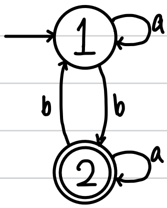
CS452 HW3

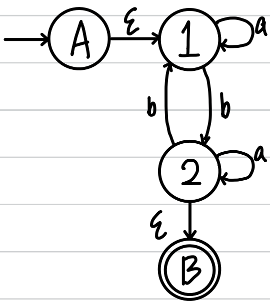
Yuting Chen

1.21

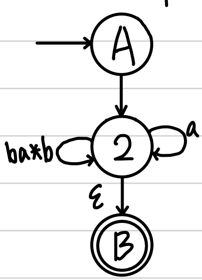
a.



Add A as new initial state. add B as new accept state. then let 2 be non-accept state.

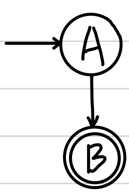


Remove 1 and add 1's loop to state 2.



a ∪ ba*b

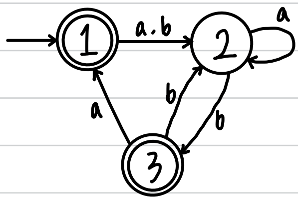
Then remove state 2



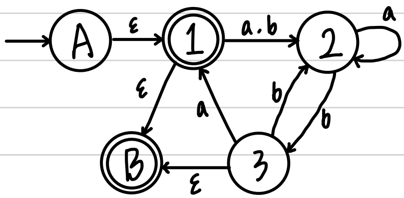
a*b(a∪ba*b)

Thus, the regular expression is a*b(a∪ba*b).

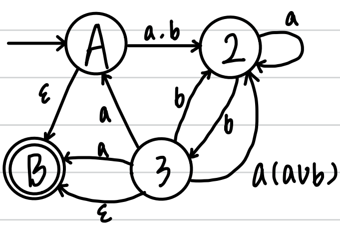
b.



Add a new initial state A and add a new accept state B, then let state 1 and 3 be non-accept state.



Remove state 1

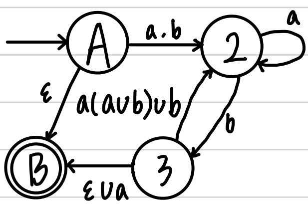


state A to 2 : $a \cup b$

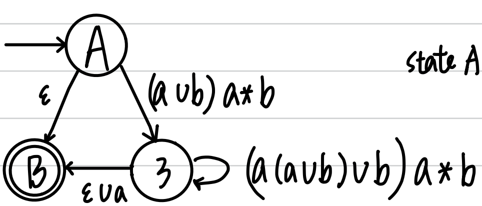
state 3 to 1, state 1 to B : a

state 3 to 1, state 1 to 2 : $a(a \cup b)$

Combine those unions in states.

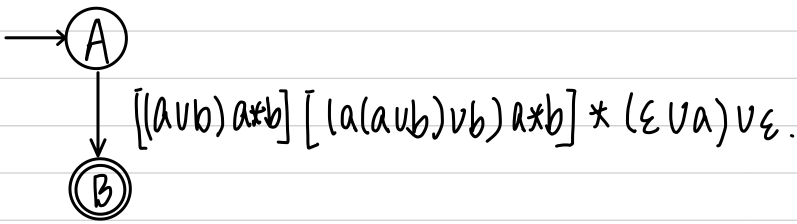


Remove state 2



state A to 2, state 2 to 3 : $(a(a \cup b) \cup b)a^*b$

Remove state 3. combine all the expression.



Thus, the regular expression is $[(a\cup b)a^*b][(a(a\cup b)\cup b)a^*b]^*(\varepsilon\cup a)\cup\varepsilon$

$\boxed{1.29}$

b. Prove $A_2=\{www \mid w\in(a,b)^*\}$ is not regular.

Assume $A_2=\{www \mid w\in(a,b)^*\}$ is regular.

Let p be the pumping length.

String $S=a^pba^pba^pb$. that $S\in A_2$ and $w=a^pb$. $|S|\geq p$.

Since the pumping lemma

$(i\geq 0)$

We could divide into $S=xyz$, $(|y|>0, |xy|\leq p, xy^iz\in A_2)$

$\Rightarrow$ y must lie in the first set of a's.

$\Rightarrow$ We have that $x=a^{|x|}$ where $|x|\geq 0$, $y=a^{|y|}$ where $|y|>0$.

