

36225 – Introduction to Probability Theory

Fall 2020

Homework 1

Due Friday, September 11th at noon

The following section and problem numbers refer to the book page images posted on Canvas, which may differ from the book version you have. Be sure you are working on the right problems. Answering the wrong homework problem lead to you receiving zero points for that problem.

You will upload your homework assignment to Gradescope. PDFs generated via scan or via export from, e.g., LaTeX are preferred, although JPG files will also be accepted. Late submissions are not allowed.

1. (10 points) Exercise 2.6
2. (10 points) Exercise 2.8
3. (10 points) A partition of the sample space S is a collection of disjoint events S_1, S_2, \dots, S_n such that $S = \bigcup_{i=1}^n S_i$.

(a) Show that for any event A , we have

$$P(A) = \sum_{i=1}^n P(A \cap S_i)$$

(b) Use part (a) to show that for any event A, B, C we have

$$P(A) = P(A \cap B) + P(A \cap C) + P(A \cap \bar{B} \cap \bar{C}) - P(A \cap B \cap C)$$

4. (10 points) A four-sided die is rolled repeatedly, until the first time (if ever) that an even number is obtained. What is the sample space for this experiments?
5. (10 points) A factory operates three different shifts. Over the last year, 200 accidents have occurred at the factory. Some of these can be attributed to unsafe working conditions, whereas the others are unrelated to working conditions. The table gives the percentage of accidents falling in each type of accident-shift category.

	Unsafe	Unrelated to
Shift	Conditions	Conditions
Day	10%	35%
Swing	8%	20%
Night	5%	22%

Suppose one of the 200 accident reports is randomly selected and the shift and type of accident are determined.

- (a) What are the simple events?
 - (b) What is the probability that the selected accident was attributed to unsafe conditions?
 - (c) What is the probability that the selected accident did not occur on the day shift?
6. (10 points) Exercise 2.18
 7. (10 points) Exercise 2.74
 8. (10 points) Exercise 2.76
 9. (10 points) We are given three fair coins: one has heads in both faces, the second has tails in both faces, and the third has a head in one face and a tail in the other. We choose a coin at random, toss it, and the result is heads. What is the probability that the opposite face is tails?
 10. (10 points) The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The table gives the proportions of individuals in the various ethnic group-blood group combinations.

Ethnic Group	Blood Group			
	O	A	B	AB
1	.082	.106	.008	.004
2	.135	.141	.018	.006
3	.215	.200	.065	.020

Suppose that an individual is randomly selected from the population, and define events by $A = \{\text{type A selected}\}$, $B = \{\text{type B selected}\}$, and $C = \{\text{ethnic group 3 selected}\}$.

- (a) Calculate $P(A)$, $P(C)$, and $P(A \cap C)$.
- (b) Calculate $P(A \mid C)$ and $P(C \mid A)$, and explain in context what each of these probabilities represents.
- (c) If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?

Be sure:

- To provide a clean, easily readable scan of your assignment, either through the use of a scanner or your phone.
- To make this work your own even if you work with others! Do not blindly copy the solutions; this is plagiarism, and furthermore it will only hurt you at exam time.
- To properly label all plots (axes, main title, units, etc.) for full credit, if applicable, and to include a printout of any code used in solutions, if applicable.