Project Report
Baysien Option Pricing Using Mixed Normal Heteroskedasticity Models

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11/28/2017

Introduction

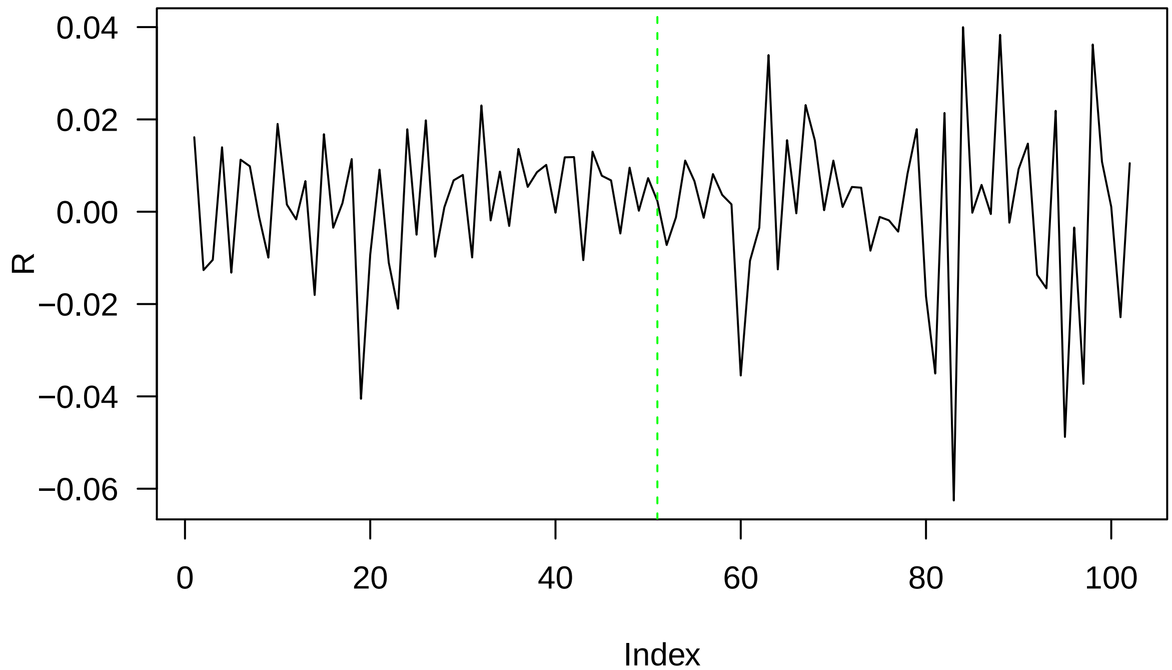
The option pricing using asymmetric mixed normal heteroscedasticitic models helps us better fit actual observed prices. We consider performing inferences and price options in a Baysian framework through computing posterior moments of the model parameters by sampling from the poterior density. Unlike classical inference that need conditions on maximum likelihood estimation, the MCMC (Markov Chain Monte Carlo) method utilizes the risk neutral predicitve price densities by product of the Gibbs sampler. An application is using the real data on the S&P 500 index and index options. We plan to perform Baysien inferences on a two-component asymmetric mixture normal by using the data between 2006 and 2007.

