

第5回レポート解答

34.

$$\frac{\frac{\frac{2}{\exists x A(x)} \frac{\frac{1}{\forall x \neg A(x)}}{\frac{3}{\neg A(a) \quad A(a)}} \frac{3}{\perp}}{\frac{3}{\perp}} \frac{2}{\perp}}{\frac{1}{\neg \exists x A(x)}} \frac{1}{\forall x \neg A(x) \supset \neg \exists x A(x)}$$

50.

$\forall x \neg A(x) \vdash \neg \exists x A(x)$ の応用

$$\frac{\frac{\frac{1}{\forall x A(x)} \quad 3}{A(a) \quad \neg A(a)} \quad 3}{\perp} \quad 3$$

$$\frac{\frac{2}{\exists x \neg A(x)} \quad \perp}{\perp} \quad 2$$

$$\frac{\frac{2}{\neg \exists x \neg A(x)}}{1}$$

$$\frac{\forall x A(x) \vdash \neg \exists x \neg A(x)}{1}$$

33再掲.

$$\frac{\frac{\frac{\neg \exists x A(x) \quad \frac{A(a)}{\exists x A(x)}}{\perp} 2}{\neg A(a)} 2}{\forall x \neg A(x)} 1}{\neg \exists x A(x) \supset \forall x \neg A(x)}$$

46再掲.

$$\begin{array}{c} \text{3} \\ \neg \exists x \neg \text{と } \forall x \text{ の} \\ \text{同値性} \end{array} \frac{\neg \exists x \neg A(x)}{\forall x A(x) \quad \neg \forall x A(x)} \quad 1$$
$$\frac{}{\perp} \quad 3$$
$$\frac{\neg \neg \exists x \neg A(x)}{\exists x \neg A(x)} \quad \text{二重否定の除去}$$
$$\frac{\exists x \neg A(x)}{\neg \forall x A(x) \cup \exists x \neg A(x)} \quad 1$$

51.

$\neg \forall x A(x) \vdash \exists x \neg A(x)$ の応用

$$\begin{array}{c}
 & 3 \\
 & \neg \exists x A(x) \\
 \neg \exists x \text{と} \forall x \neg \text{の} & \hline & 1 \\
 \text{同値性} & & \\
 & \forall x \neg A(x) & \neg \forall x \neg A(x) \\
 & \hline & \\
 & \perp & \\
 & \hline & 3 \\
 & \neg \neg \exists x A(x) & \\
 & \hline & \text{二重否定の除去} \\
 & \exists x A(x) & \\
 & \hline & 1 \\
 & \neg \forall x \neg A(x) \vdash \exists x A(x) &
 \end{array}$$

51. 誤答？例

$$\begin{array}{c} \text{3} \\ \neg \exists x \neg \text{と } \forall x \text{ の} \\ \text{同値性} \end{array} \frac{\neg \exists x \neg \neg A(x)}{\begin{array}{c} \forall x \neg A(x) \quad \neg \forall x \neg A(x) \\ \hline \perp \end{array}} \quad 1$$
$$\frac{\neg \neg \exists x \neg \neg A(x)}{\begin{array}{c} \hline \end{array}} \quad 3$$

二重否定の除去

$$\begin{array}{c} \exists x \neg \neg A(x) \\ \hline \end{array} \quad \text{二重否定の除去}$$
$$\frac{\exists x A(x)}{\begin{array}{c} \hline \end{array}} \quad 1$$
$$\frac{\neg \forall x \neg A(x) \cup \exists x A(x)}{\begin{array}{c} \hline \end{array}}$$

51. 誤答？例

直接は除去できない
↓
今回示す
置換定理を用いれば
証明できる

$$\begin{array}{c} \text{3} \\ \neg \exists x \neg \neg A(x) \\ \hline \neg \exists x \neg \neg A(x) \\ \text{1} \\ \forall x \neg A(x) \quad \neg \forall x \neg A(x) \\ \hline \perp \\ \hline \text{3} \\ \neg \neg \exists x \neg \neg A(x) \\ \hline \text{二重否定の除去} \\ \exists x \neg \neg A(x) \\ \hline \text{二重否定の除去？} \\ \exists x A(x) \\ \hline \text{1} \\ \neg \forall x \neg A(x) \cup \exists x A(x) \end{array}$$