Digital Image Processing

Lecture #4

Ming-Sui (Amy) Lee

Morphological Image Processing

Morphological Processing

- Morphology
  - Morpho-: shape/form/structure
  - -ology: study
- Morphological image processing
  - Post-processing
  - Binary images → gray-level image

Morphological Processing

- For some applications
  - Structures of objects composed by lines or arcs
  - Care about the pattern connectivity
  - Independent of width

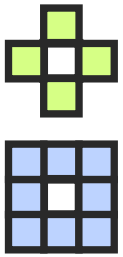
A A *A*

Hand-written characters

Fingerprint patterns

Morphological Processing

- Binary image connectivity
  - Pixel bond
    - Specify the connectivity of a pixel with its neighbors
    - Four-connected neighbor → bond = 2
    - Eight-connected neighbor → bond = 1



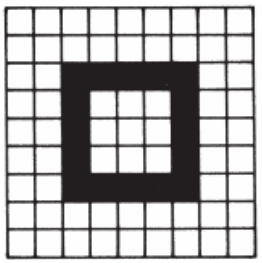
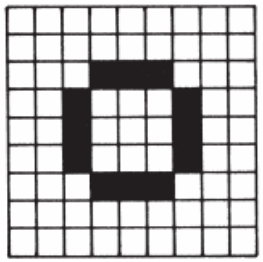
  - Minimally connected
    - Elimination of any black (object) pixel (except boundary pixels) results in disconnection of the remaining black (object) pixels

Morphological Processing

- **Binary image connectivity**
  - **Example**

0 0 0 0 1 1 0 1 0 Four-connected $B = 4$	0 0 0 0 1 1 0 0 1 Eight-connected $B = 3$	0 0 0 0 1 0 0 0 0 Isolated $B = 0$	0 0 0 0 1 1 0 1 1 Corner $B = 5$	0 1 1 1 1 1 1 1 1 Interior $B = 11$
0 0 0 0 1 0 0 0 1 Spur $B = 1$	1 0 0 1 1 1 1 0 1 Bridge $B = 7$	1 1 1 0 1 0 1 1 1 *H*-connected $B = 8$	0 1 1 0 1 1 0 1 1 Exterior $B = 8$	

  - **Another example**

Morphological Processing

- Binary hit or miss transformations
  - Select a nxn hit pattern (odd-sized mask)
  - Compare with a nxn image window
    - Match → hit → change the central pixel value
    - Otherwise → miss → do nothing
  - Example
    - To clean the isolated binary noise

0	0	0
0	1	0
0	0	0

Hit or miss?

Morphological Processing

- **Binary hit or miss transformations**
  - 0 → background
  - 1 → object (black)

0	0	0
0	1	0
0	0	0

Hit or miss?

  - **Logical expression**

$$\begin{bmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{bmatrix}$$

$$G(j,k) = X \cap (X_0 \cup X_1 \cup \cdots \cup X_7)$$

    - **Example**
      - If $G(j,k) = X \cap 1$ → do nothing
      - If $G(j,k) = X \cap 0$
        - If X=0 → G(j,k)=0 → do nothing
        - If X=1 → hit → G(j,k)=0

Morphological Processing

- **Binary hit or miss transformations**

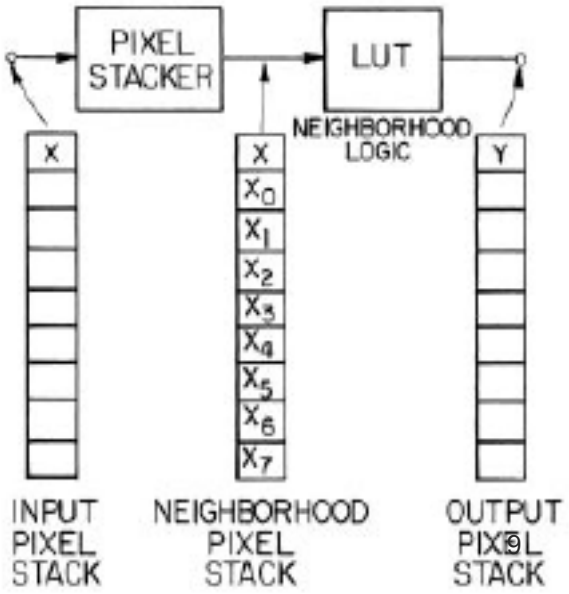
$$G(j,k) = X \cap (X_0 \cup X_1 \cup \cdots \cup X_7)$$

$$\Rightarrow 2^9 \text{ possible mask patterns}$$

  - **Implementation**
    - **Pixel stack**
      - **Treat the 8 neighboring pixels as a "byte"**

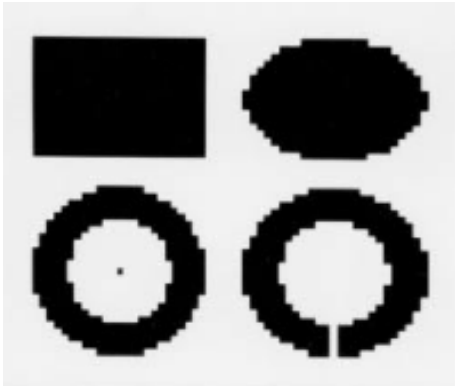
$$\begin{bmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{bmatrix} \otimes \begin{bmatrix} 2^{-4} & 2^{-3} & 2^{-2} \\ 2^{-5} & 2^{0} & 2^{-1} \\ 2^{-6} & 2^{-7} & 2^{-8} \end{bmatrix}$$

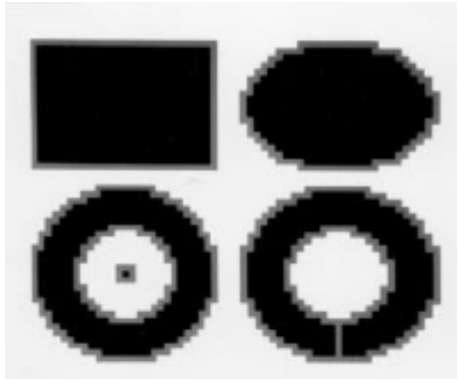
    - **Look-Up-Table (LUT)**

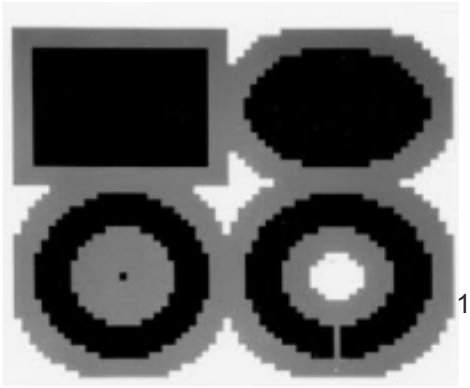


Morphological Processing

- Simple morphological processing based on binary hit or miss rules
  - Additive operators (0→1)
    - Interior fill
    - Diagonal fill
    - Bridge
    - 8-neighbor dilate

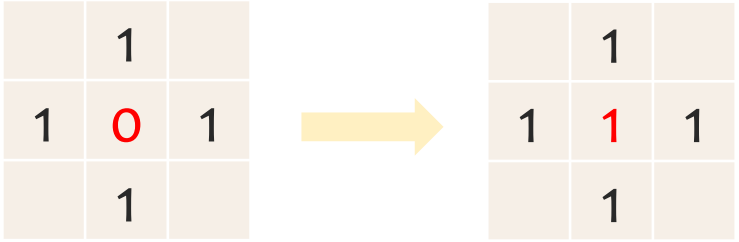






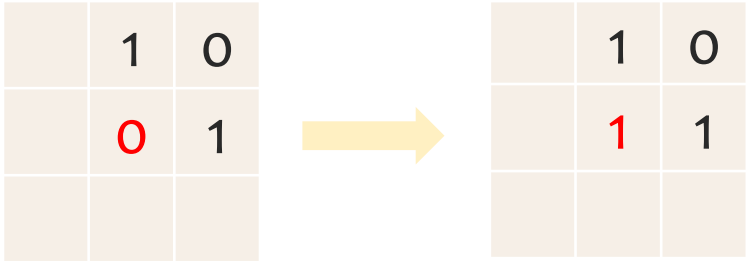
Morphological Processing

- **Interior fill**
  - Create a black pixel if all four-connected neighbor pixels are black



0 → background
1 → object (black)

- **Diagonal fill**
  - Create a black pixel if creation eliminates the eight-connectivity of the background



Morphological Processing

- **Bridge**
  - Create a black pixel if creation results in connectivity of previously unconnected neighboring black pixels

1	0	0
0	0	1
0	0	0

→

1	0	0
0	1	1
0	0	0

0 → background
1 → object (black)

- **8-neighbor dilate**
  - Create a black pixel if at least one eight-connected neighbor pixel is black

