Digital Image Processing

Lecture #10

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Announcement

- **The following schedule**

02/17	Lecture 1	04/14	Lecture 8
02/24	Lecture 2	04/21	Proposal
03/03	Lecture 3	04/28	Lecture 9
03/10	Lecture 4	05/05	Lecture 10
03/17	Lecture 5	05/12	Lecture 11
03/24	Lecture 6	05/19	Demo
03/31	Lecture 7	05/26	Demo
04/07	Midterm	06/02	Final Package Due

Image Restoration

Image Restoration

- Attempt to reconstruct or recover an image that has been degraded
- Model the degradation and apply the inverse process

- A priori modeling
  - measurements on the physical imaging system, digitizer and display
- A posteriori modeling
  - measurements of a particular image to be restored

Image Restoration

- Forward Problem
  - Given X & H → find Y
- System Identification Problem
  - Given X & Y → find H
- Inverse Problem
  - Given H & Y → find X

⇨ Image enhancement v.s. image restoration

Image Restoration

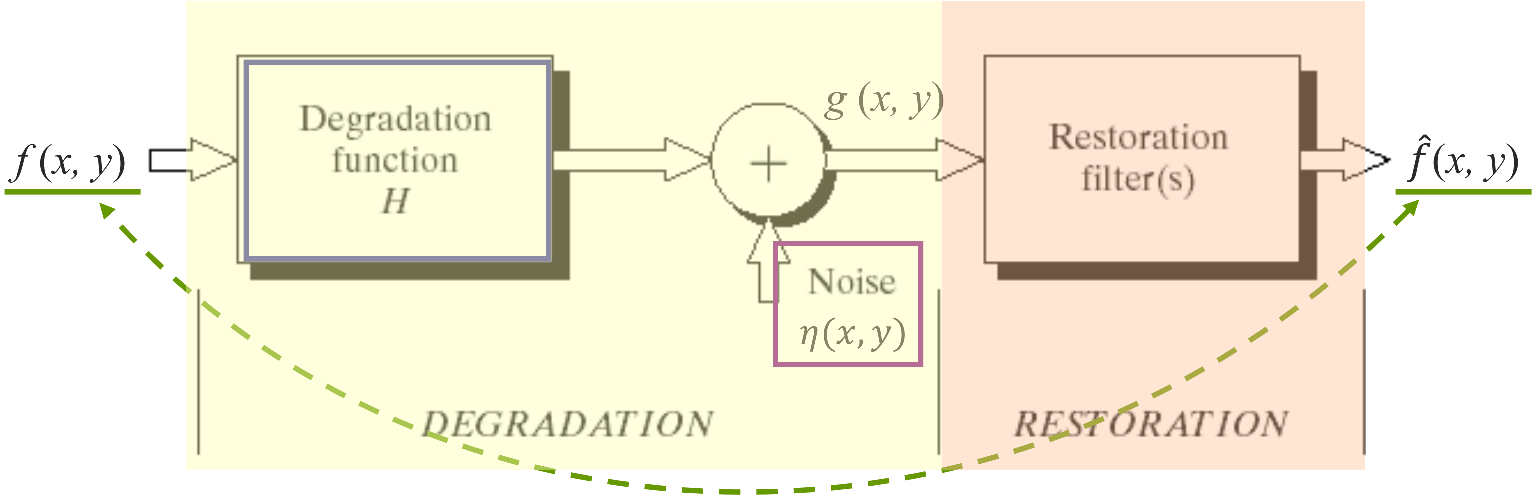
- Image Enhancement v.s. Image Restoration
  - Image enhancement
    - Largely subjective
    - No benchmark
    - Basically a heuristic procedure
  - Image restoration
    - More objective
    - With a ground truth
    - Usually formulate a criterion of goodness

Image Restoration

- Image observation models
  - An acquired image could be degraded by the sensing environment
    - Sensor noise
    - Optical system aberrations
    - Image motion blur
    - Atmospheric turbulence effects
  - Model the degradation and apply the inverse process
    - A priori modeling
    - A posteriori modeling

Image Restoration

- **Image observation models**



$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

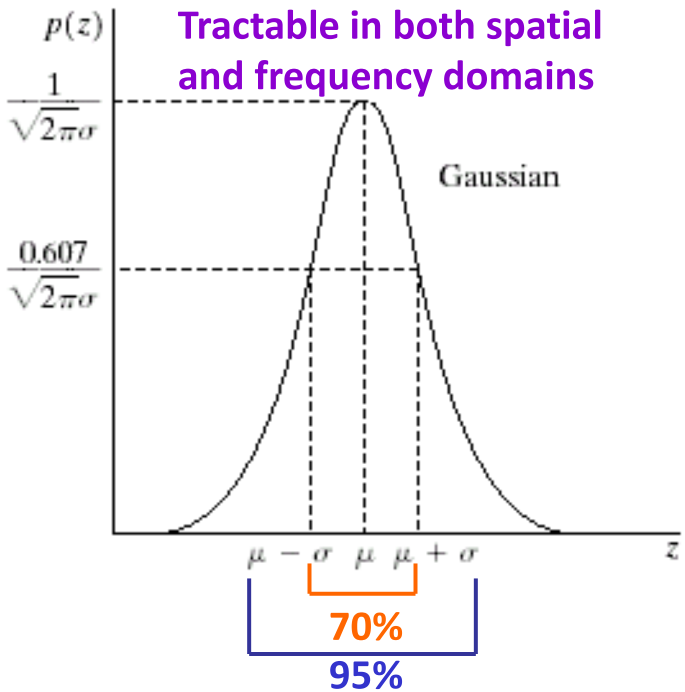
Image Restoration

- **Noise models**
  - Gaussian Noise
  - Rayleigh Noise
  - Gamma (Erlang) Noise
  - Exponential Noise
  - Uniform Noise
  - Impulse (Salt-and-Pepper) Noise

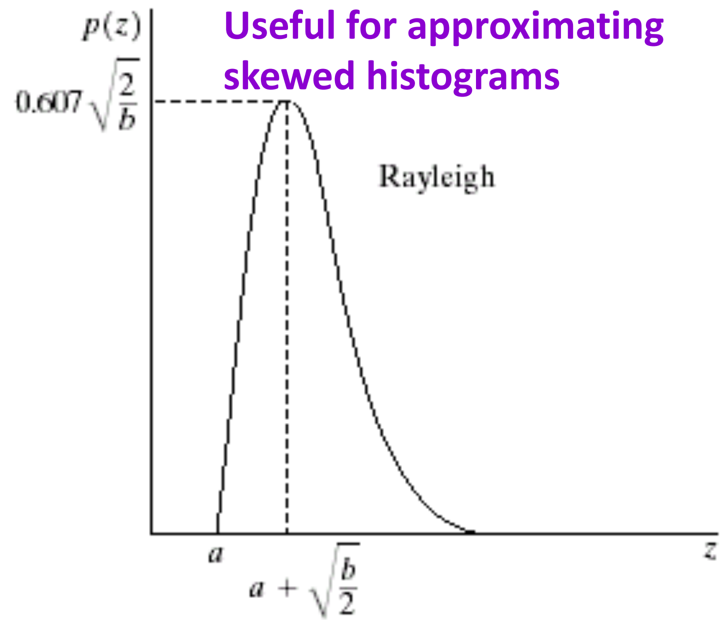
//Note// assume that noise is independent of spatial coordinates and uncorrelated w.r.t. the image itself (except periodic noise)

Image Restoration

- **Gaussian noise v.s. Rayleigh noise**



$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

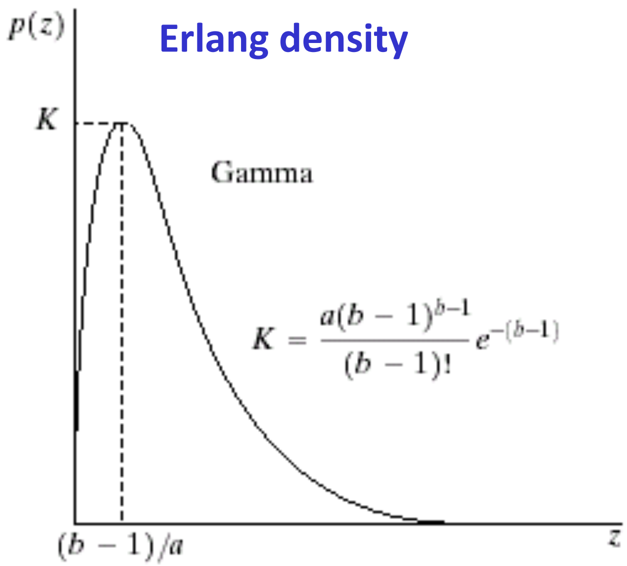


$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Image Restoration

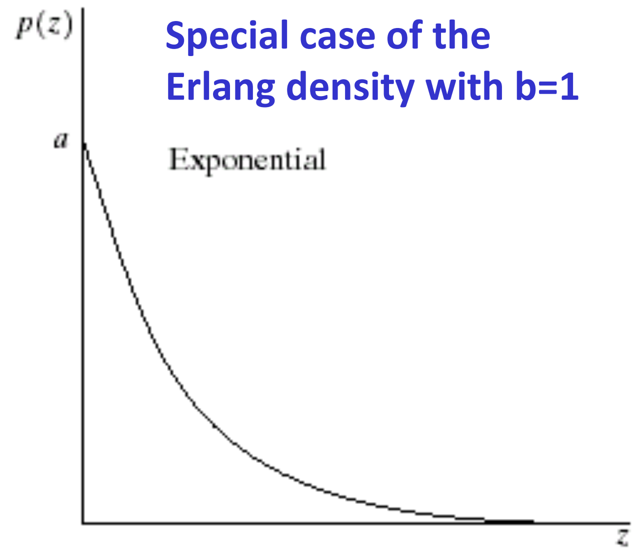
- **Gamma noise v.s. Exponential noise**

