 2.5$

Summary

We study the system of active matter by using Vicsek model to simulate the nematic order. Which the nematic order only occurs at a system with a high enough density and low enough density. From different analysis method, we can find some characteristic in different phases.

MSE analysis:

Position: non-stationary time series with long-range dependencies, the complexity of the data decreases at longer time scale.

Angle : stationary time series, the complexity of the data is similar at all time scales.

DFA analysis:

The slope is different in long-range scale of different noise and density.

Low density: weaker long-range dependence and more random or uncorrelated behavior at longer time scales.

EMD analysis:

The lower noise shows a more simple and pure periodic wave.

The higher noise shows more different frequency's IMFs, indicates that there are more groups of different direction particles.

Correlation analysis:

The order parameter reflects the degree of alignment and collective behavior of the particles.

The order drops slower in higher density of different noise strength

In a fixed 2.0 noise, after density>4.0, it could show a nematic order.

Phase transition:

Phase transition : point that order parameter undergoes a significant change

For a fixed noise $\eta = 2.0$ the phase transition occurs at density $\rho \sim 2.5$

For different density under noise scan, a larger density can delay the phase transition point.