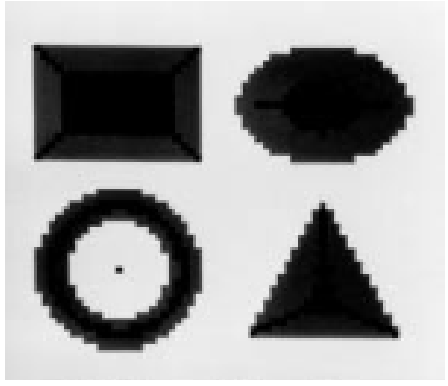
0	0	0
0	0	0
1	0	0

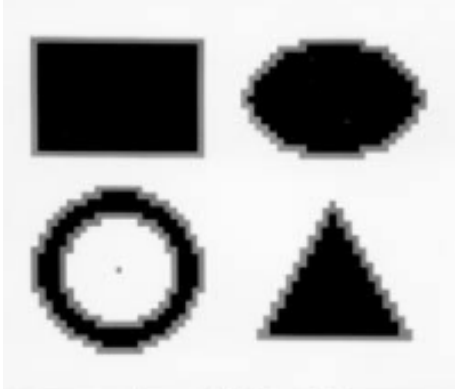
→

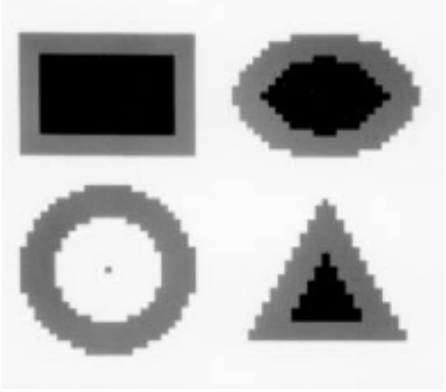
0	0	0
0	1	0
1	0	0

Morphological Processing

- Simple morphological processing based on binary hit or miss rules
  - Subtractive operators (1→0)
    - Isolated pixel removal
    - Spur removal
    - Interior pixel removal
    - H-break / Eight-neighbor erode







Morphological Processing

- **Isolated pixel removal**
  - Erase a black pixel with eight white neighbors

0	0	0
0	1	0
0	0	0

0	0	0
0	0	0
0	0	0

0 → background
1 → object (black)

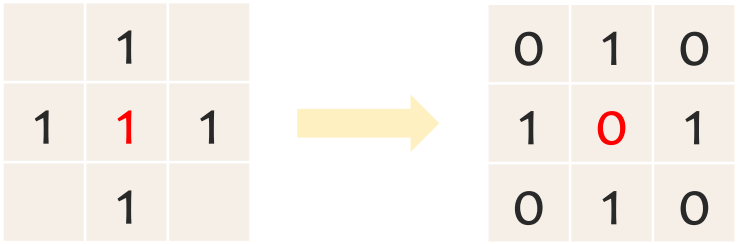
- **Spur removal**
  - Erase a black pixel with a single eight-connected neighbor

1	0	0
0	1	0
0	0	0

1	0	0
0	0	0
0	0	0

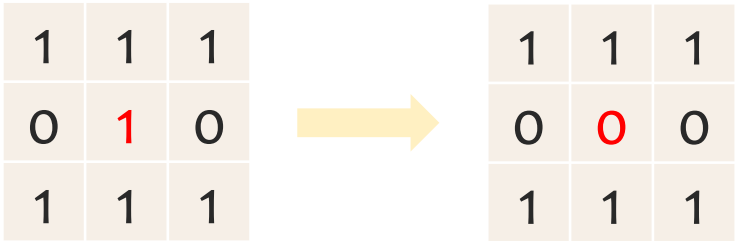
Morphological Processing

- **Interior pixel removal**
  - Erase a black pixel if all four-connected neighbors are black



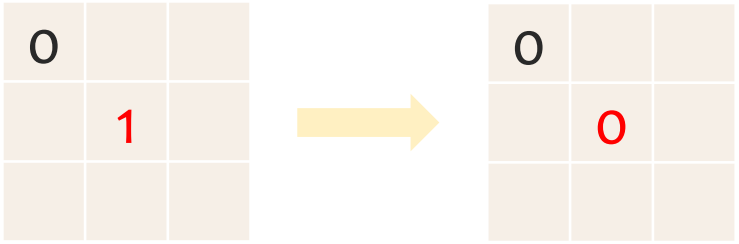
0 → background
1 → object (black)

- **H-break**
  - Erase a black pixel that is H-connected



Morphological Processing

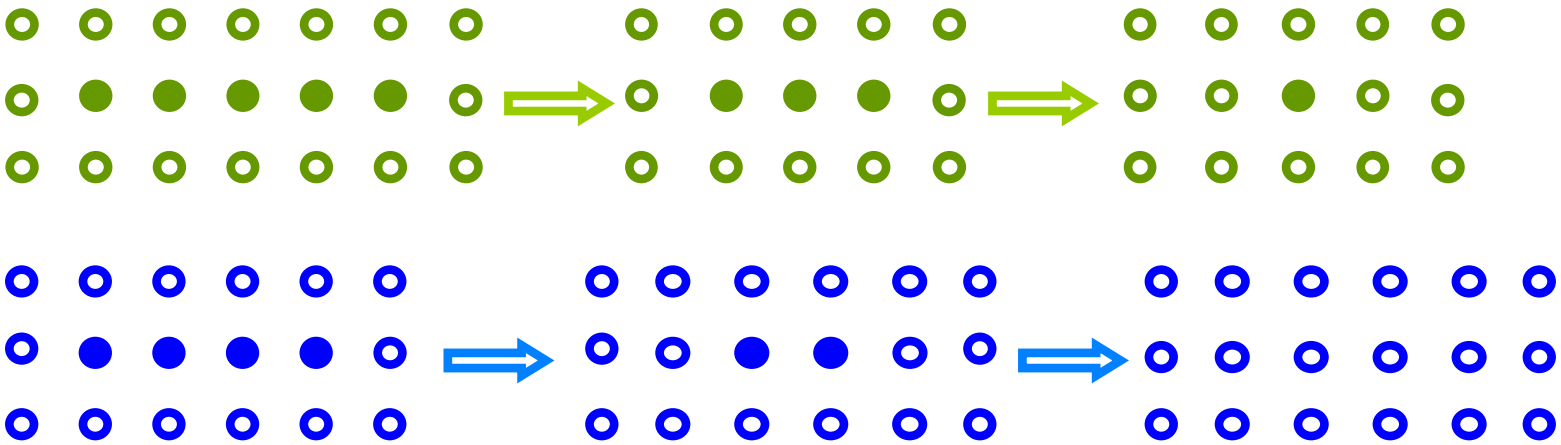
- **Eight-neighbor erode**
  - Erase a black pixel if at least one eight-connected neighbor pixel is white



0 → background
1 → object (black)

Morphological Processing

- Example
  - Subtractive operator



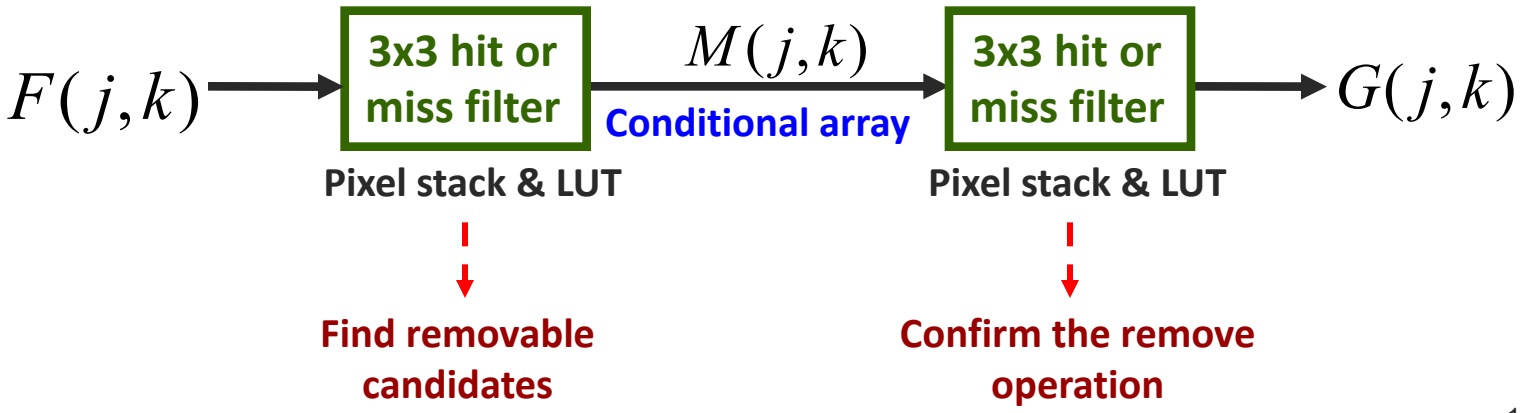
- doesn't prevent total erasure and ensure connectivity
- In this case, only a 3x3 window does not sufficient to tell whether the final stage of iteration is reached or not

Morphological Processing

- Solutions
  - Approach I
    - Apply a filter with larger size
      - “fairly complicated patterns”, “many combinations”
  - Approach II
    - Consider a structural (composite) design with 3x3 filters: two-stage approach
      - Application dependent
      - Thinning, shrinking, skeletonizing
      - Share the same structure but vary in some modular details

Morphological Processing

- **Advanced morphological processing**
  - **Shrinking/Thinning/Skeletonizing**
    - **Conditional erosion**
    - **Prevent total erasure & Ensure connectivity**