Model's Framework

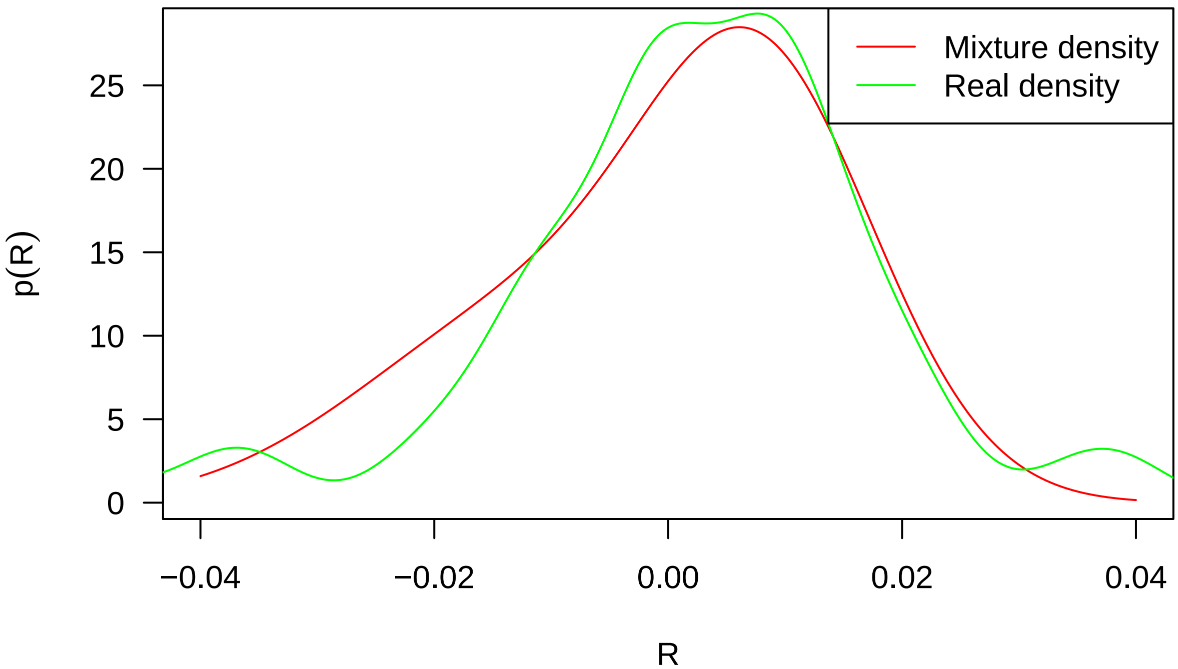
3. Data for S&P 500 Index

In this project, we use data on call options on the S&P 500 index. Our data covers 2 years period from Dec,31,2006 to Dec,31,2007 and compute the return. We also impose the following restriction on our sample: First, we only consider weekly data and choose the option prices on every Wednesday since it minimizes the impact from weekend trading. Second, we include

Daily Treasury Yield Curve Rates as the risk free rates from US Department of the Treasury corresponding to the days of option prices. Since the yield cruve rates are measured in an annual basis. We need to convert them to a daily basis by dividing 365 which is the number of days in a year. We end up the return R with 102 observations.



The green dotted line represents the cut-off between the years of 2006 and 2007. Due to the Subprime mortgage crisis that happened in US in the year of 2007, the fluctations of return are much larger than the return in 2006, which strengthens the difficulty on the precision of model prediction and inference.



4 Initial rough estimate

Bayesian inference

The likelihood function given G^T and R is

$$\mathscr{L}(\xi|G^T, R) = \prod_{t=1}^{T} \pi_{G_t} \phi(R_t|\mu_{G_t} + \rho_t(\nu_t), \theta_{G_t})$$

where $\rho_t(\nu_t) = r_t - \Psi_t(\nu_t - 1) + \Psi_t(\nu_t)$ and $\phi()$ is the component of each individual normal. We will estimate the parameters $(G^T, \nu_t, \pi, \mu, \theta)$ by using the Gibbs Sampler. The join posterior distribution based on the MN-NGARCH model is given by

