Massive Graph Management & Analytics

Community Detection Approaches

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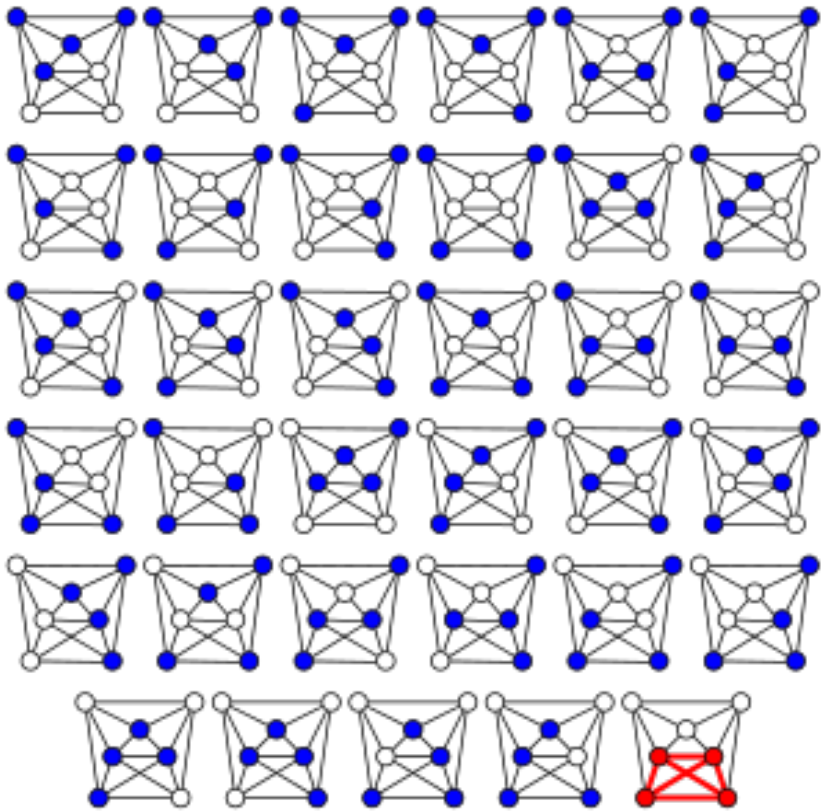
Outline

- ☞ k-Clique and Maximal Cliques
  - → Bron-Kerbosch Algorithm
- ☞ k-Core Subgraphs Decomposition (Relaxation of a clique: k-core)
  - → k-Core Subgraphs Decomposition
- ☞ Community Detection Problem
  - → Clique-based approach
  - → Modularity maximisation approach
  - → Random walk-based approach
  - → Edge betweenness separation-based approach

Typical large network structure: k-Clique

- A clique is a complete subgraph.
- k-clique is a complete subgraph with k nodes
- Brute force algorithm finds all k-cliques: C_n^k combinations for completeness.
- A maximal clique cannot be extended by including one more adjacent vertex.
- A maximum clique is a clique, such that there is no clique with more vertices.



C_7^4 4-cliques

Typical large network structure: k-Clique

☞ Finding a single maximal clique is easy, linear time

$K \leftarrow K \cup random(1, n)$,
for $v \in V$ and $v \notin K$ **do**
    $isNext \leftarrow true$
    for $u \in K$ **do**
        if $v \notin \mathcal{N}_u$ and $v \notin K$ **then**
            $isNext \leftarrow false$
        end if
    end for
    if $isNext$ **then**
        $K \leftarrow K \cup \{v\}$
    end if
end for

Typical large network structure: k-Clique

- ☞ Different formulations of clique problem (listing maximal cliques, or k-cliques, ...).
  - ✓ Clique decision problem is NP-Complete: input is an undirected graph and a number k; output is true if the graph contains a k-clique, false otherwise.
- ☞ Common problem formulation: finding maximal cliques, i.e. cliques with the largest number of nodes.
  - ✓ Many algorithms: Bron–Kerbosch algorithm to find all maximal cliques (different variants, $O(3^{\frac{n}{3}})$ in the worst case), Tarjan & Trojanowski algorithm and more recently Janez Konc algorithm.
  - ✓ Many algorithms with better theoretical complexity to fin all maximal cliques, Bron–Kerbosch (and improvements) are more efficient.

Bron-Kerbosch Algorithm: All Maximal Cliques

☞ Basic form of Bron–Kerbosch algorithm:

R: nodes of a potential clique; P: nodes to be added to a potential clique (neighbours of the current v); X: nodes already processed or already belong to a maximale clique

$P = V, X = \emptyset, R = \emptyset$

Function BronKerbosch(R, P, X)

if $P = \emptyset$ and $X = \emptyset$ **then**

  R is a maximal clique

end if

for $v \in P$ **do**

  BronKerbosch($R \cup \{v\}, P \cap \mathcal{N}_v, X \cap \mathcal{N}_v$)

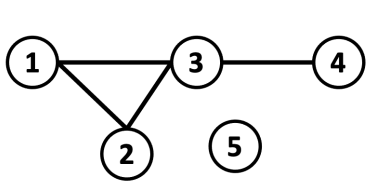
  $P \leftarrow P \setminus \{v\}$

  $X \leftarrow X \cup \{v\}$

end for

it makes a recursive call for every clique, maximal or not.

Bron-Kerbosch Algorithm: All Maximal Cliques - Example



R={};
P={1,2,3,4, 5};
X={}

v=1

P={2,3,4,5};
X={1};v=2

P={3,4,5};
X={1,2}; v=3

P={4,5};
X={1,2,3}; v=4

P={5};
X={1,2,3, 4}; v=5

R={1};
P={2,3};
X={}

R={2};
P={3};
X={1}

R={3};
P={4};
X={1,2}

R={4};
P={};
X={3}

R={5};
P={};
X={}

R is a maximal clique

P={3}; X={2};
v=3

v=2

v=3

v=4

R={1,2};
P={3};
X={}

R={1,3};
P={};
X={2}

R={2,3};
P={};
X={1}

R={3,4};
P={};
X={}

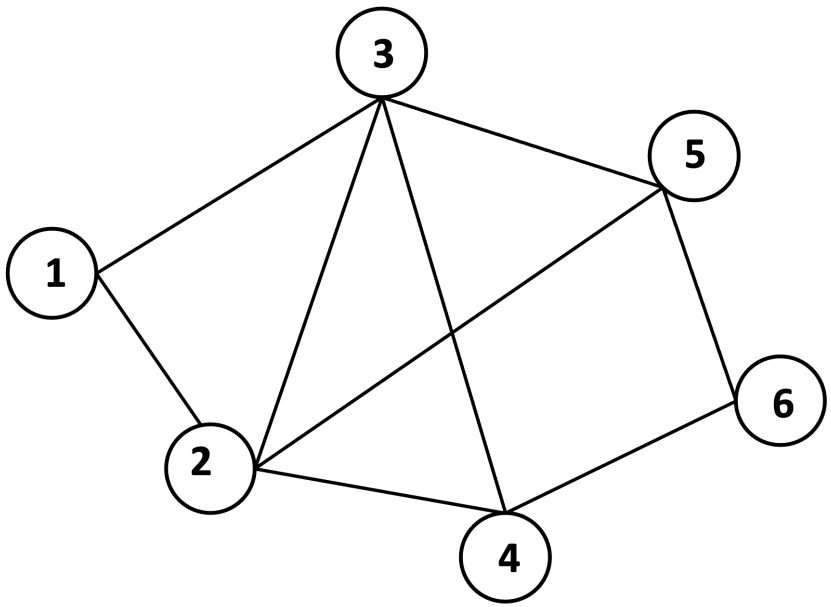
R is a maximal clique

v=3

R={1,2, 3};
P={};
X={}

R is a maximal clique

Bron-Kerbosch Algorithm: All Maximal Cliques - Exercise



Bronkerbosh(R={}; P={1,2,3,4,5,6}; X={})
v=1
Bronkerbosh(R={1}; P={2,3}; X={})
v=2
Bronkerbosh(R={1,2}; P={3}; X={})
v=3
Bronkerbosh(R={1,2,3}; P={}; X={})
R={1,2,3} is a maximal clique
v=3
Bronkerbosh(R={1,3}; P={2}; X={})
v=2
Bronkerbosh(R={1,3, 2}; P={}; X={})
R={1,2,3} is a maximal clique

Bron-Kerbosch Algorithm with pivot: All Maximal Cliques

- Bron and Kerbosch also introduced an optimization to decrease the number of recursive calls and therefore enable faster backtracking.
- Any pivot u chosen $\in P$ or later from $P \cup X$, then neighbors of u are not recursively tested. Any maximal clique potentially found in neighbors of u would also be found when testing u or one of its non-neighbors.

$P = V, X = \emptyset, R = \emptyset$
Function BronKerbosch(R, P, X)
if $P = \emptyset$ and $X = \emptyset$ **then**
    R is a maximal clique
end if
Choose pivot $u \in P \cup X$
for $v \in P \setminus \mathcal{N}_u$ **do**
    BronKerbosch($R \cup \{v\}, P \cap \mathcal{N}_v, X \cap \mathcal{N}_v$)
    $P \leftarrow P \setminus \{v\}$
    $X \leftarrow X \cup \{v\}$
end for

- Show why the number of recursive calls decreases with pivot (without missing any maximal clique). Use an example.

