Project Proposal
Baysien Option Pricing Using Mixed Normal Heteroskedasticity Models

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1. Introduction

The option pricing using asymmetric mixed normal heteroscedasticity models helps us better fit actual observed prices. We consider performing inferences and price options in a Baysian framework through compute posterior moments of the model parameters by sampling from the poterior density. Unlike classical inference that need conditions on maximum likelihood estimates, the method utilizes the risk neutral predicitve price densities by product of the Bayesian sampler. An application is using the real data on the S&P 500 index and index options. We plan to perform Baysien inference on a two-component asymennetric normal mixture model by using the available data. In section 2, we will describe the model and explain how to use it to simulate data. In section 3, we will introduce what data we use and what conditions we set up. Finally, in section 4, we outline the goals and anticipations of this project.

2. Model's Framework

Let $\mathscr{F}_t$ denote the information set up to time t. Then the underlying return process $R_t = ln(\frac{S_t}{S_{t-1}})$, where S_t is the index level on each day t. Then

$$R_t = r_t - \Psi_t(\nu_t - 1) + \Psi_t(\nu_t) + \epsilon_t$$

Since we also know each year's risk free rate r_t, and ν_t is the unit risk premium, Ψ_t is conditional cumulant generating function of ϵ_t, so we can compute their conditional distribution given by combination of K distributions

$$P(\epsilon_t|\mathscr{F}_{t-1}) = \sum_{k=1}^{K} \pi_k \Phi(\frac{\epsilon_t - \mu_k}{\sigma_{k,t}})$$

where

$$\sigma_{k,t} = \sqrt{\omega_k + \alpha(\epsilon_{t-1} + \gamma_k \sigma_{k,t-1})^2 + \beta_k \sigma_{k,t-1}^2}$$

For the above model, the conditional cumulant generating function $\Psi_t(u)$ the model of asymmetric heteroskedastic normal mixture(MN-NGARCH), given by,

$$\Psi_t(u) = ln(\sum_{k=1}^{K} \pi_i * exp(-u\mu_k + \frac{u^2 \sigma_{k,t}^2}{2}))$$

Also, the likelihood function given G^T and R is

$$\mathscr{L}(\xi|G^T, R) = \prod_{t=1}^{T} \pi_{G_t} \phi(R_t|\mu_{G_t} + \rho_t(\nu_t), \theta_{G_t})$$

We will estimate the parameters by using the Gibbs Sampler. The join posterior distribution based on the MN-NGARCH model is given by

$$\varphi(G^T, \nu_t, \mu, \theta, \pi|R) \propto \varphi(\nu_t)\varphi(\mu)\varphi(\theta)\varphi(\pi)\mathscr{L}(\xi|G^T, R)$$

where $\varphi(\nu_t), \varphi(\mu), \varphi(\theta), \varphi(\pi)$ are the corresponding prior densities. Suppose the parameters are independent to each other. We will use full conditionals of each parameter to compute the Gibbs Sampler. Through plots and diagnostic we want to verify the convergence of data and how long it will possibly converge to the target posterior distributions. Meanwhile, we will try to minimize the impact of starting values in the final result.

3. Data for S&P 500 Index

In this project, we use data on call options on the S&P 500 index. Our data covers 10 years period from Dec,31,2006 to Dec,31,2016. We also impose the following restriction on our sample: First, we only consider weekly data and choose the option prices on every Wednesday since it minimizes the impact from weekend trading. Second, we include
Daily Treasury Yield Curve Rates as the risk free rates from US Department of the Treasury corresponding to the days of option prices. In total, we end up with a sample of 252 call options.

4. Conclusion

In this project, we plan to explore the mixed normal heteroscedasticity models and use Bayesian inference to approximate the coefficients of S&P 500 across 10 years. Specifically, we will estimate the parameters by using Gibss Sampler and we anticipate the Bayesian methods yield similar pricing errors measured in dollar and implied standard deviation losses when pricing a rich sample of options on the index, and it turns out that the impact of parameter uncertainty is minor. Therefore, when large amount of data are available, the choice of the inference method to predict the price of options is unimportant.

References

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[3] Daily Treasury Yield Curve Rates. US Department of the Treasury. Retrived from https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldYear&year=2017