When will the power tower converge?

rule :

$$\begin{cases} a_1 = a \\ a_n = a^{a_{n-1}} \{n \in \mathbb{N} \mid n > 1\} \end{cases} \tag{1}$$

Then evaluate each term based on (1)

$$a_2 = a^{a_1} = a^a,\ a_3 = a^{a_2} = a^{a^a},\ a_4 = a^{a_3} = a^{a^{a^a}}$$

When the number of terms get big enough,
the value of a_1 that a_n either converges or diverges can be assumed.
For instance, although $a_1 = 1$ leads a_∞ to converge,
$a_1 = 2$ let it diverge. And a_∞ converges in the range of $0 < a_1 \leq 1$.
But it doesn't mean a_∞ diverges in the range of $1 < a_1$.
Therefore, there would be a border that the a_∞ converges or diverges
in the range of $1 < a_1 < 2$.

$a^{a^{a^{\cdots}}}$ can be given by repeating the function $y = a^x$.Whether $a^{a^{a^{\cdots}}}$ will be converge or not depends on common points.

Then let's evaluate a that the function $y = a^x$ tangent to the function $y = x$.

First, differentiate $y = a^x$

$$y' = a^x \log a$$

And seems to appear a point that the derivative would be 1, on function

$y = x$. Therefore, evaluate x that shows $y' = 1$.

$$\begin{aligned} a^x \log a &= 1 \\ e^{a^x \log a} &= e^1 \\ a^{a^x} &= e \\ x &= \log_a(\log_a e) \end{aligned}$$

The point that the slope of function $y = a^x$ becomes to be 1, is

$$(\log_a(\log_a e),\ a^{\log_a(\log_a e)}) = (\log_a(\log_a e),\ \log_a e)$$

Then, if this point is on the $y = x$ graph,

$$\log_a(\log_a e) = \log_a e$$

And solve for a.

$$\begin{aligned} \log_a(\log_a e) &= \log_a e \\ \log_a e &= e \\ e &= a^e \\ a &= e^{1/e} \end{aligned}$$