Backtracking Notes

Every backtracking problem can be solved by the following strategy:

"choose" → "explore" → "unchoose"

- Use an iterator to list all the possible starting points for our recursion.
- Select one number by adding it to a stack that holds the current branch.
- Recursively call the 'explore_helper' function which will carry on the recursion and pass along the stack which contains the numbers chosen in the current branch.
- Remove the recently added number and go back to step 1 to explore another sub-branch.
  The termination condition that will stop the recursion and add the current branch to the 'results' list is given by the comparison between the length of the branch and the length of the original list to permute.

BACKTRACKING

Subset (LC 78)

Given an integer array nums of **unique elements**, **return all possible subsets** (the power set).

The solution set **must not contain duplicate subsets.**
Return the solution in any order.

```
lst = ["a", "b", "c"]

answer = [
    ['a', 'b', 'c'],
    ['a', 'b'],
    ['a', 'c'],
    ['a'],
    ['b', 'c'],
    ['b'],
    ['c'],
    []]
```

```
class SolutionSubset:
    def subsets(self, nums: List[any]) -> List[List[any]]:
        answer = []
        subset = []

        def backtrack(i):
            if i >= len(nums):
                answer.append(subset.copy())
                return

            subset.append(nums[i])
            backtrack(i + 1)

            subset.pop()
            backtrack(i + 1)

        backtrack(0)

        return answer
```

"choose" → "explore" → "unchoose"

- add to the subset, try the backtrack by incrementing i;
- remove from subset, try the backtrack by incrementing i

Subset II (LC 90)

Given an integer array nums that **may contain duplicates**, **return all possible subsets** (the power set).

The solution set **must not contain duplicate subsets**.
Return the solution in any order.

```
lst = ["a", "b", "c", "c"]

answer = [
    ['a', 'b', 'c', 'c'],       ['b', 'c', 'c'],
    ['a', 'b', 'c'],            ['b', 'c'],
    ['a', 'b'],                 ['b'],
    ['a', 'c', 'c'],            ['c', 'c'],
    ['a', 'c'],                 ['c'],
    ['a'],                      []]
```

```
class SolutionSubsetII:
    def subsetWithDup(self, nums: List[any]) -> List[List[any]]:
        answer = []
        subset = []
        nums.sort()

        def backtrack(i):
            if i == len(nums):
                answer.append(subset.copy())
                return

            subset.append(nums[i])
            backtrack(i + 1)
            subset.pop()
            while (i + 1 < len(nums) and
                   nums[i] == nums[i + 1]):
                i += 1
            backtrack(i + 1)

        backtrack(0)

        return answer
```

- sort the nums first
- add to the subset, try the backtrack by incrementing i;
- remove from subset, try the backtrack by incrementing i
- keep incrementing i if the nums[i] is a duplicate of the previous.
- then try the backtrack by incrementing i;

this allows getting all duplicated values.
without it, returns a set without duplicates (incorrect).

```
answer = [['a', 'b', 'c'],
          ['a', 'b'],
          ['a', 'c'],
          ['a'],
          ['b', 'c'],
          ['b'],
          ['c'],
          []]
```

Combination Sum (LC 39)

Given a collection of candidate numbers (`candidates`) and a target number (`target`), **return a list of all unique combinations of candidates** where the chosen numbers sum to target.
The same number may be chosen from `candidates` **an unlimited number of times**.
The solution set **must not** contain duplicate combinations.

```
candidates = [2, 3, 5]
target = 8

answer = [
    [2, 2, 2, 2],
    [2, 3, 3],
    [3, 5]]
```

```
class SolutionCombinationSum:
    def combinationSum(self, candidates: List[int], target: int) -> List[List[int]]:
        answer = []
        permutation = []

        def backtrack(i):
            if sum(permutation) == target:
                answer.append(permutation.copy())
                return
            if i >= len(candidates) or sum(permutation) > target:
                return

            permutation.append(candidates[i])
            backtrack(i)
            permutation.pop()
            backtrack(i + 1)

        backtrack(0)

        return answer
```

- check if the sum() of permutation == target
  also check that it doesn't hit edge cases

"choose" → "explore" → "unchoose"

- add to the permutation, try the backtrack with i;
- remove last item from permutation, try the backtrack by incrementing i

"same number may be chosen from candidates **an unlimited number of times**"

Combination Sum II (LC 40)