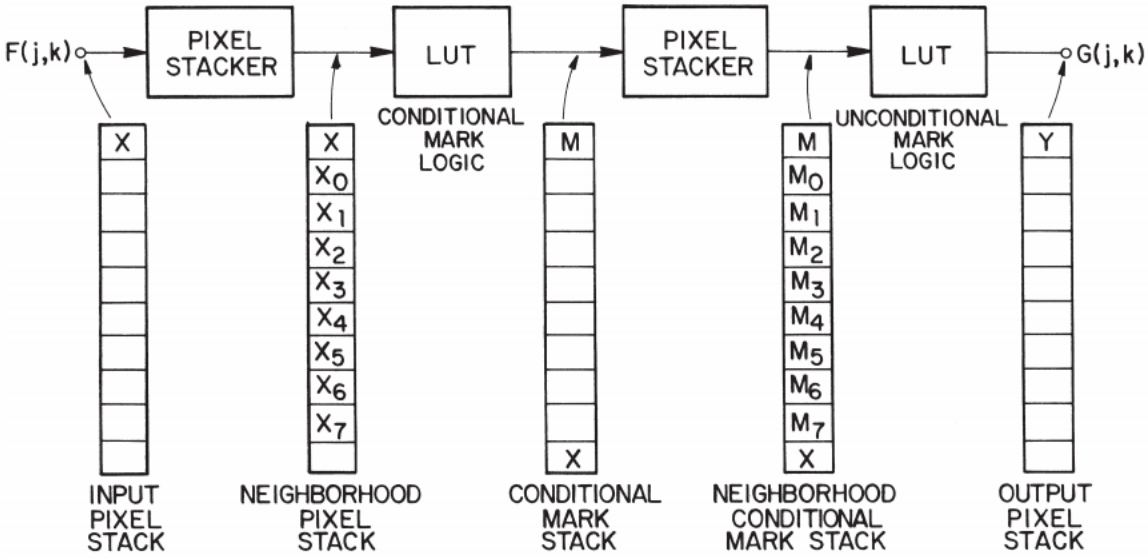


Morphological Processing

- **Advanced morphological processing**
  - **Shrinking/Thinning/Skeletonizing**
    - **Conditional erosion**
    - **Prevent total erasure & Ensure connectivity**

$$\begin{bmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{bmatrix}$$

$$\begin{bmatrix} M_3 & M_2 & M_1 \\ M_4 & M & M_0 \\ M_5 & M_6 & M_7 \end{bmatrix}$$



Morphological Processing

- **Shrinking/Thinning/Skeletonizing**
  - **Stage I**
    - **Generate a binary image M(j,k) called the conditional array (or mask)**
      - **If M(j,k)=1, it means (j,k) is a candidate for erasure**
      - **If M(j,k)=0, it means no further operation is needed on (j,k)**
  - **Stage II**
    - **Based on the center pixel, X, and M(j,k) pattern, we decide whether to erase X or not in the output G(j,k)**
      - **If there's a hit → do nothing**
      - **If there's a miss → erase the center pixel**

Morphological Processing

- **Stage I → Part of Table 14.3-1**

TABLE 14.3-1. Shrink, Thin and Skeletonize Conditional Mark Patterns [M = 1 if hit]

Table	Bond	Pattern							
		0 0 1	1 0 0	0 0 0	0 0 0				
S	1	0 1 0	0 1 0	0 1 0	0 1 0				
		0 0 0	0 0 0	1 0 0	0 0 1				
		0 0 0	0 1 0	0 0 0	0 0 0				
S	2	0 1 1	0 1 0	1 1 0	0 1 0				
		0 0 0	0 0 0	0 0 0	0 1 0				
		0 0 1	0 1 1	1 1 0	1 0 0	0 0 0	0 0 0	0 0 0	0 0 0
S	3	0 1 1	0 1 0	0 1 0	1 1 0	1 1 0	0 1 0	0 1 0	0 1 1
		0 0 0	0 0 0	0 0 0	0 0 0	1 0 0	1 1 0	0 1 1	0 0 1
		0 1 0	0 1 0	0 0 0	0 0 0				
TK	4	0 1 1	1 1 0	1 1 0	0 1 1				
		0 0 0	0 0 0	0 1 0	0 1 0				
		0 0 1	1 1 1	1 0 0	0 0 0				
STK	4	0 1 1	0 1 0	1 1 0	0 1 0				
		0 0 1	0 0 0	1 0 0	1 1 1				

Table: Shrink (S), Thin (T), Skeletonize (K)
Bond: classification, narrow down the search space
Pattern: coded as an 8-bit symbol for a filter

$$\begin{bmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{bmatrix} \otimes \begin{bmatrix} 2^{-4} & 2^{-3} & 2^{-2} \\ 2^{-5} & 2^{0} & 2^{-1} \\ 2^{-6} & 2^{-7} & 2^{-8} \end{bmatrix}$$

Morphological Processing

- **Stage II → Part of Table 14.3-2**

TABLE 14.3-2. Shrink and Thin Unconditional Mark Patterns
[$P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit] [a]

Pattern			
Spur		Single 4-connection	
0 0 *M*	*M*0 0	0 0 0	0 0 0
0 *M*0	0 *M*0	0 *M*0	0 *MM*
0 0 0	0 0 0	0 *M*0	0 0 0

$$G(j,k) = X \cap [\overline{M} \cup P(M, M_1, \ldots, M_7)]$$

where $P(M, M_1, \ldots, M_7)$ is an erasure inhibiting logical variable

L Cluster

0 0 *M*	0 *MM*	*MM*0	*M*0 0	0 0 0	0 0 0	0 0 0	0 0 0
0 *MM*	0 *M*0	0 *M*0	*MM*0	*MM*0	0 *M*0	0 *M*0	0 *MM*
0 0 0	0 0 0	0 0 0	0 0 0	*M*0 0	*MM*0	0 *MM*	0 0 *M*

4-Connected offset

0 *MM*	*MM*0	0 *M*0	0 0 *M*
*MM*0	0 *MM*	0 *MM*	0 *MM*
0 0 0	0 0 0	0 0 *M*	0 *M*0

$$\begin{bmatrix} M_3 & M_2 & M_1 \\ M_4 & M & M_0 \\ M_5 & M_6 & M_7 \end{bmatrix} \otimes \begin{bmatrix} 2^{-4} & 2^{-3} & 2^{-2} \\ 2^{-5} & 2^{0} & 2^{-1} \\ 2^{-6} & 2^{-7} & 2^{-8} \end{bmatrix}$$

Morphological Processing

- **Stage II → Part of Table 14.3-2 (cont'd)**

Spur corner cluster

$$\begin{matrix} 0\ A\ M \\ 0\ MB \\ M0\ 0 \end{matrix} \quad \begin{matrix} MB\ 0 \\ A\ M0 \\ 0\ 0\ M \end{matrix} \quad \begin{matrix} 0\ 0\ M \\ A\ M0 \\ MB\ 0 \end{matrix} \quad \begin{matrix} M0\ 0 \\ 0\ MB \\ 0\ A\ M \end{matrix}$$

Corner cluster

$$\begin{matrix} M\,M\,D \\ M\,M\,D \\ D\,D\,D \end{matrix}$$

Tee branch

$$\begin{matrix} D\,M0 \\ M\,M\,M \\ D\,0\ 0 \end{matrix} \quad \begin{matrix} 0\ M\,D \\ M\,M\,M \\ 0\ 0\ D \end{matrix} \quad \begin{matrix} 0\ 0\ D \\ M\,M\,M \\ 0\ M\,D \end{matrix} \quad \begin{matrix} D\,0\ 0 \\ M\,M\,M \\ D\,M0 \end{matrix} \quad \begin{matrix} D\,M\,D \\ M\,M0 \\ 0\ M0 \end{matrix} \quad \begin{matrix} 0\ M0 \\ M\,M0 \\ D\,M\,D \end{matrix} \quad \begin{matrix} 0\ M0 \\ 0\ M\,M \\ D\,M\,D \end{matrix} \quad \begin{matrix} D\,M\,D \\ 0\ M\,M \\ 0\ M0 \end{matrix}$$

$$A \cup B \cup C = 1, \quad D = 0 \cup 1, \quad A \cup B = 1$$

Morphological Processing

- **Stage II → Part of Table 14.3-3**

TABLE 14.3-3. Skeletonize Unconditional Mark Patterns
[$P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit][a]

$$A \cup B \cup C = 1, \quad D = 0 \cup 1$$

Pattern											
Spur											
0	0	0	0	0	0	0	0	M	M	0	0
0	M	0	0	M	0	0	M	0	0	M	0
0	0	M	M	0	0	0	0	0	0	0	0
Single 4-connection											
0	0	0	0	0	0	0	0	0	0	M	0
0	M	0	0	M	M	M	M	0	0	M	0
0	M	0	0	0	0	0	0	0	0	0	0
L corner											
0	M	0	0	M	0	0	0	0	0	0	0
0	M	M	M	M	0	0	M	M	M	M	0
0	0	0	0	0	0	0	M	0	0	M	0