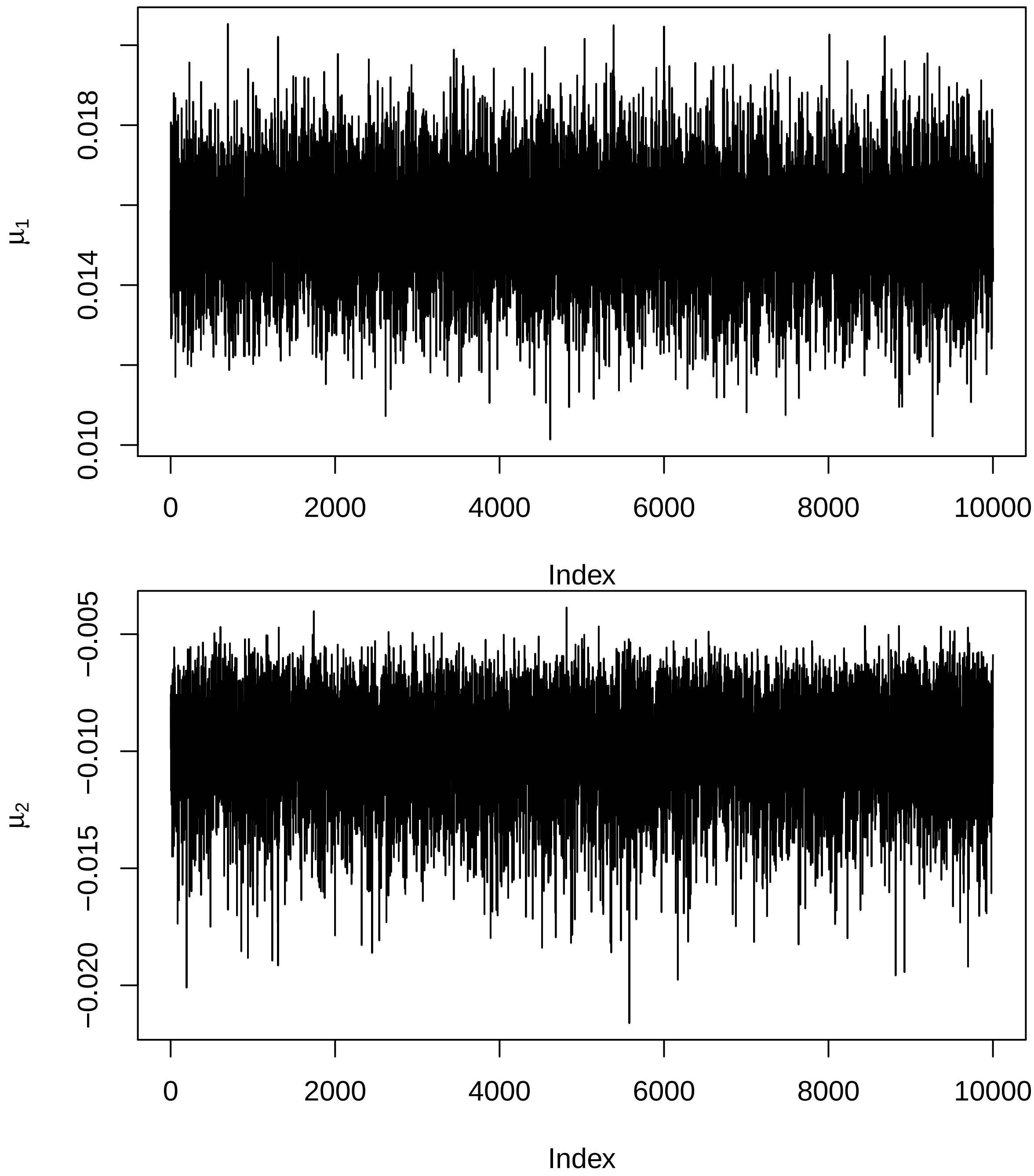
$$\varphi(G^T, \nu_t, \mu, \theta, \pi|R) \propto \varphi(\nu_t)\varphi(\mu)\varphi(\theta)\varphi(\pi)\mathscr{L}(\xi|G^T, R)$$

where $\varphi(\nu_t), \varphi(\mu), \varphi(\theta), \varphi(\pi)$ are the corresponding prior densities. Suppose the parameters are independent to each other. We will use full conditionals of each parameter to compute the Gibbs Sampler.