Given a collection of candidate numbers (`candidates`) and a target number (`targett`), **find all unique combinations** in candidates where the candidate numbers sum to target.
Number in `candidates` may only be **used once**.

```
candidates = [10,1,2,7,6,1,5]
target = 8

answer = [
    [1,1,6],
    [1,2,5],
    [1,7],
    [2,6]]
```

```
class SolutionCombinationSumII:
    def combinationSum2(self, candidates: List[int], target: int) -> List[List[int]]:
        answer = []
        permutation = []
        candidates.sort()

        def backtrack(i):
            if sum(permutation) == target:
                answer.append(permutation.copy())
            if sum(permutation) >= target:
                return

            prev = -1
            for idx in range(i, len(candidates)):
                if candidates[idx] == prev:
                    continue
                permutation.append(candidates[idx])
                backtrack(idx + 1)
                permutation.pop()
                prev = candidates[idx]

        backtrack(0)

        return answer
```

- sort the candidates first
- for loop; idx starting from i -> end of length of candidates
- this SKIPS the idx if there is a duplicate, runs continue
- add to the permutation, try the backtrack with i + 1;

candidates.sort() and this code allows skipping duplicate values
"Number in candidates may only be **used once**"

the for-loop inside of the `backtrack()` function ensures that all indices that are left in `candidates` is tested, and also allows skipping duplicates.

Inside the for-loop, the algorithm iterates through the candidates list, starting from the i index. The loop allows the algorithm to consider different candidates for the next element in the combination.

Permutations (LC 46)

Given an **array nums of distinct integers**, return all the **possible permutations**.
You can return the answer in any order.

```
lst = [1,2,3]

answer = [
    [1,2,3],
    [1,3,2],
    [2,1,3],
    [2,3,1],
    [3,1,2],
    [3,2,1]]
```

```
class SolutionPermutations:
    def permute(self, nums: List[any]) -> List[List[any]]:
        answer = []
        permutation = []

        def backtrack():
            if len(permutation) == len(nums):
                answer.append(permutation.copy())
                return

            for num in nums:
                if num not in permutation:
                    permutation.append(num)
                    backtrack()
                    permutation.pop()
        backtrack()

        return answer
```

"choose" → "explore" → "unchoose"

- for loop; goes through each element in nums
- append()
- backtrack()
- pop()

The for-loop iterates through each element (num) in the nums list. For each element, it checks if that element is not already in the permutation list. This check ensures that the same element is not added multiple times to the same permutation

Letter Combinations of a Phone Number (LC 17)

Given a **string containing digits from 2-9** inclusive, **return all possible letter combinations** that the number could represent. Return the answer in any order.

```
Input: digits = "23"

Output: [
    "ad", "be",
    "ae", "bf",
    "af", "cd",
    "bd", "ce",
          "cf"]
```


```
class Solution:
    def letterCombinations(self, digits: str) -> List[str]:
        keys = {
            "2": "abc",
            "3": "def",
            "4": "ghi",
            "5": "jkl",
            "6": "mno",
            "7": "pqrs",
            "8": "tuv",
            "9": "wxyz",
        }
        answer = []

        def backtrack(i, permutation):
            if len(digits) == len(permutation):
                answer.append("".join(permutation))
                return

            key_choices = keys[digits[i]]
            for k in key_choices:
                backtrack(i + 1, permutation + [k])

        if digits:
            backtrack(0, [])

        return answer
```

- when length of permutation is same as length of digits, joing as string, then add to the answer, and return
- use keys to obtain the possible key_choices

i=0 → "23" → keys[digits[i]]

```
keys = {
    "2": "abc"
    3": "def",
    ...
}
```

key_choices

The for-loop acts to iterate over key_choices.
The `backtrack()` function is iteratively called as i increases, to explore each option

Palindrome Partitioning (LC 131)

Given a string s, **partition s** such that every substring of the partition is a palindrome.
Return all possible palindrome partitioning of s.

```
s = "aab"

answer = [
    ["a","a","b"],
    ["aa","b"]]
```