$z=a^{p-|xy|}ba^pba^pb$.

Consider $S_1=xy^1z=xz$

$\Rightarrow$ String should be $a^{|x|}a^{p-|xy|}ba^pba^pb=a^{p-|y|}ba^pba^pb$.

It is three b's, so the string can not be A_2.

The first set of a's follow by a b has fewer a's than the second and third. $\Rightarrow S_0\notin A_2$.

It's contradiction.

Thus, $A_2=\{www \mid w\in(a,b)^*\}$ is not regular.

1.32

Given $\Sigma_3 = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix} \cdots \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$

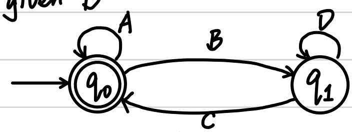
Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Each row to be a binary number.

$B = \{ w \in \Sigma_3^* \mid$ the bottom row of w is the sum of the top two rows.$\}$

let $M = \begin{bmatrix}0\\0\\1\end{bmatrix} \begin{bmatrix}1\\1\\1\end{bmatrix} \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\0\end{bmatrix}$

$\Rightarrow$ first row is binary respresentation of 7. Second row corresponds to 5. Third row corresponds to 12.

For given B



That represents NFA.

$A = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix} \right.$ $B = \left\{ \begin{bmatrix}1\\1\\0\end{bmatrix} \right\}$ $C = \left\{ \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$

$D = \left\{ \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix} \right\}$

when the previous bit does not generate a carry, it is in state q_0

eg: $\begin{bmatrix}0\\0\\0\end{bmatrix}$. which is no carry in or out.

So, the language B^R is regular.

As we know if B^R is regular, then automatically B is also regular and vice versa.

Thus, As B^R is regular, B is also regular.

1.35 Given $\Sigma_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$

Assume $E = \{ w \in \Sigma_2^* \mid$ the bottom row of w is the reveres of the top row of w$\}$ is regular language.

The constant p is associated with expression E.

Let the String $s = \begin{bmatrix}1\\0\end{bmatrix}^p \begin{bmatrix}0\\1\end{bmatrix}^p = 1^p 0^p = (0^p 1^p)^R$ where $|s| = 2p \geq p$.

Let $s = xyz$ where $p \geq q$.

$y = \begin{bmatrix}1\\0\end{bmatrix}^p$, $z = \begin{bmatrix}0\\1\end{bmatrix}^p$.

Any division $y = abc$, $\Rightarrow$ $b = \begin{bmatrix}1\\0\end{bmatrix}^n$ but $0 < n \leq p$.

valu $i \geq 0$ $\Rightarrow$ value of i is assumed to be 2.

Then, put the value y

$s = xab^i cz = xab^2 cz$

$s = \begin{bmatrix}1\\0\end{bmatrix}^{p+n} \begin{bmatrix}0\\1\end{bmatrix}^p$ since $m > 0$ and $1^{p+n} 0^p \neq (0^{p+n} 1^p)^R$.

So, it's contradiction. $xab^2cz \notin E$.

Thus, E is not a regular language.

1.40

b. Relevant computations are those which go through at least 2 accepting states, we need to force on last accepting state in $G \subseteq F$ which are those accepting states from other accepting state is reachable, go along as arrows of M, then let these states to be non-accepting in new DFA M' accomplishes the desired result, because any computation produces a sting of language A couldn't reach other accepting state.

Let $M = (Q, \Sigma, \sigma, q_0, F)$ to be a DFA recognizing the language A.

A is regular language.

Machine M could be transformed into DFA M' which recognizing NOEXTEND(A)

$M' = (Q, \Sigma, \delta, q_0, F/G)$, $G \subseteq F$.

1.46

a. $\{0^n 1^m 0^n \mid m, n \geq 0\}$

Assume the language A is regular.

Using the pumping Lemma for regular language.

There exists a pumping length p for A for any string $s \in A$, $|s| \geq p$, $s = xyz$.
We must satisfy three condictions.

① $|y| > 0$
② $|xy| \leq p$
③ $\forall i > 0$, $xy^iz \in A$

Let $s = 0^p 1 0^p$

From ①②: x, y are composed of 0, $y = 0^k$ $(k > 0)$
From ③: when $i = 0$, string will still in A which means xy^0z will still in A.

$xy^0z = xz = 0^{p-k}$ $\Rightarrow$ it doesn't belong to A.

