IEDA 3010 Project A Portfolio Construction

CAI, Yunhui
LAM, Sum Yi
TSANG, Yu
WONG, Man Hei
YEE, Ka Ying

Introduction

What is Portfolio?

- A set of financial assets
- Portfolio Construction is all about investing in a range of financial assets that work together to create an investment solution for investors.
- In this project, we will only consider the historical data of stocks.

Stock data that we have:

Learn From Reference Paper

1. Investment Decision
2. Transaction Cost
3. Portfolio Construction

Project Objective

Using the historical data to predict how to construct a portfolio in the future with the trade-off between return and risk.

Objective I

Maximize the portfolio returns

Dynamic Programming

DP formulation:

- Variables: Stage n: days $= 1, 2, \ldots, 252$

  Decision variable x_n: no. of stocks to invest on day n.

  State s_n: remaining percentage of weighting remaining at stage n (%)

  - $s_1 = 100$
  - $s_2 = s_1 - x$
  - $s_3 = s_2 - x$

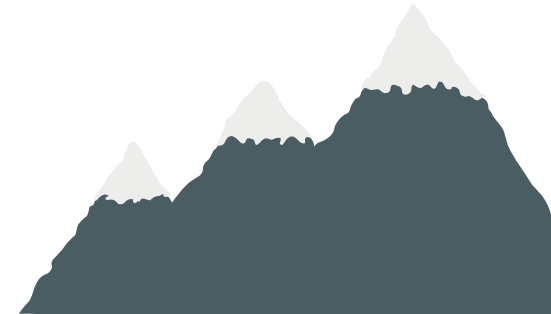
- DP Recursion:

Probability function $p_n(x_n)$

Cost function $c_n(x_n)$

$$f_n^*(s_n) = \max_{x_n = 0, 1, \ldots, s_n} \{ p_n(x_n) \cdot f_{n+1}^*[s_n - c_n(x_n)] \}$$

Return function $f_n(s_n, x_n)$



What is sharpe ratio?

Formula >

$$S_a = \frac{E[R_a - R_b]}{\sigma_a}$$

S_a = Sharpe ratio

E = expected value

R_a = asset return

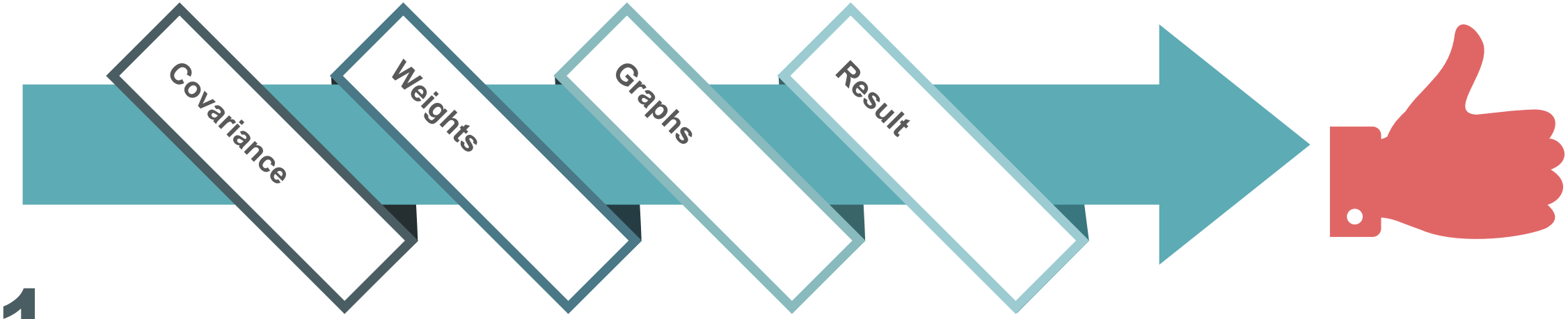
R_b = risk free return

σ_a = standard deviation of the asset excess return

High sharpe ratio = Higher, better portfolio.
Low sharpe ratio = Getting poor returns
Negative sharpe ratio = risk-free rate is greater than the portfolio's return, or the portfolio's return is expected to be negative
Limitation = Resulting returns would be right-skewed or log-normal