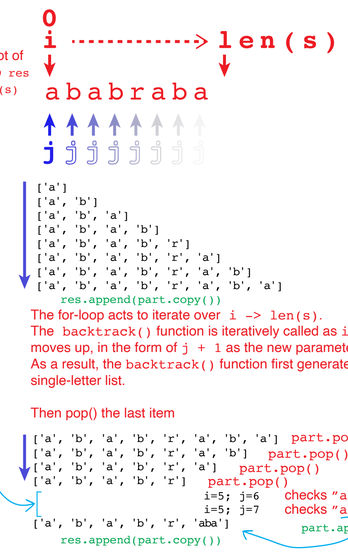
```
s2 = "ababraba"

answer2 = [
    ['a', 'b', 'a', 'b', 'r', 'a', 'b', 'a'],
    ['a', 'b', 'a', 'b', 'r', 'aba'],
    ['a', 'bab', 'r', 'a', 'b', 'a'],
    ['a', 'bab', 'r', 'aba'],
    ['aba', 'b', 'r', 'a', 'b', 'a'],
    ['aba', 'b', 'r', 'aba']]
```

```
class Solution:
    def partition(self, s: str) -> List[List[str]]:
        res, part = [], []

        def backtrack(i):
            if i >= len(s):
                res.append(part.copy())
                return

            for j in range(i, len(s)):
                if self.isPalindrome(s, i, j):
                    part.append(s[i : j + 1])
                    backtrack(j + 1)
                    part.pop()

        backtrack(0)
        return res

    def isPalindrome(self, s, l, r):
        while l < r:
            if s[l] != s[r]:
                return False
            l, r = l + 1, r - 1
        return True
```

- takes a snapshot of part, append to res when i >= len(s)

"choose" → "explore" → "unchoose"

- add to the part, try the backtrack with j + 1;
- remove last item from part, try the backtrack by incrementing j




```
['a']
['a', 'b']
['a', 'b', 'a']
['a', 'b', 'a', 'b']
['a', 'b', 'a', 'b', 'r']
['a', 'b', 'a', 'b', 'r', 'a']
['a', 'b', 'a', 'b', 'r', 'a', 'b']
['a', 'b', 'a', 'b', 'r', 'a', 'b', 'a']
    res.append(part.copy())
```

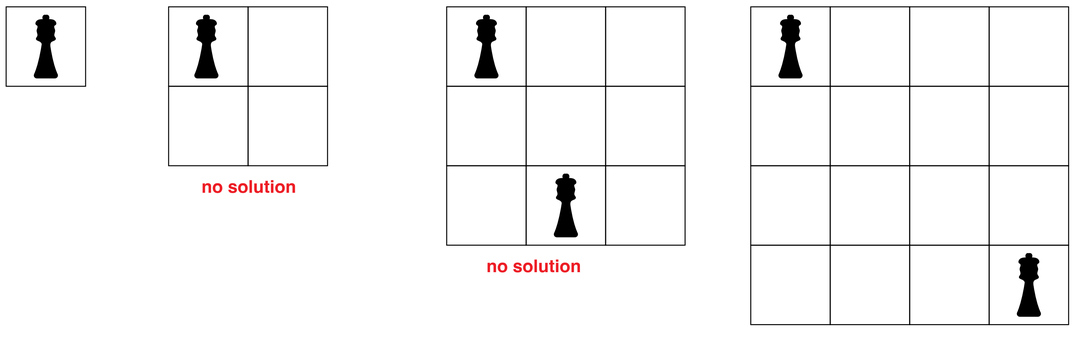
The for-loop acts to iterate over i -> len(s).
The `backtrack()` function is iteratively called as i moves up, in the form of `j + 1` as the new parameter.
As a result, the `backtrack()` function first generates a single-letter list.

Then pop() the last item

```
['a', 'b', 'a', 'b', 'r', 'a', 'b', 'a']  part.pop()
['a', 'b', 'a', 'b', 'r', 'a', 'b']   part.pop()
['a', 'b', 'a', 'b', 'r', 'a']   part.pop()
['a', 'b', 'a', 'b', 'r']   part.pop()
                              i=5; j=6    checks "ab" for palindrome
                              i=5; j=7    checks "aba" for palindrome
['a', 'b', 'a', 'b', 'r', 'aba']          part.append(s[i : j + 1])
    res.append(part.copy())
```

Inside the for-loop, the variable j ranges from i to the end of the string s. This loop iterates over all possible substrings starting from the current position i.

N-Queens (LC 51)




```
for i in range(n):
    q_ct = 0
    for j in range(n):
        if board[i] == "Q":
            q_ct += 1
        if q_ct > 1:
            return False
```