CS452 HW5

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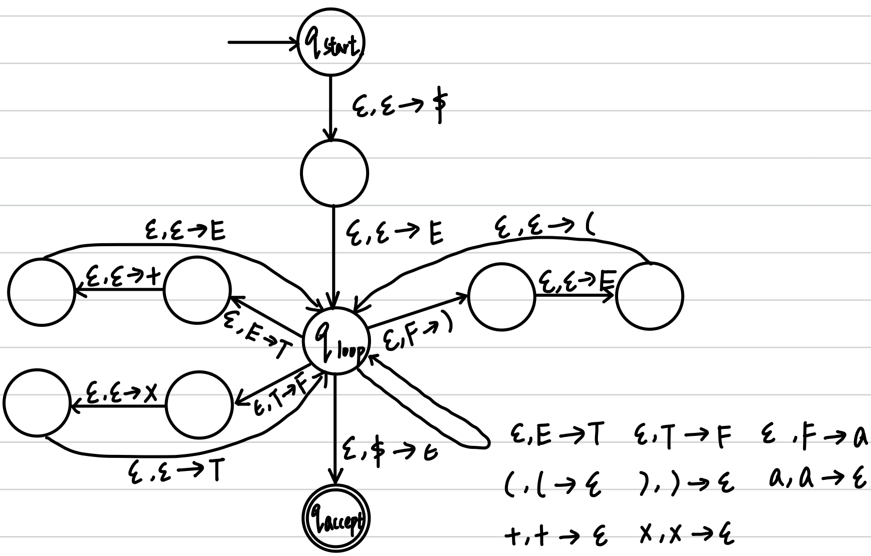
2.11

Theorem 2.20: A language is context free iff some pushdown automaton recognizes it.

G_4: $E \to E+T \mid T$

$T \to T \times F \mid F$

$F \to (E) \mid a$



2.25

For any language A, Let SUFFIX(A) = {v | uv ∈ A for some string u}. Show that the class of context-free languages is closed under the SUFFIX operation.

We would prove this problem by construct a PDA which would accept SUFFIX(A).

Language A is context-free.

Let the PDA for language A be M_1.

The PDA for SUFFIX(A) be M.

Follow the step to construct a PDA M.

① Making a copy of M_1 and call it as M_2. The PDA M_2 has the same transitions as M_1. its a replica of M_1. M_1 and M_2 will form the whole PDA M.

② Changing the input transitions of M_2 to ε. If the input transition has $0,1 \to \varepsilon$, change it to $\varepsilon,1 \to \varepsilon$. If the input transition $0,1 \to \varepsilon$ is 0 and it is changed $\varepsilon,1 \to \varepsilon$ where the stack symbol ε is unchanged.

③ For each state in M_2, adding a transition $\varepsilon,\varepsilon \to \varepsilon$ to corresponding state in M_1.

④ Let the start of M_2 be the start of the whole PDA.

Therefore, we can see M will construct the stack and ignore the input.

M transitions indefinitely from the state in M_2 to the corresponding state in M_1, then start the first character of the suffix as input and then transition from M_1.

Thus, all the suffixes of the string belongs to A will accepted by M.

2.30

a. $\{0^n1^n0^n1^n \mid n \geq 0\}$

Let $A = \{0^n1^n0^n1^n \mid n \geq 0\}$

Assume the language A is context free.

Let P be the pumping length

Let string $s = 0^P1^P0^P1^P \in A$

The pumping lemma says that for some split $s = uvxyz$ all following condition hold $|vxy| \leq P$, $|vy| > 0$.

Case 1: If vxy is in the first half of string, 0^P1^P is the suffix of uv^0xy^0z.

The length of uv^0xy^0z is less than 4P.

$\Rightarrow$ It could not be of the $0^n1^n0^n1^n$.

Similar, it could not be of $0^n1^n0^n1$, if vxy is in another half of string.

Case 2: If vxy is in the middle. $|vxy| \leq P$. 01^P is the suffix of uv^0xy^0z.

The length of uv^0xy^0z is less than 4P

$\Rightarrow$ It also could not be of the $0^n1^n0^n1^n$.

Thus, it's a contradiction. The language A is not context free.

d. $\{t_1\#t_2\#\cdots\#t_k \mid k \geq 2$, each $t_i \in \{a,b\}^*$, and $t_i = t_j$ for some $i \neq j\}$.

Let $D = \{t_1\#t_2\#\cdots\#t_k \mid k \geq 2$, each $t_i \in \{a,b\}^*$, and $t_i = t_j$ for some $i \neq j\}$.

Assume the language D is context free.

Let P be the pumping length

Let string $s = 0^P 1^P \# 0^P 1^P \in D$.

The pumping lemma says that for some split $s = uvxyz$ all following condition hold $|vxy| \leq P$, $|vy| > 0$.

Case 1: vxy contains #.

$\Rightarrow$ The pumping uv^0xy^0z will remove # from string.

That is not in the language D.

Case 2: vxy not contains #.

then the vxy only can be the one side of #.

$\Rightarrow$ uv^2xy^2z result in string.

