CS452 HW4

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2.4

e. $\{w \mid w = w^R$, that is, w is a palindrome.$\}$.

$$S \to 1S1 \mid 0S0 \mid 0 \mid 1 \mid \varepsilon$$

List following string:

$S \to 0S0$

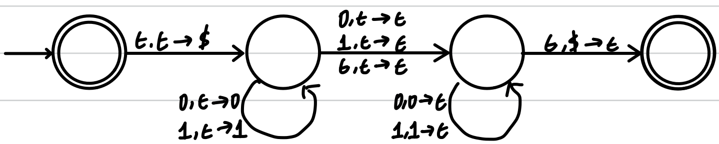
$S \to 01S10$

$S \to 010S010$

$S \to 0100010$ Therefore, it is palindrome.

2.5

e. When we push the symbols to read onto the stack. At each point, we should know if the next symbol read is the middle string and it would input empty. and if the symbol matches the read input symbol, all the symbol would pop from stack.
If all the popped symbol is exactly the same as inserted symbol from stack, and the stack the empty after all the input is complete, then it would accept. Otherwise, refuse.
for example, 11011 is accept.



2.9

$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$.

Given: $i = j$ or $j = k$

So, when $i = j$, it's no contraint on k.

when $j = k$, it's no contraint on i.

① when $i=j$ ② when $j=k$

$S_1 \to XY$ $S_2 \to PQ$

$X \to aXb \mid \varepsilon$ $P \to aP \mid \varepsilon$

$Y \to cY \mid \varepsilon$ $Q \to bQc \mid \varepsilon$

There exist two parse trees left or right derivations for single strigen.

Above Two. $A \to S_1 / S_2$

$S_1 \to XY$

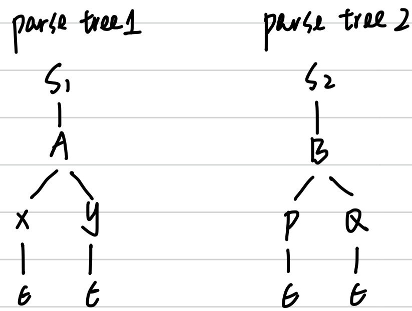
$X \to aXb \mid \varepsilon$

$Y \to cY \mid \varepsilon$

$S_2 \to PQ$

$P \to aP \mid \varepsilon$

$Q \to bQc \mid \varepsilon$



∵ Two parse tree exist for ε.

∴ The context free grammar for the language A is ambiguous.

2.10

Let $S_1 = \{a^i b^j c^k \mid i=k,\ i,k>0\}$, $S_2 = \{a^i b^j c^k \mid i=k,\ i,k>0\}$.

The informal description of PDA that recognizes the S_1.

First, Read and push a's. then read b's, pop a's. if stack is empty after b's complete. we would skip c's on input. then it's accept. otherwise, refuse.

The informal description of PDA that recognizes the S_2.

First, skip a's on input. then read and push b's. After that, read c's and pop b's. If stack is empty after c's complete. it's accept. Otherwise, refuse.

Combinition of those two.

First, we need to know the non-deterministic decision needs to be made to push a's or not. As we know, there will 2 branches. checking the equality of a's and b's or equality of b's and c's. Then, the machine would keep on push a's and see a's. The machine see a, b the a's are popped on each b, and see c's, then there is no operation is performed.

Contents of stack is updated on see a's and b's that are pushed on see b's and c's, then b's will be popped. If the stack is empty. It's accept. Otherwise, refuse.

2.14

Using the theorem 2.9: Any context-free language is generated by a context-free grammar in Chomsky normal form.

Given: $A \to BAB \mid B \mid \epsilon$

$B \to 00 \mid \epsilon$

① ∵ The start symbol A occurs on R.H.S

∴ let S be the start variable, $S \to A$.

$\Rightarrow$ $S \to A$

$A \to BAB \mid B \mid \epsilon$

$B \to 00 \mid \epsilon$

② Remove null production. $A \to \epsilon$ and $B \to \epsilon$

Eliminate $A \to \epsilon$, finding whose R.H.S contains A.

$A \to BAB$ $\quad$ $S \to A$

$\Rightarrow A \to BB$ $\quad$ $\Rightarrow S \to \epsilon$.

$\Rightarrow$ $S \to A \mid \epsilon$

$A \to BAB \mid B \mid BB$

$B \to 00 \mid \epsilon$

Eliminate $B \to \epsilon$, finding whose R.H.S contains B.

$A \to BAB$

$\Rightarrow A \to AB \mid BA \mid A$

$\Rightarrow S \to A \mid \epsilon$

$A \to BAB \mid B \mid BB \mid AB \mid BA \mid A$

$B \to 00$

③ Remove all unit production

Remove $S \to A$, $A \to A$, $A \to B$

1st : remove $A \to A$.

