Problem 1

- **First Question (Python Approach):**

We use Python to download data for AMZN, NVDA, TSLA, and SPY from yfinance. Then we calculate μ and Σ, resulting in the values shown below.

```
Mean daily returns (μ):
Ticker
AMZN    0.001594
NVDA    0.003206
TSLA    0.000728
dtype: float64

Covariance matrix (Σ):
Ticker      AMZN      NVDA      TSLA
Ticker
AMZN    0.000334  0.000241  0.000230
NVDA    0.000241  0.001033  0.000316
TSLA    0.000230  0.000316  0.001278
```

- **Calculating Weights (Zero Interest Rate):**

Using a zero interest rate, we apply the Analytical Solution formula and obtain the weights.

$$\omega^* = \frac{\Sigma^{-1}\mu}{\mathbf{1}^T\Sigma^{-1}\mu}$$

The solution is:

```
Optimal portfolio weights:
AMZN: 0.6497
NVDA: 0.4773
TSLA: -0.1271
```

- **No Short-Selling Constraint:**

If we add a "No Short-Selling" constraint, we define our objective function to maximize the Sharpe Ratio and use scipy.optimize.minimize to perform the constrained optimization. We get:

```
Optimal portfolio weights (no short-selling):
AMZN: 0.5572
NVDA: 0.4428
TSLA: 0.0000

Optimal portfolio expected daily return: 0.0023
Optimal portfolio daily volatility: 0.0206
Optimal portfolio Sharpe Ratio: 0.1119
```

- **Perpare for Problem 2:**

We calculated ωeq in Python and added this row of data to MATLAB

```
wtsMarket = [0.344533; 0.193961; 0.461505];
```

Problem 2

In this section, we use MATLAB to complete the following steps:

- **Data Prepare:**

First, we import the μ and Σ obtained from Python into the code. We use SPY as the market portfolio to calculate the Sharpe ratio and the equilibrium risk premium Π.

- **Investor Views:**

```
#1 Absolute view: The expected annualized return of AMZN is greater than 0.005.
#2 Relative view: NVDA has an annualized return 0.002 higher than TSLA.
#3 Absolute view: The expected annualized return of TSLA is greater than 0.0012.
```

We use matrix P and vector q to define these absolute and relative views and use the Ω matrix to represent the uncertainty of these views. The three omega values are set to 0.001, 0.02, and 0.05, indicating that we have more confidence in the first view.
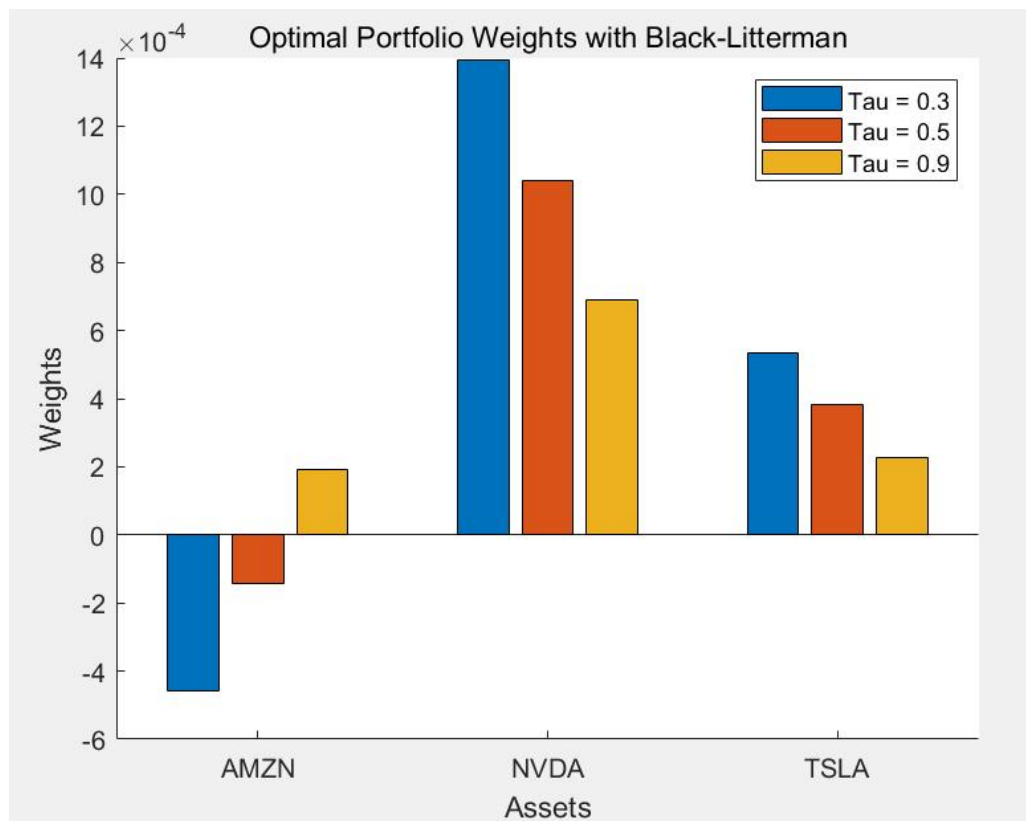
- **Posterior Mean and Covariance of BL:**

Using the investor views and the prior mean Π, we compute the adjusted mean M and covariance V under the Black-Litterman framework.

$$M = \left((\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right)^{-1}\left((\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}q\right)$$

$$V = \left((\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right)^{-1}$$

- **Optimal Weights with Short-Selling:**

We calculate the optimal portfolio weights based on the analytical solution, which allows for short-selling.

● **Optimal Weights with No Short-Selling and Maximum Sharpe Ratio:**

Using quadprog and the Portfolio object in MATLAB, we add a non-negativity constraint to compute the optimal weights without allowing short-selling.

Asset_Name	Mean_Variance	BL_Tau_0.3	BL_Tau_0.5	BL_Tau_0.9
{'AMZN'}	0.56001	6.4503e-23	2.523e-12	0.17153
{'NVDA'}	0.43999	0.7335	0.7345	0.62341
{'TSLA'}	5.146e-12	0.2665	0.2655	0.20506

● **Visualization:**

The results show that as the τ value increases, the portfolio weights under the Black-Litterman model gradually deviate from those of the mean-variance model. Specifically, the weight allocated to NVDA increases significantly, while the weight for AMZN decreases, indicating that investor views have a stronger impact at higher τ values.