ECE 470 Project Update 3
Forward Kinematics Derivation

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Figure illustrates the locations of the joints on the UR3 robot.

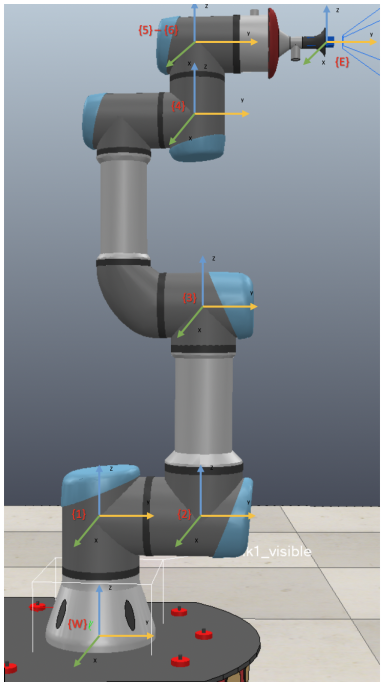

Figure 1: Location of Coordinate Frames (Arm)

The locations of the coordinate frames are given by,

$$W = [-0.0454 \quad 0.000073 \quad 0.2700]^T \tag{1}$$
$$1 = [-0.0454 \quad 0.000073 \quad 0.3745]^T \tag{2}$$
$$2 = [-0.1566 \quad 0.0098 \quad 0.3789]^T \tag{3}$$
$$3 = [-0.1566 \quad 0.0099 \quad 0.6225]^T \tag{4}$$
$$4 = [-0.1566 \quad 0.0098 \quad 0.8358]^T \tag{5}$$
$$5 = [-0.1573 \quad 0.0099 \quad 0.9200]^T \tag{6}$$
$$6 = [-0.1566 \quad 0.0098 \quad 0.9211]^T \tag{7}$$
$$E = [-0.3022 \quad 0.0225 \quad 0.9211]^T \tag{8}$$

The rotational vectors, w_i, are given by

$$w_1 = [0 \quad 0 \quad 1]^T \tag{9}$$
$$w_2 = [0 \quad 1 \quad 0]^T \tag{10}$$
$$w_3 = [0 \quad -1 \quad 0]^T \tag{11}$$
$$w_4 = [0 \quad 1 \quad 0]^T \tag{12}$$
$$w_5 = [0 \quad 0 \quad 1]^T \tag{13}$$
$$w_6 = [0 \quad 1 \quad 0]^T \tag{14}$$
$$\tag{15}$$

The vectors, q_i, to the corresponding rotational axes are given by

$$q_i = \{i\} - \{W\} \tag{16}$$

The velocity vectors for each joint are then given by

$$v_i = -w_i \times q_i \tag{17}$$

For each joint, the matrix exponential of the screw axis matrix can be written as,

$$e^{[\mathcal{S}_i]\theta_i} = \begin{bmatrix} e^{[w_i]\theta_i} & (I\theta_i + (1 - cos\theta_i)[w_i] \\ 0 & 1 + (\theta_i - sin\theta_i)[w_i]^2)v_i \end{bmatrix} \tag{18}$$

where

$$e^{[w_i]\theta_i} = I + sin\theta_i[w_i] + (1 - cos\theta_i)[w_i]^2 \tag{19}$$

Then the transformation from the end-effector to the base of the robot arm is given by

$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_3]\theta_3} e^{[\mathcal{S}_4]\theta_4} e^{[\mathcal{S}_5]\theta_5} e^{[\mathcal{S}_6]\theta_6} M \tag{20}$$