Morphological Processing

- **Example - shrinking**

0	0	0	0	0	0
0	0	1	1	0	0
0	0	0	0	0	0

$F(j,k)$

0	0	0	0	0	0
0	0	M	M	0	0
0	0	0	0	0	0

$M(j,k)$

0	0	0	0	0	0
0	0	P	0	0	0
0	0	0	0	0	0

$P(j,k)$

0	0	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

$G(j,k)$

Morphological Processing

- **Example - shrinking**

0	0	0	0
0	1	1	0
0	1	1	0
0	0	0	0

$F(j,k)$

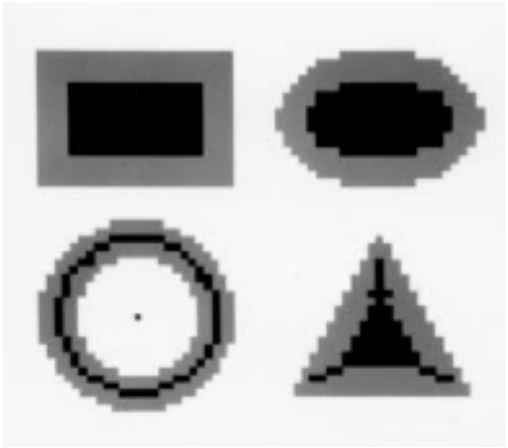
$M(j,k)$

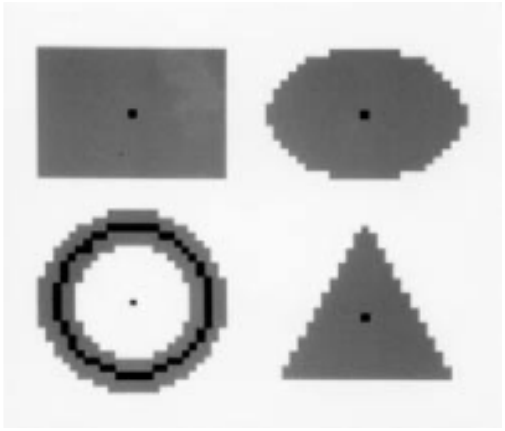
$P(j,k)$

$G(j,k)$

Morphological Processing

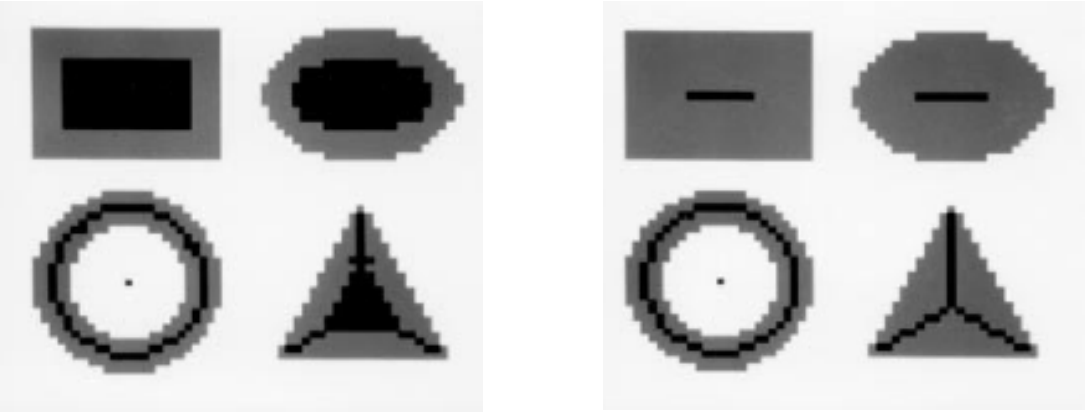
- **Shrinking**
  - Erase black pixels such that an object without holes erodes to a single pixel at or near its center of mass, and an object with holes erodes to a connected ring lying midway between each hole and its nearest outer boundary





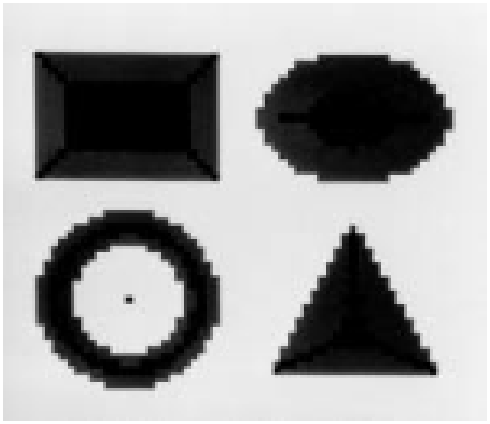
Morphological Processing

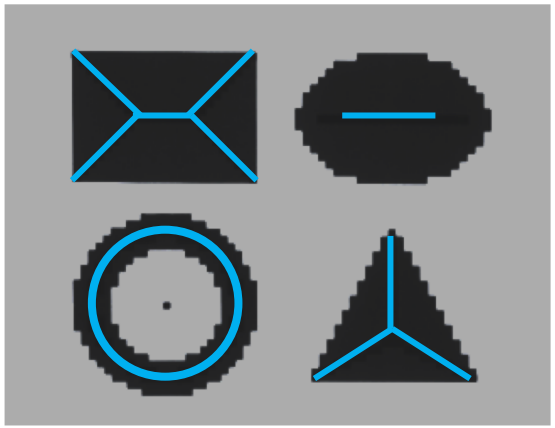
- **Thinning**
  - Erase black pixels such that an object without holes erodes to a minimally connected stroke located equidistant from its nearest outer boundaries, and an object with holes erodes to a minimally connected ring midway between each hole and its nearest outer boundary



Morphological Processing

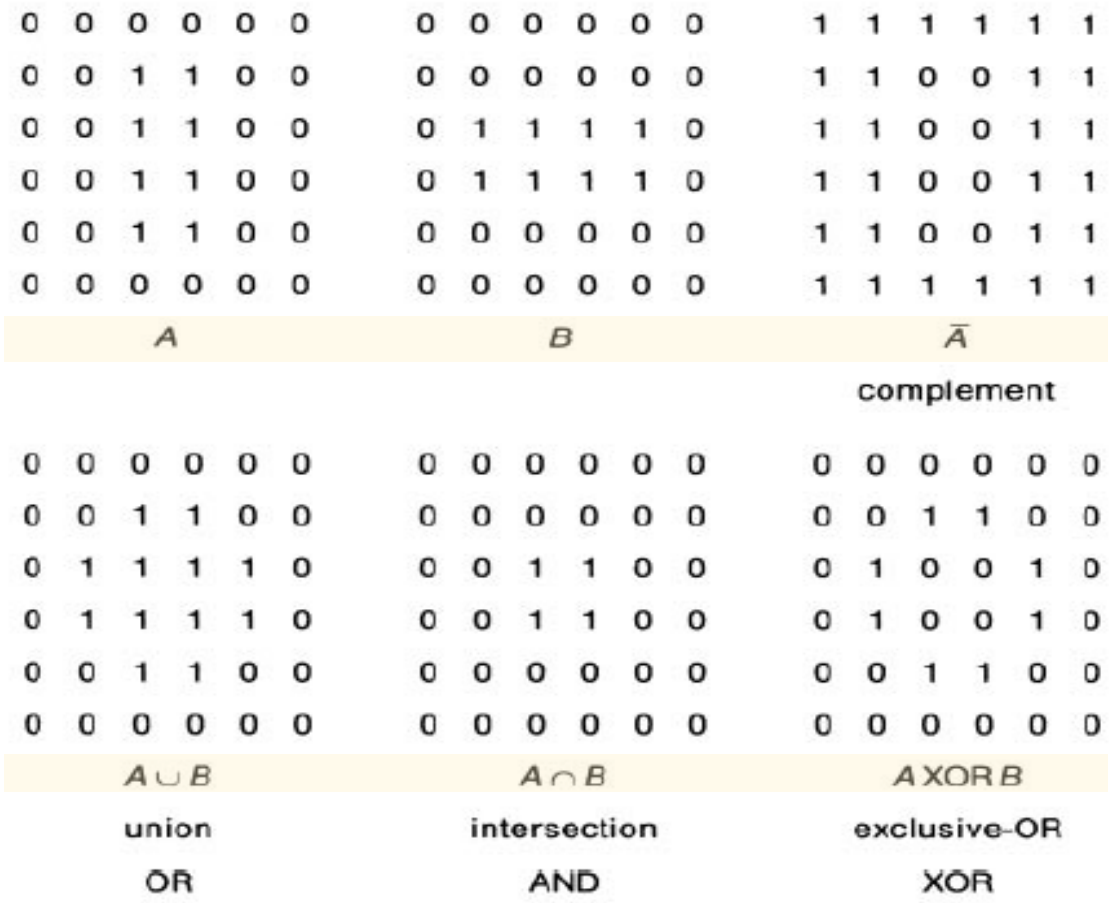
- **Skeletonizing**
  - The medial axis skeleton consists of the set of points that are **equally distant** from **two closest** points of an object boundary