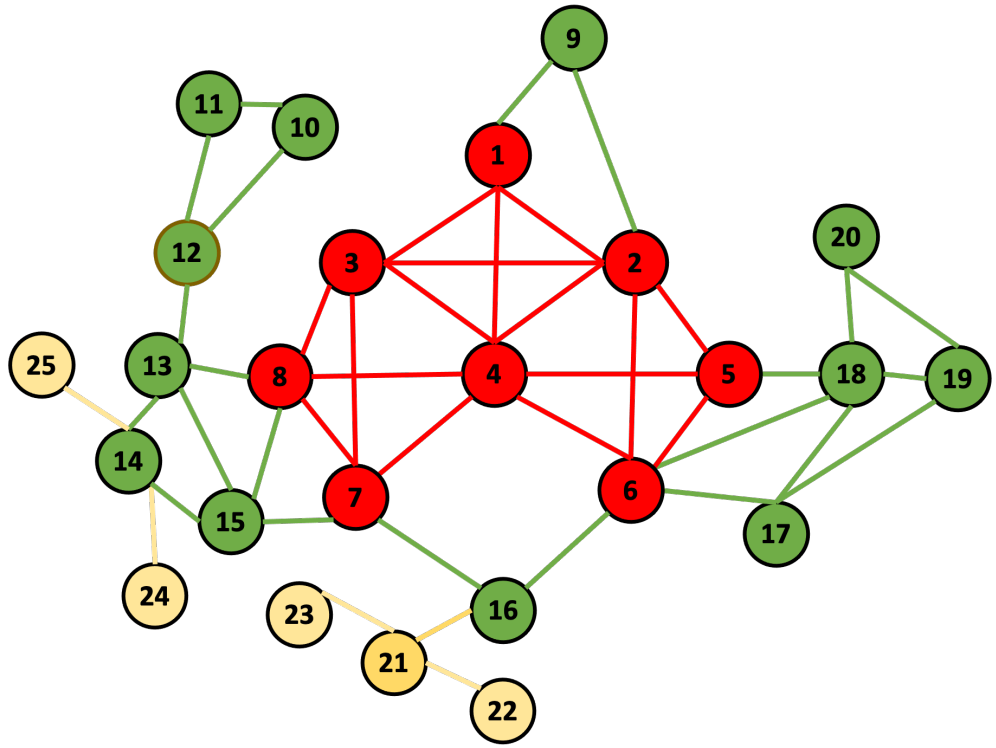
Bron-Kerbosch Algorithm with degeneracy: All Maximal Cliques

- The degeneracy of $\mathcal{G}$ is the smallest degree d s.t. each subgraph of $\mathcal{G}$ has a vertex with degree $\leq d$. A degeneracy ordering of vertices may be determined in linear time (seen later).
- Bron–Kerbosch algorithm loops through a degeneracy ordering, P of candidate vertices in each call (the neighbors of v that are later in the ordering) will be guaranteed to have size at most d.
- Compare the algorithm with and without degeneracy ordering.

$P = V, X = \emptyset, R = \emptyset$
for $v \in$ degeneracy ordered V **do**
    BronKerbosch($\{v\}, P \cap \mathcal{N}_v, X \cap \mathcal{N}_v$)
    $P \leftarrow P \setminus \{v\}$
    $X \leftarrow X \cup \{v\}$
end for

Typical large network structure: k-Core

- Core-periphery structure of a network: internal core where a lot of nodes are connected and peripherical nodes called whiskers (Leskovec et al, 2009).



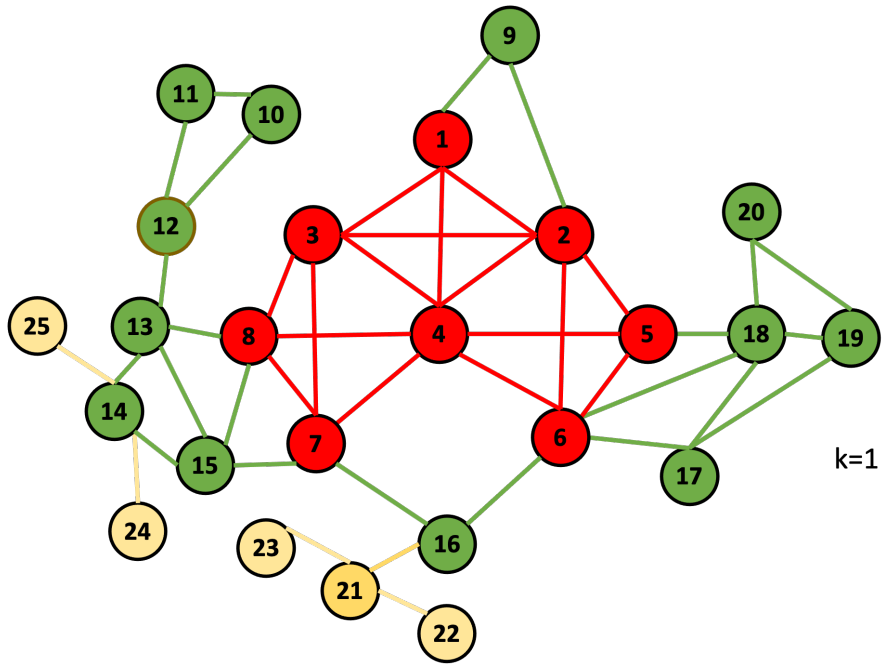
- k-core of a graph G is a maximal connected subgraph of G where vertices have at least k-degree. Also called k-degenerate.

Typical large network structure: k-Core

- k-core decomposition: Given a graph $\mathcal{G}(V, E)$, delete recursively all vertices, and edges connecting them, of degree less than k to extract the k-core graph such that:
  - ✓ Each v_i in a k-core graph has $d_i \geq k$
  - ✓ $(k+1)$-core is a subgraph of a k-core graph
- Matula & Beck, 1983;

```
1:  procedure kCore(G(V, E) )
2:      L[ ]; d[ ]; D[ {}]; d_max = max_{v_i ∈ V} d_{v_i}
3:  ** L list of ranked k by node, d list of degrees by node, D list of set of nodes by degree
4:      for v_i ∈ V do
5:          d[i] ← d_{v_i}; D[d[i]].insert(v_i)
6:      end for
7:      for k ← 0 to d_max do
8:          while D[k].notempty() do
9:              i ← D[k].remove(); L[i] ← k
10:             for v_j ∈ N_{v_i} do
11:                 if d[j] > k then
12:                     D[d[j]].remove(j); D[d[j] − 1].insert(j); d[j] − −
13:                 end if
14:             end for
15:         end while
16:     end for
17: end procedure
```

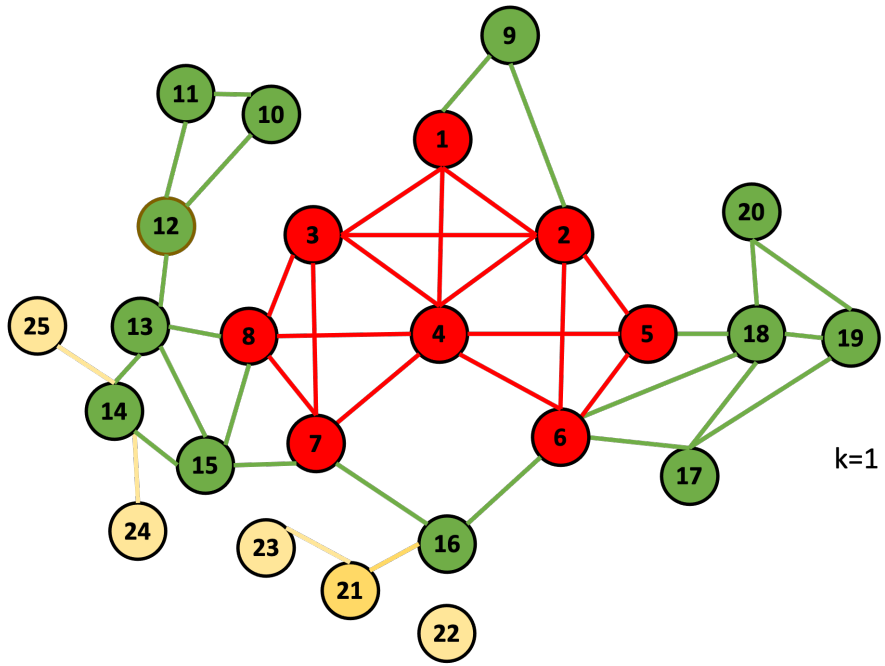
Typical large network structure: *k*-Core



i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
$d[v_i]$	4	6	5	7	4	6	5	5	2	2	2	3	4	4	4	3	3	5	3	2	3	1	1	1	1

$d[v_i]$	7	6	5	4	3	2	1
$D[d[v_i]]$	{4}	{2,6}	{3,7,8, 18}	{1, 5,13,14,15}	{12, 16, 17, 19, 21}	{9, 10, 11, 20}	{22, 23, 24, 25}

Typical large network structure: *k*-Core



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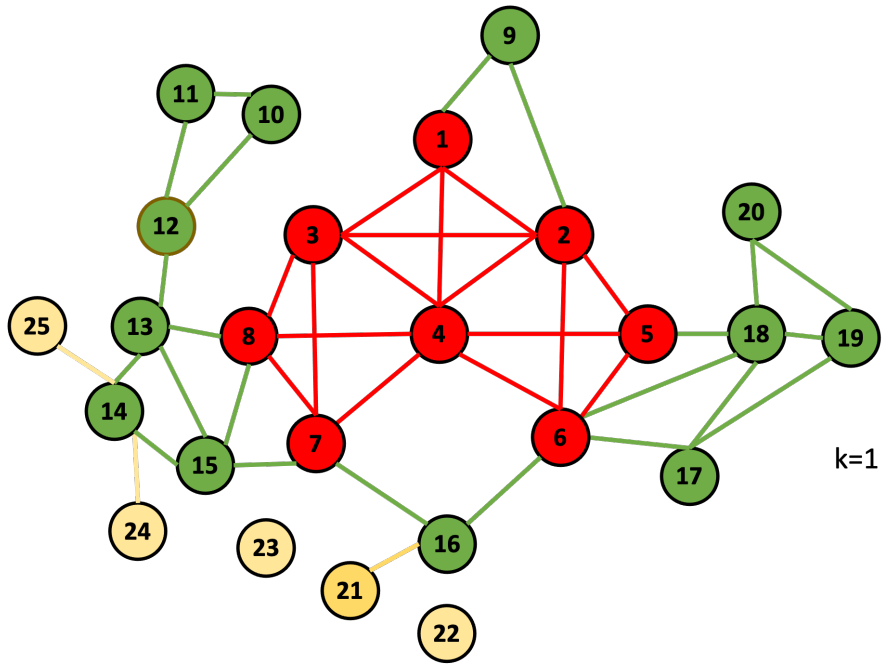
$d[v_i]$	4	6	5	7	4	6	5	5	2	2	2	3	4	4	4	3	3	5	3	2	2	1	1	1	1

