

## Problem 1

for  $n \times n$  checkerboard copula:

$$C_{ij} = \int_{(i-1)/n}^{i/n} \int_{(j-1)/n}^{j/n} c(u_1, u_2) du_1 du_2$$
$$= \left(\frac{1}{n}\right)^2 C_{ij} = \frac{1}{4} C_{ij}$$
$$\therefore C = \frac{1}{4} \begin{bmatrix} 1.6 & 0.4 \\ 0.4 & 1.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}$$

## Problem 2

### Checkerboard Copula Defined by Sums of Random Variables

In this section, we use MATLAB to complete the following steps:

- 1. Compare MAE and MSE Code

We first reviewed the example code for both MAE and MSE methods. And find the only difference lies in the `%% Construct Problem` section:

MAE uses: `' meanabs_err(matrix_scenarios_05)'`

MSE uses: `' meansquare_err(matrix_scenarios_05)'`

- 2. Implement the Mixed Error Objective

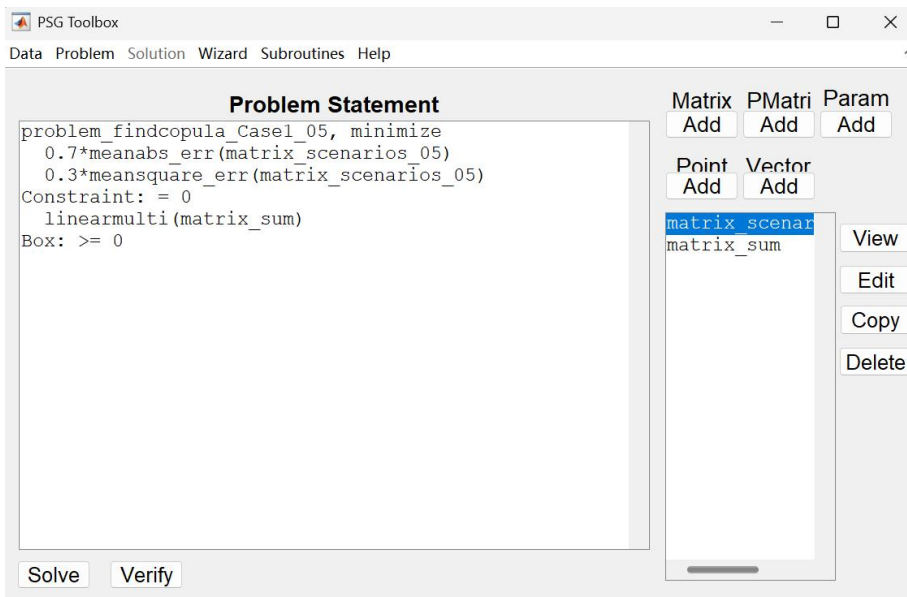
Since the problem requires the following mixed error:

$$0.7 \times \text{meanabs err} + 0.3 \times \text{meansquare err}$$

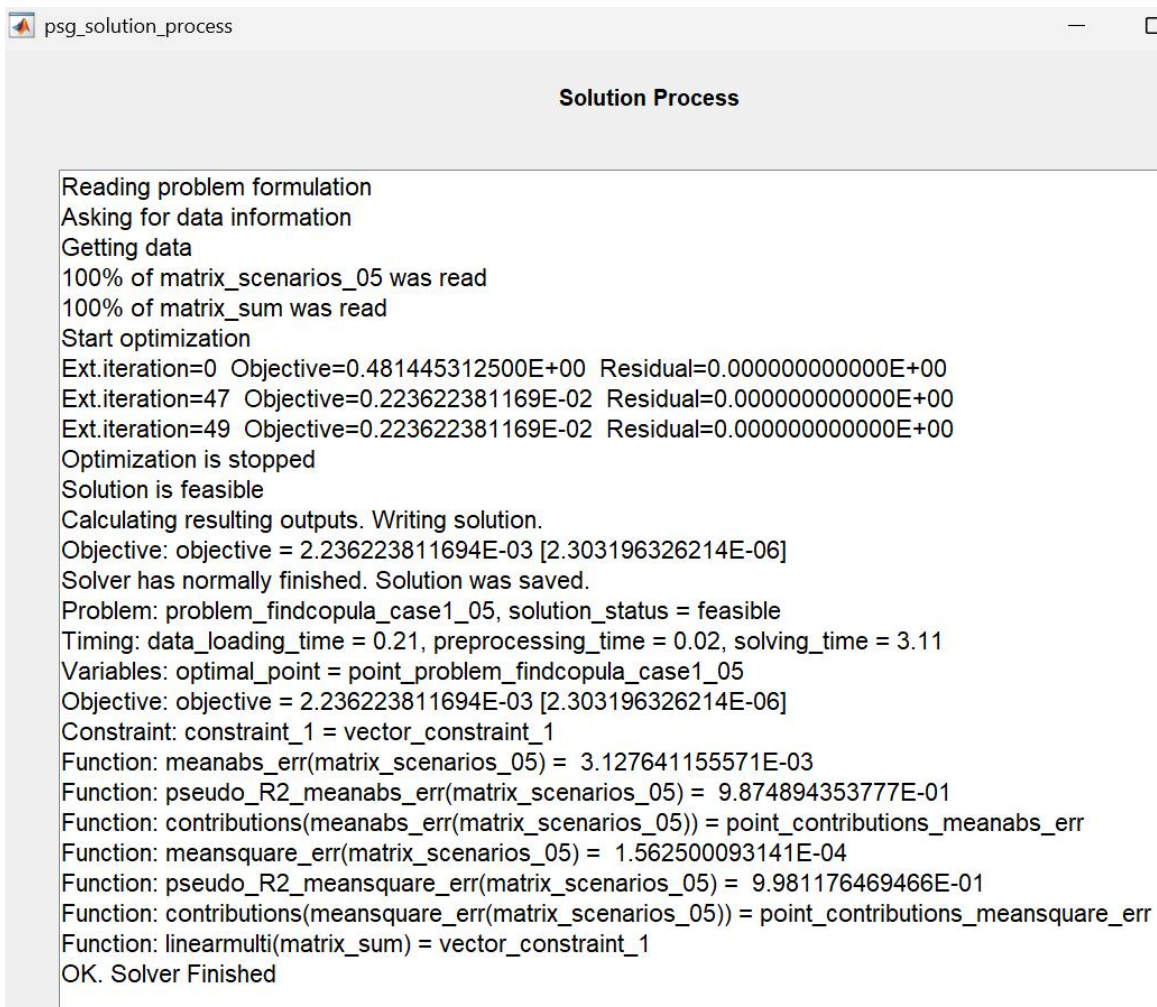
So we just need to follow the hint and modify the `%% Construct Problem` section.

```
problem_statement=sprintf('%s\n', ...  
'problem_findcopula_Case1_05, minimize', ...  
' 0.7*meanabs_err(matrix_scenarios_05)', ...  
' 0.3*meansquare_err(matrix_scenarios_05)', ...  
'Constraint: = 0', ...  
' linearmulti(matrix_sum)', ...  
'Box: >= 0');
```

Then we ran the problem using the PSG toolbox.



### ● 3. Solution Process& Results



```
Problem: problem_findcopula_case1_05, solution_status = feasible
Timing: data_loading_time = 0.21, preprocessing_time = 0.02, solving_time = 3.11
Variables: optimal_point = point_problem_findcopula_case1_05
Objective: objective = 2.236223811694E-03 [2.303196326214E-06]
Constraint: constraint_1 = vector_constraint_1
Function: meanabs_err(matrix_scenarios_05) = 3.127641155571E-03
Function: pseudo_R2_meanabs_err(matrix_scenarios_05) = 9.874894353777E-01
Function: contributions(meanabs_err(matrix_scenarios_05)) = point_contributions_meanabs_err
Function: meansquare_err(matrix_scenarios_05) = 1.562500093141E-04
Function: pseudo_R2_meansquare_err(matrix_scenarios_05) = 9.981176469466E-01
Function: contributions(meansquare_err(matrix_scenarios_05)) = point_contributions_meansquare_err
Function: linearmulti(matrix_sum) = vector_constraint_1
```

The solver successfully returned a feasible solution with objective value  $\approx 0.002236$ .

The individual error components were:

MAE = 0.00313

MSE = 0.000156

pseudo- $R^2$ (MAE)  $\approx 0.987$

pseudo- $R^2$ (MSE)  $\approx 0.998$

This confirms that the optimization was carried out correctly with the desired mixed-error objective.