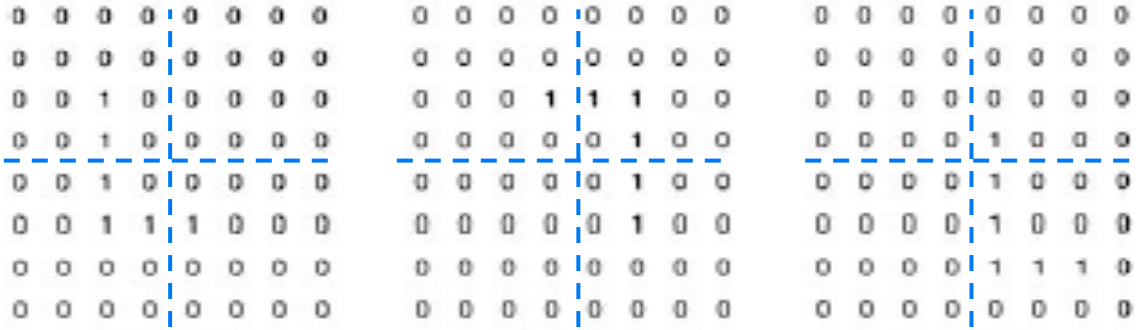
Morphological Processing

- Algebraic operations on binary arrays



Morphological Processing

- **Generalized dilation and erosion**
  - **Reflection and translation of a binary image**



$F(j,k)$ $\quad \widetilde{F}(j,k)$ $\quad T_{1,2}\{F(j,k)\}$

  - **Dilation**

$$G(j,k) = F(j,k) \oplus H(j,k)$$

Structuring element

  - **Erosion**

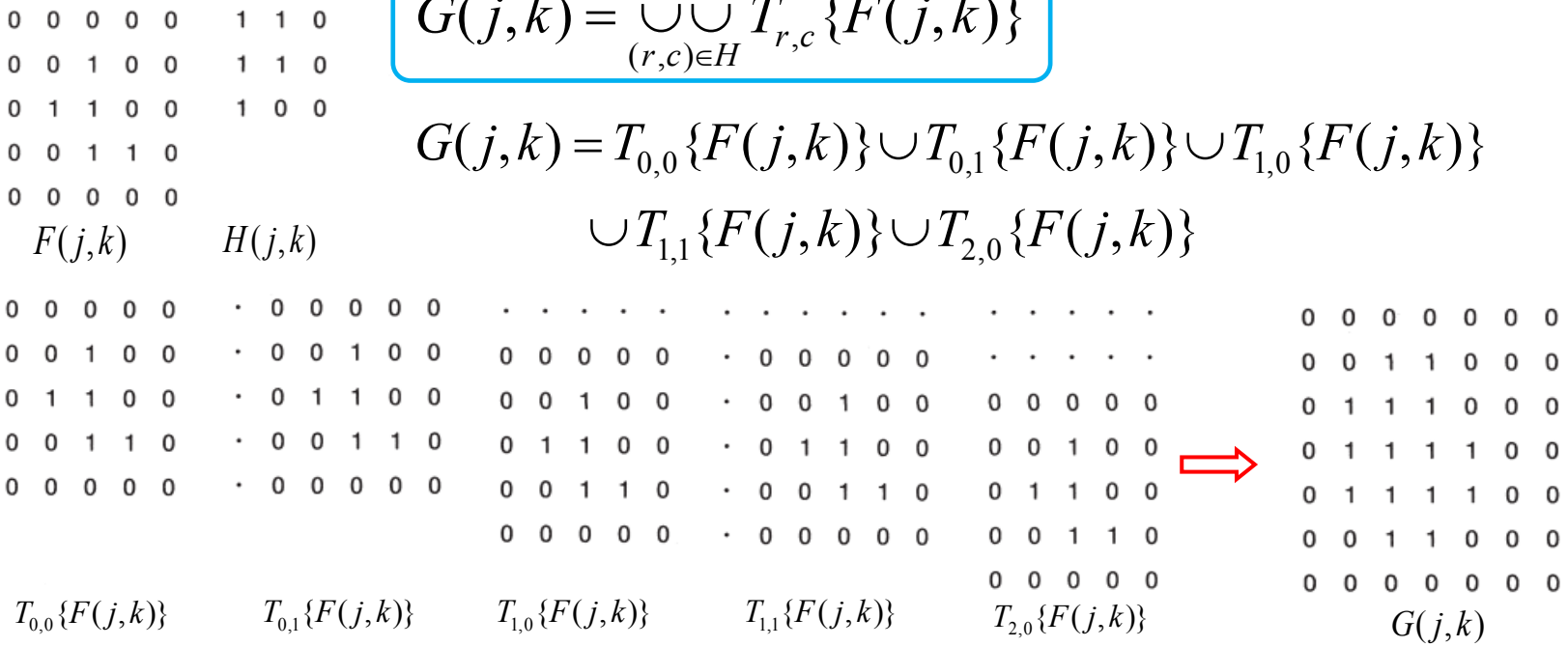
$$G(j,k) = F(j,k) \Theta H(j,k)$$

Morphological Processing

- **Dilation** $G(j,k) = F(j,k) \oplus H(j,k)$
  - Can be implemented in several ways
  - Minkowski addition definition

$$G(j,k) = \bigcup\bigcup_{(r,c)\in H} T_{r,c}\{F(j,k)\}$$

$$G(j,k) = T_{0,0}\{F(j,k)\} \cup T_{0,1}\{F(j,k)\} \cup T_{1,0}\{F(j,k)\} \cup T_{1,1}\{F(j,k)\} \cup T_{2,0}\{F(j,k)\}$$



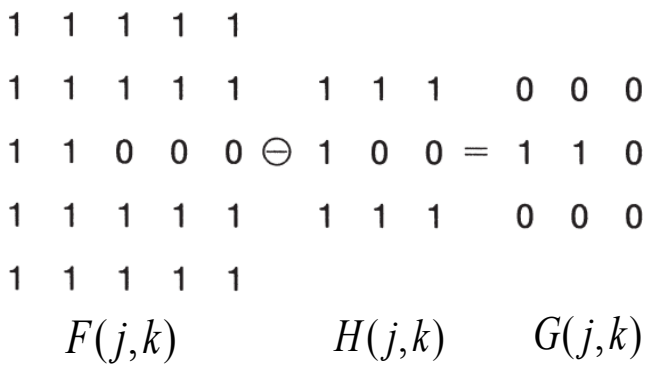
Morphological Processing

- **Erosion** $G(j,k) = F(j,k) \Theta H(j,k)$
  - Can be implemented in several ways
  - Dual relationship of Minkowski addition

