

35' $\forall x(A(x) \vee B(x)) \supset \forall xA(x) \vee \forall xB(x)$ の反例

$A(x) : x=0, B(x) : x=1$ とする.

モデル $M = \langle D, I \rangle$ を以下のように定める.

(記述を簡単にするため, 記号の $0, 1, =$ とモデルの要素の $0, 1, =$ を同一視する)

$D = \{0, 1\}, I(=)(0, 0) = I(=)(1, 1) = \top, I(=)(1, 0) = I(=)(0, 1) = \perp$

これが反例になる, すなわち解釈したら \perp になることをしめす,

F_1 を $\forall x(x=0 \vee x=1)$, F_2 を $\forall x(x=0)$, F_3 を $\forall x(x=1)$ とする. 与式 F は $F_1 \supset F_2 \vee F_3$ である.

F_1, F_2, F_3 を解釈する.

$I_\rho(F_1) = \forall^*(\{I_\rho[a/x][x=0 \vee x=1] \mid a \in \{0, 1\}\})$.

ここで $I_\rho[0/x][x=0 \vee x=1] = \vee^*(I(0=0), I(0=1)) = \vee^*(\top, \perp) = \top$.

同様に $I_\rho[1/x][x=0 \vee x=1] = \top$.

ゆえに $I_\rho(F_1) = \forall^*(\{I_\rho[a/x][x=0 \vee x=1] \mid a \in \{0, 1\}\}) = \forall^*(\{\top\}) = \top$.

一方, $I_\rho(F_2) = \forall^*(\{I_\rho[a/x][x=0] \mid a \in \{0, 1\}\}) = \forall^*(\{\top, \perp\}) = \perp$.

$I_\rho(F_3) = \forall^*(\{I_\rho[a/x][x=1] \mid a \in \{0, 1\}\}) = \perp$.

したがって, $I_\rho(F) = \supset^*(\top, \vee^*(\perp, \perp)) = \perp$.

36' は各自で解いてほしい.

37.

1
 $\neg A$

2
 A

\perp
2
 $A \supset \perp$

$\neg A \supset (A \supset \perp)$ 1

38.

$$\begin{array}{c} \begin{array}{cc} 1 & 2 \\ A \supset \bot & A \end{array} \\ \hline \bot \\ \hline 2 \\ \neg A \\ \hline 1 \\ (A \supset \bot) \supset \neg A \end{array}$$

39.

$$\begin{array}{c}
 \begin{array}{c}
 \begin{array}{c}
 \text{1} \\
 \exists x \neg A(x)
 \end{array}
 \end{array}
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 \text{2} \\
 \neg A(a)
 \end{array}
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 \text{3} \\
 \forall x A(x)
 \end{array} \\
 \hline
 A(a)
 \end{array}
 \end{array}
 \begin{array}{c}
 \hline
 \perp
 \end{array}
 \begin{array}{c}
 \text{2}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 \begin{array}{c}
 \perp
 \end{array}
 \end{array}
 \begin{array}{c}
 \text{3} \\
 \neg \forall x A(x)
 \end{array}
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 \text{1} \\
 \exists x \neg A(x) \supset \neg \forall x A(x)
 \end{array}
 \end{array}$$

40.

$$\frac{\frac{\frac{1}{\exists x(A(x) \wedge B)} \quad \frac{2}{\exists x A(x)}}{\exists x A(x)} \quad \frac{2}{A(a) \wedge B}}{A(a)}$$
$$\frac{\frac{1 \quad \frac{A(a) \wedge B}{\quad}}{\exists x(A(x) \wedge B) \quad B}}{B} 3$$
$$\frac{\exists x A(x) \wedge B}{\exists x (A(x) \wedge B) \supset \exists x A(x) \wedge B} 1$$

41.

1

$$\exists x A(x) \wedge B$$

2

$$A(a)$$

B

1

$$\exists x A(x) \wedge B$$
$$A(a) \wedge B$$
$$\exists x A(x)$$
$$\exists x(A(x) \wedge B)$$

2

$$\exists x(A(x) \wedge B)$$

1

$$\exists x A(x) \wedge B \supset \exists x (A(x) \wedge B)$$

42.