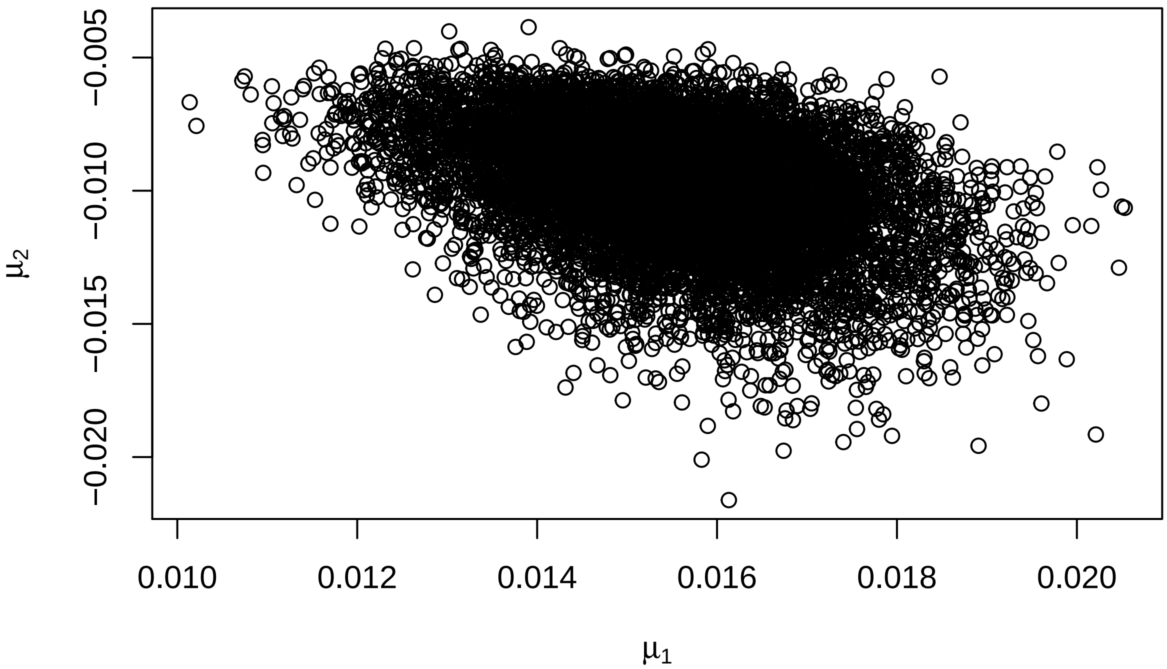
(1) $\varphi(G^T|\nu_t, \mu, \theta, \pi, R)$ For K $= 2$, the posterior distribution of $G^T = i, i = 1, 2$ is just a Bernoulli process. Therefore,

$$P(G^T = i|\nu_t, \mu, \theta, \pi, R) = \frac{\pi_i \phi(R_t|\mu_i + \rho_t(\nu_t), \theta_i)}{\pi_1 \phi(R_t|\mu_1 + \rho_t(\nu_t), \theta_1 + \pi_2 \phi(R_t|\mu_2 + \rho_t(\nu_t), \theta_2)}$$

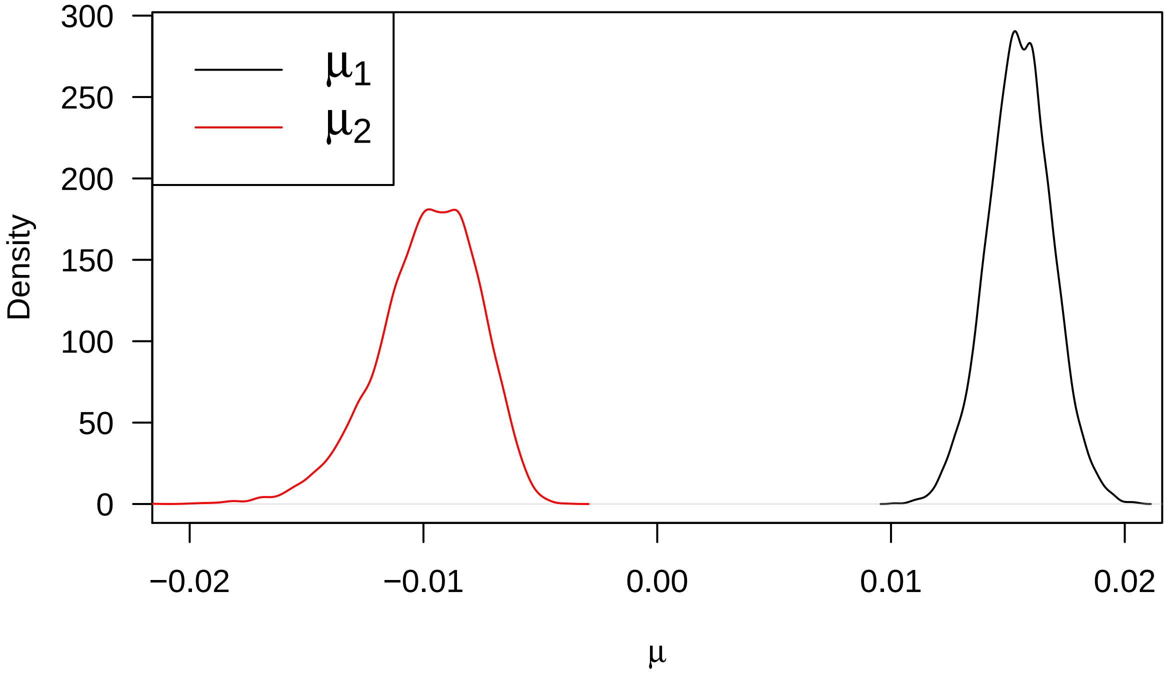
Using the information above and compute the probability for all data, 39 data belong to the first normal model and 63 data belong to the second normal model.