$$G(j,k) = \bigcap_{(r,c)\in H} \bigcap T_{r,c}\{F(j,k)\}$$

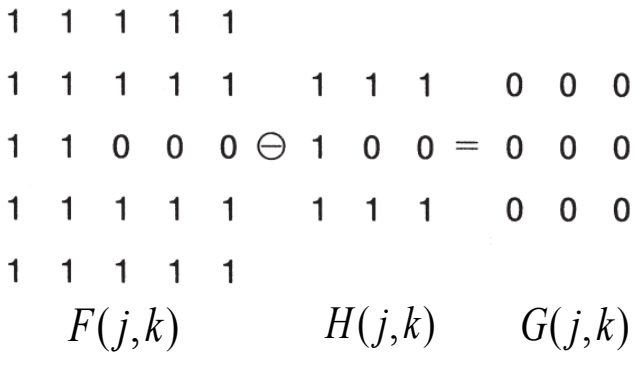
//Sternberg definition//

$$G(j,k) = \bigcap \bigcap_{(r,c)\in H} T_{r,c}\{F(j,k)\}$$



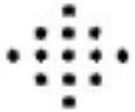
//Serra definition//

$$G(j,k) = \bigcap \bigcap_{(r,c)\in \tilde{H}} T_{r,c}\{F(j,k)\}$$



Morphological Processing

- **Example**



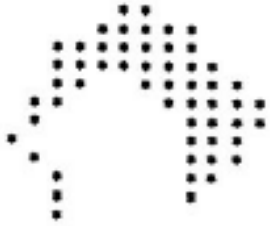
Structuring element

original

Dilation

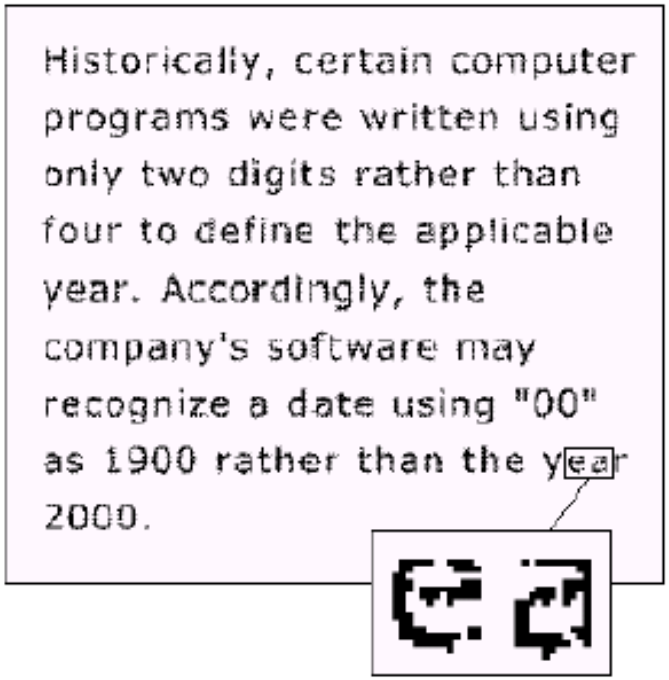
original

Erosion



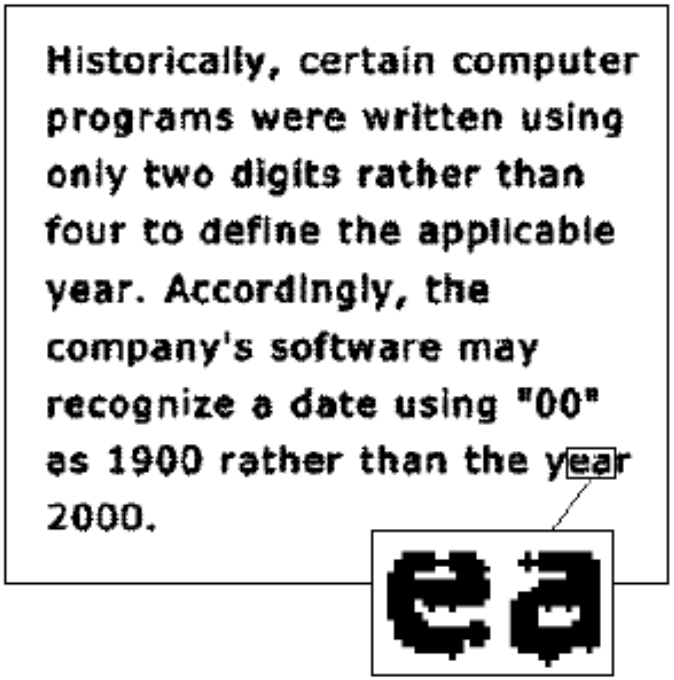
Morphological Processing

- **Example**



Structuring element

0	1	0
1	1	1
0	1	0



Morphological Processing

- **Example**

Original fingerprint

Skeletoninzed fingerprint

The original fingerprint contains ridges with width of several pixels.
The skeletonized fingerprint contains ridges only a single pixel wide.

http://www.dspguide.com/ch25/4.htm

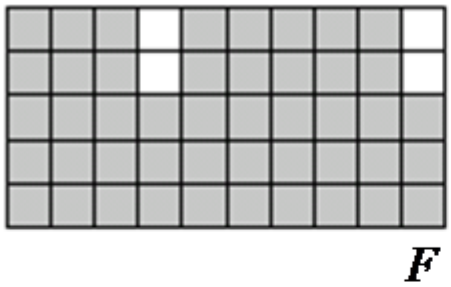
Morphological Processing

- Applications
  - Boundary Extraction
    - Extract the boundary (or outline) of an object
  - Hole Filling
    - Given a pixel inside a boundary, hole filling attempts to fill that boundary with object pixels
  - Connected Component Labeling
    - Scan an image and groups its pixels into components based on pixel connectivity

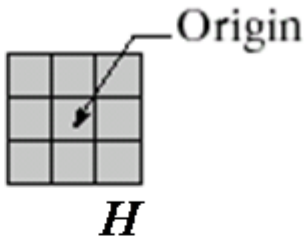
Morphological Processing

- **Boundary Extraction**

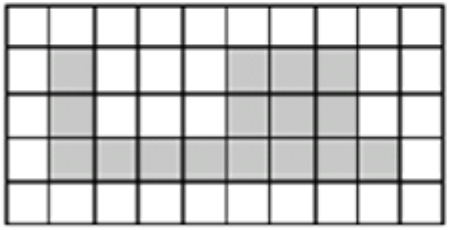
$$\beta(F(j,k)) = F(j,k) - (F(j,k)\Theta H(j,k))$$



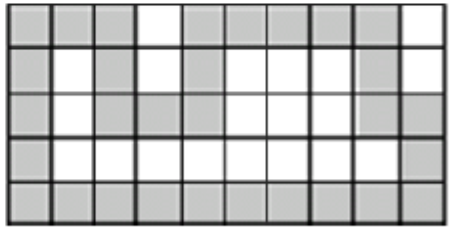
F



H



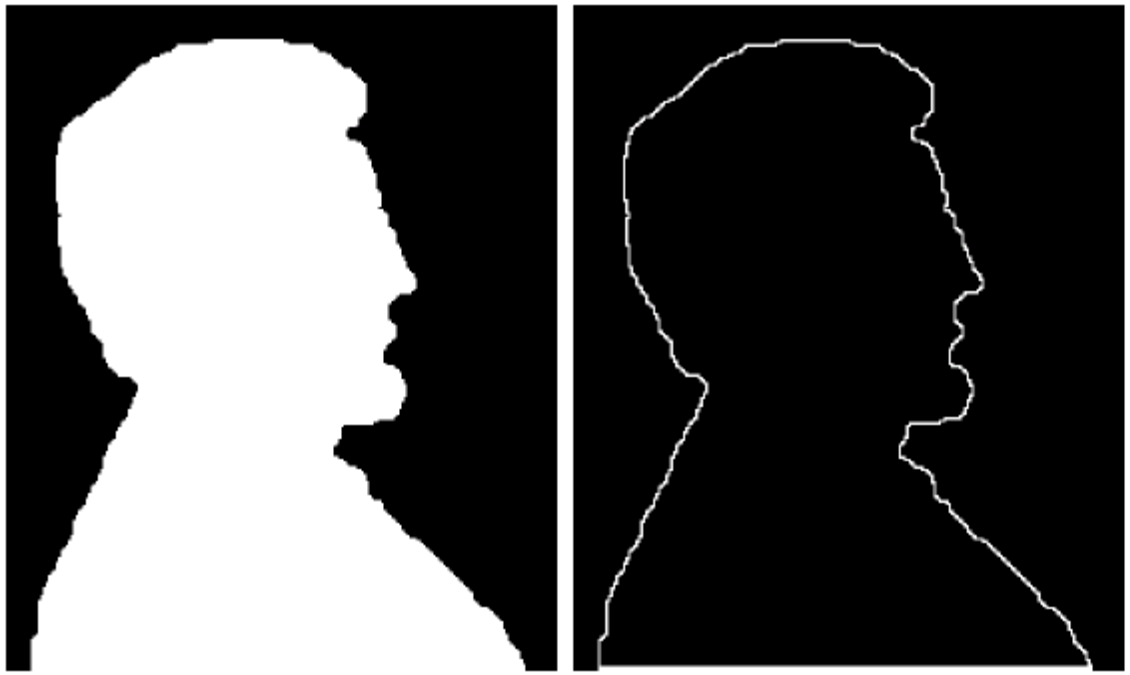
$F \Theta H$



$\beta(F)$

Morphological Processing

- Example



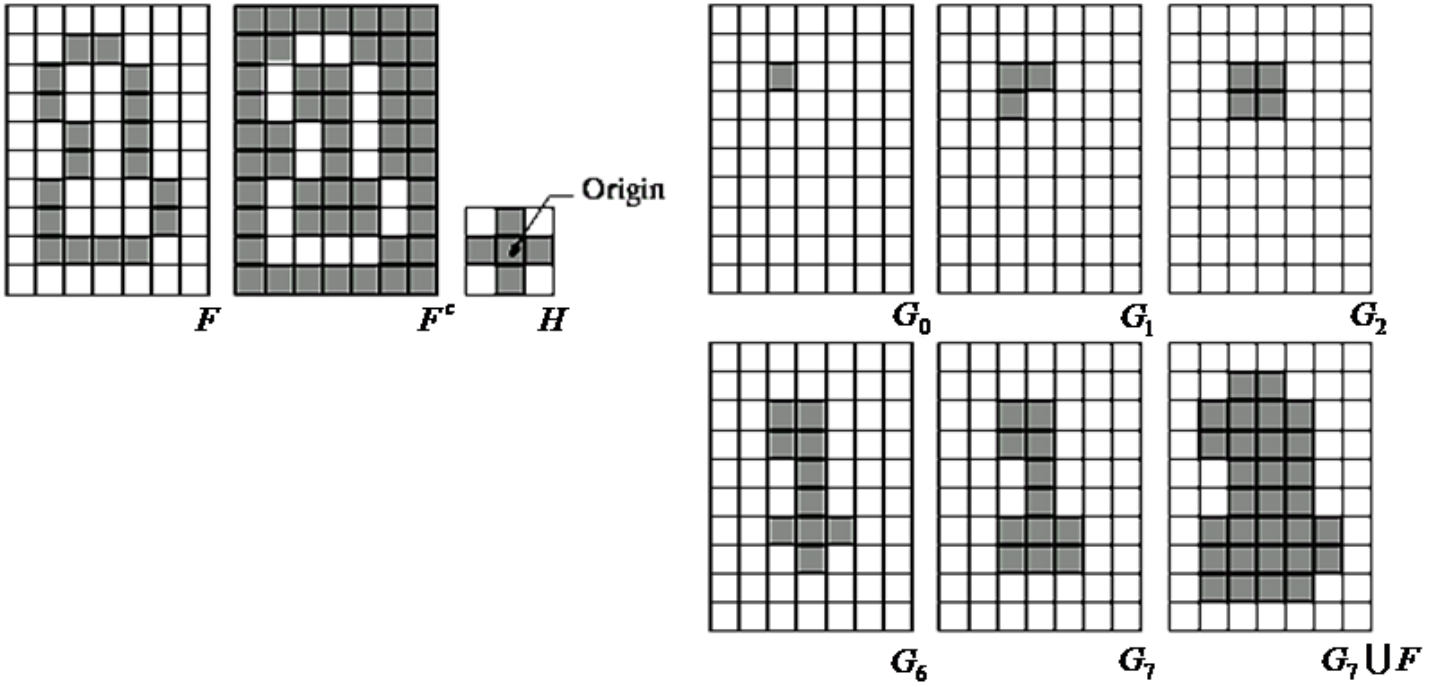
Original Image Extracted Boundary

Morphological Processing

- **Hole Filling**

$$G_i(j,k) = (G_{i-1}(j,k) \oplus H(j,k)) \cap F^c(\mathrm{j},\mathrm{k}) \qquad i = 1,2,3\ldots$$