$d[v_i]$	7	6	5	4	3	2	1
$D[d[v_i]]$	{4}	{2,6}	{3,7,8, 18}	{1, 5,13,14,15}	{12, 16, 17, 19, ~~21~~}	{9, 10, 11, 20, 21}	{~~22~~, 23, 24, 25,}

v_i	22
$L[v_i]$	1

Typical large network structure: *k*-Core



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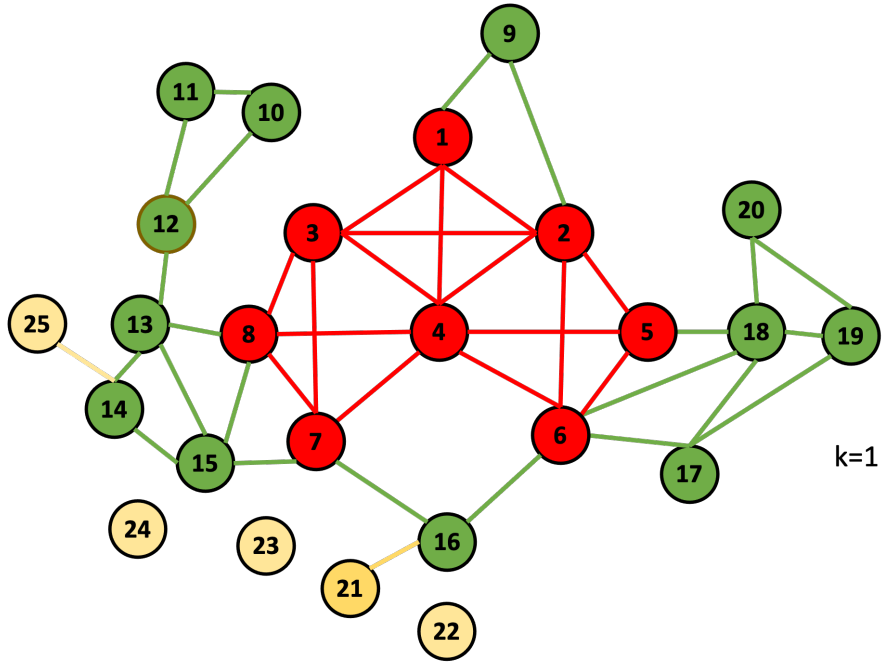
$d[v_i]$	7	6	5	4	3	2	1
$D[d[v_i]]$	{4}	{2,6}	{3,7,8, 18}	{1, 5,13,14,15}	{12, 16, 17, 19}	{9, 10, 11, 20, ~~21~~}	{~~23~~, 24, 25, 21}

v_i	22	23
$L[v_i]$	1	1

Typical large network structure: *k*-Core



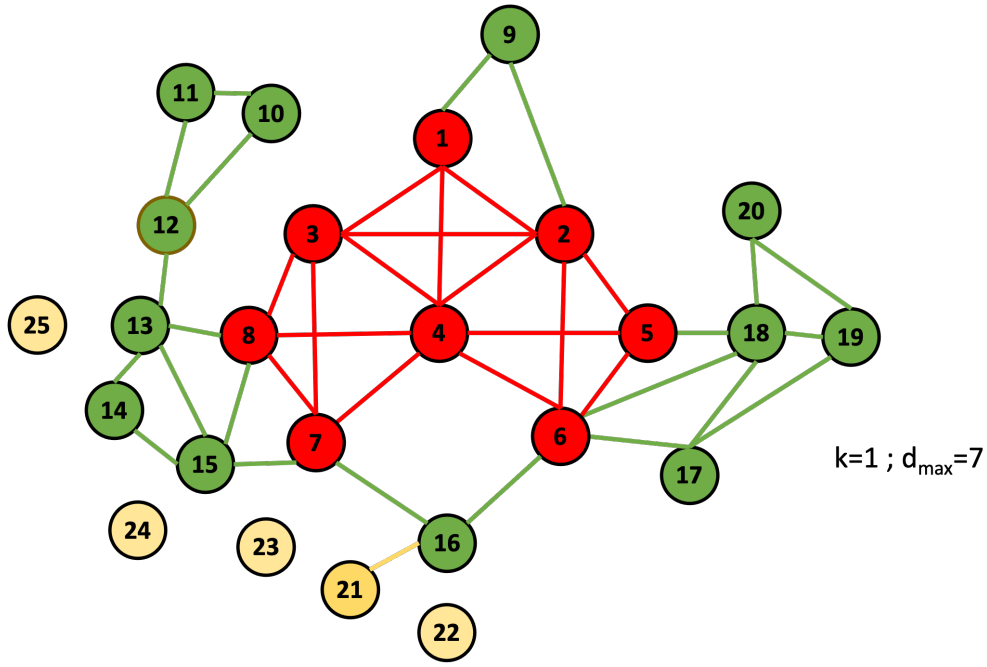
$k=1$; $d_{max}=7$

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
$d[v_i]$	4	6	5	7	4	6	5	5	2	2	2	3	4	3	4	3	3	5	3	2	1	1	1	1	1

$d[v_i]$	7	6	5	4	3	2	1
$D[d[v_i]]$	{4}	{2,6}	{3,7,8, 18}	{1, 5,13,14,15}	{12, 16, 17, 19, 14}	{9, 10, 11, 20}	{24, 25, 21}

v_i	22	23	24
$L[v_i]$	1	1	1

Typical large network structure: *k*-Core



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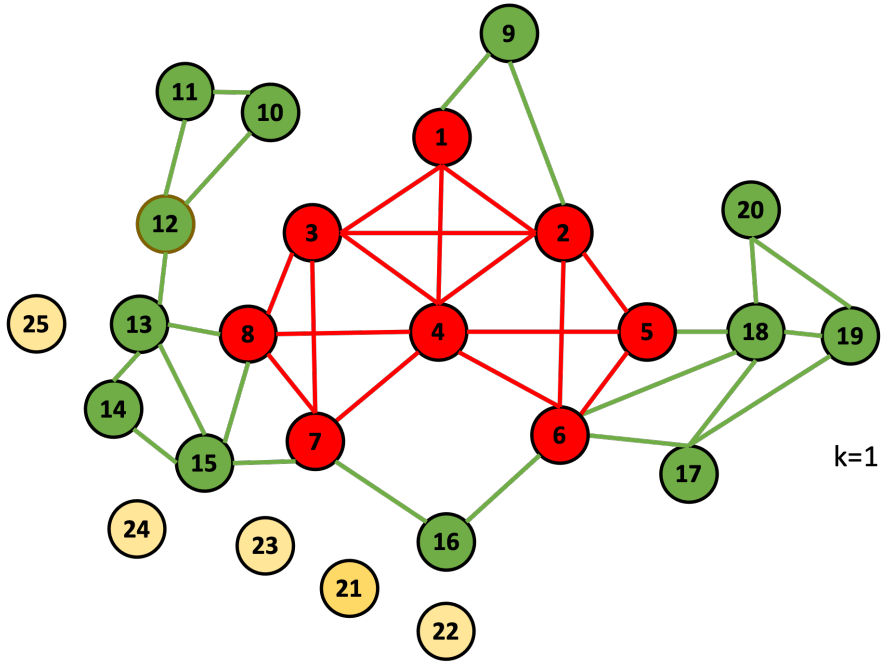
$d[v_i]$	4	6	5	7	4	6	5	5	2	2	2	3	4	2	4	3	3	5	3	2	1	1	1	1	1

$d[v_i]$	7	6	5	4	3	2	1
$D[d[v_i]]$	{4}	{2,6}	{3,7,8, 18}	{1, 5,13,15}	{12, 16, 17, 19, ~~14~~}	{9, 10, 11, 20, 14}	{~~25~~, ~~21~~}

v_i	22	23	24	25
$L[v_i]$	1	1	1	1

Typical large network structure: *k*-Core



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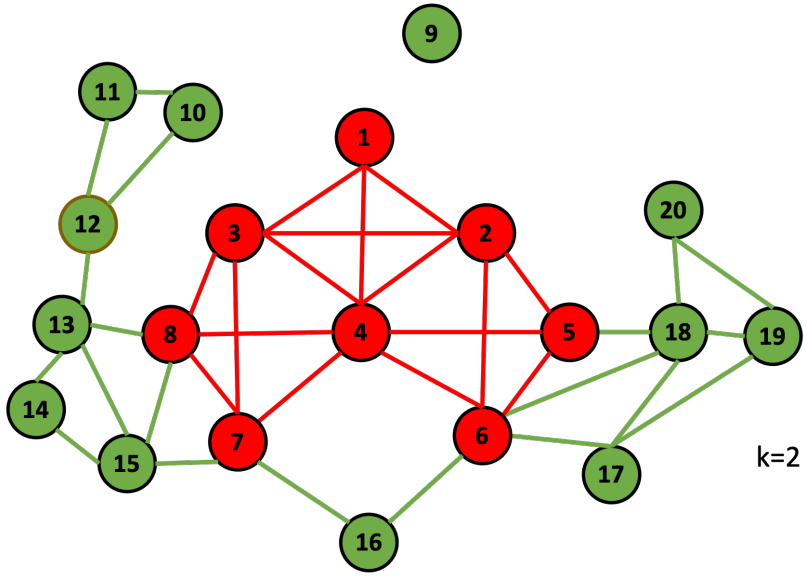
$d[v_i]$	4	6	5	7	4	6	5	5	2	2	2	3	4	2	4	2	2	5	3	2	1	1	1	1	1

$d[v_i]$	7	6	5	4	3	2	1
$D[d[v_i]]$	{4}	{2,6}	{3,7,8, 18}	{1, 5,13,15}	{12, ~~16~~, 17, 19}	{9, 10, 11, 20, **14, 16**}	{~~21~~}

v_i	22	23	24	25	21
$L[v_i]$	1	1	1	1	1

Typical large network structure: *k*-Core



i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
$d[v_i]$	3	5	5	7	4	6	5	5	2	2	2	3	4	2	4	3	2	5	3	2	1	1	1	1	1

