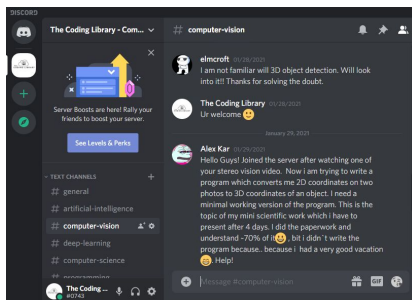




Computer Vision

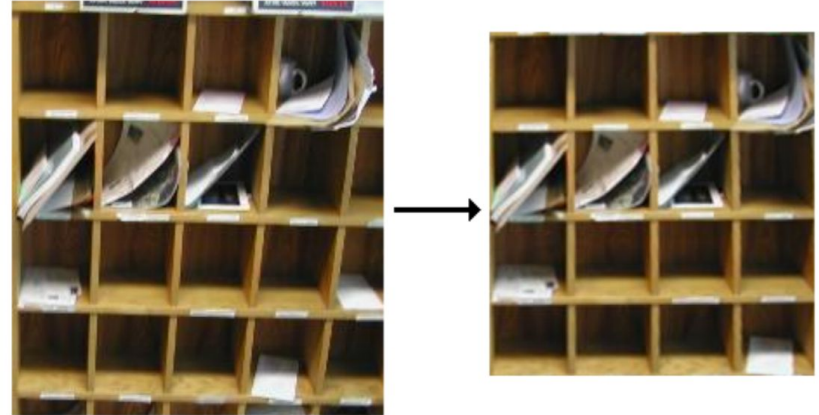
Geometric Transformations



Discord Link in Description

Geometric Transformations

- Types of Geometric Transformations:
 - Geometric
 - Pixel coordinate
 - Brightness interpolation
- Applications:
 - Computer graphics
 - Distortion - Introduce / Eliminate
 - Image processing / Preprocessing
 - Text recognition
 - Recognition of signs, numbers, etc.

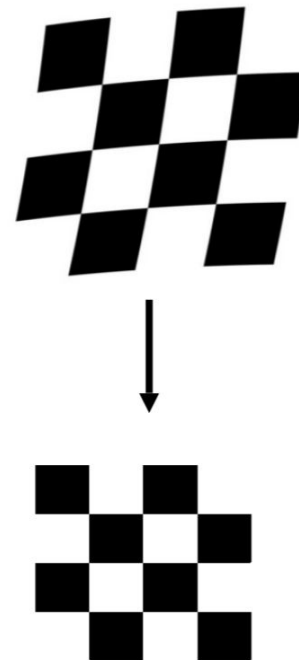


Formulation of the problem

- Distorted image $f(i, j)$
- Corrected image $f'(i', j')$
- Mapping between the images

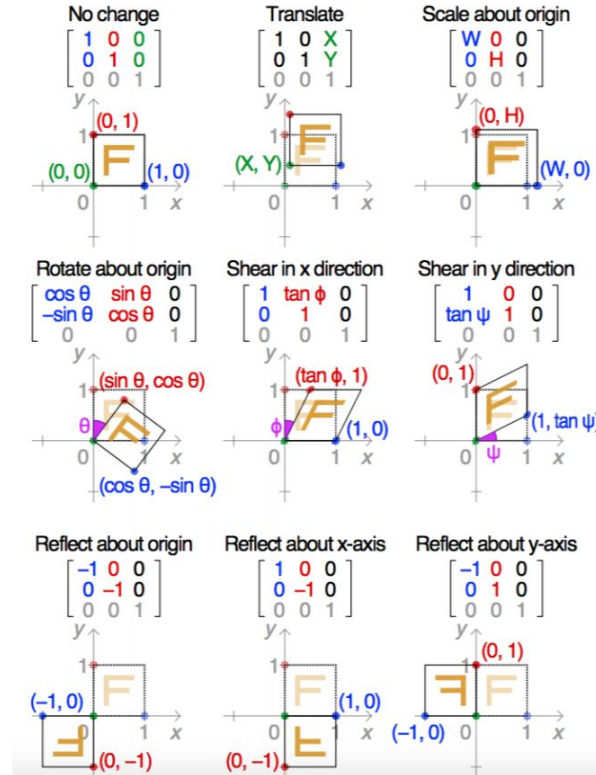
$$i = T_i(i', j') \quad j = T_j(i', j')$$

