

演習

以下を証明せよ.

$$27. \quad \forall x \forall y A(x, y) \supset \forall y \forall x A(x, y)$$

$$28. \quad \forall x (A(x) \wedge B(x)) \supset \forall x A(x) \wedge \forall x B(x)$$

$$29. \quad \forall x A(x) \wedge \forall x B(x) \supset \forall x (A(x) \wedge B(x))$$

27.

1

$$\forall x \forall y A(x, y)$$

$$\forall y A(a, y)$$

$$A(a, b)$$

$$\forall x A(x, b)$$

$$\forall y \forall x A(x, y)$$

1

$$\forall x \forall y A(x, y) \supset \forall y \forall x A(x, y)$$

28.

1

$$\forall x(A(x) \wedge B(x))$$

$$A(a) \wedge B(a)$$

$$A(a)$$

$$\forall xA(x)$$

$$\forall xA(x) \wedge \forall xB(x)$$

$$\forall x(A(x) \wedge B(x)) \supset \forall xA(x) \wedge \forall xB(x)$$

1

$$\forall x(A(x) \wedge B(x))$$

$$A(a) \wedge B(a)$$

$$B(a)$$

$$\forall xB(x)$$

1

29.

$$\begin{array}{c}
 \begin{array}{c}
 1 \\
 \hline
 \forall x A(x) \wedge \forall x B(x) \\
 \hline
 \forall x A(x) \\
 \hline
 A(a) \\
 \hline
 \end{array}
 \qquad
 \begin{array}{c}
 1 \\
 \hline
 \forall x A(x) \wedge \forall x B(x) \\
 \hline
 \forall x B(x) \\
 \hline
 B(a) \\
 \hline
 \end{array} \\
 \\
 \begin{array}{c}
 A(a) \wedge B(a) \\
 \hline
 \forall x (A(x) \wedge B(x)) \\
 \hline
 \forall x A(x) \wedge \forall x B(x) \supset \forall x (A(x) \wedge B(x)) \quad 1
 \end{array}
 \end{array}$$

演習

以下を証明せよ.

- 30. $\exists x \exists y A(x, y) \supset \exists y \exists x A(x, y)$
- 31. $\exists x (A(x) \vee B(x)) \supset \exists x A(x) \vee \exists x B(x)$
- 32. $\exists x A(x) \vee \exists x B(x) \supset \exists x (A(x) \vee B(x))$

30.

3

$$A(a, b)$$
$$\exists x A(x, b)$$

2

$$\exists y A(a, y)$$
$$\exists y \exists x A(x, y)$$

1

$$\exists x \exists y A(x, y)$$
$$\exists y \exists x A(x, y)$$

3

2

$$\exists y \exists x A(x, y)$$

1

$$\exists x \exists y A(x, y) \supset \exists y \exists x A(x, y)$$

31.

3
A(a)

4
B(a)

$\exists xA(x)$

$\exists xB(x)$

2

$A(a) \vee B(a)$

$\exists xA(x) \vee \exists xB(x)$

$\exists xA(x) \vee \exists xB(x)$

3,4

1

$\exists x(A(x) \vee B(x))$

$\exists xA(x) \vee \exists xB(x)$

2

$\exists xA(x) \vee \exists xB(x)$

1

$\exists x(A(x) \vee B(x)) \supset \exists xA(x) \vee \exists xB(x)$

32.

[illegible]

演習

以下を証明せよ.

33. $\neg \exists x A(x) \supset \forall x \neg A(x)$ (ド・モルガン)

34. $\forall x \neg A(x) \supset \neg \exists x A(x)$ (ド・モルガン)

33.

$$\begin{array}{c}
 \begin{array}{cc}
 \begin{array}{c} 1 \\ \neg \exists x A(x) \end{array} & \begin{array}{c} 2 \\ A(a) \\ \hline \exists x A(x) \end{array} \\
 \hline
 \perp \\
 \hline
 \neg A(a) \quad 2 \\
 \hline
 \forall x \neg A(x) \\
 \hline
 \neg \exists x A(x) \supset \forall x \neg A(x) \quad 1
 \end{array}
 \end{array}$$

34.

$$\begin{array}{c}
 \begin{array}{c}
 1 \\
 \hline
 \forall x \neg A(x)
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 2 \\
 A(a)
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 3 \\
 \exists x A(x)
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \hline
 \perp
 \end{array}$$

$$\begin{array}{c}
 2
 \end{array}$$

$$\begin{array}{c}
 \hline
 \perp
 \end{array}$$

$$\begin{array}{c}
 3 \\
 \neg \exists x A(x)
 \end{array}$$

$$\begin{array}{c}
 \hline
 1
 \end{array}$$

$$\forall x \neg A(x) \supset \neg \exists x A(x)$$