数値実験の補足説明

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ϕ_a^i, w_a^i に関する以下の方程式を考える。

$$
\begin{aligned}
\phi_a^i &= \gamma_a^i \left(\frac{w_a^i}{1 - \sum_j \alpha_a^{ji}} \right)^{1 - \sum_j \alpha_a^{ji}} \prod_{j \in \mathbb{I}} \left(\frac{\hat{\rho}_a^j}{\alpha_a^{ji}} \right)^{\alpha_a^{ji}}, \\
\sigma^i (\phi_a^i)^{\sigma^i} &= \sum_{b \in \mathbb{A}} \left(\frac{\tau_{ab}^i}{\hat{\rho}_b^i} \right)^{1-\sigma^i} \sum_{j \in \mathbb{I}} (\mu^i N_b^j \hat{w}_b^j + \sigma^j \alpha_b^{ij} n_b^j \phi_b^j) \\
&= \sum_{b \in \mathbb{A}} \left(\frac{\tau_{ab}^i}{\hat{\rho}_b^i} \right)^{1-\sigma^i} \sum_{j \in \mathbb{I}} \left(\mu^i (1 - \lambda_b) + \frac{\alpha_b^{ij}}{1 - \sum_k \alpha_b^{kj}} \right) N_b^j w_b^j.
\end{aligned}
$$

ここで、

$$
\begin{aligned}
\psi^i &= \frac{\sigma^i \beta^i}{\sigma^i - 1}, \\
\gamma_a^i &= \prod_{j \in \mathbb{I}} (\psi^j)^{\alpha_a^{ji}}, \\
\hat{w}_a^i &= (1 - \lambda_a) w_a^i, \\
N_a^i &: \text{定数}, \\
n_a^i &= \frac{N_a^i w_a^i}{\sigma^i (1 - \sum_j \alpha_a^{ji}) \phi_a^i}, \\
(\hat{\rho}_a^i)^{1-\sigma^i} &= \sum_{b \in \mathbb{A}} n_b^i (\tau_{ba}^i \phi_b^i)^{1-\sigma^i} = \sum_{b \in \mathbb{A}} \frac{N_b^i (\tau_{ba}^i)^{1-\sigma^i}}{\sigma^i (1 - \sum_j \alpha_b^{ji})} w_b^i (\phi_b^i)^{-\sigma^i}
\end{aligned}
$$

このとき、

$$
\frac{\partial n_a^i}{\partial \phi_b^j} = \begin{cases} 0 & (a,i) \neq (b,j) \\ -\dfrac{n_a^i}{\phi_a^i} & (a,i) = (b,j) \end{cases} \qquad \frac{\partial n_a^i}{\partial w_b^j} = \begin{cases} 0 & (a,i) \neq (b,j) \\ \dfrac{n_a^i}{w_a^i} & (a,i) = (b,j) \end{cases}
$$

$$
\frac{\partial \hat{\rho}_a^i}{\partial \phi_b^j} = \begin{cases} 0 & i \neq j \\ \dfrac{\sigma^i (\hat{\rho}_a^i)^{\sigma^i}}{(\sigma^i - 1) \phi_b^i} n_b^i (\tau_{ba}^i \phi_b^i)^{1-\sigma^i} & i = j \end{cases} \qquad \frac{\partial \hat{\rho}_a^i}{\partial w_b^j} = \begin{cases} 0 & i \neq j \\ \dfrac{(\hat{\rho}_a^i)^{\sigma^i}}{(1 - \sigma^i) w_b^i} n_b^i (\tau_{ba}^i \phi_b^i)^{1-\sigma^i} & i = j \end{cases}
$$