$\Rightarrow S \to A \mid \epsilon$

$A \to BAB \mid B \mid BB \mid AB \mid BA$

$B \to 00$

2nd : remove $A \to B$. $\because B \to 00$, then $A \to 00$

$\Rightarrow S \to A \mid \epsilon$

$A \to BAB \mid 00 \mid BB \mid AB \mid BA$

$B \to 00$

3rd : remove $S \to A$. $\because A \to BAB \mid 00 \mid BB \mid AB \mid BA$.

$\Rightarrow S \to BAB \mid 00 \mid BB \mid AB \mid BA \mid \epsilon$.

$A \to BAB \mid 00 \mid BB \mid AB \mid BA$.

$B \to 00$.

④ Find the production that has more than 2 variables in R.H.S.

$S \to BAB$, $A \to BAB$

Add a new production $P \to AB$.

$S \to BP \mid 00 \mid BB \mid AB \mid BA \mid \epsilon$

$A \to BP \mid 00 \mid BB \mid AB \mid BA$.

$B \to 00$

$X \to AB$

$\because$ 00 also is terminate variable.

$\therefore$ We add another new production $Q \to 00$.

$\Rightarrow S \to BP \mid Q \mid BB \mid AB \mid BA \mid \epsilon$

$A \to BP \mid Q \mid BB \mid AB \mid BA$

$B \to 00$

$P \to AB$

$Q \to 00$

Thus, this CFG has been converted to CFG in Chomsky normal form.

2.35

Given, G is in CNF.

$\because$ A string of length X should have dervation of length $\leq 2^X - 1$.

$\therefore$ if dorivation of length $= 2^b$

then, G must derive a string of length $b+1$.

The parse tree must have depth $\geq b+1$.

$\because$ The substitution in CNF only increases the length by 1.

$\therefore$ The parse tree of length $= b+1$.

According to the pigeonhole principle, the path should contain some variable A twice.

$\Rightarrow$ A derives a string of form uAv. ($u, v \in \Sigma^*$)

A derives some string $x \in \Sigma^*$.

$\Rightarrow$ A yield $u^i x v^i$ ($i \in \mathbb{N}$) by substituting uAv for A for i times, substituing x for A.

Since A is in the tree below start variable S.

S derives some aAb.

$\Rightarrow$ $av^i x v^i b$ is in the string for any i.

Thus, there are infinitely many such strings.

$\therefore$ $L(G)$ is infinite.

Using CYK to determine if the following string are generated by the grammar.

Given $S \rightarrow AB \mid CB \mid a \mid b$

$B \rightarrow AS \mid SC$

$A \rightarrow a$

$C \rightarrow c$.

① c a a a c

5	S				
4	-	B			
3	S	S	S		
2	-	B	B	B	
1	C	S, A	S, A	S, A	C
	c	a	a	a	c

2: ca : C S,A. ⇒ CS X

CA X

ac : S,A C ⇒ SC B

AC X

aa : S,A S,A. ⇒ SS X

SA X

AA X

AS B

3: c, aa : C B ⇒ CB S

ca, a : – S,A ⇒ X

a, aa : S,A B ⇒ SB X

AB S

aa, a : B S,A ⇒ BS X

BA X

aa, c : B C ⇒ BC X

a, ac : S,A B ⇒ SB X

AB S

4: c, aaa : C S ⇒ CS X

ca, aa : – B ⇒ X

caa, a : S S,A ⇒ SS X

SA X

a, aac : S,A S ⇒ SS X

SA X

aa, ac B B ⇒ BB X

aaa, c S C ⇒ SC B

5: c, aaac C B ⇒ CB S

ca, aac – ⇒ X

caa, ac S B ⇒ SB X

caaa, c – ⇒ X

5	S				
4	–	B			
3	S	S	S		
2	–	B	B	B	
1	C	S,A	S,A	S,A	C
	c	a	a	a	c

Accepted.

Given $S \to AB \mid CB \mid a \mid b$

$B \to AS \mid SC$

$A \to a$

$C \to c.$

① b a a a c

5	–				
4	–	B			
3	–	S	S		
2	–	B	B	B	
1	S	S,A	S,A	S,A	C
	b	a	a	a	c

2: b , a : S S,A ⇒ SS X

SA X

a , a : S,A S,A ⇒ SS X

SA X

AS B

AA X

a , c : S,A C ⇒ SC B

AC X

3: b , aa : S B ⇒ SB X

ba , a : – ⇒ X

4: b , aaa : S S ⇒ SS X

ba , aa : – ⇒ X

baa , a : X ⇒ X

5: b , aaac : S B ⇒ SB X

ba , aac : – ⇒ X

baa , ac : X ⇒ X

baaa , c : X ⇒ X

Not accepted.

$\boxed{\text{Extra 2.23}}$

The length of String Q must be even. Q ∈ D.

Its two halves must differ in at least one bit.

Written as xy with $|x| = |y|$. for some i.

So, the ith character of x different from ith character of y.

Generating the corresponding ith characters and filling the remains up.

CFG for Q: $S \to AB \mid BA$

$A \to XAX \mid 0$

$B \to XBX \mid 0$

$X \to 0 \mid 1$.