

# ECE 470 Project Update 3

## Forward Kinematics Derivation

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Figure illustrates the locations of the joints on the UR3 robot.

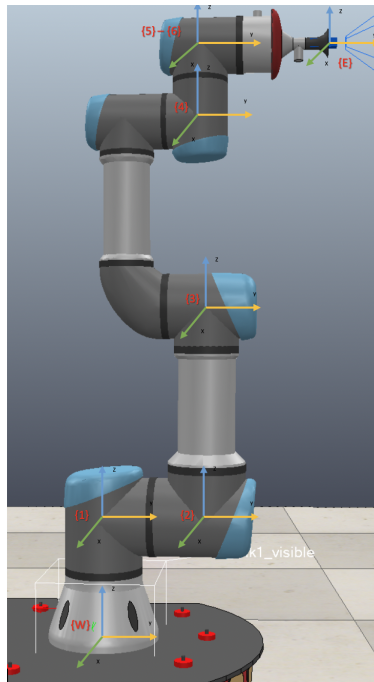


Figure 1: Location of Coordinate Frames (Arm)

The locations of the coordinate frames are given by,

$$W = [-0.0454 \quad 0.000073 \quad 0.2700]^T \quad (1)$$

$$1 = [-0.0454 \quad 0.000073 \quad 0.3745]^T \quad (2)$$

$$2 = [-0.1566 \quad 0.0098 \quad 0.3789]^T \quad (3)$$

$$3 = [-0.1566 \quad 0.0099 \quad 0.6225]^T \quad (4)$$

$$4 = [-0.1566 \quad 0.0098 \quad 0.8358]^T \quad (5)$$

$$5 = [-0.1573 \quad 0.0099 \quad 0.9200]^T \quad (6)$$

$$6 = [-0.1566 \quad 0.0098 \quad 0.9211]^T \quad (7)$$

$$E = [-0.3022 \quad 0.0225 \quad 0.9211]^T \quad (8)$$

The rotational vectors,  $w_i$ , are given by

$$w_1 = [0 \quad 0 \quad 1]^T \quad (9)$$

$$w_2 = [0 \quad 1 \quad 0]^T \quad (10)$$

$$w_3 = [0 \quad -1 \quad 0]^T \quad (11)$$

$$w_4 = [0 \quad 1 \quad 0]^T \quad (12)$$

$$w_5 = [0 \quad 0 \quad 1]^T \quad (13)$$

$$w_6 = [0 \quad 1 \quad 0]^T \quad (14)$$

$$(15)$$

The vectors,  $q_i$ , to the corresponding rotational axes are given by

$$q_i = \{i\} - \{W\} \quad (16)$$

The velocity vectors for each joint are then given by

$$v_i = -w_i \times q_i \quad (17)$$

For each joint, the matrix exponential of the screw axis matrix can be written as,

$$e^{[S_i]\theta_i} = \begin{bmatrix} e^{[w_i]\theta_i} & (I\theta_i + (1 - \cos\theta_i)[w_i]) \\ 0 & 1 + (\theta_i - \sin\theta_i)[w_i]^2 v_i \end{bmatrix} \quad (18)$$

where

$$e^{[w_i]\theta_i} = I + \sin\theta_i[w_i] + (1 - \cos\theta_i)[w_i]^2 \quad (19)$$

Then the transformation from the end-effector to the base of the robot arm is given by

$$T(\theta) = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4} e^{[S_5]\theta_5} e^{[S_6]\theta_6} M \quad (20)$$