演習

以下を証明せよ.

27. $\forall x \forall y A(x, y) \supset \forall y \forall x A(x, y)$

28. $\forall x (A(x) \wedge B(x)) \supset \forall x A(x) \wedge \forall x B(x)$

29. $\forall x A(x) \wedge \forall x B(x) \supset \forall x (A(x) \wedge B(x))$

27.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{1}{\forall x \forall y A(x, y)}}{\forall y A(a, y)}}{A(a, b)}}{\forall x\, A(x, b)}}{\forall y \forall x A(x, y)}}{\forall x \forall y A(x, y) \supset \forall y \forall x A(x, y)}\,1$$

28.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{1}{\forall x(A(x)\wedge B(x))}}{A(a)\wedge B(a)}}{A(a)}}{\forall xA(x)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{1}{\forall x(A(x)\wedge B(x))}}{A(a)\wedge B(a)}}{B(a)}}{\forall xB(x)}}{\dfrac{\forall xA(x)\wedge\forall xB(x)}{\forall x(A(x)\wedge B(x))\supset\forall xA(x)\wedge\forall xB(x)}\ 1}$$

29.

$$\dfrac{\dfrac{\dfrac{\dfrac{1}{\forall xA(x)\wedge\forall xB(x)}}{\forall xA(x)}}{A(a)} \qquad \dfrac{\dfrac{\dfrac{1}{\forall xA(x)\wedge\forall xB(x)}}{\forall xB(x)}}{B(a)}}{\dfrac{A(a)\wedge B(a)}{\forall x(A(x)\wedge B(x))}}\,1$$

$$\forall xA(x)\wedge\forall xB(x)\supset\forall x(A(x)\wedge B(x))$$

演習

以下を証明せよ.

30. $\exists x \exists y A(x, y) \supset \exists y \exists x A(x, y)$
31. $\exists x (A(x) \vee B(x)) \supset \exists x A(x) \vee \exists x B(x)$
32. $\exists x A(x) \vee \exists x B(x) \supset \exists x (A(x) \vee B(x))$

30.

$$\dfrac{\dfrac{\exists x \exists y A(x, y) \qquad \dfrac{\dfrac{\dfrac{A(a, b)}{\exists x A(x, b)}\,3}{\exists y \exists x A(x, y)}\,2}{\exists y \exists x A(x, y)}\,3}{\exists y \exists x A(x, y)}\,2}{\exists x \exists y A(x, y) \supset \exists y \exists x A(x, y)}\,1$$

31.

$$\dfrac{\dfrac{\exists x(A(x)\vee B(x)) \qquad \dfrac{\dfrac{\dfrac{}{A(a)}\,3}{\exists xA(x)} \qquad \dfrac{\dfrac{}{B(a)}\,4}{\exists xB(x)}}{\exists xA(x)\vee\exists xB(x)}\,3,4}{\exists xA(x)\vee\exists xB(x)}\,2}{\exists x(A(x)\vee B(x))\supset\exists xA(x)\vee\exists xB(x)}\,1$$

32.

$$\dfrac{\exists xA(x)\vee\exists xB(x)\qquad\dfrac{\exists xA(x)\qquad\dfrac{\dfrac{A(a)}{A(a)\vee B(a)}}{\exists x(A(x)\vee B(x))}\,4}{\exists x(A(x)\vee B(x))}\qquad\dfrac{\exists xB(x)\qquad\dfrac{\dfrac{B(a)}{A(a)\vee B(a)}}{\exists x(A(x)\vee B(x))}\,5}{\exists x(A(x)\vee B(x))}\,2,3}{\exists x(A(x)\vee B(x))}\,1$$

$$\exists xA(x)\vee\exists xB(x)\supset\exists x(A(x)\vee B(x))$$

演習

以下を証明せよ.

33. $\neg\exists x A(x) \supset \forall x \neg A(x)$ （ド・モルガン）

34. $\forall x \neg A(x) \supset \neg\exists x A(x)$ （ド・モルガン）

33.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg\exists xA(x)}\,1 \qquad \dfrac{}{A(a)}\,2}{\exists xA(x)}}{\bot}}{\neg A(a)}\,2}{\forall x\neg A(x)}}{\neg\exists xA(x)\supset\forall x\neg A(x)}\,1$$

34.