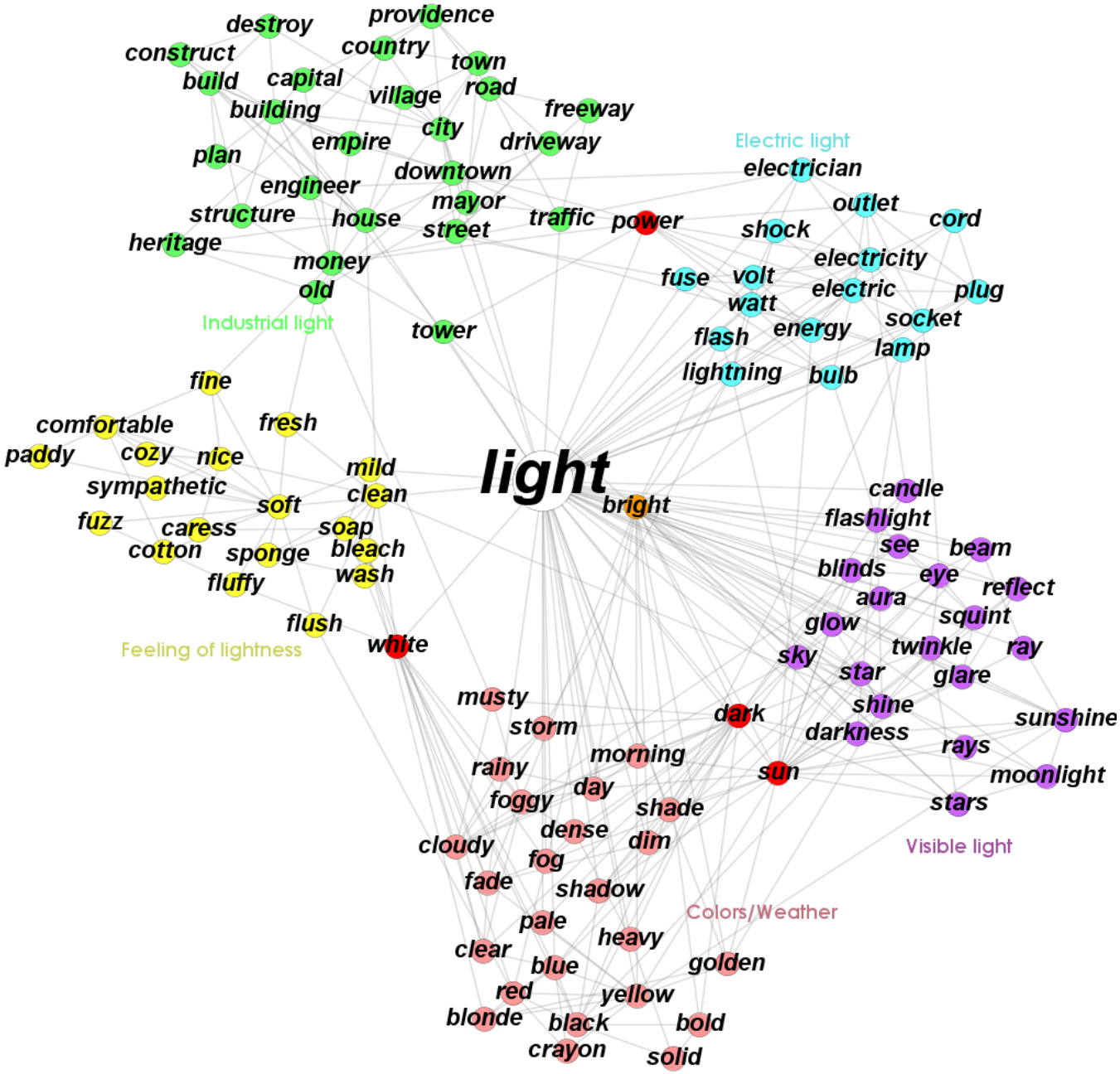
$d[v_i]$	7	6	5	4	3	2
$D[d[v_i]]$	{4}	~~{2,6}~~	{3,7,8, 18, 2}	~~{1,~~ 5,13,15}	{12, 17, 19, 1}	~~{9,~~ 10, 11, 20, 14, 16}

v_i	22	23	24	25	21	9
$L[v_i]$	1	1	1	1	1	2

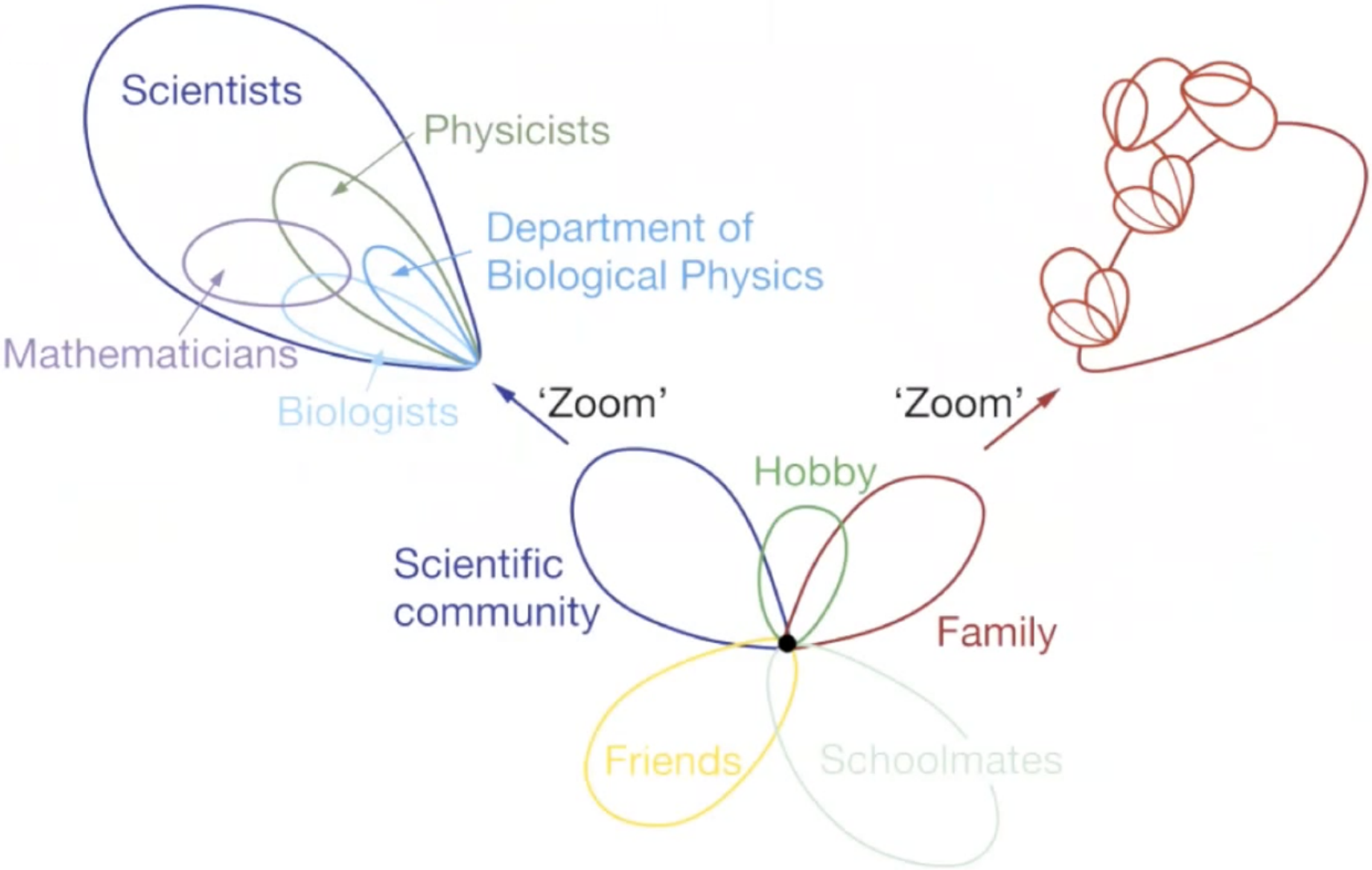
Community Detection Problem

- Community in network science refers to the occurrence of clusters (groups) of nodes in a network that are more densely connected internally than with the other nodes
- Two cases:
  - ✓ Non-overlapping community: divides into clusters of nodes with dense connections internally and sparser connections between clusters. Each node assigned to one community.
  - ✓ Overlapping community: assumes that pairs of nodes are more likely to be connected if they are both members of the same communities, and less likely to be connected if they do not share any community.

Non-overlapping Example



Overlapping Example



Community Detection Problem

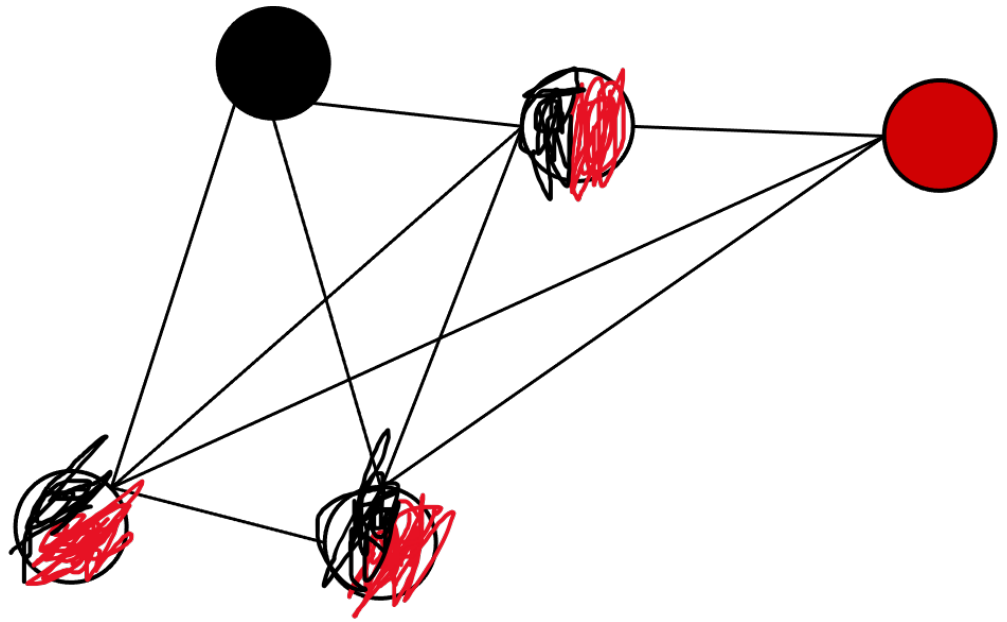
- ✓ Mutuality of ties: every node in the group has ties to one another
- ✓ Compactness: closeness or reachability of group members
- ✓ Density of edges: high number of edges within a group
- ✓ Separation higher frequency of ties among group members compared to non members
- ✓ Usual metrics: Graph density, internal and external density, graph cut, modularity score

Community Detection Problem

- ☞ Combinatorial optimisation problem, according to some metrics. Exact solution NP-hard
- ☞ Many approaches by greedy or approximate heuristics for community detection in the literature: agglomerative/hierarchical (ex. Dendogram) or partitioning (Newman-Girvan 2004)
  - ✓ Clique-based approach
  - ✓ Modularity maximisation approach
  - ✓ Random walk-based approach
  - ✓ Edge betweenness separation-based approach