$$\begin{array}{c}
 \begin{array}{c}
 1 \\
 \hline
 \forall x(A(x) \supset B)
 \end{array} \\
 \begin{array}{c}
 3 \\
 A(a)
 \end{array}
 \quad
 \begin{array}{c}
 \hline
 A(a) \supset B
 \end{array} \\
 \hline
 \begin{array}{c}
 2 \\
 \exists x A(x)
 \end{array}
 \quad
 \begin{array}{c}
 \hline
 B
 \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \hline
 B
 \end{array}
 \quad
 3$$

$$\begin{array}{c}
 \hline
 \begin{array}{c}
 2 \\
 \exists x A(x) \supset B
 \end{array}
 \end{array}$$

$$\hline
 \begin{array}{c}
 1 \\
 \forall x(A(x) \supset B) \supset (\exists x A(x) \supset B)
 \end{array}$$

43.

$$\begin{array}{c}
 \begin{array}{cc}
 1 & 2 \\
 \exists x A(x) \supset B & \frac{A(a)}{\exists x A(x)} \\
 \hline
 \end{array} \\
 \\
 \begin{array}{c}
 \frac{B}{A(a) \supset B} \quad 2 \\
 \hline
 \forall x (A(x) \supset B)
 \end{array} \\
 \hline
 1 \\
 (\exists x A(x) \supset B) \supset \forall x (A(x) \supset B)
 \end{array}$$

44.

$$\begin{array}{c} \begin{array}{ccc} & 1 & 3 \\ & \neg A \supset B & \neg A \\ \hline & B & \\ \hline \end{array} \\ \begin{array}{ccc} 2 & & \\ \hline A & & \\ \hline \end{array} \\ \begin{array}{ccc} A \vee \neg A & A \vee B & A \vee B \\ \hline & & 2,3 \\ & A \vee B & \\ \hline & & 1 \\ & (\neg A \supset B) \supset A \vee B & \end{array} \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\frac{\neg \exists x \neg A(x) \quad 1}{\exists x \neg A(x)} \quad \frac{A \neg(a)}{\exists x \neg A(x)} \quad 2}{\perp} \\
 \frac{\perp}{\neg \neg A(a)} \quad 2 \\
 \frac{\neg \neg A(a)}{A(a)} \\
 \frac{A(a)}{\forall x A(x)} \\
 \frac{\forall x A(x)}{\neg \exists x A(x) \subset \forall x A(x)} \quad 1
 \end{array}$$

45.

$$\begin{array}{c}
 \begin{array}{c}
 \frac{\frac{1}{\neg \exists x \neg A(x)} \quad \frac{\frac{2}{\neg A(a)}}{\exists x \neg A(x)}}{\perp} \quad 2 \\
 \hline
 \neg \neg A(a)
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{\frac{\frac{\neg \neg A(a) \quad \neg A(a)}{\perp}}{A(a)} \quad 3 \quad \frac{A(a) \vee \neg A(a)}{A(a)} \quad 3}{A(a)} \quad 4 \\
 \hline
 \neg \neg A(a) \supset A(a)
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{A(a)}{\forall x A(x)} \quad 1}{\neg \exists x \neg A(x) \supset \forall x A(x)}
 \end{array}$$

45別解

$$\begin{array}{c}
 \begin{array}{c} 1 \\ \neg \exists x \neg A(x) \end{array}
 \qquad
 \begin{array}{c} 2 \\ \neg A(a) \end{array} \\
 \hline
 \exists x \neg A(x)
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c} \neg \neg A(a) \\ \hline A(a) \end{array}
 \qquad
 \begin{array}{c} \perp \\ \hline \neg \neg A(a) \end{array}
 \qquad
 \begin{array}{c} 4 \\ \neg A(a) \end{array} \\
 \hline
 \begin{array}{c} A(a) \vee \neg A(a) \end{array}
 \qquad
 \begin{array}{c} 3 \\ A(a) \end{array}
 \qquad
 \begin{array}{c} \perp \\ \hline A(a) \end{array}
 \end{array}$$