Erlang density



$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

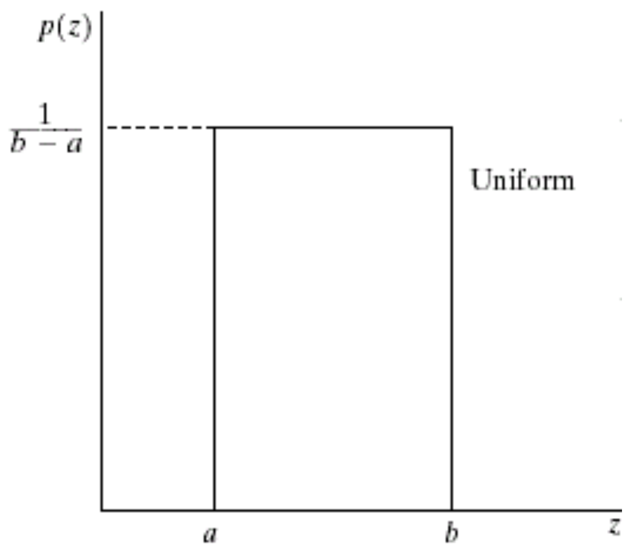
Special case of the Erlang density with b=1

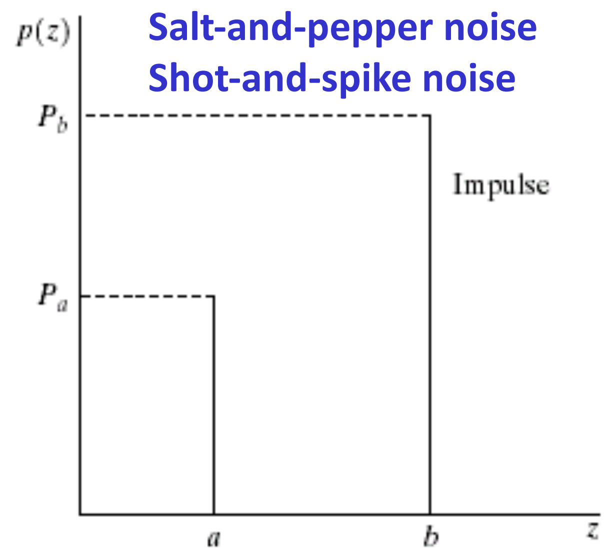


$$p(z) = \begin{cases} ae^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

Image Restoration

- **Uniform noise v.s. Impulse noise**





$$p(z) = \begin{cases} \dfrac{1}{b-a} & if\ a \le z \le b \\ 0 & otherwise \end{cases}$$

$$p(z) = \begin{cases} P_a & for\ z = a \\ P_b & for\ z = b \\ 0 & otherwise \end{cases}$$

Image Restoration

- Noise models
  - Gaussian Noise
    - Electronic noise and sensor noise due to poor illumination and/or high temperature
  - Rayleigh Noise
    - Helpful in characterizing noise phenomena in range imaging
  - Gamma (Erlang) Noise & Exponential Noise
    - Applications in laser imaging
  - Uniform Noise
    - As basis for random number generators
  - Impulse (Salt-and-Pepper) Noise
    - Quick transient (faulty switching) during imaging