- Define the Transformation
 - Known in advance / determine through correspondences
 - Image to know
 - Image to image
- Apply the defined Transformation
 - For every point in the output image
 - Determine where it came from using T from mapping
 - Interpolate a value for the output point



Transformations

- Linear Transformations in 3D
 - Affine Transformation
 - Euclidean (6 DoF)
- Higher Order Transformations
 - Parameterized
 - B-splines
 - Freeform
 - Warp field



Affine Transformations in 2D

Affine Transformations

- Known Transformations
 - E.g. Translation, Rotation, etc
- Unknown Transformations
 - Would require at least 3 observations
 - Could be points in an image
 - E.g. Corners of objects, etc.

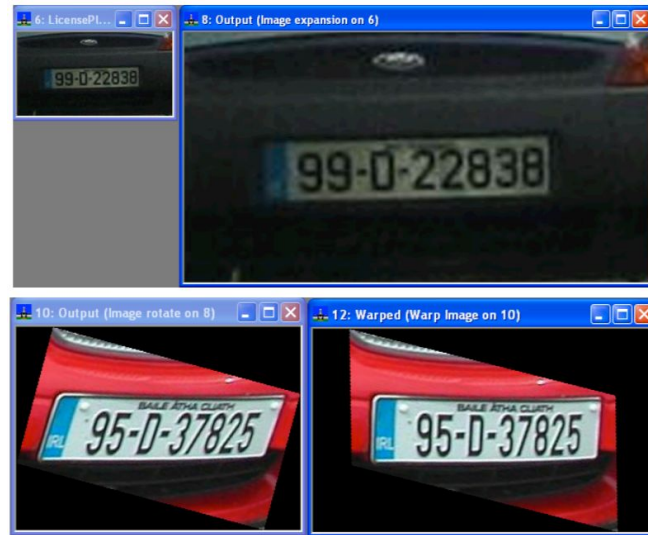
$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} 1 & 0 & m \\ 0 & 1 & n \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

```
Mat affine_matrix( 2, 3, CV_32FC1 );  
...  
warpAffine( image, result, affine_matrix, image.size() );
```

Example of a Simple Affine Transformation

1. Rotating the image
2. Scaling the image
3. Skewing or shearing the image



Unknown Affine Transformation

- More observations - We need at least 3
 - Better estimate of the coefficients
- Uses pseudo inverse
 - For unknown transformations

```
Point2f source [3], destination [3];  
...  
affine_matrix = getAffineTransform(source,destination);
```



Perspective Transformations

- Perspective projection
- Planar surface
- Not parallel to the image plane
- Can't be corrected with the affine trans
- Therefore we need a perspective trans

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} i.w \\ j.w \\ w \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & 1 \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$



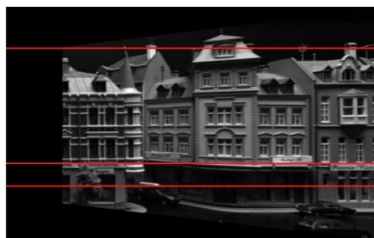
Perspective Transformations in OpenCV



```
Point2f source [4], destination [4];  
// Assign values to source and destination points.  
perspective_matrix = getPerspectiveTransform( source, destination );  
warpPerspective( image, result, perspective_matrix,  
result.size() );
```

Rectifying Homographies

- Image transformation can be computed such that scanlines can be directly matched on images

