解

$M[\ \forall z \exists w(z=2w+x \vee z=2w+y)\ ]_{\rho}$ =true

$\Leftrightarrow M[\ \exists w(z=2w+x \vee z=2w+y)\ ]_{\rho[v/z]}$ =true for every v∈N

$\Leftrightarrow M[\ z=2w+x \vee z=2w+y\ ]_{(\rho[v/z])[u/w]}$ =true for every v∈N, for some u∈N

$\Leftrightarrow M[\ z=2w+x\ ]_{(\rho[v/z])[u/w]}$ =true or $M[\ z=2w+y\ ]_{(\rho[v/z])[u/w]}$ =true

for every v∈N, for some u∈N

$\Leftrightarrow =^{M}((\rho[v/z])_{[u/w](z)}, +^{M}(\cdot^{M}(2^{M}, (\rho[v/z])[u/w](w)), (\rho[v/z])[u/w](x)))$ =true

or

$=^{M}((\rho[v/z])[u/w](z), +^{M}(\cdot^{M}(2^{M}, (\rho[v/z])[u/w](w)), (\rho[v/z])[u/w](y)))$ =true

for every v∈N, for some u∈N

$\Leftrightarrow =^{M}(v, +^{M}(\cdot^{M}(2, u), 2))$ =true or $=^{M}(v, +^{M}(\cdot^{M}(2, u), 3))$ =true for every v∈N,

for some u∈N

⇔ vが2以上の偶数のときu=(v-2)/2, 3以上の奇数のときu=(v-3)/2をとればtrue

vが0, 1のときはuをどんな(0以上の)値にしてもfalse

⇔ false

解

全てのvに対し，その1つ1つに条件を満たすuが存在する

$M[\ \forall z \exists w(z=2w+x \vee z=2w+y)\ ]_\rho = \text{true}$

$\Leftrightarrow M[\ \exists w(z=2w+x \vee z=2w+y)\ ]_{\rho[v/z]} = \text{true for every } v \in N$

$\Leftrightarrow M[\ z=2w+x \vee z=2w+y\ ]_{(\rho[v/z])[u/w]} = \text{true for every } v \in N, \text{ for some } u \in N$

$\Leftrightarrow M[\ z=2w+x\ ]_{(\rho[v/z])[u/w]} = \text{true or } M[\ z=2w+y\ ]_{(\rho[v/z])[u/w]} = \text{true}$

for every $v \in N$, for some $u \in N$

$\Leftrightarrow =^M((\rho[v/z])_{[u/w](z)}, +^M(\cdot^M(2^M, (\rho[v/z])[u/w](w)), (\rho[v/z])[u/w](x))) = \text{true}$

or

$=^M((\rho[v/z])[u/w](z), +^M(\cdot^M(2^M, (\rho[v/z])[u/w](w)), (\rho[v/z])[u/w](y))) = \text{true}$

for every $v \in N$, for some $u \in N$

$\Leftrightarrow =^M(v, +^M(\cdot^M(2, u), 0)) = \text{true or } =^M(v, +^M(\cdot^M(2, u), 1)) = \text{true for every } v \in N,$

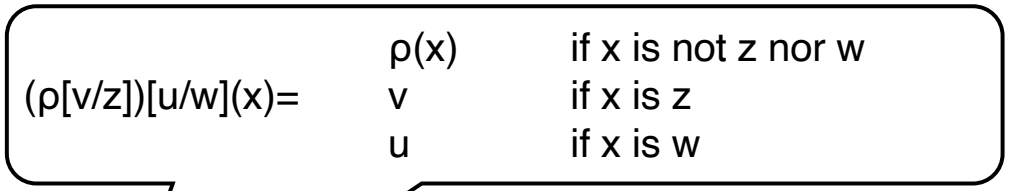
for some $u \in N$

$\Leftrightarrow$ vが2以上の偶数のときu=(v-2)/2, 3以上の奇数のときu=(v-3)/2をとればtrue

vが0, 1のときはuをどんな(0以上の)値にしてもfalse

$\Leftrightarrow$ false

解



M[∀z∃w(z=2w+x ∨ z=2w+y)]

⇔ M[∃w(z=2w+x ∨ z=2w+y)]$_{\rho[v/z]}$ =true for every v∈N

⇔ M[z=2w+x ∨ z=2w+y]$_{(\rho[v/z])[u/w]}$ =true for every v∈N, for some u∈N

⇔ M[z=2w+x]$_{(\rho[v/z])[u/w]}$ =true or M[z=2w+y]$_{(\rho[v/z])[u/w]}$ =true
for every v∈N, for some u∈N

⇔ $=^M((\rho[v/z])_{[u/w](z)}, +^M(\cdot^M(2^M, (\rho[v/z])[u/w](w)), (\rho[v/z])[u/w](x))$ =true
or
$=^M((\rho[v/z])[u/w](z), +^M(\cdot^M(2^M, (\rho[v/z])[u/w](w)), (\rho[v/z])[u/w](y))$ =true
for every v∈N, for some u∈N

⇔ $=^M(v, +^M(\cdot^M(2, u), 0))$ =true or $=^M(v, +^M(\cdot^M(2, u), 1))$ =true for every v∈N,
for some u∈N

⇔ vが2以上の偶数のときu=(v-2)/2, 3以上の奇数のときu=(v-3)/2をとればtrue
vが0, 1のときはuをどんな(0以上の)値にしてもfalse

⇔ false