Image Restoration

- Test pattern
  - Three-leveled gray-scale image

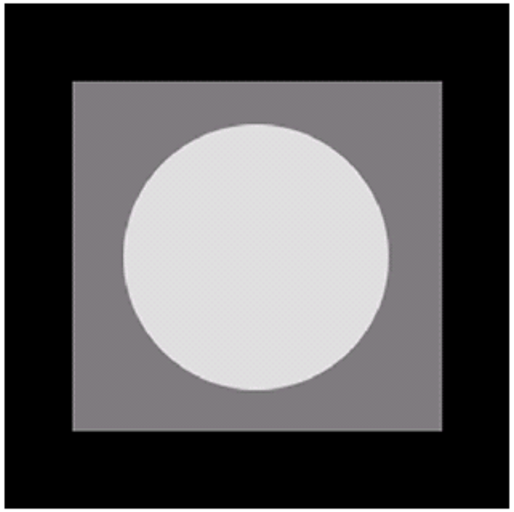


Image Restoration

- **Experimental Results**

image

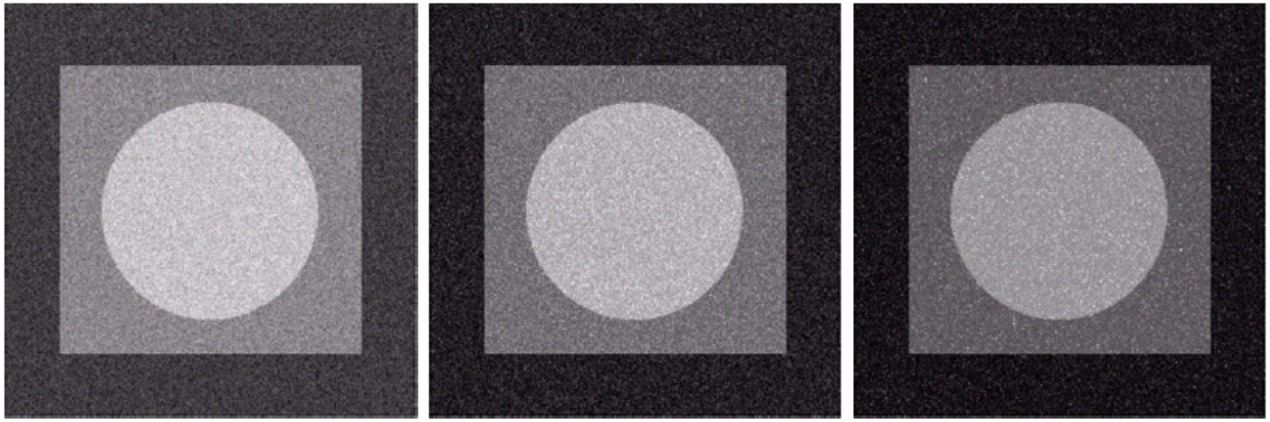


Image Restoration

- **Experimental Results**

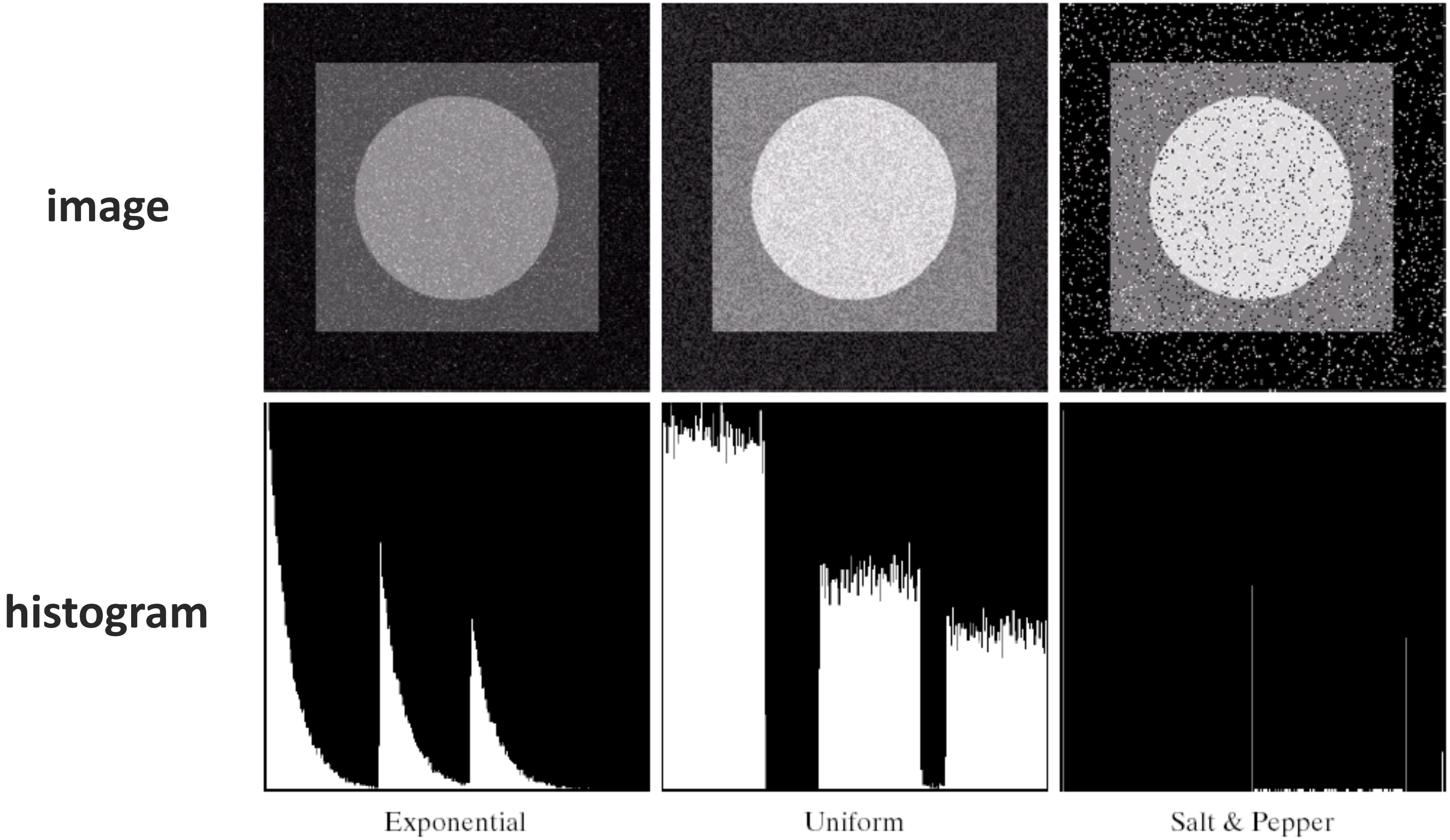
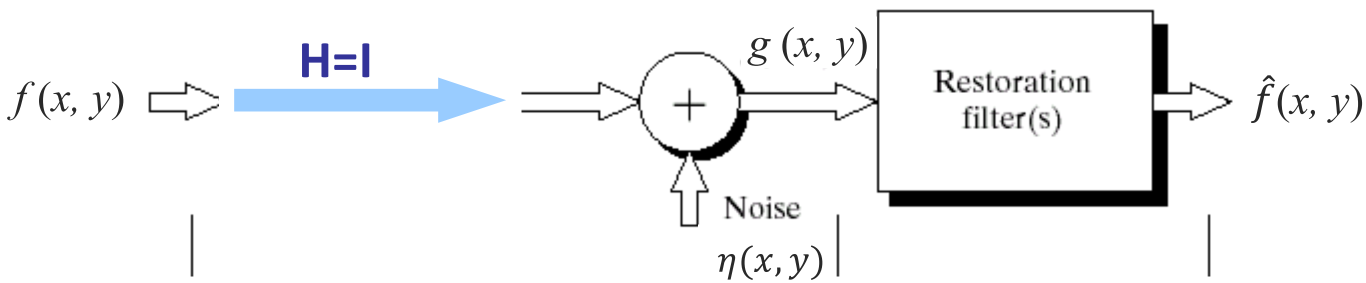


Image Restoration

- Restoration with only additive noise



$$g(x,y) = f(x,y) + \eta(x,y)$$

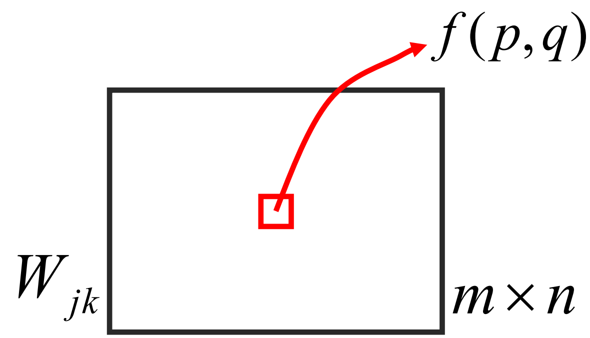
$$G(u,v) = F(u,v) + N(u,v)$$

- Spatial filtering: three solutions
  - Mean filters
  - Order-statistics filters
  - Adaptive filters

Image Restoration

- **Mean filters**
  - **Arithmetic mean filter**

$$g(j,k)=\frac{1}{mn}\sum_{(p,q)\in W_{jk}} f(p,q)$$



Smooth local variations as a result of blurring

  - **Geometric mean filter**

$$g(j,k)=\left[\prod_{(p,q)\in W_{jk}} f(p,q)\right]^{\frac{1}{mn}}$$

Tend to lose less image detail in the process

Image Restoration

- Example
  - Arithmetic mean filter v.s. Geometric mean filter

lose less image detail

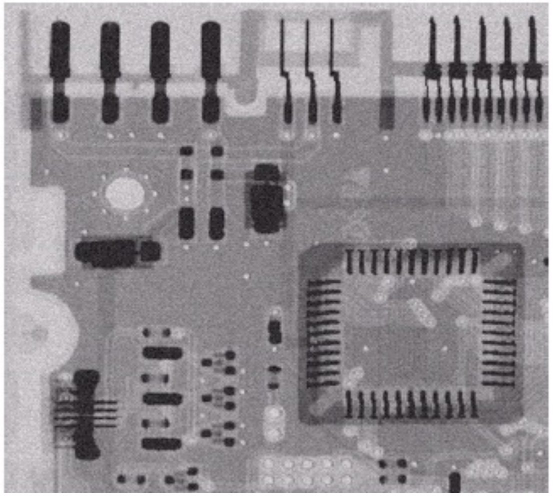
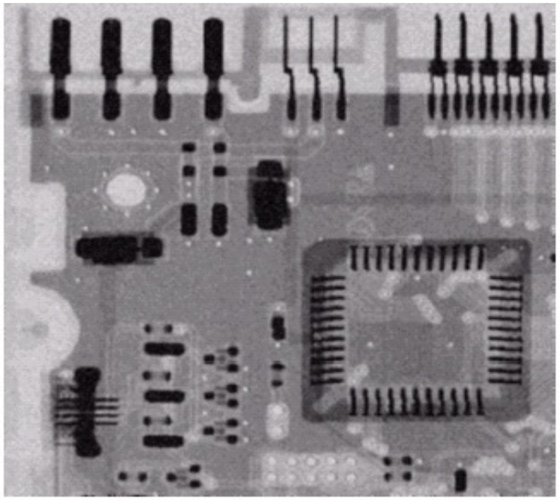
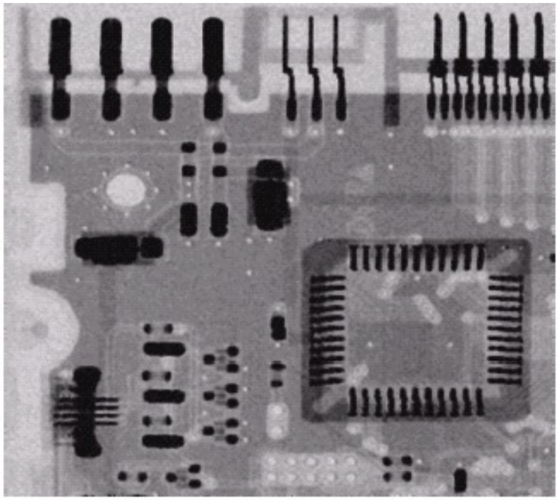


Image corrupted by
additive Gaussian noise



Arithmetic mean filter
of size 3x3



Geometric mean filter
of size 3x3

Image Restoration

- **Mean filters**
  - **Harmonic mean filter**

$$g(j,k) = \frac{mn}{\sum_{(p,q)\in W_{jk}} \frac{1}{f(p,q)}}$$

→Good for salt noise
→Gaussian noise
→Fails for pepper noise

  - **Contraharmonic mean filter**

$$g(j,k) = \frac{\sum_{(p,q)\in W_{jk}} f(p,q)^{Q+1}}{\sum_{(p,q)\in W_{jk}} f(p,q)^{Q}}$$

order

→Q>0: pepper noise
Q<0: salt noise
→Q=0? →Arithmetic
→Q=-1? →Harmonic

Image Restoration

- **Example**
  - **Contraharmonic mean filter with order=1.5 (eliminate pepper noise)**