よって、以下がわかる。

$$
\begin{aligned}
\frac{\partial F_a^i}{\partial \phi_b^j} &= \delta_{ab}^{ij} - \frac{\alpha_a^{ji}}{\hat{\rho}_a^j}\frac{\partial \hat{\rho}_a^j}{\partial \phi_b^j}\gamma_a^i\left(\frac{w_a^i}{1-\sum_k \alpha_a^{ki}}\right)^{1-\sum_k \alpha_a^{ki}}\prod_{k\in\mathbb{I}}\left(\frac{\hat{\rho}_a^k}{\alpha_a^{ki}}\right)^{\alpha_a^{ki}} \\
\frac{\partial F_a^i}{\partial w_b^j} &= -\left(\delta_{ab}^{ij}\frac{1-\sum_k \alpha_a^{ki}}{w_a^i} + \frac{\alpha_a^{ji}}{\hat{\rho}_a^j}\frac{\partial \hat{\rho}_a^j}{\partial w_b^j}\right)\gamma_a^i\left(\frac{w_a^i}{1-\sum_k \alpha_a^{ki}}\right)^{1-\sum_k \alpha_a^{ki}}\prod_{k\in\mathbb{I}}\left(\frac{\hat{\rho}_a^k}{\alpha_a^{ki}}\right)^{\alpha_a^{ki}} \\
\frac{\partial G_a^i}{\partial \phi_b^j} &= \begin{cases} 0 & i\neq j \\ \delta_{ab}(\sigma^i)^2(\phi_a^i)^{\sigma^i-1} - \sum_{c,k}\frac{\sigma^i-1}{\hat{\rho}_c^i}\frac{\partial \hat{\rho}_c^i}{\partial \phi_b^i}\left(\frac{\tau_{ac}^i}{\hat{\rho}_c^i}\right)^{1-\sigma^i}\left(\mu^i(1-\lambda_c)+\frac{\alpha_c^{ik}}{1-\sum_l \alpha_c^{lk}}\right)N_c^k w_c^k & i=j \end{cases} \\
\frac{\partial G_a^i}{\partial w_b^j} &= \begin{cases} 0 & i\neq j \\ -\sum_{c,k}\left(\frac{\tau_{ac}^i}{\hat{\rho}_c^i}\right)^{1-\sigma^i}\left(\mu^i(1-\lambda_c)+\frac{\alpha_c^{ik}}{1-\sum_l \alpha_c^{lk}}\right)N_c^k w_c^k\left(\frac{\sigma^i-1}{\hat{\rho}_c^i}\frac{\partial \hat{\rho}_c^i}{\partial w_b^i}+\delta_{bc}^{ik}\frac{1}{w_b^i}\right) & i=j \end{cases}
\end{aligned}
$$

次に、N_a^i, n_a^i が定数であり、関係式 $\sigma^i = 1+\beta^i s_a^i$ がない場合を考える。このとき、ϕ_a^i, w_a^i に関する以下の方程式を考えることになる。

$$
\begin{aligned}
\phi_a^i &= \gamma_a^i\left(\frac{w_a^i}{1-\sum_j \alpha_a^{ji}}\right)^{1-\sum_j \alpha_a^{ji}}\prod_{j\in\mathbb{I}}\left(\frac{\hat{\rho}_a^j}{\alpha_a^{ji}}\right)^{\alpha_a^{ji}}, \\
\psi^i s_a^i(\phi_a^i)^{\sigma^i} &= \sum_{b\in\mathbb{A}}\left(\frac{\tau_{ab}^i}{\hat{\rho}_b^i}\right)^{1-\sigma^i}\sum_{j\in\mathbb{I}}\left(\mu^i(1-\lambda_b)+\frac{\alpha_b^{ij}}{1-\sum_k \alpha_b^{kj}}\right)N_b^j w_b^j.
\end{aligned}
$$

ここで、

$$
\begin{aligned}
\psi^i &= \frac{\sigma^i\beta^i}{\sigma^i-1}, \\
\hat{w}_a^i &= (1-\lambda_a)w_a^i, \\
(\hat{\rho}_a^i)^{1-\sigma^i} &= \sum_{b\in\mathbb{A}} n_b^i(\tau_{ba}^i\phi_b^i)^{1-\sigma^i}, \\
s_a^i &= \frac{1}{\beta^i}\left(\frac{N_a^i w_a^i}{(1-\sum_j \alpha_a^{ji})n_a^i\phi_a^i}-1\right)
\end{aligned}
$$