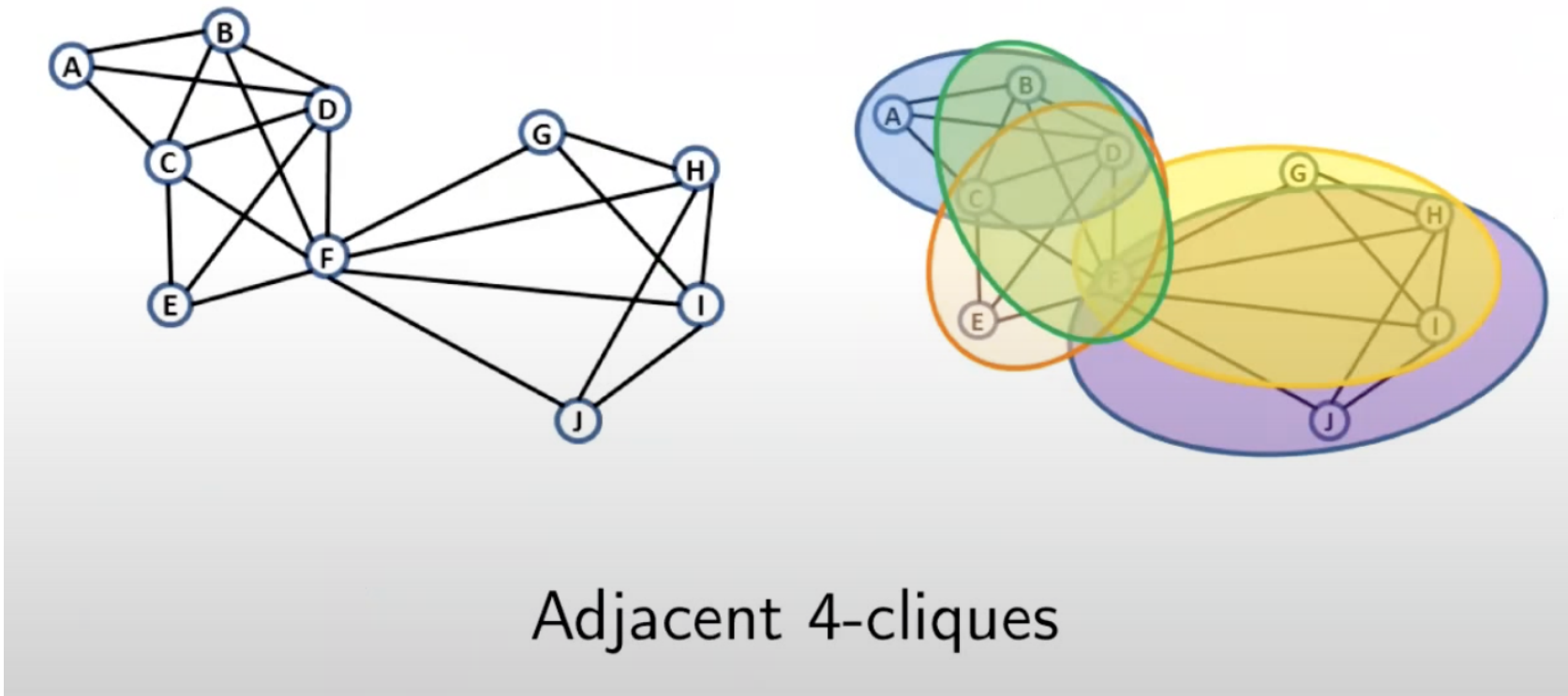
k-Clique Community

- A $k-$clique community is the union of all $k-$cliques that can be reached from one to the other through a sequence of adjacent k-cliques.
- Two $k-$cliques are adjacent if they share $k-1$ nodes.



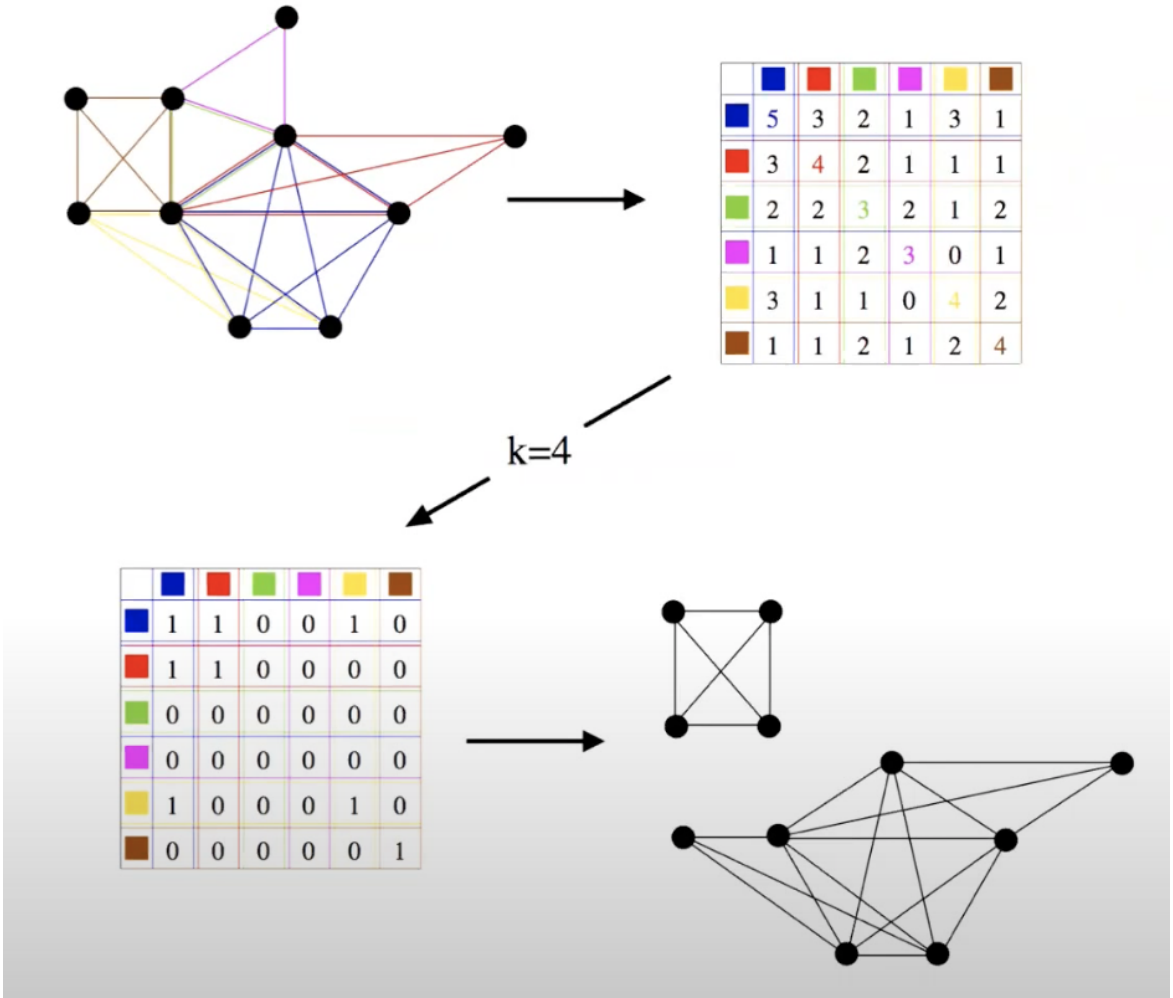
k-Clique Community

- ☞ Blue community $\{A, B, C, D\}$ is adjacent to green community $\{B, C, D, F\}$
- ☞ Green community $\{C, D, F, B\}$ is adjacent to orange community $\{C, D, F, E\}$
- ☞ Purple community is adjacent $\{F, H, I, J\}$ to yellow community $\{F, H, I, G\}$

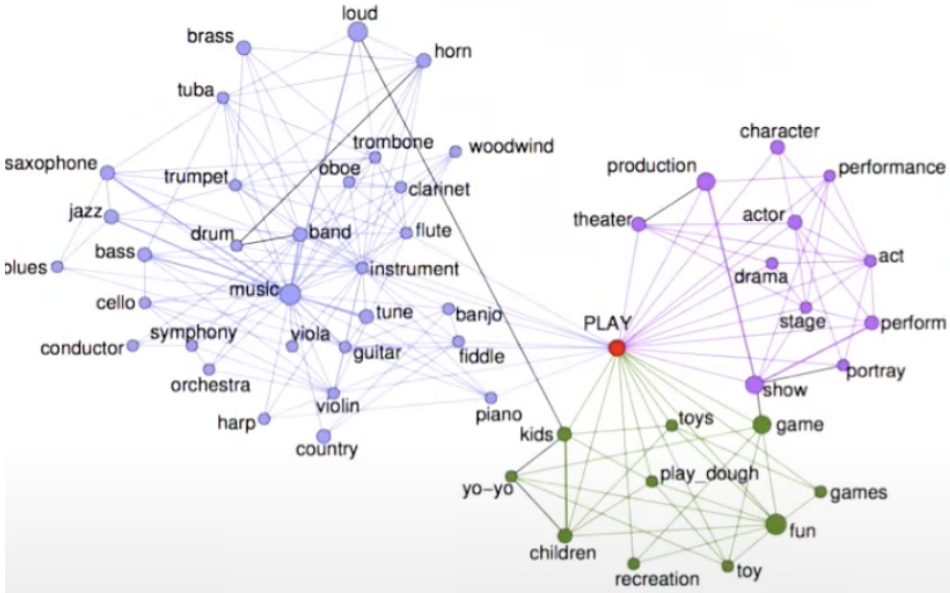


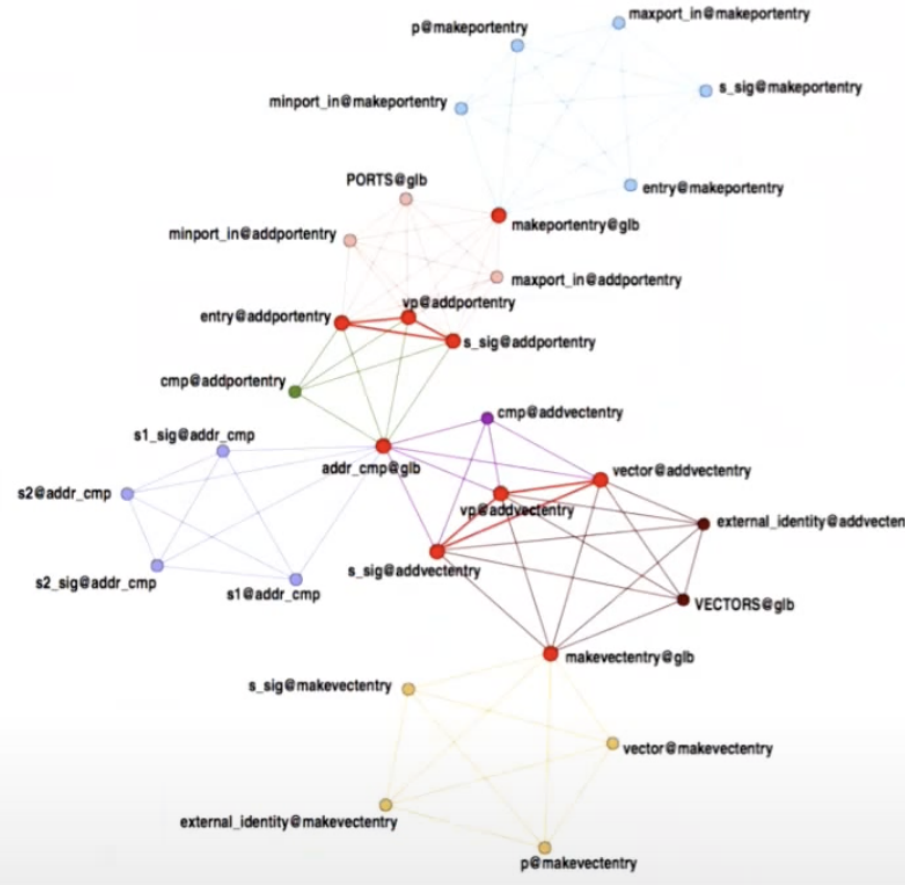
Clique Percolation Method: Algorithm

- ✓ Palla et *al.* 2005
- ✓ Find all maximal cliques, sorting them based on degrees.
- ✓ Create clique overlap matrix at a given $k-1$ how many nodes are overlapping.
- ✓ Communities are connected components: blue+red+yellow and brown.