Maximum the portfolio returns with sharpe ratio

Methodology



01 Find out the covariance between each stocks.

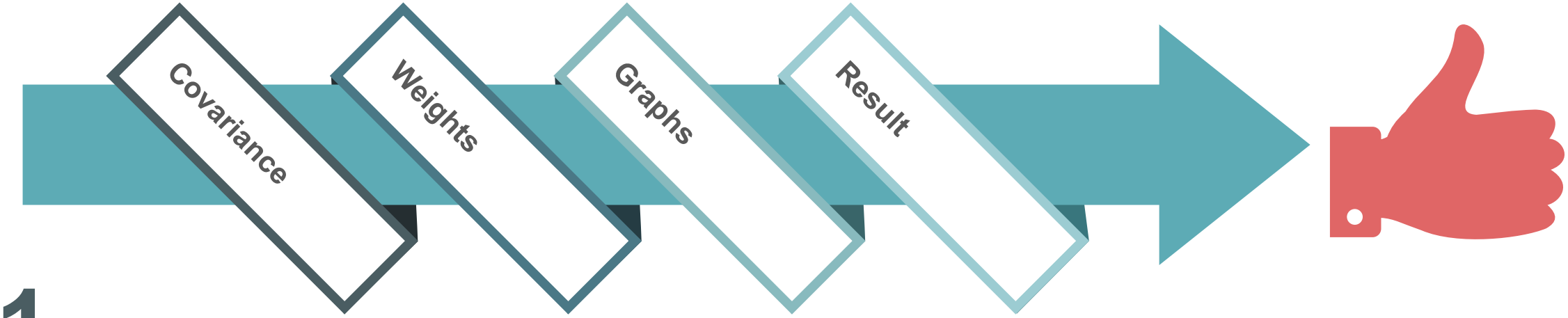
02 Generate a graph with variance and standard deviation.

03 Optimal result.

	0003.HK	0857.HK	1398.HK	0941.HK	0066.HK	1093.HK	2319.HK	0914.HK	192
2015-01-02	11.0276	8.69	5.77	91.400002	31.900000	6.68	16.025000	29.299999	37.70
2015-01-05	10.9034	8.74	5.80	90.099998	31.799999	6.64	15.825000	29.100000	37.25
2015-01-06	10.7916	8.53	5.71	88.750000	31.500000	6.68	15.800000	28.700001	35.95
2015-01-07	10.8289	8.57	5.75	91.750000	31.700001	6.78	16.075001	28.450001	37.04
2015-01-08	11.0151	8.78	5.72	93.599998	32.250000	6.65	16.400000	28.900000	37.15