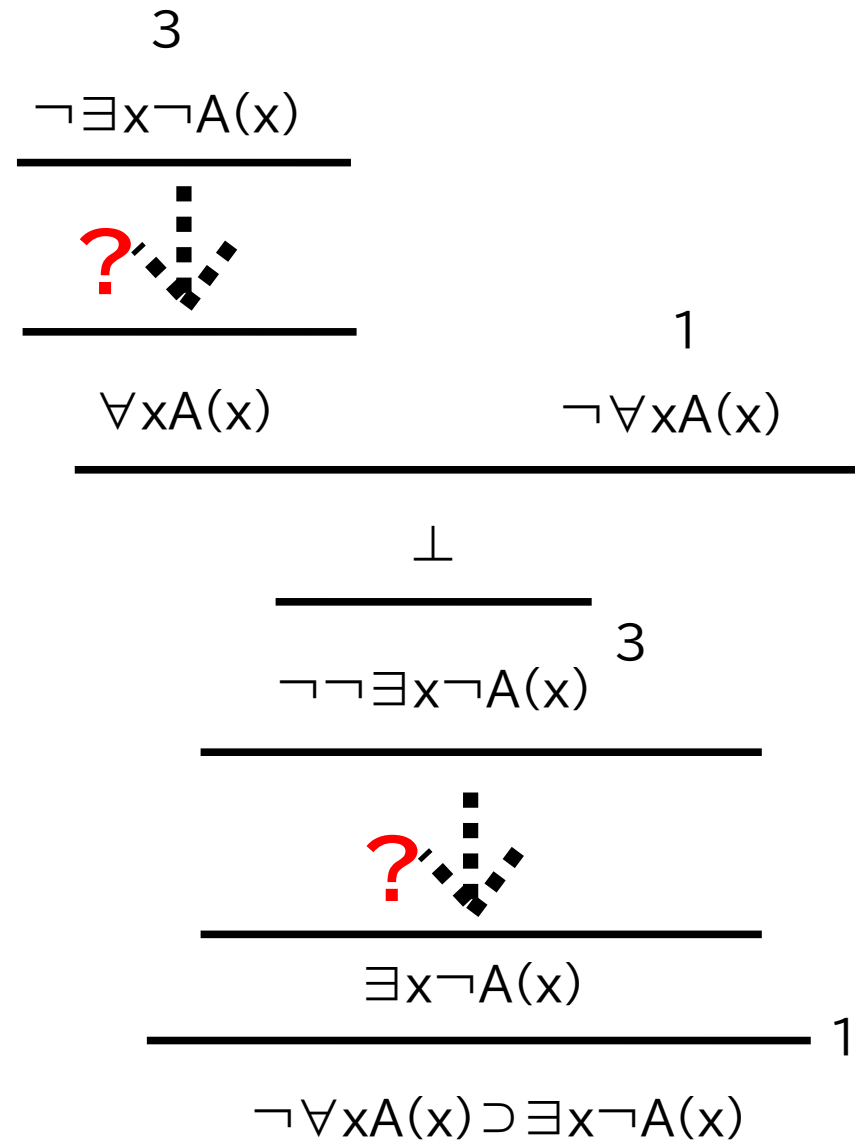
$$\begin{array}{c}
 \hline
 A(a)
 \end{array}$$

$$\begin{array}{c}
 \forall x A(x) \\
 \hline
 \neg \exists x \neg A(x) \supset \forall x A(x) \qquad 1
 \end{array}$$

45別解

$$\begin{array}{c}
 \frac{\frac{\frac{}{A(a) \vee \neg A(a)}}{A(a)} \quad \frac{\frac{\frac{\frac{}{\neg \exists x \neg A(x)}}{\neg \exists x \neg A(x)} \quad \frac{\frac{\frac{}{\neg A(a)}}{\exists x \neg A(x)}}{\perp}}{A(a)}}{3,4} \\
 \frac{\frac{}{A(a)}}{\forall x A(x)} \\
 \frac{}{\neg \exists x \neg A(x) \supset \forall x A(x)} \quad 1
 \end{array}$$

46.



46.

$$\begin{array}{c} 3 \\ \neg \exists x \neg A(x) \end{array}$$

$\neg \exists x \neg$ と $\forall x$ の同値性

$$\begin{array}{c} 1 \\ \forall x A(x) \quad \neg \forall x A(x) \end{array}$$

\perp

$$\begin{array}{c} 3 \\ \neg \neg \exists x \neg A(x) \end{array}$$

二重否定の除去

$$\exists x \neg A(x)$$

$$\neg \forall x A(x) \supset \exists x \neg A(x) \quad 1$$

[illegible]

推論規則を

例 二重否定の除去

$$\begin{array}{c}
 \\
 \\
 \frac{\frac{\frac{A \vee \neg A}{\quad} \quad A}{\quad} \quad \frac{\frac{\frac{\neg\neg A}{\quad} \quad \neg A}{\quad}}{\perp}}{A} \quad 1,2 \\
 \frac{\neg\neg A \quad \frac{A}{\neg\neg A \supset A} \quad 3}{A}
 \end{array}$$

