

Problem 1 continuous entropy pooling

1. Notation

- ① X : market state, pdf: $f_X(x)$
- ② $\tilde{f}_X(x)$: posterior distribution after considering views
- ③ V : Investor's views
- ④ $D(\cdot, \cdot)$: relative entropy (K-L divergence)

2. Objective Function

$$D(\tilde{f}_X, f_X) = \int \tilde{f}_X(x) \ln \frac{\tilde{f}_X(x)}{f_X(x)} dx$$

$$\tilde{f}_X^* \equiv \arg \min_{f \in V} D(f, f_X)$$

3. Constraints

$$E_{\tilde{f}_X}[g(X)] = \int g(x) \tilde{f}_X(x) dx = b \quad (\text{from views})$$

$$\int \tilde{f}_X(x) dx = 1, \quad \tilde{f}_X(x) \geq 0 \quad \forall x$$

4. About views

To be more specific, there are 6 kinds of views we can set

- ① location measure ($D(X)$ or $E(X)$) e.g. $\tilde{m}[V_k] \geq m_k$
- ② ordering information e.g. $E(V_1) = E(X_1 - X_2) \geq E(V_2) = E(X_2 - X_3)$
- ③ volatilities e.g. $\tilde{\sigma}(V_k) \geq 1.2 \sigma(V_k)$
- ④ correlation stress-tests e.g. $\tilde{C}(V) = 0.2 I + 0.5 C(V) + 0.3 1.1'$
- ⑤ tail behavior e.g. $\tilde{Q}_V(0.05) \leq Q_V(0.05)$, more risky \downarrow
- ⑥ tail co-dependence e.g. $\tilde{C}_V(0.05) \geq 1.2 C_V(0.05)$

5. Confidence

now we get $f_x(x)$ & $\tilde{f}_x(x)$, we can set different confidence level

$$\tilde{f}_x^c = (1-c)f_x + c\tilde{f}_x = \sum_{s=1}^S C_s \cdot \hat{f}_x^{(s)} \quad , \quad c \in [0,1]$$

6. Conclusion & Interpretation

- full optimization problem:

$$\min_{\tilde{f}_x(x)} \int \tilde{f}_x(x) \ln \frac{\tilde{f}_x(x)}{f_x(x)} dx$$

$$\text{s.t.} \quad \int g(x) \tilde{f}_x(x) dx = b$$

$$\int \tilde{f}_x(x) dx = 1 \quad , \quad \tilde{f}_x(x) \geq 0 \quad \forall x$$

- our target:

view \rightarrow adjust pdf of market \rightarrow preserve as much as information from $f_x(x)$

Therefore, we need to minimize relative entropy \rightarrow minimize information loss.

Problem 2 discrete case

1. objective function.

same as problem 1, we min. relative entropy as target

$$D(\tilde{p}, p) = \sum_{j=1}^J \tilde{p}_j \ln \frac{\tilde{p}_j}{p_j}$$

$$\tilde{p} \equiv \arg\min_{p \in \mathcal{V}} D(\tilde{p}, p)$$

$$\sum_{j=1}^J \tilde{p}_j = 1 \quad , \quad \tilde{p}_j \geq 0 \quad \forall j$$

2. views

$$\textcircled{1} E(X) = \frac{1}{2} E(Y)$$

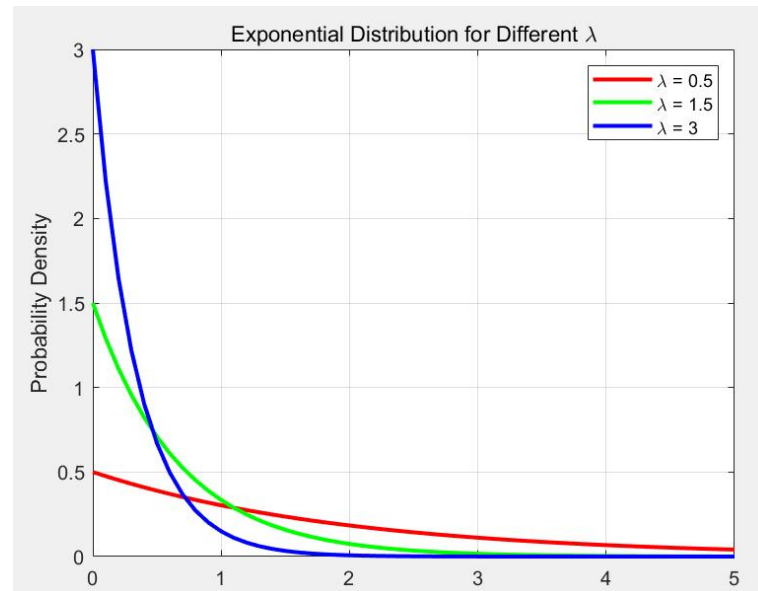
$$\sum_{j=1}^J \tilde{p}_j X_j = \frac{1}{2} \sum_{j=1}^J \tilde{p}_j Y_j$$

$$\textcircled{2} \sigma_X = \frac{1}{2} \sigma_Y$$

$$\sqrt{\sum_{j=1}^J \tilde{p}_j (X_j - E(X))^2} = \frac{1}{2} \sqrt{\sum_{j=1}^J \tilde{p}_j (Y_j - E(Y))^2}$$

Problem 3

- We chose the exponential distribution. By observing the graph below, we found that a decay rate of 1.5 is moderate, so we selected 1.5 as the parameter for our exponential distribution.