That is also not in the language D

Thus, it's a contradiction. The language D is not context free.

2.32 $\Sigma = \{1, 2, 3, 4\}$

$C = \{w \in \Sigma^* \mid$ in w, the number of 1s equal numbers of 1s equals the number of 2s, and the number of 3s equals the number of 4s.$\}$

Assume the language C is context free.

Let P be the pumping length

Let string $s = 1^P 3^P 2^P 4^P \in C$

The pumping lemma says that for some split $s = uvxyz$ all following condition hold $|vxy| \leq P$, $|vy| > 0$

Case 1: vxy is substring of $1^P 3^P$

$\Rightarrow$ It's in uv^2xy^2z.

$\Rightarrow$ The number of 1s is greater than 2s.

or the number of 3s is greater than 4s. or both.

$\Rightarrow$ $uv^2xy^2z \notin C$.

Case 2: vxy is substring of $3^P 2^P$

$\Rightarrow$ It's in uv^2xy^2z.

$\Rightarrow$ The number of 2s is greater than 1s.

or the number of 3s is greater than 4s. or both.

$\Rightarrow$ $uv^2xy^2z \notin C$

Case 3: vxy is substring of $2^p 4^p$

$\Rightarrow$ It's in uv^2xy^2z.

$\Rightarrow$ The number of 1s is greater than 2s.

or the number of 3s is greater than 4s. or both.

$\Rightarrow uv^2xy^2z \notin C$.

Thus, it's contradictions. The language C is not context free.

$\boxed{2.48}$ $C_1 = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^*, \text{ where } |x| = |z| \geq |y|\}$

$C_2 = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^* 1 \Sigma^*, \text{ where } |x| = |z| \geq |y|\}$.

① Using pumping lemma.

For a CFL L. there exist an integer n. $X \in L$, $|X| \geq n$.

there exists $u, v, x, y, z \in \Sigma^*$. $X = uvxyz$.

then $|vxy| \leq n$, $|vy| \geq 1$, for all $i \geq 0$, $uv^ixy^iz \in L$.

Let string $S =$ abcde of language C_1.

Following pumping lemma conditions $q \geq 1$, $|s| \geq q$.

1. $|bcd| \leq q$
2. $|bd| \geq 1$
3. ab^xcd^xe is in L for all $x \geq 0$.

② Assume C_2 is CFL.

Using pumping Lemma $S = 0^{p+2} 1 0^p 1 0^{p+2} \in C_2$.

Let the pumping length is p.

The pumping lemma says that for some split $s = uvxyz$ all following condition hold $|vxy| \leq p$, $|vy| > 1$.

$uv^kxy^kz \in C_2$. for all $k \geq 0$.

Then considering several according to the possible decompositions of s.

Since $|vxy| \leq p$.

$\Rightarrow$ there are at most two of string of 0s can be pumped

So, it's contradiction. The language C_2 is not CFL.

2.24 $E = \{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$

Consider the language E as following three languages:

$E_1 = \{a^i b^j \mid i < j\}$

$E_2 = \{a^i b^j \mid i < j < 2i\}$

$E_3 = \{a^i b^j \mid 2i < j\}$

For E_1 build the grammar as follows:

$S \to aSb \mid aS_1$

$S_1 \to aS_1 \mid \varepsilon$

For E_2 build the grammar as follows.

$S \to aSb \mid aS_1b$

$S_1 \to aS_1bb \mid aS_2bb$

$S_2 \to \varepsilon$

For E_3 build the grammar as follows.

$S \to aSbb \mid S_1b$

$S_1 \to S_1b \mid \varepsilon$

Therefore, S can generate $a^n a^m b^{2m} b^n$ that is equivalent to $a^{n+m} b^{n+2m}$. $(m, n > 0)$.

2.37

Assume A is generated by a CFG $G = (V, \Sigma, R, S)$ in CNF.

$k = 2^{2|V|+1}$

Let string s be any in language A with the length $\geq k$.

The smallest size parse-tree T for s.

The depth of $T \leq 2|V|+1$.

when a path P in T has length at least $2|V|+1$.

It must be the case that a variable R appears at least 3 times in path.

Let R occurs at least 3 times, the lowest $2|V|+1$ in P.

Let t be the leaves of the subtree of T rooted at the lowest of these R's, and t is not empty. Then the leaves of the subtree of rooted at the second of these R's

Can be written as the concatenation of three string s_1, t and s_2 as $s_1 t s_2$.

The leaves subtree of T rooted at top most R can be written as the concatenation of five string r_1, s_1, t, s_2, r_2 as $r_1 s_1 t s_2 r_2$.

$\Rightarrow$ It must be the case that s_1 and s_2 are not empty.

Or, we can substitute the tree rooted at lowest R as the tree for middle R.

As the same, we also get It must be the case that r_1 and r_2 are not empty.

Case 1: r_1 and s_1 are empty.

$\Rightarrow V = t,\ X = s_2,\ y = r_2$

Case 2: r_2 and s_2 are empty.

$\Rightarrow V = r_1,\ X = s_1,\ y = t$.

As same, other cases can also show that it's satisfied conditions of pumping lemma.