$$G(j,k) = G_i(j,k) \cup F(\mathrm{j},\mathrm{k})$$



Morphological Processing

- Example

Original Image

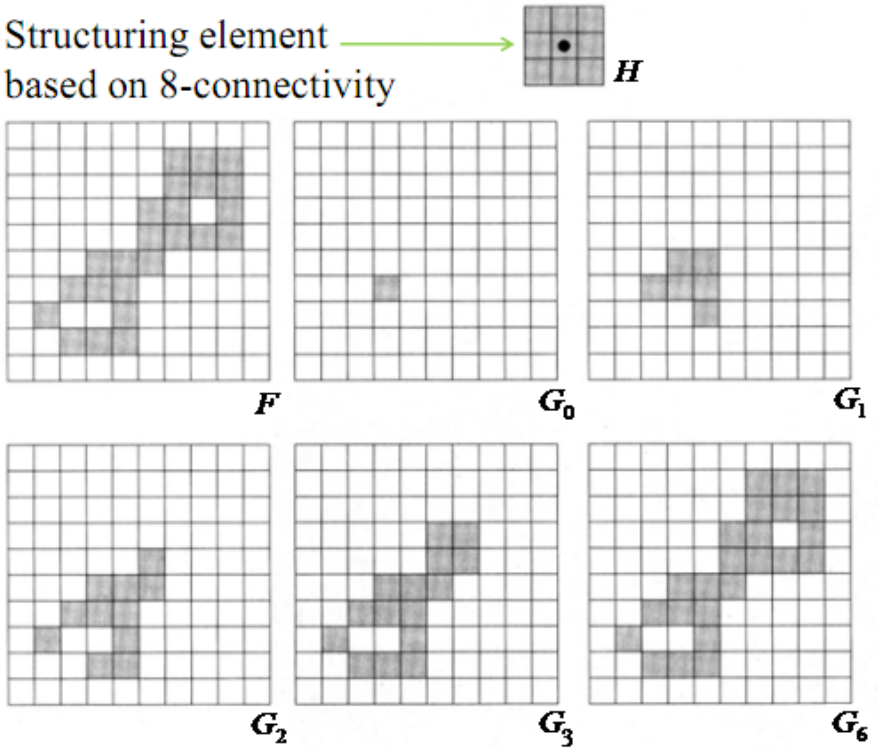
One Hole Filled

All Holes Filled

Morphological Processing

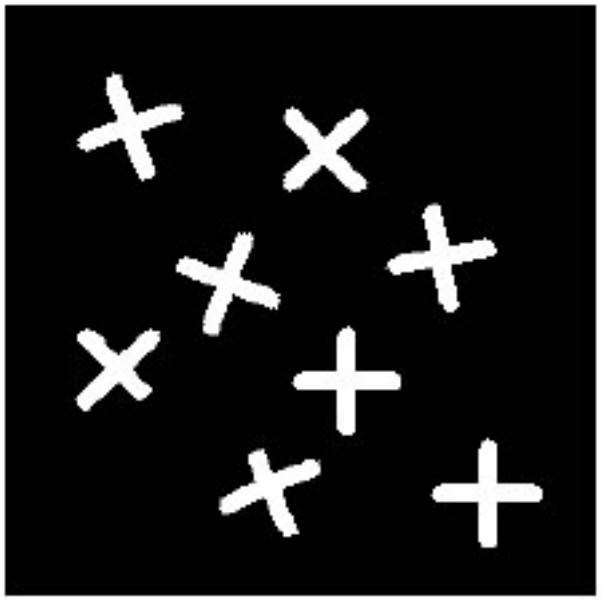
- **Connected Component Labeling**

$$G_i(j,k) = (G_{i-1}(j,k) \oplus H(j,k)) \cap F(j,k) \quad \mathrm{i} = 1,2,3,\ldots$$

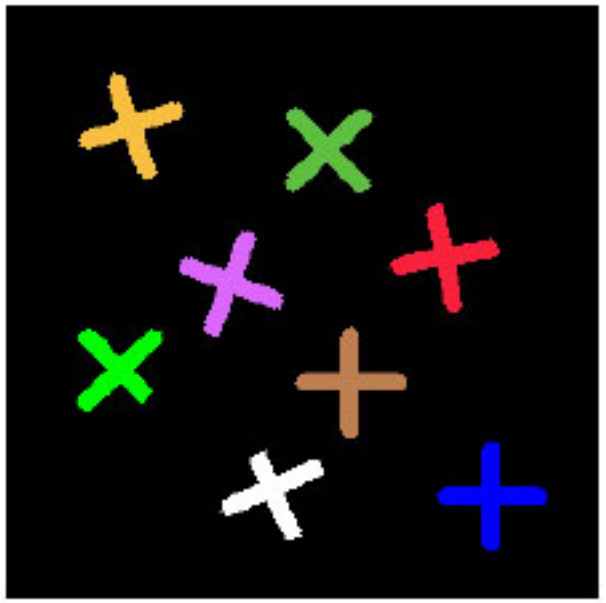


Morphological Processing

- Example



Original Image



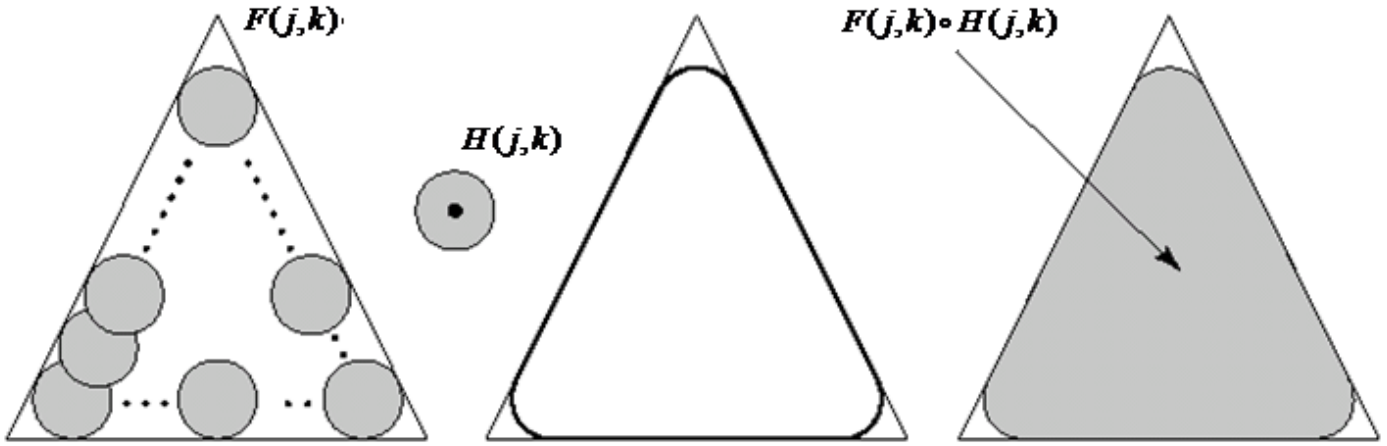
Labelled Components

Morphological Processing

- **Applications**
  - **Open operator**

$$G(j,k) = F(j,k) \circ H(j,k) = [F(j,k) \Theta \tilde{H}(j,k)] \oplus H(j,k)$$

    - **With a compact structuring element**
      - **Smoothes contours of objects**
      - **Eliminates small objects**
      - **Breaks narrow strokes**