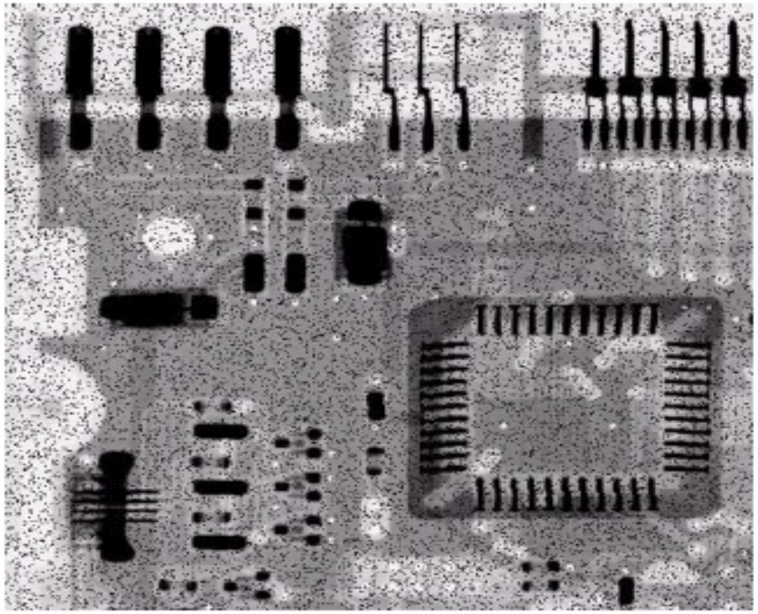
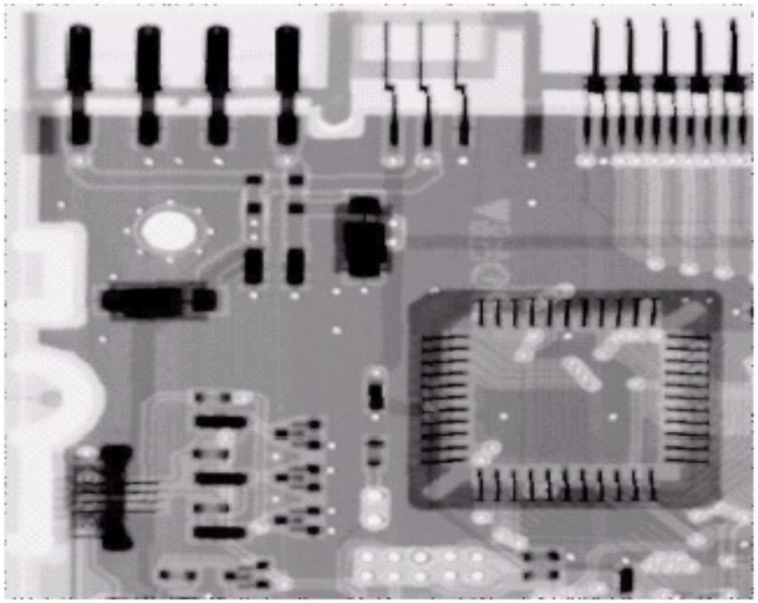


Image corrupted by pepper noise



3x3 contraharmonic filter of order 1.5

Image Restoration

- Example
  - Contraharmonic mean filter with order=-1.5 (eliminate salt noise)

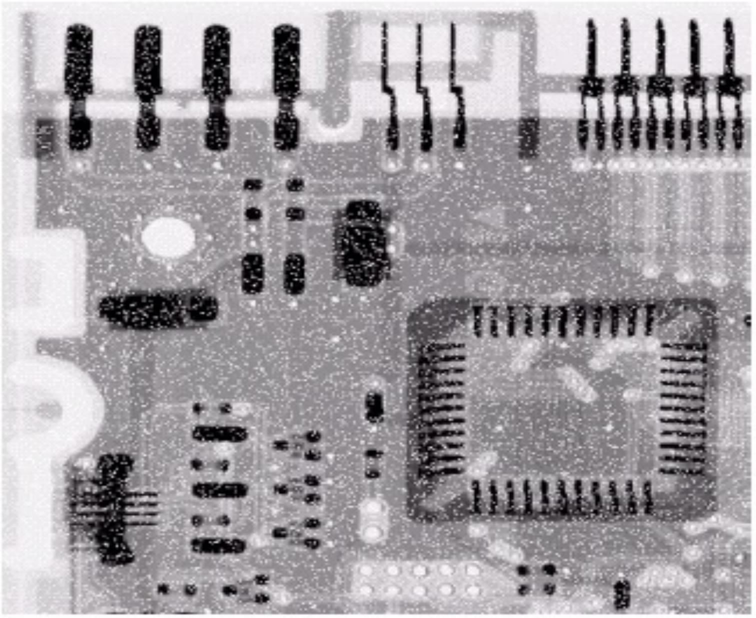
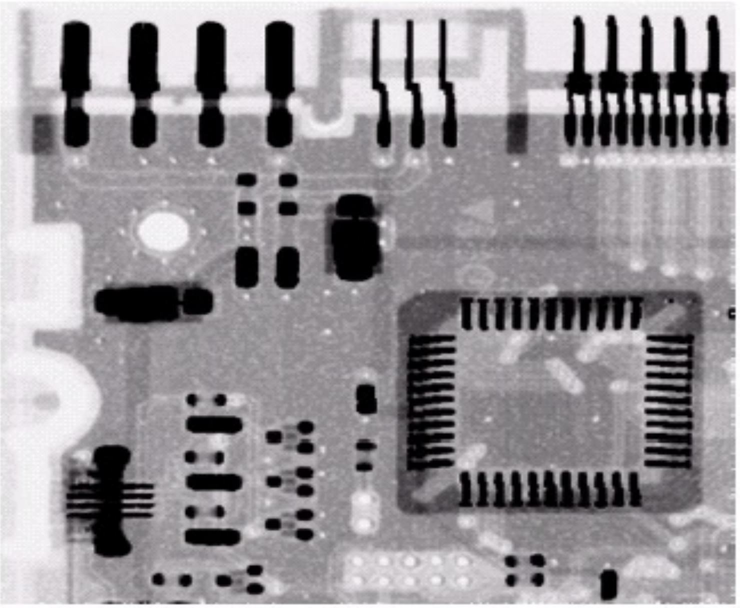


Image corrupted by salt noise

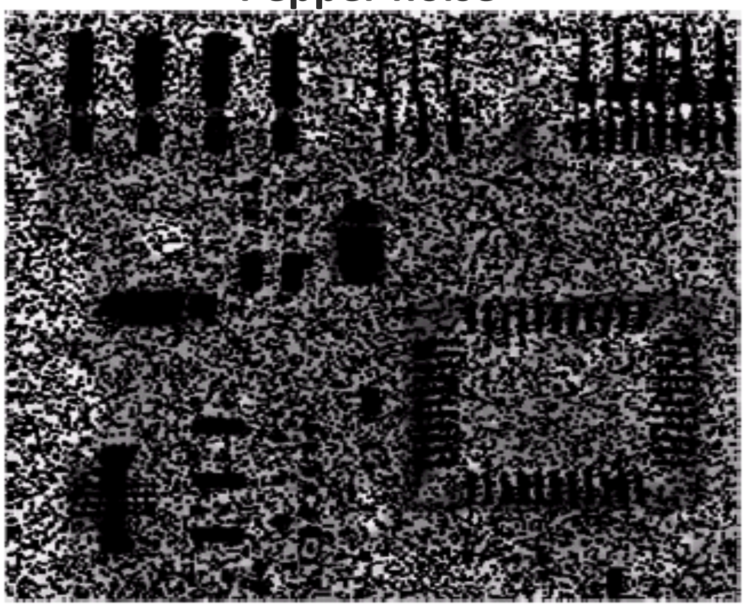


3x3 contraharmonic filter of order -1.5

Image Restoration

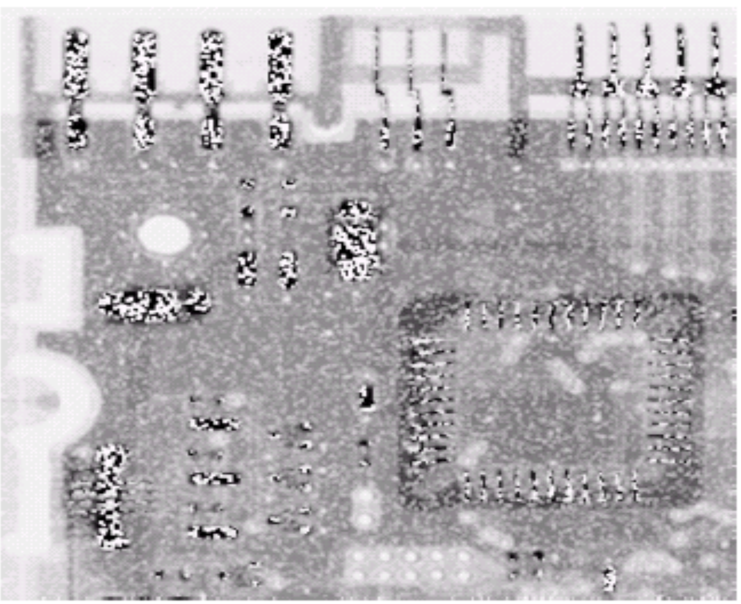
- **Contraharmonic mean filters**
  - **Choose wrong sign of the filter**