So, it's contradiction by the pumping Lemma.
Thus, $A = \{0^n 1^m 0^n \mid m, n > 0\}$ is not regular language.

c. $\{w \mid w \in \{0,1\}^+$ is not a palindrome$\}^8$

Assume language C $\{w \mid w \in \{0,1\}^+$ is a palindrome$\}^8$ is regular.
Using the pumping Lemma
There exists a pumping length p for C
Let the string $s \in C$, $s = 0^p 1 0^p$. $|s| > p$, $s = xyz$
We must satisfy three condictions.

① $|y| > 0$
② $|xy| \leq p$
③ $\forall i > 0$, $xy^iz \in C$.

From ①, ② $\Rightarrow$ $y = 0^k$ $(0 < k \leq p)$
From ③: let $i = 2$, $xy^iz = xy^2z = 0^{p+k} 1 0^p$
$\Rightarrow$ it doesn't belong to C.
$p + k > p$ $(0 < k \leq p)$
$\therefore$ The start of string 0 would always more than the end of string 0.
$\Rightarrow$ It is not a palindrome.

It's contradiction for condiction ③.
Thus, language C is not a regular language.

(d) $\{wtw \mid w, t \in \{0,1\}^+\}$.

Assume language $D = \{wtw \mid w, t \in \{0,1\}^+$ is regular.

Using the pumping Lemma

There exists a pumping length p for D.

Let $s = 0^{p+k}110^p1$, then $s \in D$, $|s| > p$. let $s = xyz$.

We must satisfy three conditions.

① $|y| > 0$

② $|xy| \le p$

③ $\forall i > 0$, $xy^iz \in C$.

From ①, ② $\Rightarrow y = 0^k \quad (0 < k \le p)$

From ③: let $i = 2$, $xy^iz = xy^2z = 0^{p+k}110^p1$.

Assume $0^{p+k}110^p1$ is in D.

We need to written as $wtw \Rightarrow w = 0^{p+k}1$, $t = 1$, $w = 0^p1$.

w must end with 1 from last part of string

So, the first part of string $w = 0^{p+k}1$.

As we know, the end part of string have p 0s.

So, it's contradiction, because w can't equal to $0^{p+k}1$.

$\Rightarrow xy^2z = 0^{p+k}110^p1 \notin D$

Thus, the language D is not regular.

$\boxed{1.49}$

a. $B = \{1^ky \mid y \in \{0,1\}^*$ and y contains at least k 1s, for $k \ge 1.\}$

The language B can be written as:

$$B = 11^ky = 1X \quad \text{where } X = 1^ky.$$

$\because$ Given. y contains at least k 1s.

$\therefore X = 1^{k-1}y$ contains $(k-1)+k$ 1s.

$\Rightarrow$ Total number of 1s in $1^{k-1}y$ is $(2k-1)$.

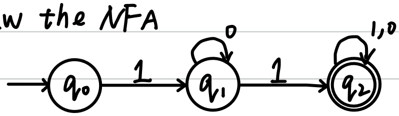
$\because$ For language B, given $k \ge 1$.

$\therefore$ when $k=1$, the $X = 1^{k-1}$ contains at least $(2k-1)$ 1s.

$2k-1 = 2(1)-1 = 1 \Rightarrow X$ contains at least 1 1s.

$\Rightarrow B = \{1X \mid X \in \{0,1\}^*$ X contain at least 1 1s.$\}$

Draw the NFA



We got that it exists NFA for B.

Thus, the language B is regular.

(b). $C = \{1^k y \mid y \in \{0,1\}^*$ and y contains at most k 1s, for $k \geq 1\}$.

Using Pumping lemma.

Assume language C is regular language.

There exists a pumping length p for D.

Let $s = 1^p 0 1^p$, then $s \in C$, $|s| > p$. let $s = xyz$.

We must satisfy three conditions.

① $|y| > 0$

② $|xy| \leq p$

③ $\forall i > 0$, $xy^iz \in C$.

$\because$ the first part of string s is made of 1's only, the xy is only made of 1s.

$\therefore$ On pumping up y part of string

The number of 1s in the first part would greater than the number of 1s in second part.

when $i=0$, $\Rightarrow xy^0z$

On pumping up y part of string.

The number of 1s in the second part would greater than the number of 1s in first part.

So, it's contradiction

Thus, the language C is not regular.