

解

$$\langle\langle \phi_1 \rangle\rangle y=y+x \langle\langle \phi_0 \rangle\rangle$$

$$\langle\langle y=(x_0-x)(x_0+x+1)/2 \wedge x>0 \rangle\rangle y=y+x \langle\langle \phi_0 \rangle\rangle$$

$$\langle\langle \phi_0 \rangle\rangle x=x-1 \langle\langle y=(x_0-x)(x_0+x+1)/2 \rangle\rangle$$

$$\langle\langle y=(x_0-x)(x_0+x+1)/2 \wedge x>0 \rangle\rangle y=y+x; x=x-1 \langle\langle y=(x_0-x)(x_0+x+1)/2 \rangle\rangle$$

$$\langle\langle y=(x_0-x)(x_0+x+1)/2 \rangle\rangle \text{while } (x>0) \{y=y+x; x=x-1\} \langle\langle y=(x_0-x)(x_0+x+1)/2 \wedge \neg(x>0) \rangle\rangle$$

$$\langle\langle y=0 \wedge x=x_0 \rangle\rangle \text{while } (x>0) \{y=y+x; x=x-1\} \langle\langle y=x_0(x_0+1)/2 \rangle\rangle$$

ただし

$$\phi_0 : y=(x_0-(x-1))(x_0+(x-1)+1)/2$$

$$\phi_1 : y+x=(x_0-(x-1))(x_0+(x-1)+1)/2$$

ループ不変表明 : $y=(x_0-x)(x_0+x+1)/2$

検証条件:

$$y=(x_0-x)(x_0+x+1)/2 \wedge x>0 \supset \phi_1$$

$$y=0 \wedge x=x_0 \supset y=(x_0-x)(x_0+x+1)/2$$

$$y=(x_0-x)(x_0+x+1)/2 \wedge \neg(x>0) \supset y=x_0(x_0+1)/2$$

(x は自然数なので $\neg(x>0) \equiv x=0$)