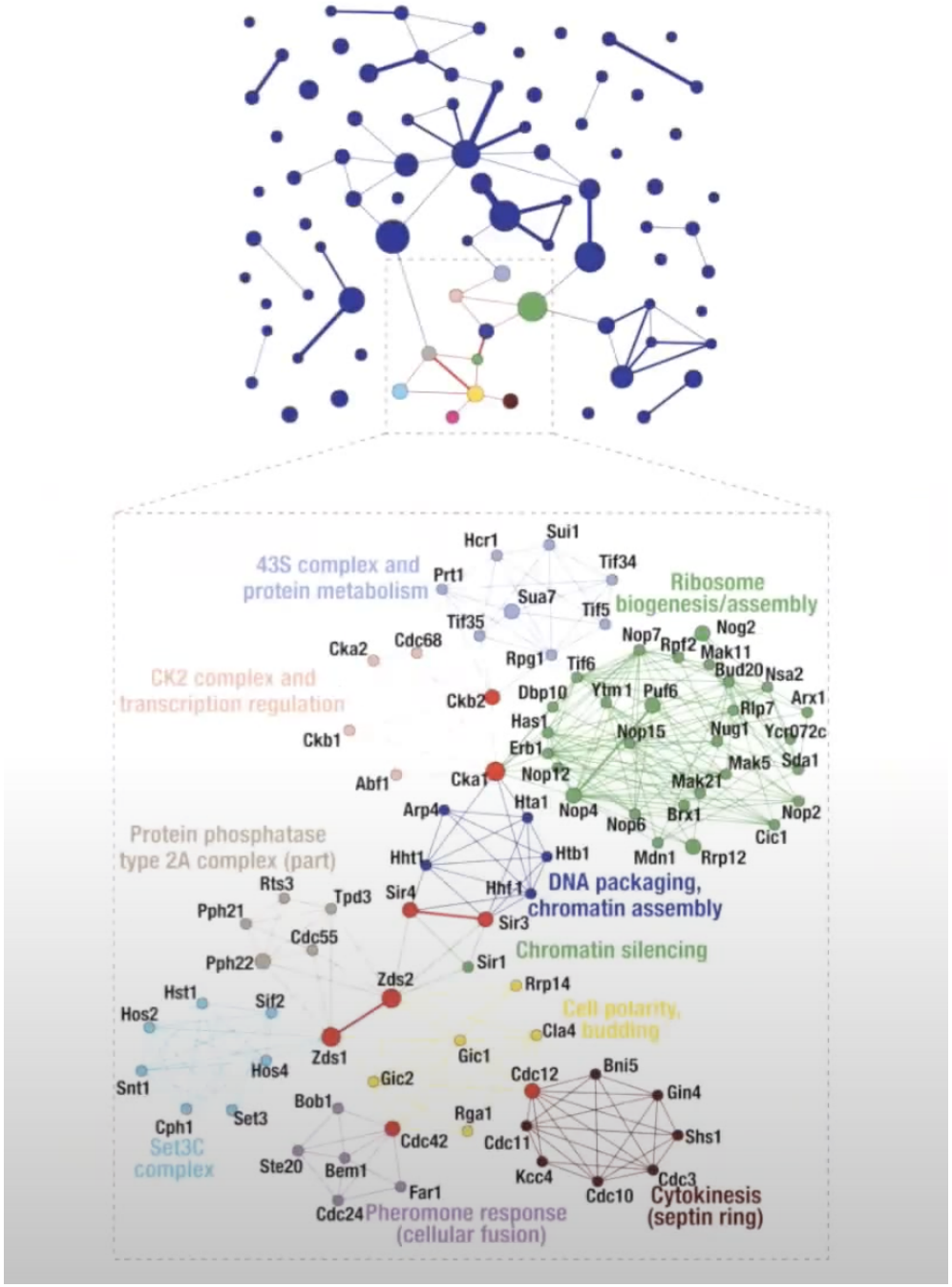


Clique Percolation Method: Example $k = 4$, $k = 5$





Clique Percolation Method:



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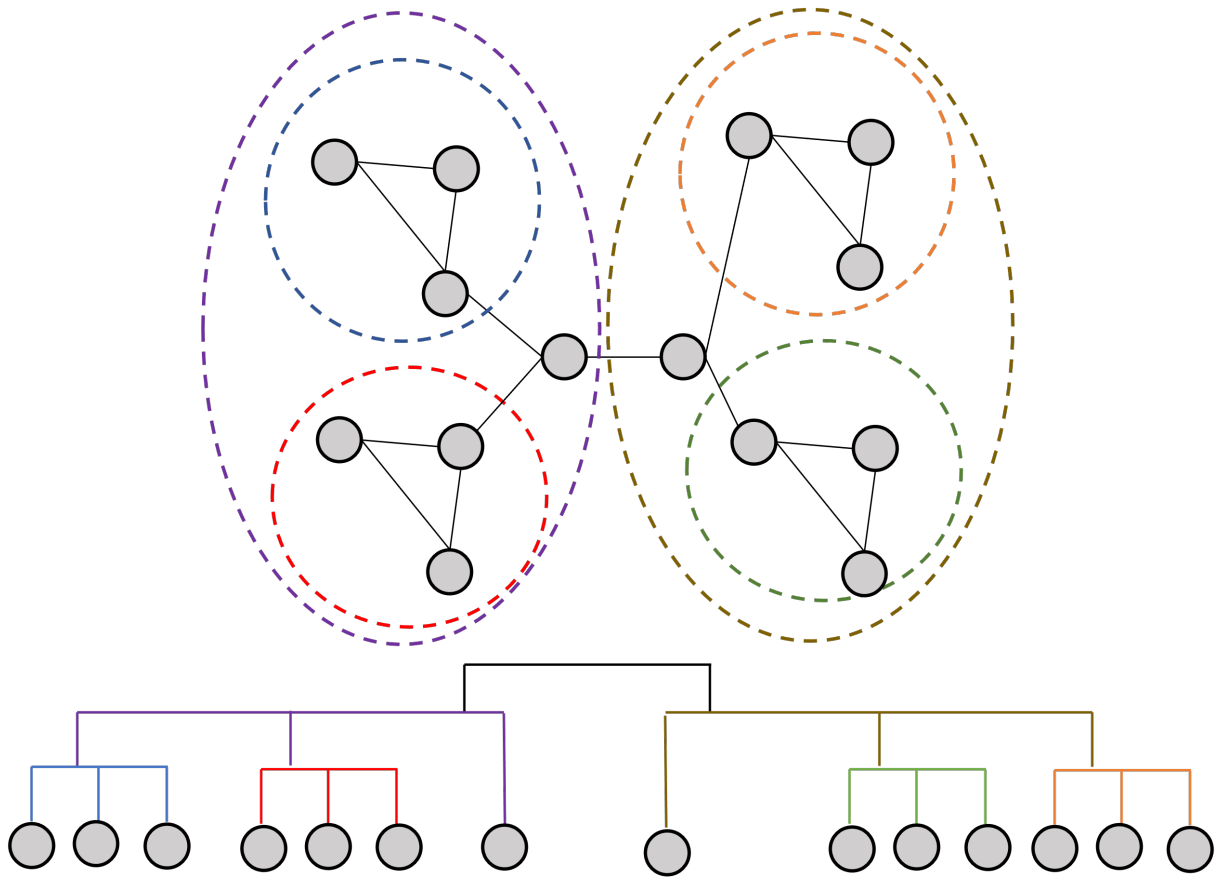
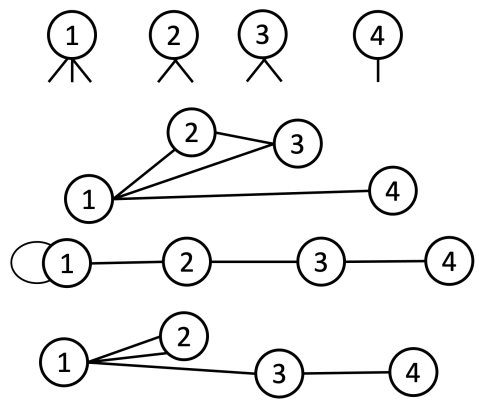
Community Detection: Louvain Algorithm

- ✓ Louvain: The most popular community detection based on modularity measure. Widely used for large networks: efficient (fast convergence), high dense communities (Blondel et *al*, 2008).
- ✓ Greedy algorithm with $\mathcal{O}(nlogn)$ in time
- ✓ Supports weighted graphs
- ✓ Provides hierarchical communities



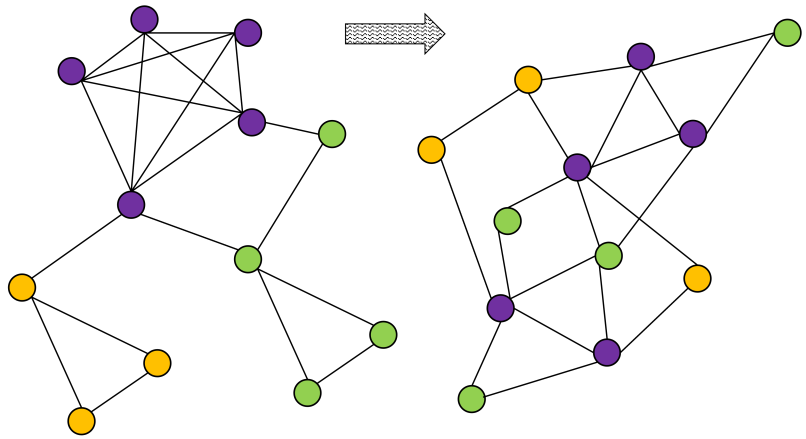
Community Detection: Modularity

- ✓ In network science, the configuration model is a method for generating random networks from a given degree sequence.
  - ☞ Widely used as a reference model for social networks, it allows the modeler to incorporate arbitrary degree distributions.
  - ☞ Assign a degree k_v to each node v. Each degree is a half-link or stub. The sum of stubs must be even to be able to build a graph $\sum_{v \in V} d_v = 2m$.
  - ☞ Choose two stubs uniformly at random and connect them by a link. Choose another pair from the remaining $2m-2$ stubs and connect them. Continue until you run out of stubs.
  - ☞ The resulting network keeps the same degrees but randomly pair up nodes.
- ✓ A realization might include cycles, self-loops or multi-links. The uniform distribution of the matching must be kept. This why it does not exclude self-loops or multi-links. Their expected number goes to zero for large networks.
- ✓ The probability of a stub of v being connected to one of w stubs is $\frac{d_w}{2m-1}$. Since node v has d_v stubs, the probability of v being connected to w is $\frac{d_v d_w}{2m-1} \approx \frac{d_v d_w}{2m}$ for large m



Community Detection: Modularity

- ✓ Modularity measures the relative density of links inside communities with respect to links outside communities. Different methods to compute the modularity. In the most common version, the randomization of the edges is done so as to preserve the degree of each vertex.
- ✓ The basic idea is to compare the number of links within communities with the number expected on the basis of chance. The generated network has less links between nodes of the same community and more between nodes of different communities.