このとき、

$$
\begin{aligned}
\frac{\partial \hat{\rho}_a^i}{\partial \phi_b^j} &= \begin{cases} 0 & i\neq j \\ \frac{(\hat{\rho}_a^i)^{\sigma^i}}{\phi_b^i} n_b^i(\tau_{ba}^i\phi_b^i)^{1-\sigma^i} & i=j \end{cases} \qquad \frac{\partial \hat{\rho}_a^i}{\partial w_b^j} = 0 \\
\frac{\partial s_a^i}{\partial \phi_j^b} &= \begin{cases} 0 & (a,i)\neq(b,j) \\ -\frac{N_a^i w_a^i}{\beta^i(1-\sum_k \alpha_a^{ki})n_a^i(\phi_a^i)^2} & (a,i)=(b,j) \end{cases} \\
\frac{\partial s_a^i}{\partial w_j^b} &= \begin{cases} 0 & (a,i)\neq(b,j) \\ \frac{N_a^i}{\beta^i(1-\sum_k \alpha_a^{ki})n_a^i\phi_a^i} & (a,i)=(b,j) \end{cases}
\end{aligned}
$$

よって、

$$\frac{\partial F_a^i}{\partial \phi_b^j} = \delta_{ab}^{ij} - \frac{\alpha_a^{ji}}{\hat{\rho}_a^j}\frac{\partial \hat{\rho}_a^j}{\partial \phi_b^j}\gamma_a^i\left(\frac{w_a^i}{1-\sum_k \alpha_a^{ki}}\right)^{1-\sum_k \alpha_a^{ki}}\prod_{k\in\mathbb{I}}\left(\frac{\hat{\rho}_a^k}{\alpha_a^{ki}}\right)^{\alpha_a^{ki}}$$

$$\frac{\partial F_a^i}{\partial w_b^j} = \begin{cases} 0 & (a,i)\neq(b,j) \\ -\dfrac{1-\sum_k \alpha_a^{ki}}{w_a^i}\gamma_a^i\left(\dfrac{w_a^i}{1-\sum_k \alpha_a^{ki}}\right)^{1-\sum_k \alpha_a^{ki}}\displaystyle\prod_{k\in\mathbb{I}}\left(\frac{\hat{\rho}_a^k}{\alpha_a^{ki}}\right)^{\alpha_a^{ki}} & (a,i)=(b,j) \end{cases}$$

$$\frac{\partial G_a^i}{\partial \phi_b^j} = \begin{cases} 0 & i\neq j \\ \delta_{ab}\psi^i\left(\dfrac{\partial s_a^i}{\partial \phi_a^i}(\phi_a^i)^{\sigma^i} + \sigma^i s_a^i(\phi_a^i)^{\sigma^i-1}\right) & \\ \quad +\displaystyle\sum_{c\in\mathbb{A}}\frac{1-\sigma^i}{\hat{\rho}_c^i}\frac{\partial \hat{\rho}_c^i}{\partial \phi_b^i}\left(\frac{\tau_{ac}^i}{\hat{\rho}_c^i}\right)^{1-\sigma^i}\sum_{k\in\mathbb{I}}\left(\mu^i(1-\lambda_c)+\frac{\alpha_c^{ik}}{1-\sum_l \alpha_c^{lk}}\right)N_c^k w_c^k & i=j \end{cases}$$

$$\frac{\partial G_a^i}{\partial w_b^j} = \delta_{ab}^{ij}\psi^i\frac{\partial s_a^i}{\partial w_a^i}(\phi_a^i)^{\sigma^i} - \left(\frac{\tau_{ab}^i}{\hat{\rho}_b^i}\right)^{1-\sigma^i}\left(\mu^i(1-\lambda_b)+\frac{\alpha_b^{ij}}{1-\sum_l \alpha_b^{lj}}\right)N_b^j$$