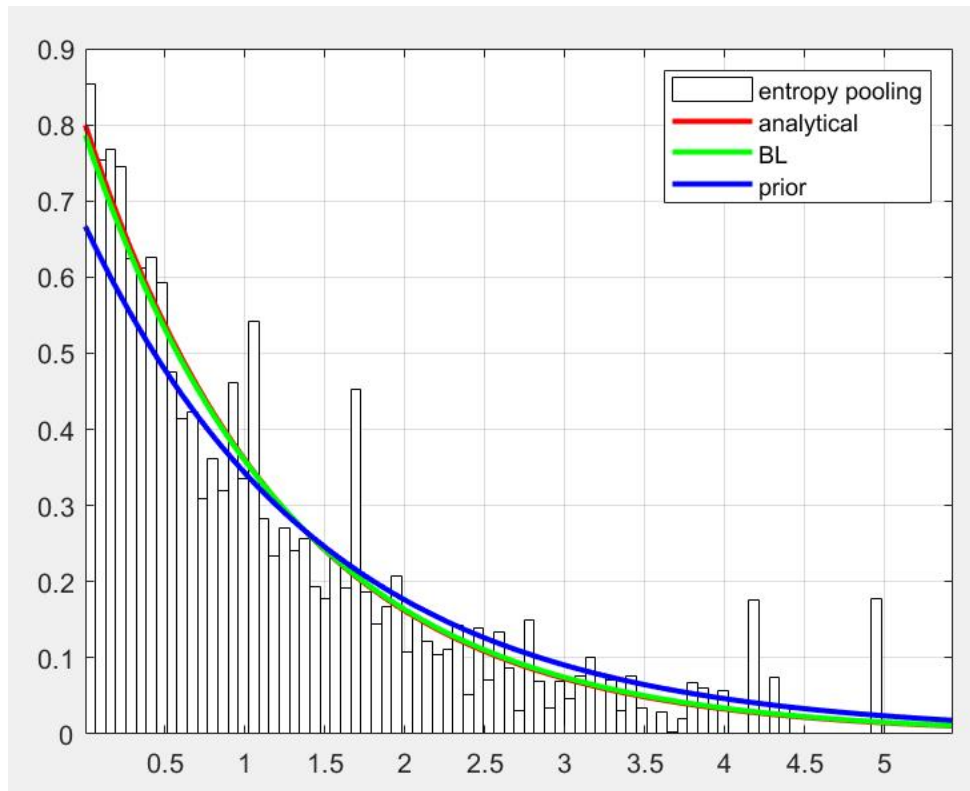


- Starting from the example MATLAB code, we need to make the following modifications:
 1. Adjust the sample size and the distribution used for data generation.
 2. Modify the method for calculating the mean and variance.
 3. Update the views.
 4. Replace the final plotting function from normpdf to exppdf.
- Other formulas and information can be referenced in the Summary Table below.

Component	Parameter / Method	Value / Equation
Prior Distribution	Exponential ($\lambda = 1.5$)	$f(x; 1.5) = 1.5e^{-1.5x}$
	Mean ($\mathbb{E}[X]$)	$\frac{1}{1.5} = 0.667$
	Variance ($\text{Var}(X)$)	$\frac{1}{(1.5)^2} = 0.444$
	Covariance Matrix (Σ)	$\text{diag}([0.444, 0.444])$
Views	Constraint $E[X] = \frac{1}{2}E[Y]$	$Q = [1, -2]$
	Constraint $\text{Var}(Y) = 4\text{Var}(X)$	$\Sigma_G = 0.25$
Posterior (Analytical)	Updated Mean	$\mu' = \mu + \Sigma Q'(Q\Sigma Q')^{-1}(Mu_Q - Q\mu)$
	Updated Variance	$\Sigma' = \Sigma + \text{correction term}$
Black-Litterman Posterior	Updated Mean	$\mu_{BL} = (\Sigma^{-1} + Q'\Sigma_G^{-1}Q)^{-1}(\Sigma^{-1}\mu + Q'\Sigma_G^{-1}Mu_Q)$
	Updated Variance	$\Sigma_{BL} = (\Sigma^{-1} + Q'\Sigma_G^{-1}Q)^{-1}$

- Finally, we obtain the probability density functions (PDFs) of X and Y under different estimation methods.

This is the distribution of X .



This is the distribution of Y .

