

# 解

$$\langle\langle \phi_1 \rangle\rangle y = y + x \langle\langle \phi_0 \rangle\rangle$$

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$$\langle\langle y = (x_0 - x)(x_0 + x + 1)/2 \wedge x > 0 \rangle\rangle y = y + x \langle\langle \phi_0 \rangle\rangle$$

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$$\langle\langle \phi_0 \rangle\rangle x = x - 1 \langle\langle y = (x_0 - x)(x_0 + x + 1)/2 \rangle\rangle$$

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$$\langle\langle y = (x_0 - x)(x_0 + x + 1)/2 \wedge x > 0 \rangle\rangle y = y + x; x = x - 1 \langle\langle y = (x_0 - x)(x_0 + x + 1)/2 \rangle\rangle$$

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$$\langle\langle y = (x_0 - x)(x_0 + x + 1)/2 \rangle\rangle \text{while } (x > 0) \{ y = y + x; x = x - 1 \} \langle\langle y = (x_0 - x)(x_0 + x + 1)/2 \wedge \neg(x > 0) \rangle\rangle$$

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$$\langle\langle y = 0 \wedge x = x_0 \rangle\rangle \text{while } (x > 0) \{ y = y + x; x = x - 1 \} \langle\langle y = x_0(x_0 + 1)/2 \rangle\rangle$$

ただし

$$\phi_0 : y = (x_0 - (x - 1))(x_0 + (x - 1) + 1)/2$$

$$\phi_1 : y + x = (x_0 - (x - 1))(x_0 + (x - 1) + 1)/2$$

ループ不变表明 :  $y = (x_0 - x)(x_0 + x + 1)/2$

検証条件:  $y = (x_0 - x)(x_0 + x + 1)/2 \wedge x > 0 \supset \phi_1$

$$y = 0 \wedge x = x_0 \supset y = (x_0 - x)(x_0 + x + 1)/2$$

$$y = (x_0 - x)(x_0 + x + 1)/2 \wedge \neg(x > 0) \supset y = x_0(x_0 + 1)/2$$

( $x$ は自然数なので  $\neg(x > 0) \equiv x = 0$ )