$$\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i-j)^2$$

= $\sum_{i=1}^{2^k}\sum_{j=1}^{2^k}(i-j)^2$ 因為是算 i-j 的差，所以平移不影響答案

$=\sum_{i=1}^{n}\sum_{j=1}^{n}(i^2-2ij+\ j^2)$ <- let $2^k=n$

$=\sum_{i=1}^{n}\sum_{j=1}^{n}i^2$ $-2\sum_{i=1}^{n}\sum_{j=1}^{n}ij$ + $\sum_{i=1}^{n}\sum_{j=1}^{n}j^2$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}j^2$$

$=\mathrm{n}(\frac{n(n+1)(2n+1)}{6})$

$=(\frac{2n^4+3n^3+n^2}{6})$

$$\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}ij$$

$=(\frac{(1+n)n}{2})*\ (\frac{(1+n)n}{2})$

$=\frac{n^4+2n^3+n^2}{4}$

$\sum_{i=1}^{n}\sum_{j=1}^{n}i^2$ + $\sum_{i=1}^{n}\sum_{j=1}^{n}j^2$ $-2\sum_{i=1}^{n}\sum_{j=1}^{n}ij$

$=\left(\frac{2n^4+3n^3+n^2}{6}\right)*2\ -\ 2*(\frac{n^4+2n^3+n^2}{4})$

$=\left(\frac{2n^4+3n^3+n^2}{3}\right)\ -\ (\frac{n^4+2n^3+n^2}{2})$

$=\frac{4n^4+6n^3+2n^2-3n^4-6n^3-3n^2}{6}$

$=\frac{n^4-n^2}{6}$

$=\frac{n^2(n^2-1)}{6}$

$=\frac{2^{2k}(2^{2k}-1)}{6}$ 證明完成

依照新 ppt 更新

$$(1/2^k)\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i-j)^2 \quad = \quad (1/2^k)\frac{2^{2k}(2^{2k}-1)}{6} \ = \ \frac{(2^{2k}-1)}{6}$$