Pepper noise



3x3 contraharmonic filter of order -1.5

Salt noise



3x3 contraharmonic filter of order 1.5

Image Restoration

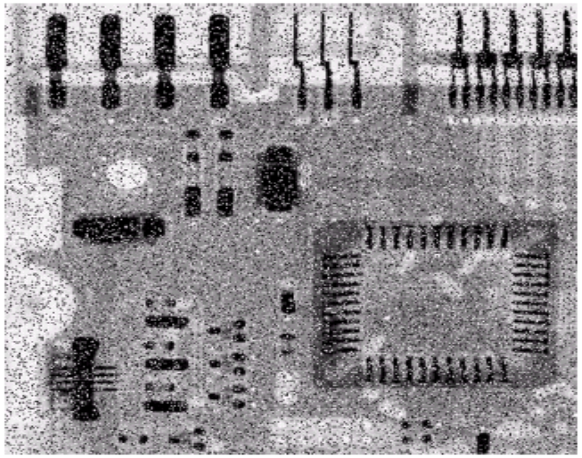
- **Order-statistics filters**
  - with response that is based on ordering the pixels contained in the image area encompassed by the filter
  - **Median filter**

$$g(j,k) = \underset{(p,q)\in W_{jk}}{median}\left\{f(p,q)\right\}$$

    - Effective in the presence of impulse noise
    - Less blurring than smoothing filters

Image Restoration

- Example
  - Median filter



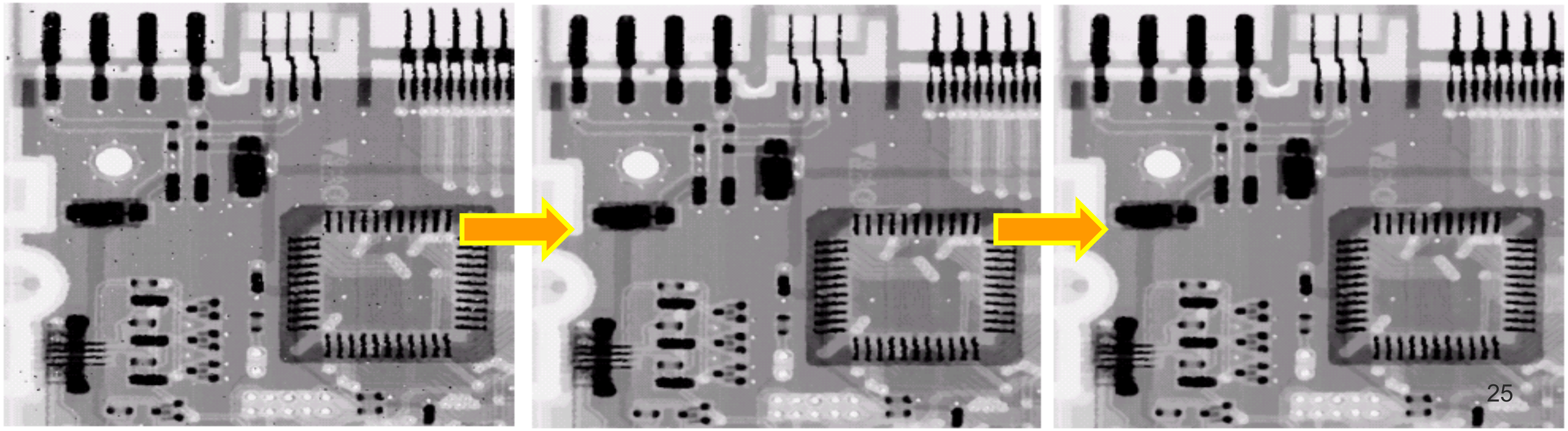


Image Restoration

- **Order-statistics filters**
  - **Max and min filters**

$$g(j,k) = \max_{(p,q)\in W_{jk}} \{f(p,q)\}$$ → **Remove pepper noise**

$$g(j,k) = \min_{(p,q)\in W_{jk}} \{f(p,q)\}$$ → **Remove salt noise**

  - **Midpoint filter**

$$g(j,k) = \frac{1}{2}\left[\max_{(p,q)\in W_{jk}} \{f(p,q)\} + \min_{(p,q)\in W_{jk}} \{f(p,q)\}\right]$$

    - **Combine order statistics and averaging**
    - **Best for randomly distributed noise, eg. Gaussian or uniform noise**

Image Restoration

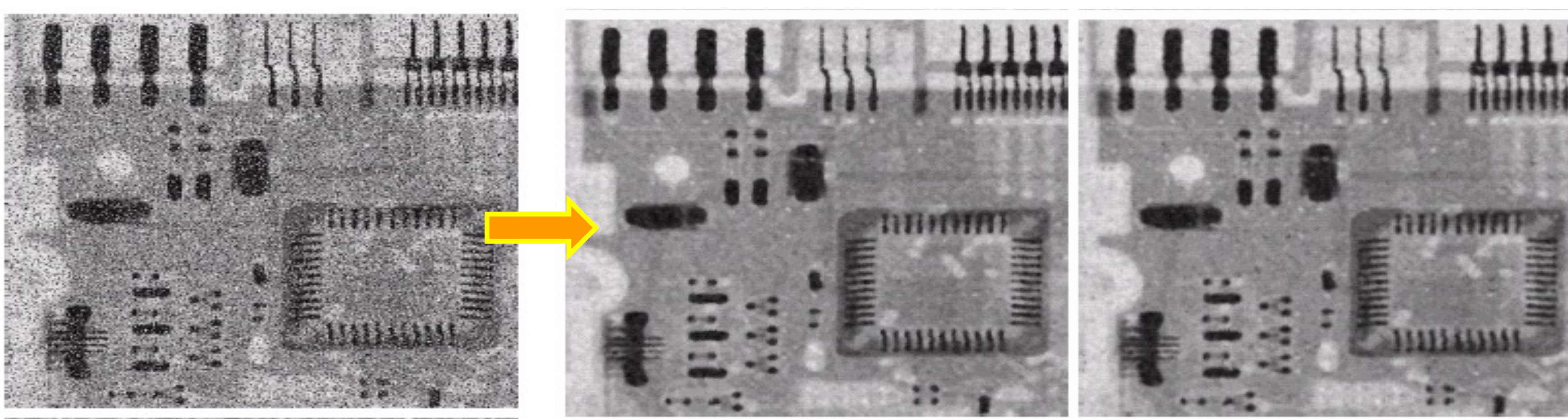
- Order-statistics filters
  - Alpha-trimmed mean filter
    - Suppose we delete d/2 lowest and d/2 highest gray-level values of $f(p,q)$ in the neighborhood W_{jk}

$$g(j,k) = \frac{1}{mn-d} \sum_{(p,q)\in W_{jk}} f_r(p,q)$$

    - where $f_r(p,q)$ represents the remaining $mn-d$ pixels
    - *d* ranges from 0 to $mn-1$
    - Useful for removing multiple types of noise

Image Restoration

- Example
  - Median and alpha-trimmed filters
    - Image corrupted by both uniform and salt-and-pepper noise



median

Alpha-trimmed

Image Restoration

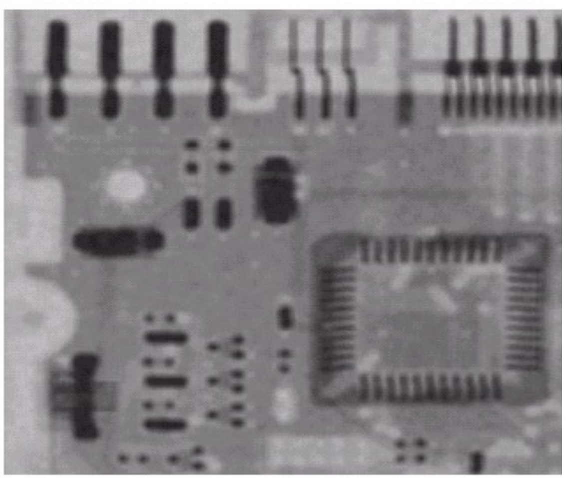
- **Adaptive filters**
  - **Adaptive, local noise reduction filter (Adaptive mean filter)**
    - σ_η^2 : noise variance
    - σ_L^2 : local variance of the pixels in S_{xy}
    - m_L : local mean of the pixels in S_{xy}
  - If $\sigma_L^2 >> \sigma_\eta^2$, return a value close to $f(j,k)$

$$g(j,k) = f(j,k) - \frac{\sigma_\eta^2}{\sigma_L^2}\left(f(j,k) - m_L\right)$$

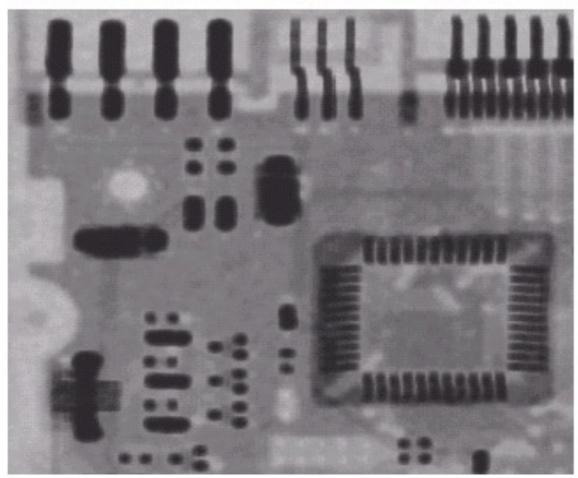
  - If $\sigma_L^2 \approx \sigma_\eta^2$, return arithmetic mean of the pixels in S_{xy}