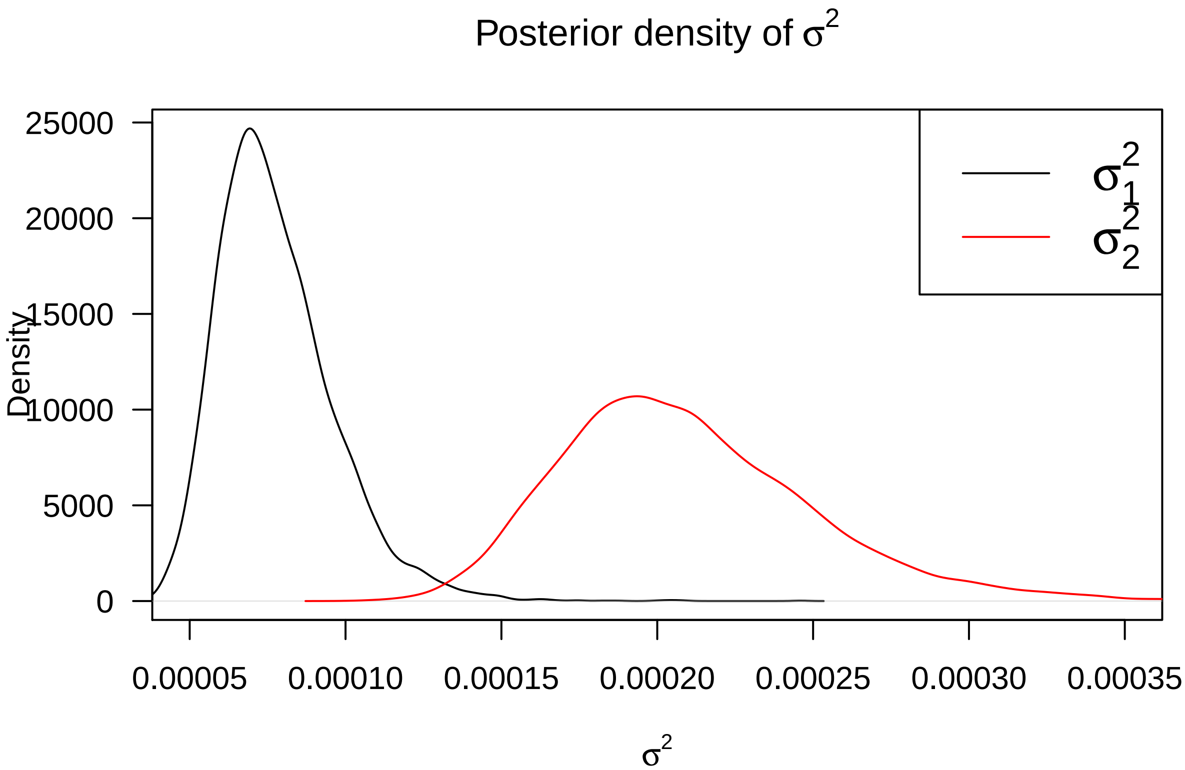


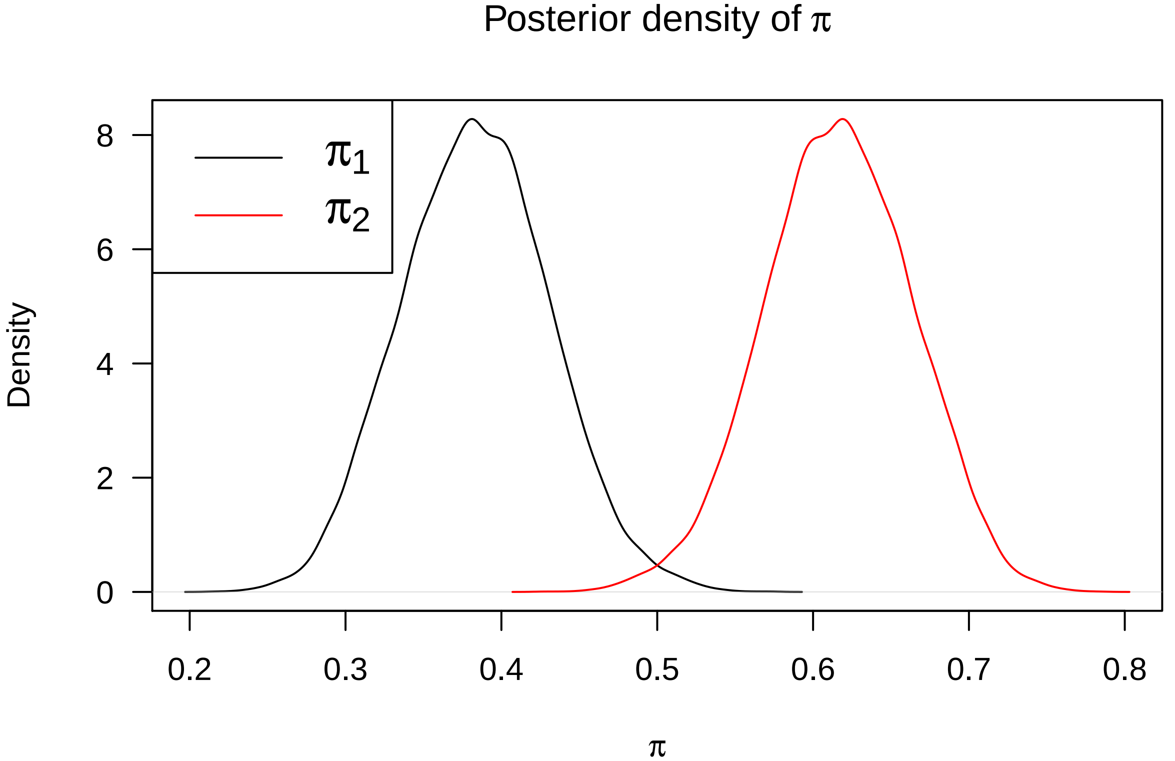
μ_1
0.010
0.014
0.018
0
2000
4000
6000
8000
10000
Index
μ_2
−0.020
−0.015
−0.010
−0.005
0
2000
4000
6000
8000
10000
Index



