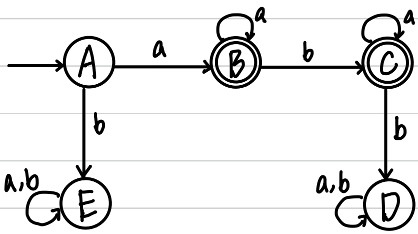
CS452 HW2

Yuting Chen

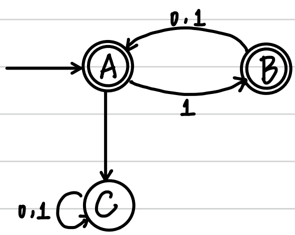
1.4

e. {w | w starts with an a and has at most one b.}

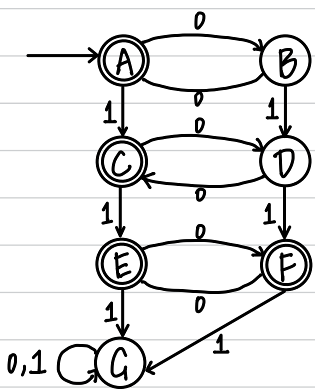


1.6

i. {w | every odd position of w is a 1.

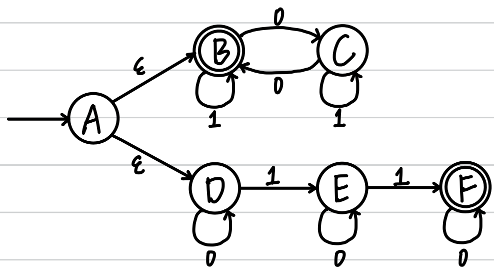


l. {w | w contains an even number of 0s, or contains exactly two 1s}



1.7

c. The language of Exercise 1.6 l with six states.

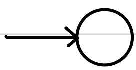


1.10

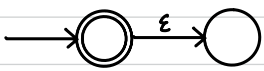
c. Exercise 1.6 m: The empty set.

Noted: Language is the empty set for exercise 1.6m.

Let A_1 = the empty set = ϕ. Let M_1 be the NFA that recognizes A_1. The state diagram as shown in the following figure.



NFA M_1 for A_1 and modify it to recognize $A*_1$, let M be the NFA that recognizes A. A is the star of A_1. So, The state diagram of M as shown in the following figure.



1.14

b. Let M' be the new NFA that has swapped accept and non-accept state in M. M' recognizes the complement of language C, where language C is recognized by M.

Consider M' accepts a string x. and the string x is acepted by automation. Swapping the accept and non-accept states of a NFA do not necessarily yield a new one.

If, we run M' on x then we would end in an accept state of M'.

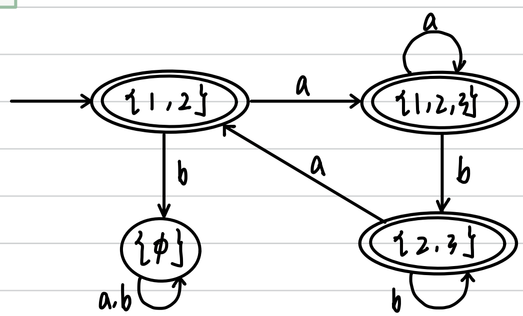
If, we run M on x then we would end in an non-accept state of M'.

Since, M' and M have swapped accept and non-accept states. $\Rightarrow x \notin C$

So, if x isn't accept by M', then it would be accept by M.
$\Rightarrow$ M' recognizes the languages which are complement of C.
Hence, the class of languages recognized by NFAs closed under complement.

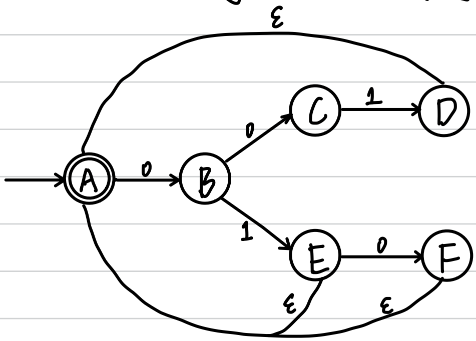
1.16

b.

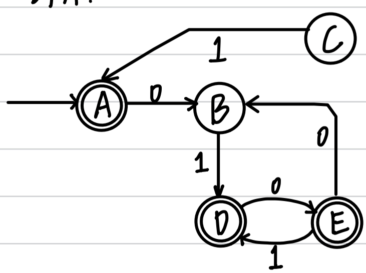


1.17

a. Given an NFA recognizing the language $(01 \cup 001 \cup 010)^*$.



b. NFA $\rightarrow$ DFA.



1.18

i. {w | every odd position of w is a 1}

$(1\Sigma)^*(\varepsilon \cup 1)$

l. {w | w contains an even number of 0s, or contains exactly two 1s}

$1^*(01^*01)^* \cup 0^*10^*10$.

1.31

Assume A is regular.

We know that it is a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes A by definition.

So if A^R is regular, it should have a DFA that recognizes A^R. Let this NFA be M'.

Reserve all arrows of M. designate the start state for M and the accept state q'_a for M'. then add a new one q'_n for M', add ε to each state of M'.

Thus, we can find any $w \in \Sigma^*$.

Only if the path follows w^R from q'_n to q'_a in M'. It would have a path follow w from state Start to accept state. $w \in A$ iff $w^R \in A^R$

Thus, if A is a regular, so is A^R.

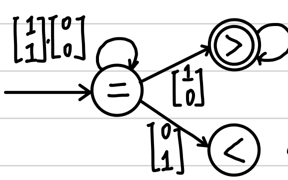
1.34

Given: $D = \{ w \in \Sigma_2^* \mid$ the top row of w is a large number than is the bottom row$\}$.

$\begin{bmatrix}0\\0\end{bmatrix} \begin{bmatrix}1\\0\end{bmatrix} \begin{bmatrix}1\\1\end{bmatrix} \begin{bmatrix}0\\0\end{bmatrix} \in D$.

We can take three states "=, >, <" to consider the language D by NFA.

The NFA for D as shown in the following figure.



We got that equivalent DFA exist for each NFA.

So, Every language is regular is accepted by a DFA. D is regular

1.51

We need to show that $\equiv_L$ is an equivalence relation.

Equivalence relation which means satisfying all three properties: Reflexivity, Symmetry, Transitivity.

Let L be a language, k be a set of string.

So, M is pairwise distinguishable by L.

If every two strings in k are distinguishable by L.

Index of L should be defined to max number of elements in any set of strings

Index of L = number of equivalence classes in L.

The L can be infinite or finite.

Reflexivity: Given $xz \in L$ iff $xz \in L$

Symmetry: Given for any set of string z, $xz \in L$, whenever $yz \in L$ is equivalent to $yz \in L$ whenever $xz \in L$.

Transitivity: Given $xz \in L$ whenever $yz \in L$, $yz \in L$ whenever $xz \in L \Rightarrow xz \in L$ whenever $yz \in L$ collectively