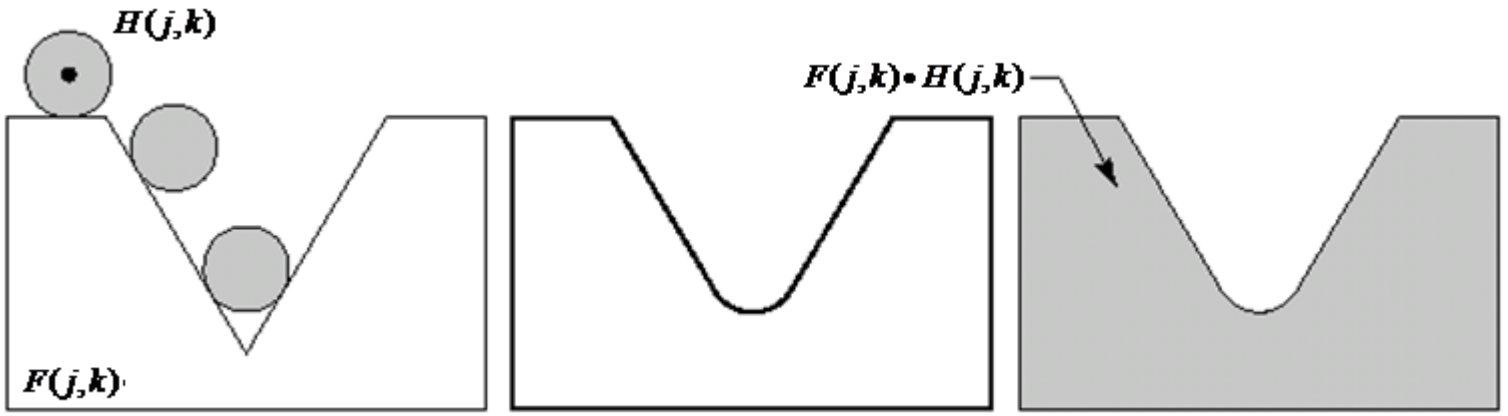


Morphological Processing

- **Applications**
  - **Close operator**

$$G(j,k) = F(j,k) \bullet H(j,k) = [F(j,k) \oplus H(j,k)] \Theta \tilde{H}(j,k)$$

    - **With a compact structuring element**
      - **Smoothes contours of objects**
      - **Eliminate small holes**
      - **Fuses short gaps between objects**



Morphological Processing

- Example

original

(a) close

(b) open

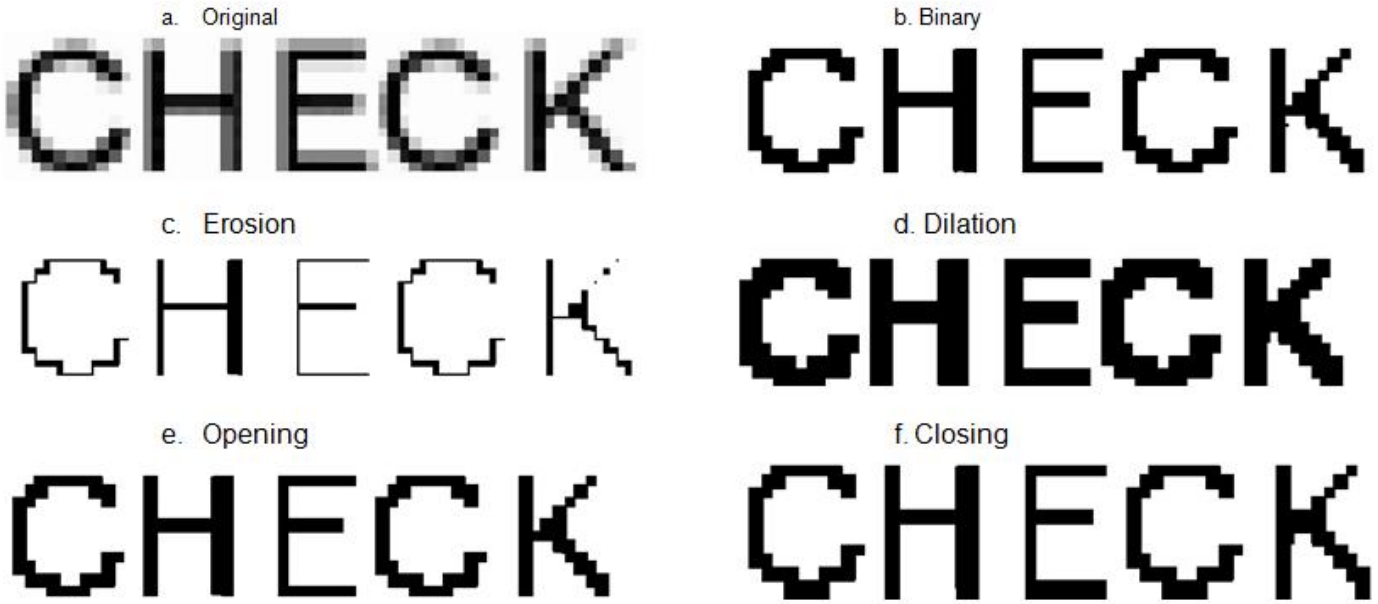
Q: repeated openings/closings?

Compare (a) with the original image

Compare (b) with the original image

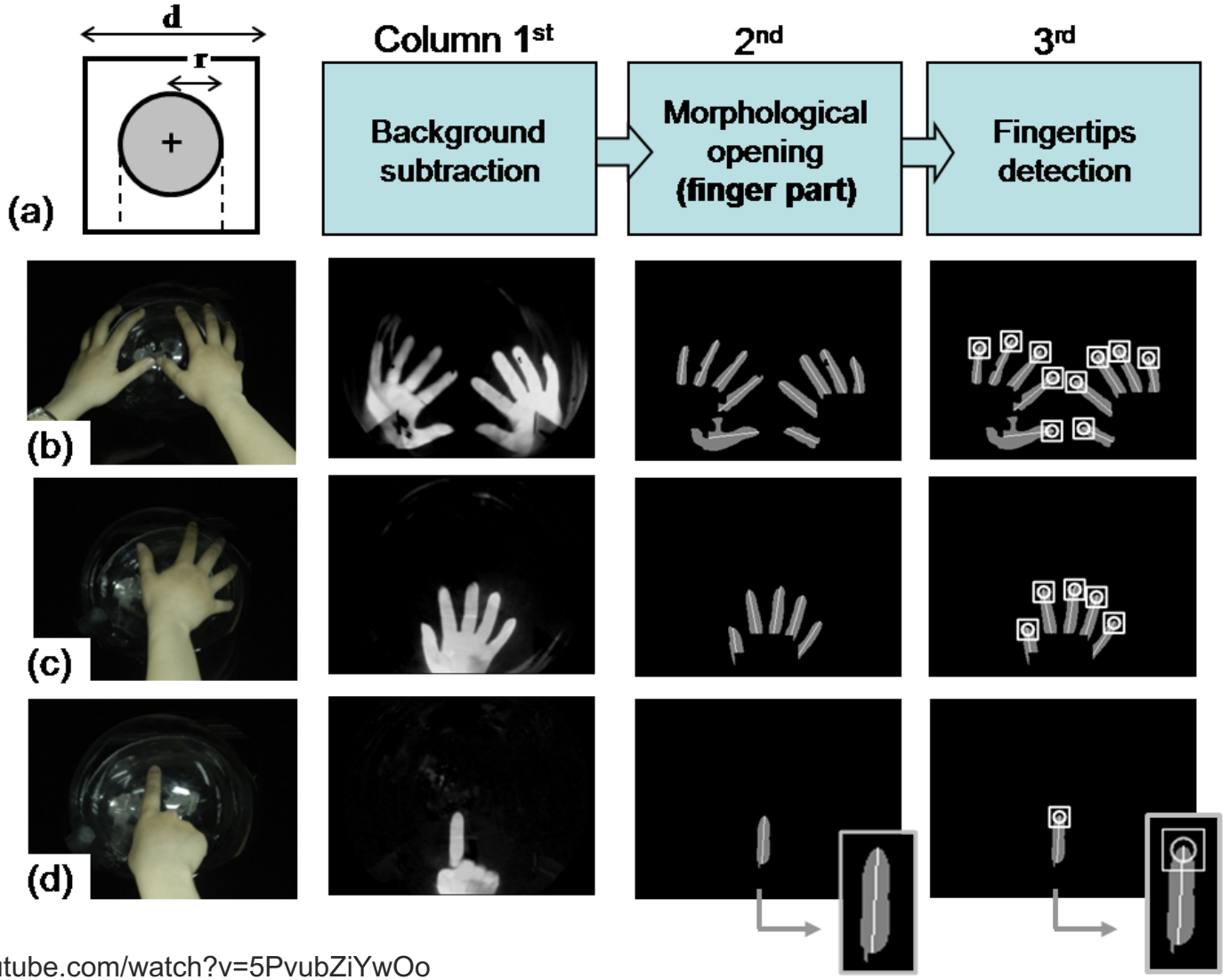
Morphological Processing

- **Example**



http://xplorimaging.com/DocumentCleanUp.html

MCBall



http://www.youtube.com/watch?v=5PvubZiYwOo

Some videos

- **Morphing**
  - https://www.youtube.com/watch?v=-rnVUzA8yMY
- **SIGGRAPH**
  - **2013** https://www.youtube.com/watch?v=JAFhkdGtHck
  - **2015** https://www.youtube.com/watch?v=XrYkEhs2FdA
  - **2017** https://www.youtube.com/watch?v=5YvIHREdVX4
  - **2018** https://www.youtube.com/watch?v=t952yS8tcg8
  - **2019** https://www.youtube.com/watch?v=EhDr3Rs5fTU
  - **2020** https://www.youtube.com/watch?v=jYdMKdRUq_8
  - **2021** https://www.youtube.com/watch?v=Ros7ZXqLbFg