Homework 29

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Minimum of Probability of Bloom Filter Error

$$P(h) = (1-\frac{1}{n})^u(1-(1-\frac{1}{m})^{uh})^h$$

$$\begin{aligned}
P'(h) &= (1-\frac{1}{n})^u(ln(1-(1-\frac{1}{m})^{uh}) * (1-(1-\frac{1}{m})^{uh})^h - uh * ln(1-\frac{1}{m}) * (1-\frac{1}{m})^{uh} * (1-(1-\frac{1}{m})^{uh})^{h-1}) \\
&= (1-\frac{1}{n})^u(ln(1-Q) * (1-Q)^h - uh * ln(1-\frac{1}{m}) * Q * (1-Q)^{h-1}) \\
& \quad for\ Q = (1-\frac{1}{m})^{uh}
\end{aligned}$$

$$\begin{gathered}
Note\ that\ e^{-x} = 1 - x + \frac{x^2}{2} - \ldots \approx 1 - x\ for\ small\ x \\
\implies Q = (1-\frac{1}{m})^{uh} \approx e^{-\frac{uh}{m}}\ and\ (1-\frac{1}{m}) \approx e^{-\frac{1}{m}} \\
If\ P'(h) = 0,\ then \\
ln(1-e^{-\frac{uh}{m}}) * (1-e^{-\frac{uh}{m}})^h = uh * ln(e^{-\frac{1}{m}}) * e^{-\frac{uh}{m}} * (1-e^{-\frac{uh}{m}})^{h-1} \\
\implies ln(1-e^{-\frac{uh}{m}}) * (1-e^{-\frac{uh}{m}}) = -\frac{uh}{m} * e^{-\frac{uh}{m}} \\
\implies ln(1-e^{-\frac{uh}{m}}) * (1-e^{-\frac{uh}{m}}) = -\frac{uhe^{-\frac{uh}{m}}}{m} \\
\implies h = \frac{m}{u} ln2
\end{gathered}$$