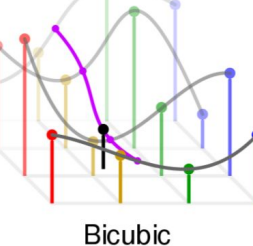
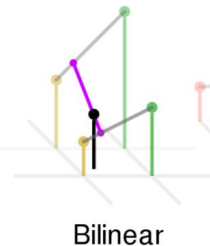
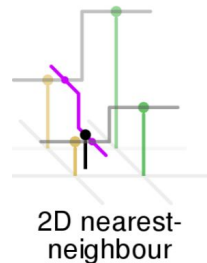
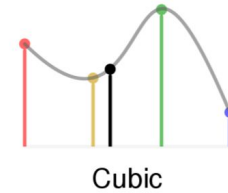
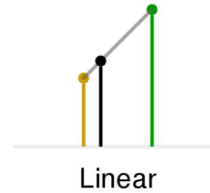
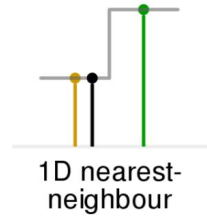


Brightness Interpolation

- Locations are not integer coordinates
- Interpolate output pixel value from the nearby pixels in the original image

- Interpolation methods

- Nearest neighbour
- Bilinear interpolation
- Bicubic interpolation



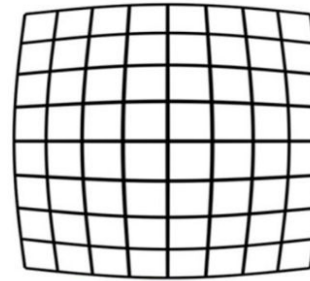
Brightness Interpolation in OpenCV



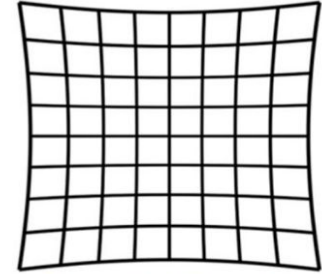
```
int interpolation_scheme = INTER_LINEAR;
// Nearest neighbour interpolation: INTER_NEAREST
// Bilinear interpolation: INTER_LINEAR
// Bicubic interpolation: INTER_CUBIC
warpAffine( image, result, affine_matrix, result_size,
            interpolation_scheme );
warpPerspective( image, result, perspective_matrix,
                 result_size,
                 interpolation_scheme );
```

Distortion - Camera Models

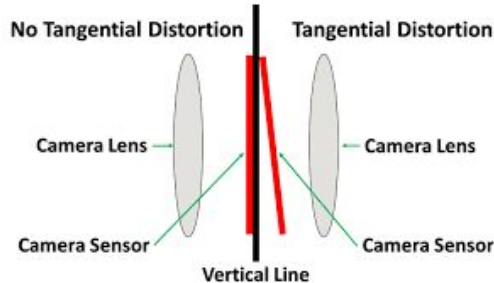
- Radial Distortion
 - Radially symmetric from the optical axis
 - Change in magnification
 - Barrel distortion
 - Pincushion distortion
 - Caused by the lenses
- Tangential Distortion
 - Uneven magnification from one side to the other
 - Lenses not parallel to the image plane



Barrel Distortion

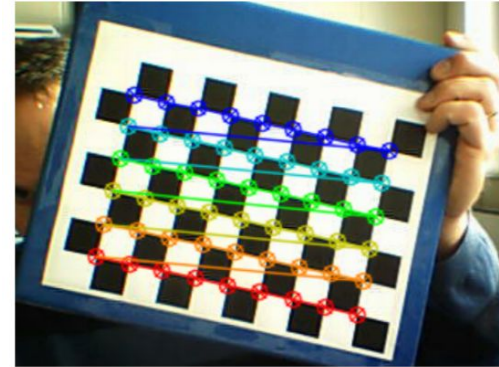


Pincushion Distortion



Removing Distortion - Camera Models

- Calibrate using multiple images
- Calibrate with known objects
- Change positions
- Compute the camera matrix and distortion parameters
- Remove distortion using the distortion parameters



```
calibrateCamera( object_points, image_points,  
                image_size, camera_matrix, distortion_coefficients,  
                rotation_vectors, translation_vectors );  
  
...  
  
undistort( camera_image, corrected_image,  
           camera_matrix, distortion_coefficients );
```