証明図内で数ステップの推論を
まとめて表すときには2本線

「 」

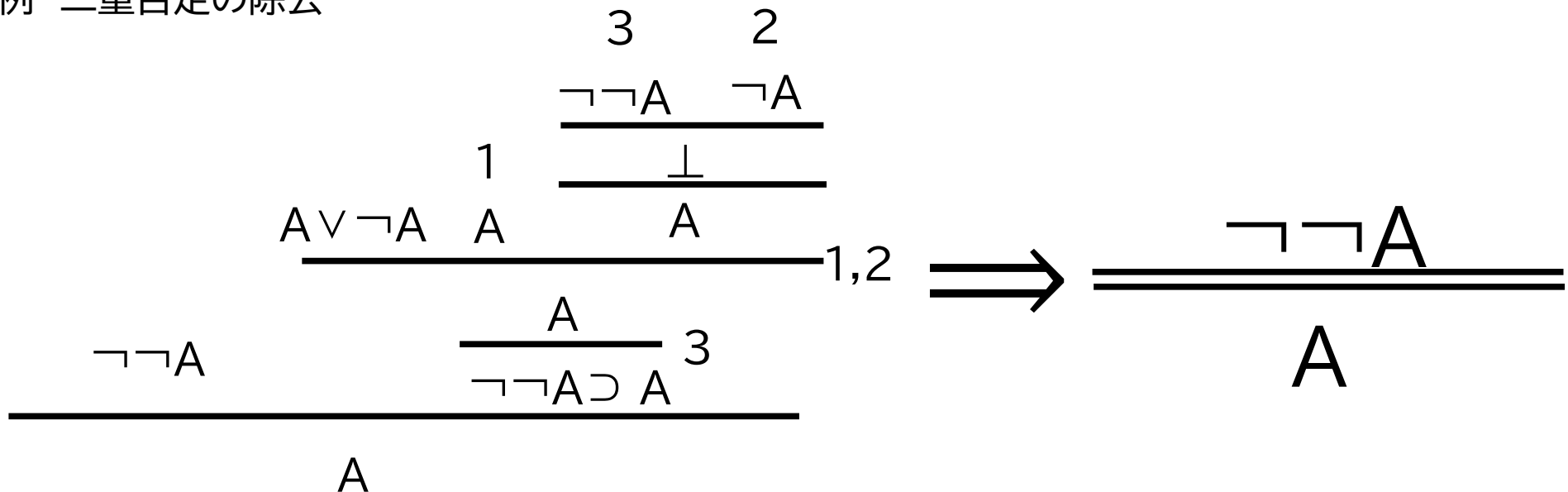
を用いる.

推論規則を組み合わせてできる

推論規則を

擬似推論規則(derived rule)と呼ぶ

例 二重否定の除去



46.

$$\begin{array}{c}
 \neg \exists x \neg \text{と } \forall \text{ の} \\
 \text{同値性}
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 3 \\
 \neg \exists x \neg A(x)
 \end{array} \\
 \hline
 \begin{array}{c}
 \forall x A(x) \qquad \neg \forall x A(x) \\
 \hline
 \perp
 \end{array} \\
 \begin{array}{c}
 \hline
 \neg \neg \exists x \neg A(x) \quad 3 \\
 \hline
 \exists x \neg A(x)
 \end{array} \\
 \hline
 \neg \forall x A(x) \supset \exists x \neg A(x) \quad 1
 \end{array}
 \begin{array}{c}
 1
 \end{array}
 \end{array}$$

証明図内で数ステップの推論を
まとめて表すときには2本線

「 」

を用いる.

推論規則を組み合わせてできる

推論規則を

擬似推論規則(derived rule)と
よぶ

47.

$$\begin{array}{c}
 \begin{array}{c}
 1 \\
 \hline
 \forall x A(x) \vee B
 \end{array}
 \qquad
 \begin{array}{c}
 \begin{array}{c}
 2 \\
 \hline
 \forall x A(x) \\
 \hline
 A(a) \\
 \hline
 A(a) \vee B \\
 \hline
 \forall x (A(x) \vee B)
 \end{array}
 \qquad
 \begin{array}{c}
 \begin{array}{c}
 2 \\
 B \\
 \hline
 A(a) \vee B \\
 \hline
 \forall x (A(x) \vee B)
 \end{array}
 \end{array}
 \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \hline
 2 \\
 \forall x (A(x) \vee B)
 \end{array}$$

$$\begin{array}{c}
 \hline
 1 \\
 \forall x A(x) \vee B \supset \forall x (A(x) \vee B)
 \end{array}$$

48.

[illegible]