Image Restoration

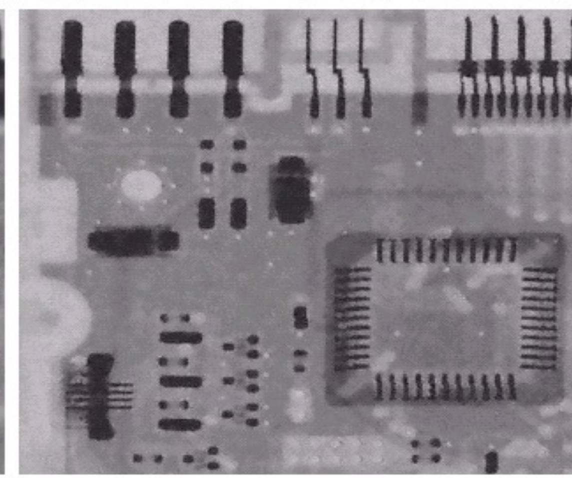
- Example
  - Adaptive noise reduction filter
    - Compared with arithmetic and geometric mean filters



arithmetic



geometric



adaptive noise reduction

Image Restoration

- **Adaptive filters**
  - **Adaptive median filter**

$z_{\min}$ = minimum gray level value in $W_{j,k}$

$z_{\max}$ = maximum gray level value in $W_{j,k}$

z_{med} = median of gray levels in $W_{j,k}$

z_{jk} = gray level at coordinates (j,k)

$W_{\max}$ = maximum allowed size of W_{jk}

$Level \quad A:$

$A_1 = z_{med} - z_{\min}$

$A_2 = z_{med} - z_{\max}$

$if \quad A_1 > 0$ and $A_2 < 0$, go to $Level \quad B$

$else$ increase the window $size$

if window size $\leq W_{\max}$, repeat Level A

$else$ output z_{jk}

$Level \quad B:$

$B_1 = z_{jk} - z_{\min}$

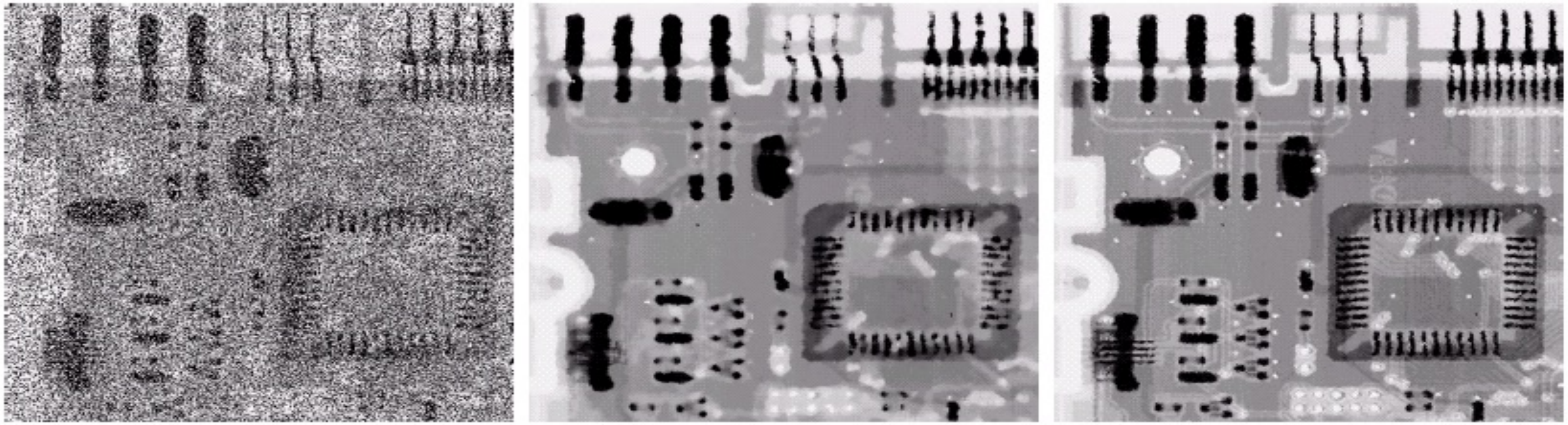
$B_2 = z_{jk} - z_{\max}$

$if \quad B_1 > 0$ and $B_2 < 0$, output z_{jk}

$else$ output z_{med}

Image Restoration

- Example
  - Adaptive median filter



salt-and-pepper　　　median　　　adaptive median

→Handle impulse noise with larger probabilities
→Preserve detail while smoothing nonimpulse noise
→Change the size of filter

Image Restoration

- **Periodic Noise Reduction by Frequency Domain Filtering**
  - **Example**

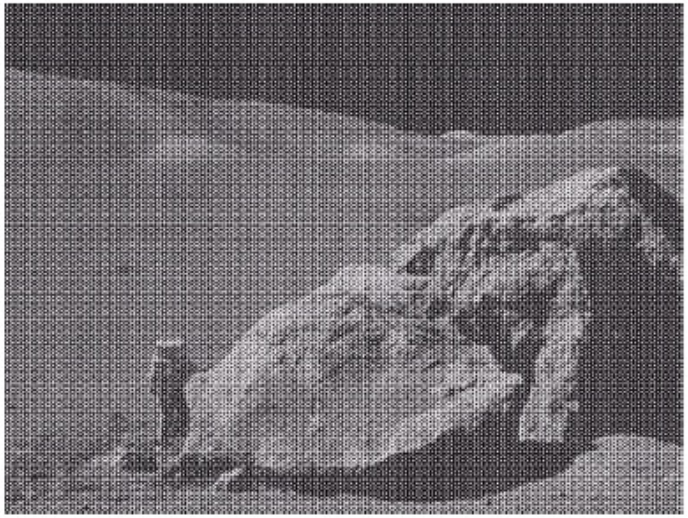
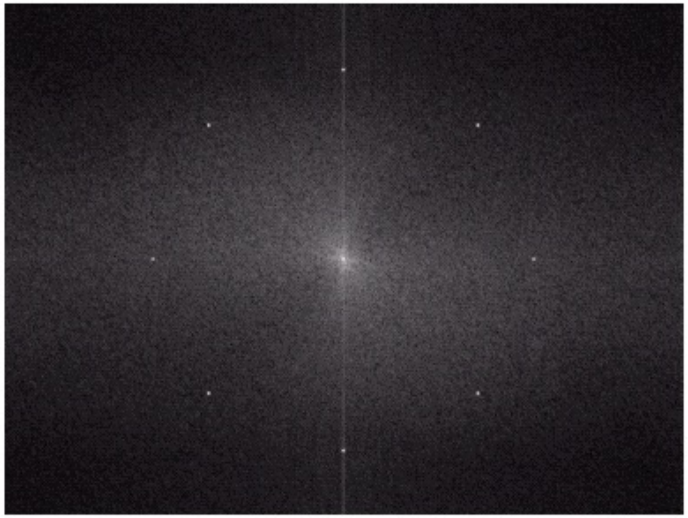


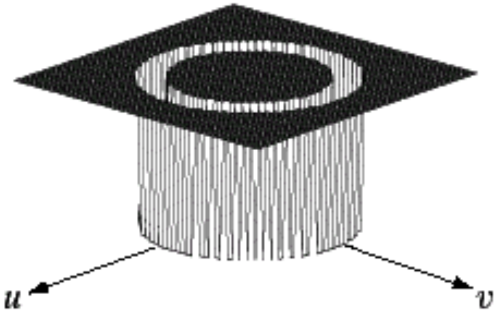
Image corrupted by sinusoidal noise



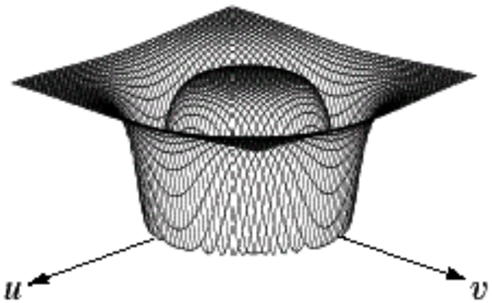
Spectrum of the corrupted image

Image Restoration

- **Periodic Noise Reduction by Frequency Domain Filtering**
  - **Band-reject Filters**



ideal

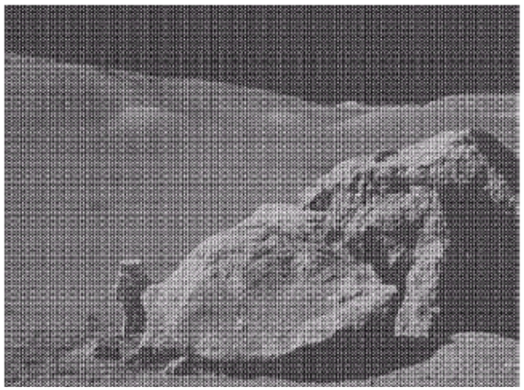


Butterworth (of order 1)