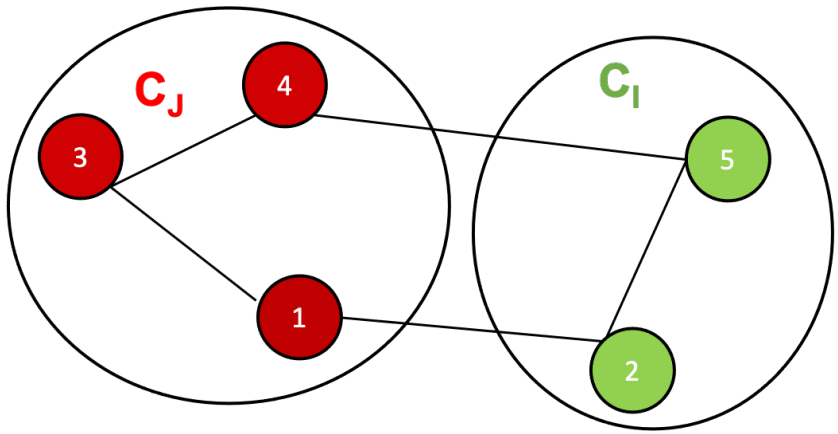


$$Q = \frac{1}{2m}\sum_i\sum_j\left[A_{ij} - \frac{k_i k_j}{2m}\right]\delta(c_i, c_j) \text{ or } Q = \left[\frac{\sum_i\sum_j A_{ij}}{2m} - \frac{\sum_i k_i \sum_j k_j}{(2m)^2}\right]\delta(c_i, c_j)$$

where k_i and k_j are resp. the sum of weights of v_i and v_j links (degree if unweighted graph), and c_i and c_j are resp. the communities of v_i and v_j. Kronecker function δ: 1 if $c_i = c_j$; 0 otherwise.
$-1 \leq Q \leq 1$. $Q > 0$: when links within communities $>$ links within communities in a randomly rewired network.

Community Detection: Modularity

Compare fraction of edges within a community to expected edges were distributed at random. $\frac{k_i k_j}{2m}$ is the random distribution of a link on the original graph. For the entire graph we sum the modularities over all communities.



δ: 1 if $c_i = c_j$; 0 otherwise

$$A_{11} - \frac{k_1 k_1}{2*5} = \left(0 - \frac{2*2}{2*5}\right) * 1 = -\frac{2}{5}; A_{33} - \frac{k_3 k_3}{2*5} = \left(0 - \frac{2*2}{2*5}\right) * 1 = -\frac{2}{5}; A_{44} - \frac{k_4 k_4}{2*5} = \left(0 - \frac{2*2}{2*5}\right) * 1 = -\frac{2}{5}$$

$$A_{13} - \frac{k_1 k_3}{2*5} = \left(1 - \frac{2*2}{2*5}\right) * 1 = \frac{3}{5}; A_{14} - \frac{k_1 k_4}{2*5} = \left(0 - \frac{2*2}{2*5}\right) * 1 = -\frac{2}{5}; A_{34} - \frac{k_1 k_3}{2*5} = \left(1 - \frac{2*2}{2*5}\right) * 1 = \frac{3}{5}$$

$$A_{55} - \frac{k_5 k_5}{2*5} = \left(0 - \frac{2*2}{2*5}\right) * 1 = -\frac{2}{5}; A_{22} - \frac{k_2 k_2}{2*5} = \left(0 - \frac{2*2}{2*5}\right) * 1 = -\frac{2}{5}; A_{25} - \frac{k_5 k_2}{2*5} = \left(1 - \frac{2*2}{2*5}\right) * 1 = \frac{3}{5};$$

$$Q = \frac{1}{2*5}\left[-\frac{2}{5} - \frac{2}{5} - \frac{2}{5} + 2*\frac{3}{5} - 2*\frac{2}{5} + 2*\frac{3}{5} - \frac{2}{5} - \frac{2}{5} + 2*\frac{3}{5}\right] = \frac{2}{25}$$

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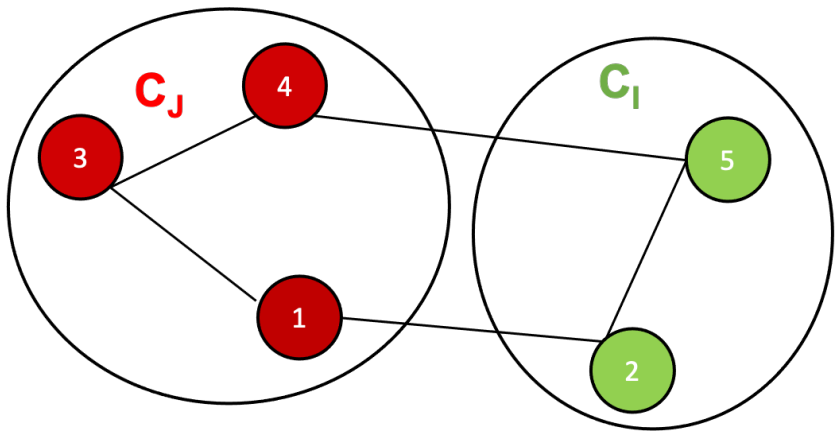
Community Detection: Modularity

The modularity of a community *c*:

$$Q_c = \frac{\sum_{i \in c} \sum_{j \in c} A_{ij}}{2m} - \frac{\sum_{i \in c} k_i \sum_{j \in c} k_j}{(2m)^2}$$

$$Q_c = \frac{\sum_{in}}{2m} - \left(\frac{\sum_{in+out}}{2m} \right)^2$$

where $\sum_{in}$ is the sum of edge weights between nodes within the community *c* (each edge is considered twice); and $\sum_{in+out}$ is the sum of all edge weights for nodes within the community; including incident ones from the other communities. If the number of communities = 1, $Q = 0$.

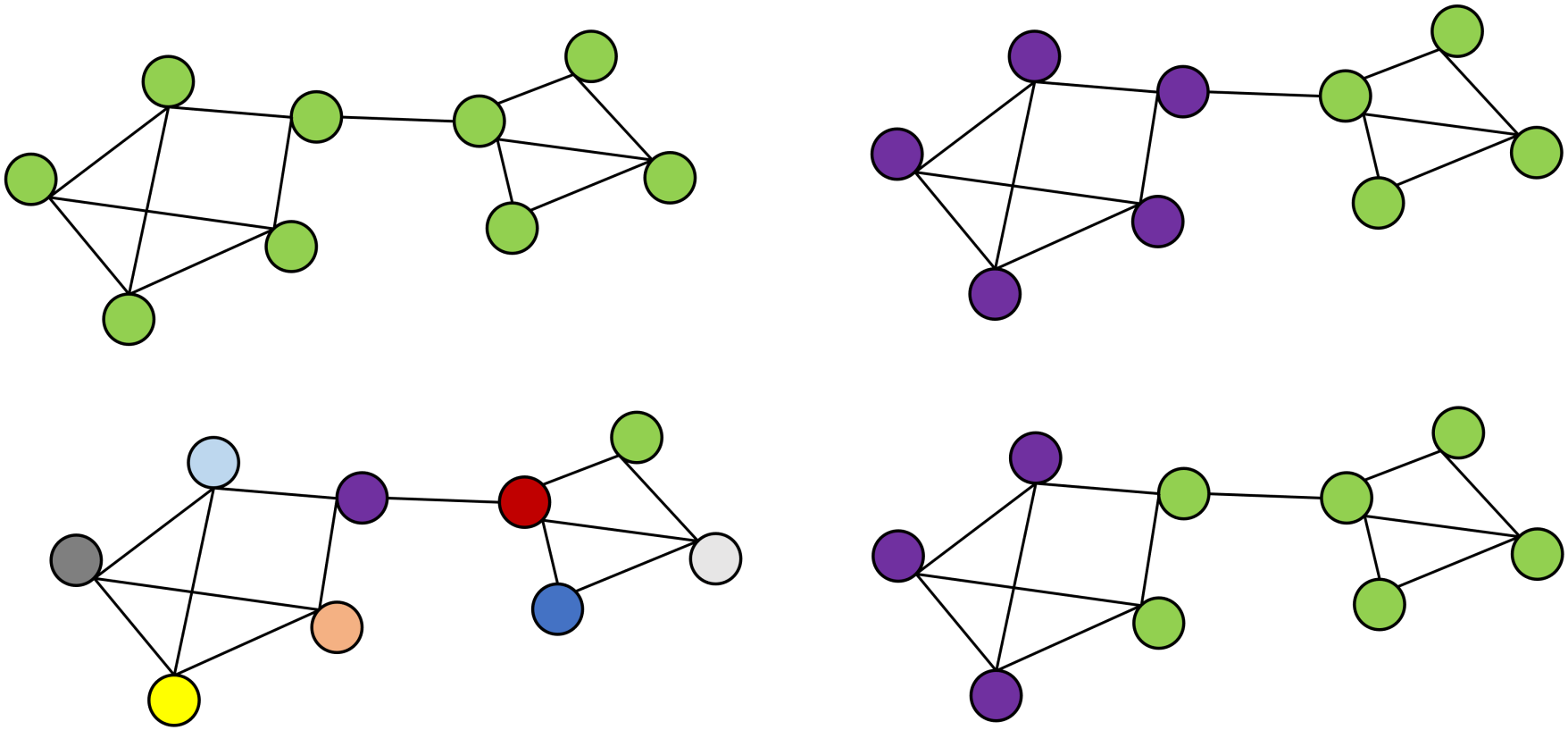


$$Q_{c_J} = \frac{2*2}{2*5} - \left(\frac{2*2+2}{2*5} \right)^2 ; Q_{c_I} = \frac{2*1}{2*5} - \left(\frac{2*1+2}{2*5} \right)^2$$

$$Q = \frac{2}{25}$$

Community Detection: Louvain Algorithm

Compare fraction of edges within a community to expected edges were distributed at random. $\frac{k_i k_j}{2m}$ is the random distribution of a link on the original graph. For the entire graph we sum the modularities over all communities.