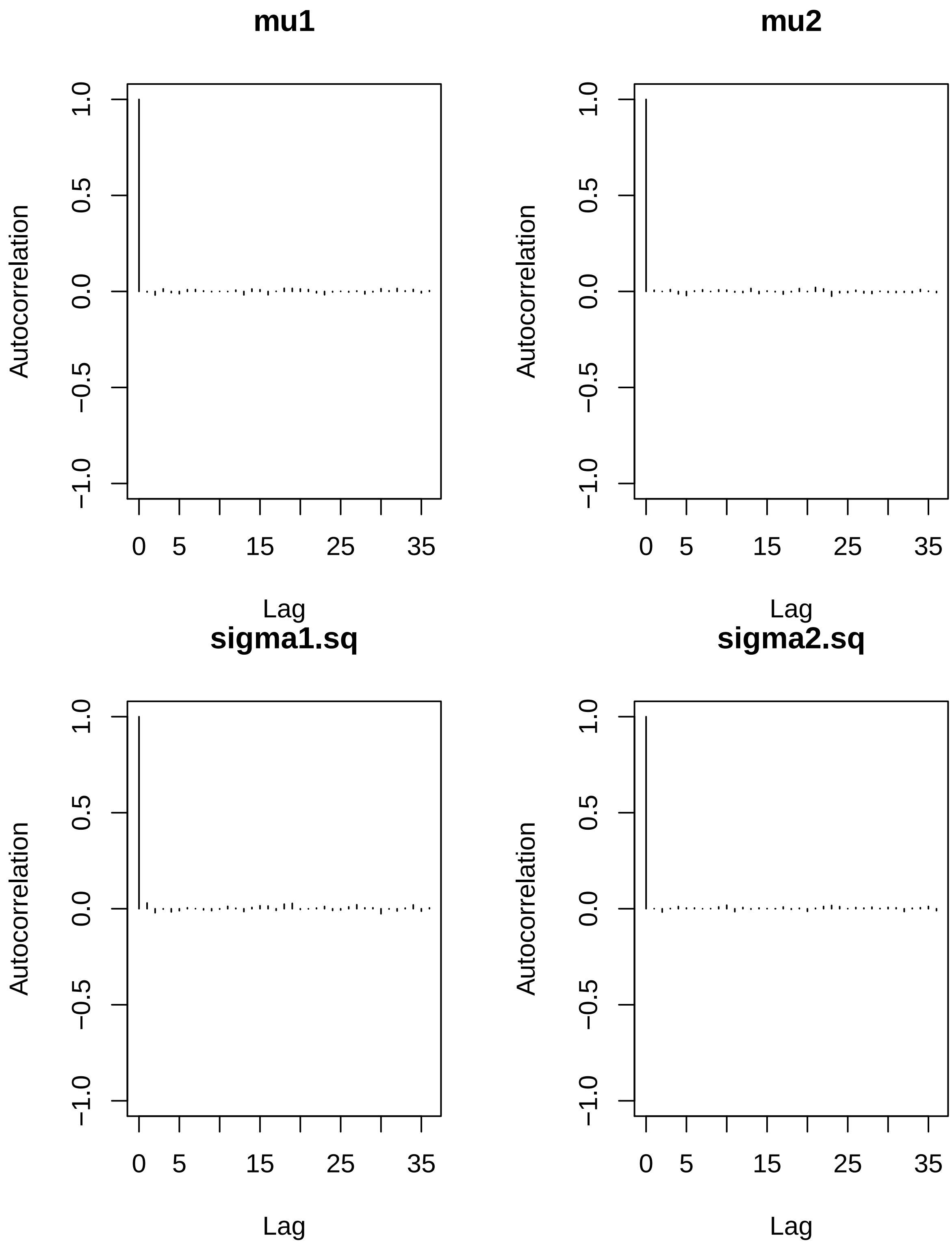
Posterior density of μ








```
##         parameters
##               mu1       mu2 sigma1.sq sigma2.sq        p
##    mean  0.015505 -0.009811  0.000078  0.000210 0.383821
##    2.5%  0.012712 -0.014668  0.000050  0.000144 0.293124
##    97.5% 0.018231 -0.006186  0.000122  0.000305 0.478213
```




```
##         mu1   mu2 sigma1.sq sigma2.sq
## [1,] 10000 10000  9835.472     10000
```

References

[1] Rombouts, J. and Stentoft, L.(2014) "Bayesian option pricing using mixed normal heteroskedasticity models" in *Computational Statistics & Data Analysis*, 76,588-605

[2] Standard & Poor's 500 index. Retrived from https://finance.google.com/finance/historical?cid=626307&startdate=Jul+2%2C+1972&enddate=Dec+28%2C+2011&num=30&ei=dxwBWqmEFozKjAG-_IyQDQ

[3] Daily Treasury Yield Curve Rates. US Department of the Treasury. Retrived from https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldYear&year=2017