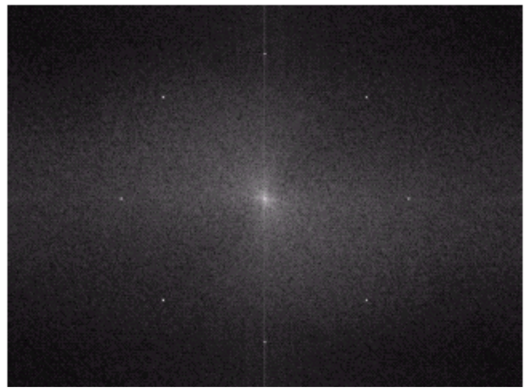
Gaussian

Image Restoration

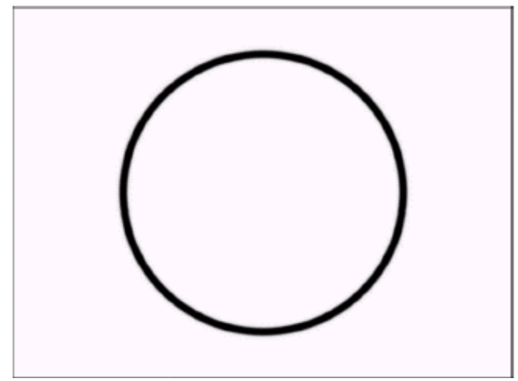
- Example



Degraded image



Spectrum



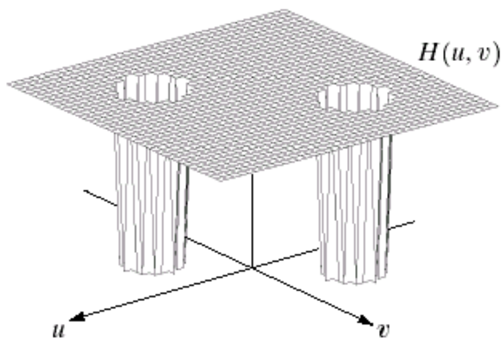
Butterworth (of order 1)

Restored image

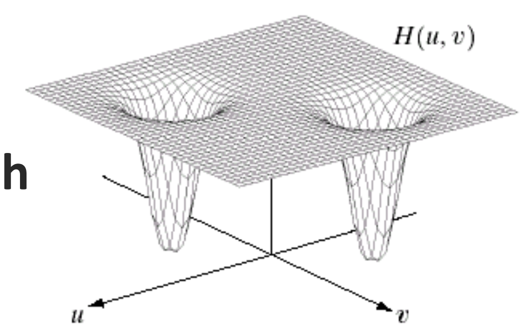
Image Restoration

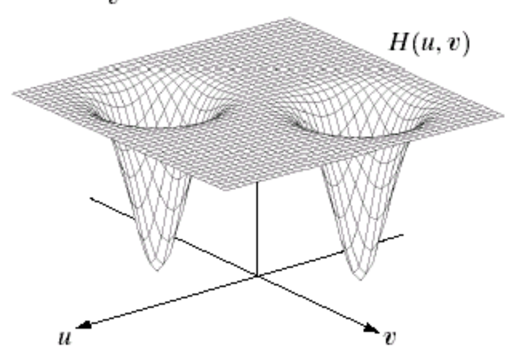
- **Periodic Noise Reduction by Frequency Domain Filtering**
  - **Notch Filters**

