CS452 HW1

Yuting Chen

0.7

Let the relation R over the set $\{x, y, z\}$

The Reflexive Property states that $(x,x) \in R$, $(y,y) \in R$, $(z,z) \in R$

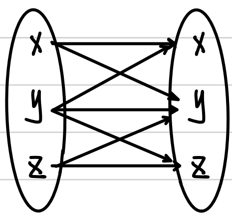
The Symmetric Property states that if $(x,y) \in R$, then $(y,x) \in R$

if $(x,z) \in R$, then $(z,x) \in R$

if $(y,z) \in R$, then $(z,y) \in R$

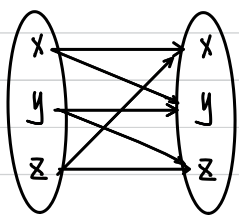
The Transitive Property states that if $(x,y) \in R$, $(y,z) \in R$, then $(x,z) \in R$

(a) Reflexive and symmetric, but not transitive.



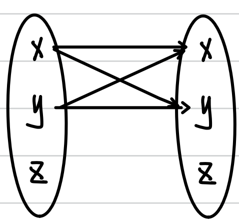
$\{(x,x), (y,y), (z,z), (x,y), (y,x), (y,z), (z,y)\}$

(b) Reflexive and transitive, but not symmetric.



$\{(x,x), (y,y), (z,z), (x,y), (y,z), (z,x)\}$

(c) Symmetric and transitive, but not reflexive.



$\{(x,y), (y,x), (x,x), (y,y)\}$

It's symmetric and transitive, but not reflexive. $\because (z,z) \notin R$.

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Fromal description of this graph is $\{1,2,3,4,5,6\}$ $A=\{1,2,3\}$ $B=\{4,5,6\}$.
$\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
For this graph $G=(V,E)$ where V is the set of nodes or vertices and E is the set of edges.

$V=\{1,2,3,4,5,6\}$

$E=\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

0.10

In this proof, we first assum the equation $a=b$, then we got the equation $(a+b)(a-b)=b(a-b)$. We need to divide each side by $a-b$. But $a=b \Rightarrow a-b=0$ As we know, the denominator can not be 0. It's a division by 0 in this proof, so it is not vaild.

0.12

This proof fails for $n=2$, We can't conclude by having only two horses. Let $h=2$ following the induction hypothesis, we will find there aren't any horses remaining in $h=2$ to apply the transitivity of "have the same color as". So if the argument works, there are at least 3 horses. There has to be a horses which was not removed in order to conclude that the other two removed horses. The error in the following proof that did not prove the proposition holds for base case. The base case must have $h=2$.

0.13

As we know, if the Set S is notempty and the integers in S are all equal or greater than X, if $|S|=n$, the S contains an element $\geq n+X-1$. if the S doesn't include an element then $S=\{X, X+1, X+2, \cdots, n+X-2\}$, the elements $< n$.

Assume G has n nodes and no two nodes that have equal degrees.

$\Rightarrow$ there is a node that is connected two every node, or equivalenty a node v of degree at least $n-1$.

the v is a node of G. the set degree(v) is a set of n that are all ≥ 0. So S contains an element should $\geq n-1$.

Every node has degree at least 1 since v has degree $n-1 \geq 1$. We could use that way again to

show that there exist a node of degree at least n.
So that is a contradiction, since only n-1 nodes it possibly be connected to.
$\Rightarrow$ every graph with two or more nodes contains two nodes that have equal degrees.

$\boxed{0.14}$ Given every graph with n nodes contains either a clique or an anti-clique.

Ramsey number $R(p,q)$ is the min number of vertices nodes where p is at least size of anti-clique or q is at least size of clique.

For this question, we need find every vertex part of clique or anti-clique. If we take the vertex v that connected to over half other vertices. then we need take the vertex v add to clique list and discard all vertices that not connected to vertex v. If we take any vertex v' that not connected to over half other vertices, then we need take the vertex v' add to anti-clique list and discard all vertices connected to vertex v'.

At most half of the nodes are discarded at each steps, so it is at least $\log_2 n$ steps and at each step discard $\leq \frac{1}{2}$ of the remaining nodes.

Thus, Every graph with n nodes contains either a clique or an anti-clique with at least $\frac{1}{2}\log_2 n$ nodes.

$\boxed{\text{Final Exercise}}$

Given $(1,1) \in S$,

if $(a,b) \in S$

then $(a+1, b+2a+1) \in S$

let $a=1, b=1$ $\quad (1+1, 1+2(1)+1) = (2,4) \in S$

let $a=2, b=4$ $\quad (2+1, 4+2(2)+1) = (3,9) \in S$

let $a=3, b=9$ $\quad (3+1, 9+2(3)+1) = (4,16) \in S$

$\vdots \quad \vdots \quad \vdots$

$$(a+1, b+2a+1) = (a+1, (a+1)^2) \in S$$

Thus, the function would be $f(x) = x^2$. $\forall x \in \mathbb{N}$

Using Induction.

Base case : let $X=1 \Rightarrow f(1)=1^2=1 \in S$

let $x=2 \Rightarrow f(2)=2^2=4 \in S$

Inductive hypothesis when $X=k$, $f(k)=k^2$ is true.

let $x=k+1$, $f(k+1)=(k+1)^2=k^2+2k+1 \in S$.

Thus, the function $f(x)=x^2$ is true.