Community Detection: Louvain method

- ✓ Louvain method: heuristic method for greedy modularity maximisation in 2 phases:
- ✓ Optimal value of modularity is impractical because it needs to go through all possible iterations of the nodes into groups

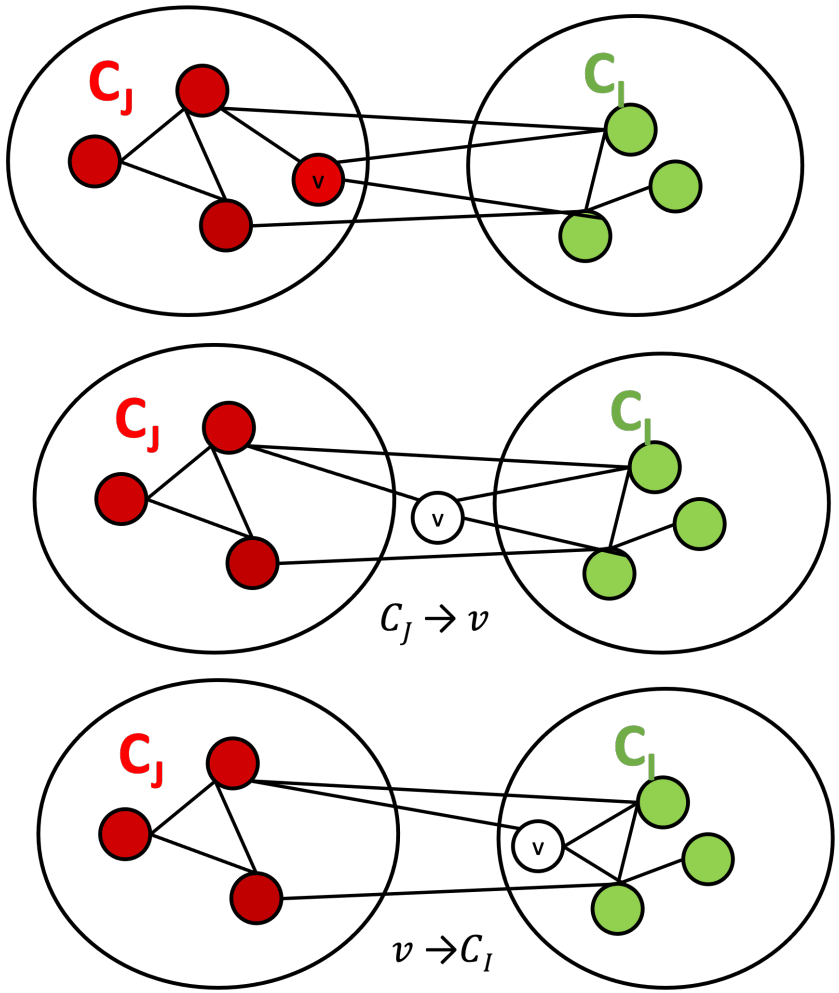
  - Greedy heuristic: first small communities are found by optimizing modularity locally on all nodes, then each small community is grouped into one node and the first step is repeated.
- ✓ Phase 1: Modularity is optimized by allowing only local changes to node communities memberships

  - Initially each node is assigned its own community.
- ✓ Phase 2: The identified communities are aggregated into super-nodes to build a new network
- ✓ Repeat Phase 1 until 1 super node

Community Detection: Louvain Algorithm

Phase 1

✓ What is $\triangle Q$ if we move a node *v* from C_J to C_I: $\triangle Q_{C_J \to v \to C_I} = \triangle Q_{C_J/\{v\}} + \triangle Q_{C_I \cup \{v\}}$



Community Detection: Louvain Algorithm

$$\triangle Q_{C_J \to v \to C_I} = \triangle Q_{C_J/\{v\}} + \triangle Q_{C_I \cup \{v\}}$$

✓ How we can derive $\triangle Q_{C_I \cup \{v\}}$? the gain in modularity doesn't depend on the original community of v. It can easily be computed by moving an isolated node v into C_I:

$$\triangle Q_{C_I \cup \{v\}} = Q_{C_I \cup \{v\}} - [Q_{C_I} + Q_{\{v\}}]$$

$$\triangle Q_{C_I \cup \{v\}} = \left[\frac{\sum_{in} + 2k_{v,in}}{2m} - \left(\frac{\sum_{in+out} + k_v}{2m}\right)^2\right] - \left[\frac{\sum_{in}}{2m} - \left(\frac{\sum_{in+out}}{2m}\right)^2 - \left(\frac{k_v}{2m}\right)^2\right]$$

$$\triangle Q_{C_J \setminus \{v\}} = [Q_{C_J \setminus \{v\}} + Q_{\{v\}}] - Q_{C_J}$$

$$\triangle Q_{C_J \setminus \{v\}} = \left[\frac{\sum_{in} - 2k_{v,in}}{2m} - \left(\frac{\sum_{in+out} - k_v}{2m}\right)^2 - \left(\frac{k_v}{2m}\right)^2\right] - \left[\frac{\sum_{in}}{2m} - \left(\frac{\sum_{in+out}}{2m}\right)^2\right]$$

✓ The gain computation depends only on $k_{v,in}$ and C

Community Detection: Louvain Algorithm

- For each node v: (i) compute modularity gain from removing v from its community and placing it in the community of its neighbor; (ii) place v in the community maximizing $\triangle Q$. One iteration is achieved for all the nodes in a sequential manner.

```
1:  procedure LOUVAIN-PHASE1-ONEITERATION(G(V, E, K), C)
2:      for v ∈ V do
3:          cv ← C.get(v); Cn ← C.get(N_v) \ cv; gmax ← −∞; cvmax ← cv
4:          for c ∈ Cn do
5:              gain ← △Q_(cv→v→c)
6:              if gain > gmax then
7:                  gmax ← gain; cvmax ← c
8:              end if
9:          end for
10:         if gmax > 0 then
11:             cv ← cv \ {v}; cvmax ← cvmax ∪ {v}
12:         end if
13:     end for
14: end procedure
```

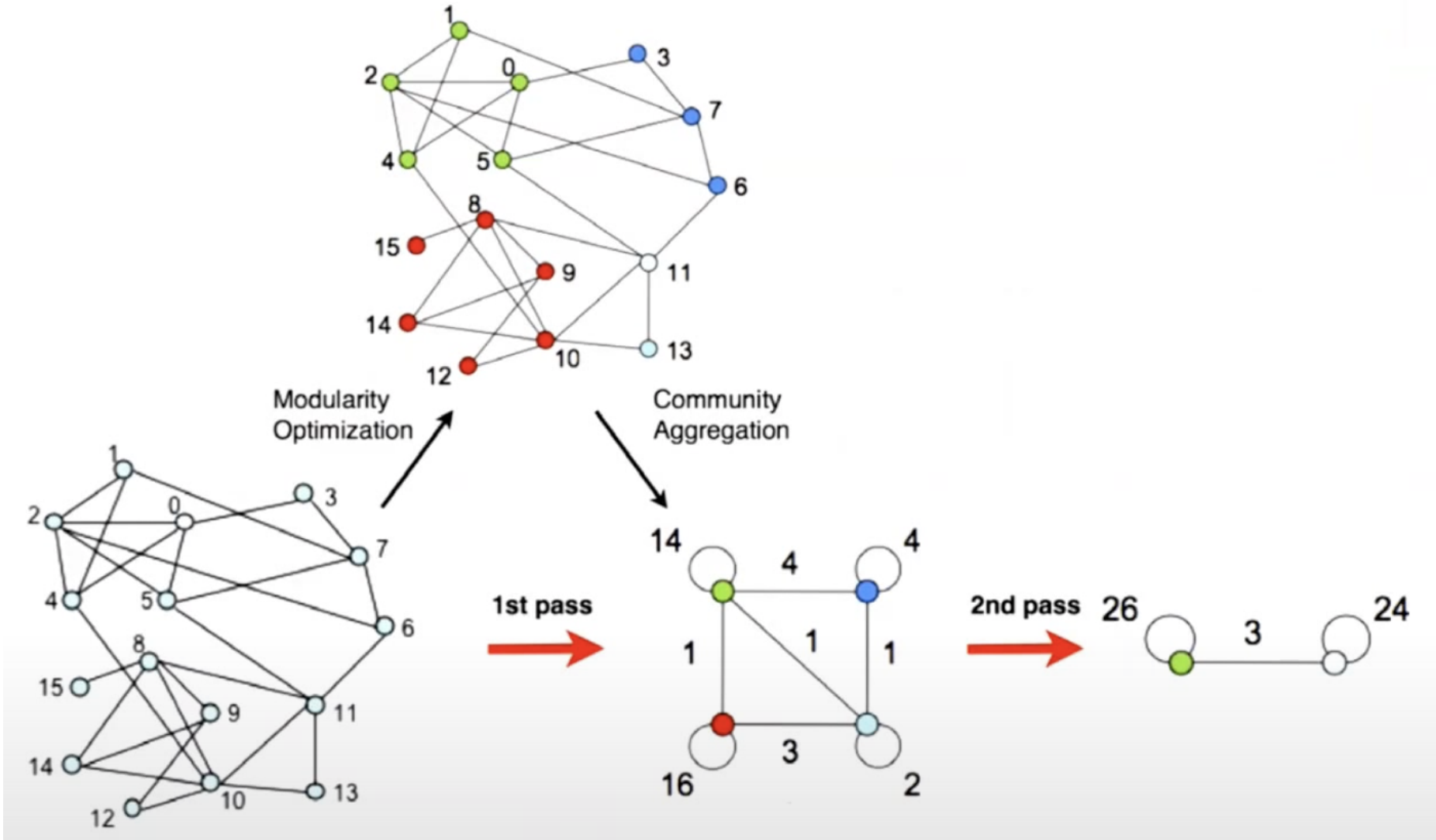
- Repeat the procedure sequentially to all nodes until no more improvement (local max of modularity).
  - The output of this phase depends on the order in which the nodes are considered -> research shows that this doesn't significantly affect the overall modularity.

Community Detection: Louvain method

✓ Phase II

- Nodes from communities grouped into super nodes
- Links between nodes of the same community c are represented by self-loops weighted by adding up the links between these nodes:
  c has a loop edge with weight $\sum_{v_i, v_j \in c} A_{i,j}$
- Links between communities weighted by adding up the links between community's nodes each (c_I, c_J) has a link with weight $\sum_{v_i \in c_I, v_j \in c_J} A_{i,j}$

Community Detection: Louvain Algorithm



Community Detection: Louvain Algorithm

✓ This figure shows many local nested communities to be easily explored