ideal



Butterworth

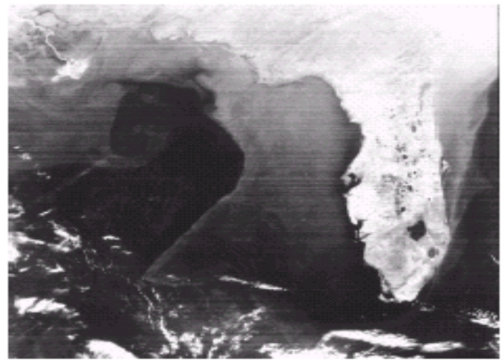




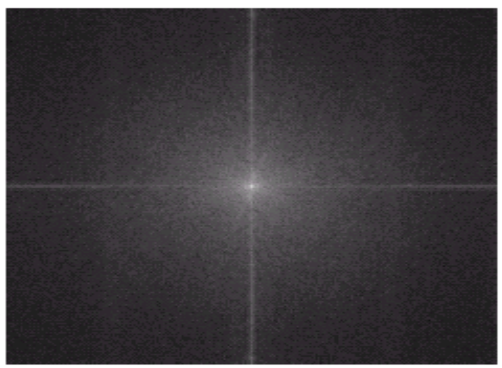
Gaussian

Image Restoration

- Example

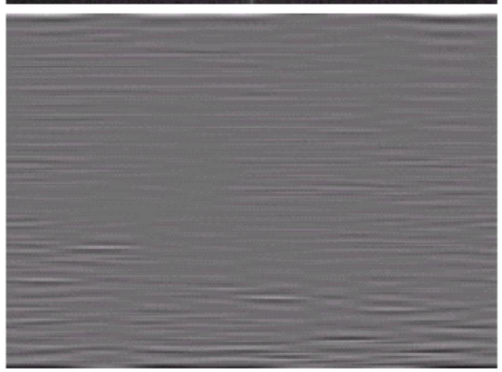


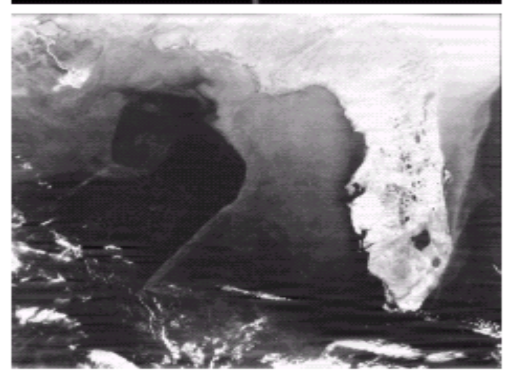
Fourier spectrum



Notch pass filter

Noise pattern



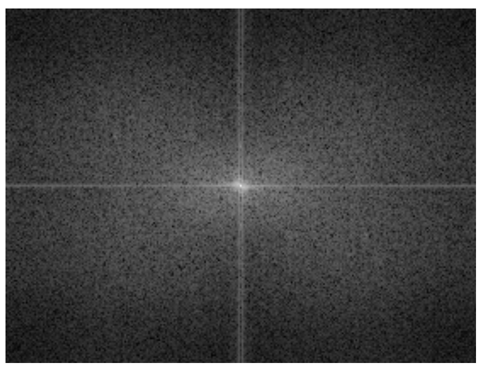


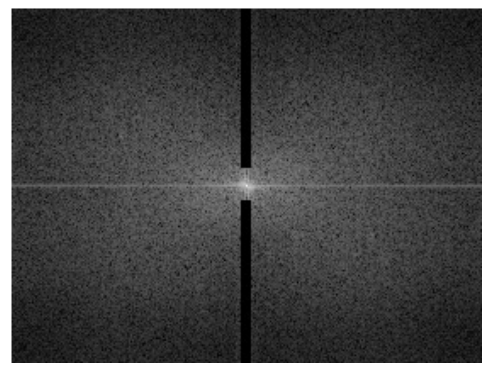
Result

Image Restoration

- Example

Fourier spectrum





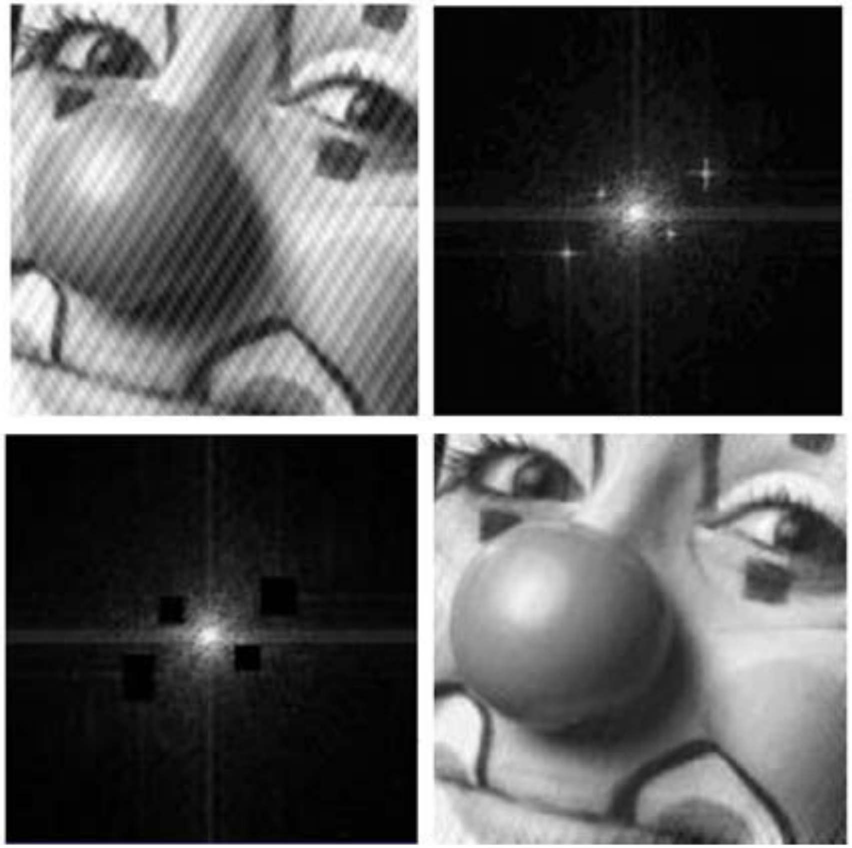
Notch reject filter

Noise pattern

Result

Image Restoration

- **Example**



Fourier spectrum

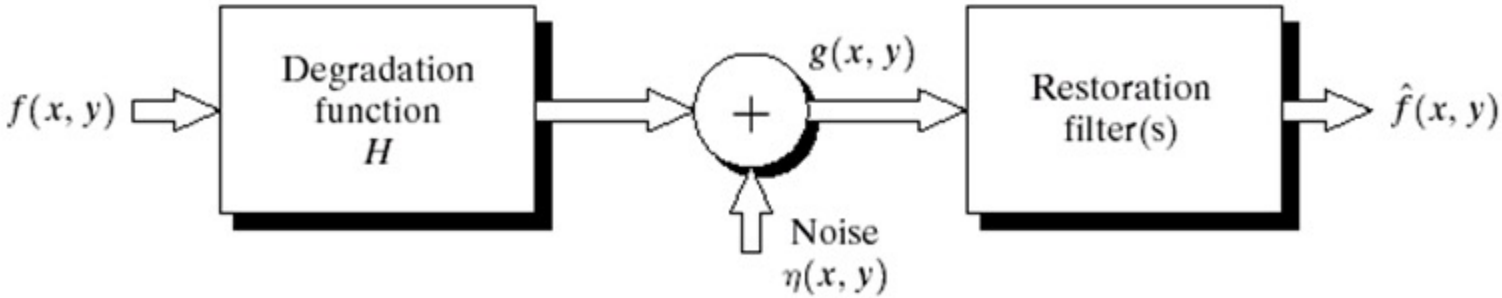
Notch filter

Result

Moallemi, Payman, and Majid Behnampourii. "Adaptive Optimum Notch Filter for Periodic Noise Reduction in Digital Images." *Amirkabir International Journal of Electrical & Electronics Engineering* 42.1 (2010): 1-7.

Image Restoration

- **Inverse Filter**



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

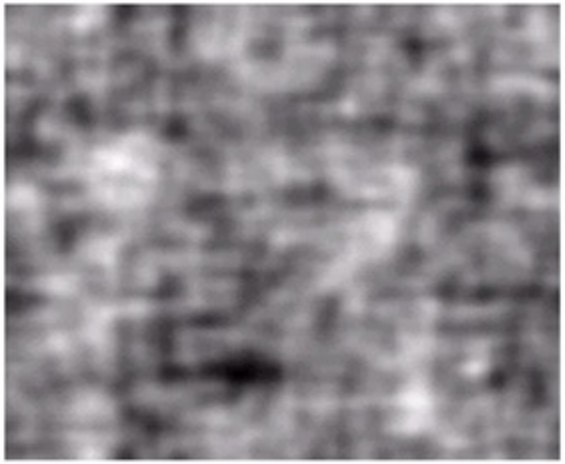
$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Image Restoration

- **Examples**

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

degraded image

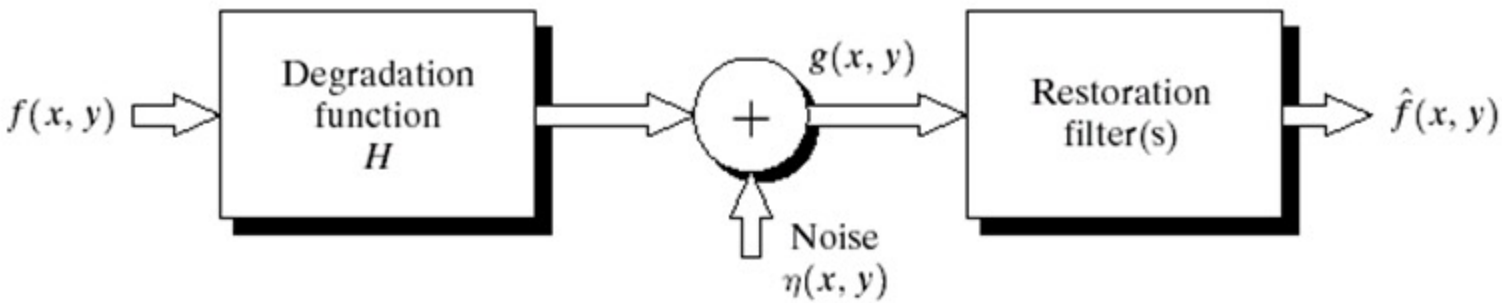


full inverse filter

limited inverse filter

Image Restoration

- **Wiener Filter**



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$
$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$
$$\hat{F}(u,v) = H_{Wiener}(u,v)\,G(u,v)$$

- **The objective is to find $H_{Wiener}(u,v)$ with minima MSE between original image and received one.**

$$\varepsilon = E[(F(u,v) - \hat{F}(u,v))^2]$$

Image Restoration

- **Wiener Filter**

$$
\begin{aligned}
\varepsilon &= E[(F-\hat{F})^2] \\
&= E[(F-H_{Wiener}G)^2] \\
&= E[(F-H_{Wiener}(HF+N))^2] \\
&= E[((1-HH_{Wiener})F-H_{Wiener}N))^2] \\
&= (1-HH_{Wiener})\overline{(1-HH_{Wiener})}E[F^2]-(1-HH_{Wiener})\overline{H_{Wiener}}E[F\overline{N}] \\
&\quad -\overline{(1-HH_{Wiener})}H_{Wiener}E[\overline{F}N]+H_{Wiener}\overline{H_{Wiener}}E[N^2]
\end{aligned}
$$

Image Restoration

- **Wiener Filter**
  - **Noise is independent to signal, and its mean is zero**

$$E[\overline{F}N] = 0 = E[F\overline{N}]$$

  - **Define power spectral densities (PSD) as follows**

$$D_F(u,v) = E[F(u,v)^2]$$

$$D_N(u,v) = E[N(u,v)^2]$$

  - **Hence**

$$\varepsilon = (1 - HH_{Wiener})\overline{(1 - HH_{Wiener})}D_F + H_{Wiener}\overline{H_{Wiener}}D_N$$

Image Restoration

- **Wiener Filter**
  - **To minimize the error, let the derivative to be zero**

$$\frac{d\varepsilon}{dH_{Wiener}} = 0 = -H\overline{(1 - HH_{Wiener})}D_F + \overline{H_{Wiener}}D_N$$

  - **Then**

$$H_{Wiener}(u,v) = \frac{\overline{H}(u,v)D_F(u,v)}{\left|H(u,v)\right|^2 D_F(u,v) + D_N(u,v)}$$

Image Restoration

- Examples

Original image

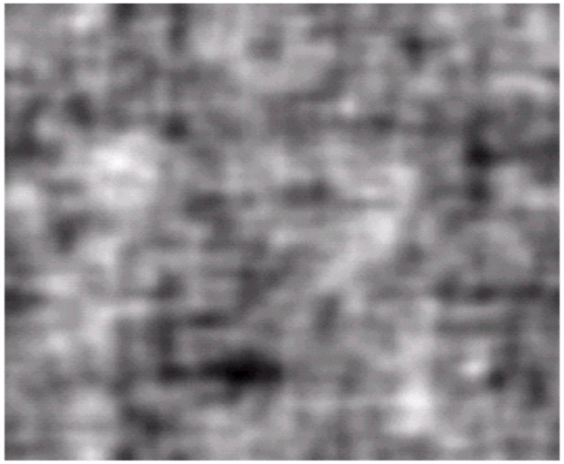
Add motion blur and Gaussian noise

Restored by Wiener filter

Image Restoration

- **Other examples**

degraded image



full inverse filter

limited inverse filter

Wiener filter

Image Restoration

- **Other examples**

with motion blur & additive noise

inverse filtering

Wiener filter