Maximum the portfolio returns with sharpe ratio

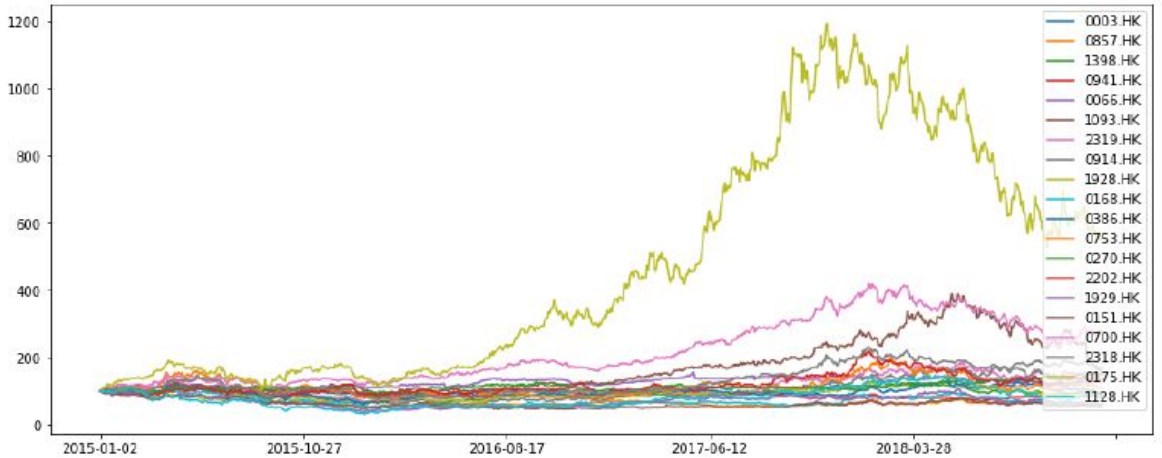
Methodology



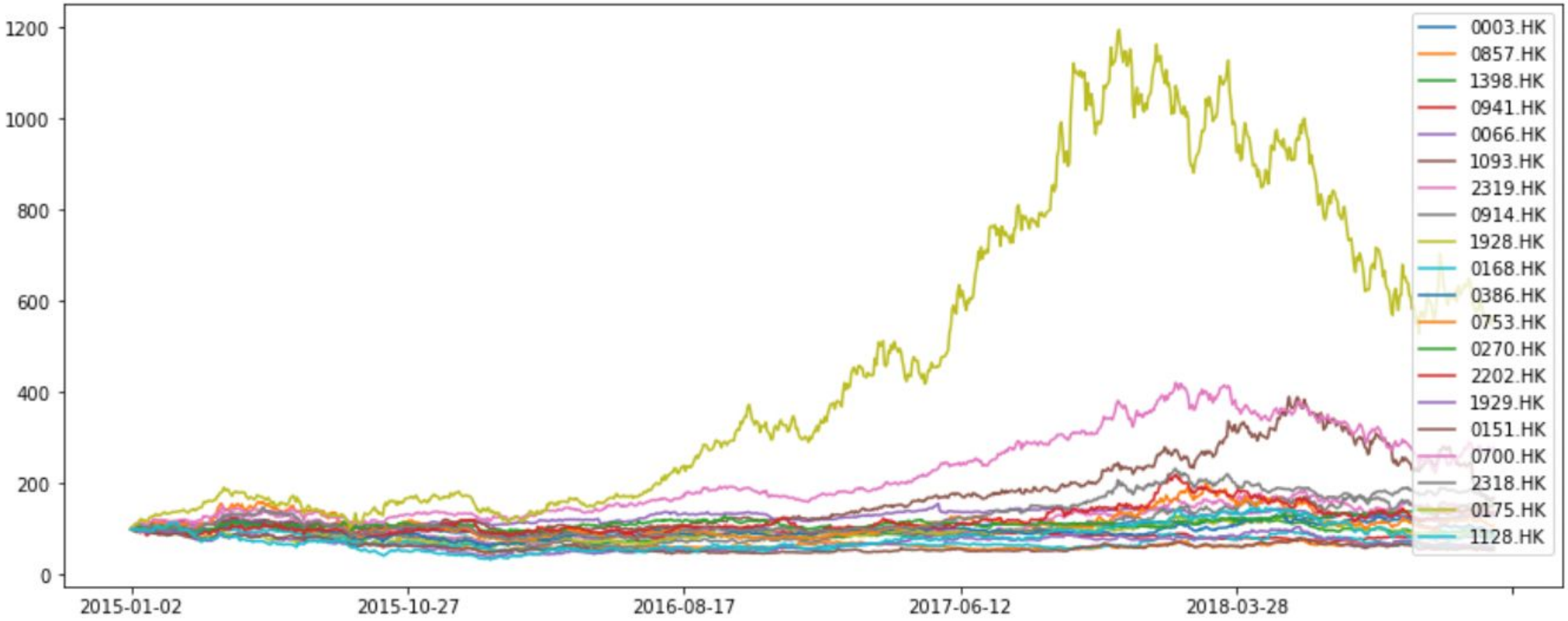
01 Find out the covariance between each stocks.

02 Generate a graph with variance and standard deviation.

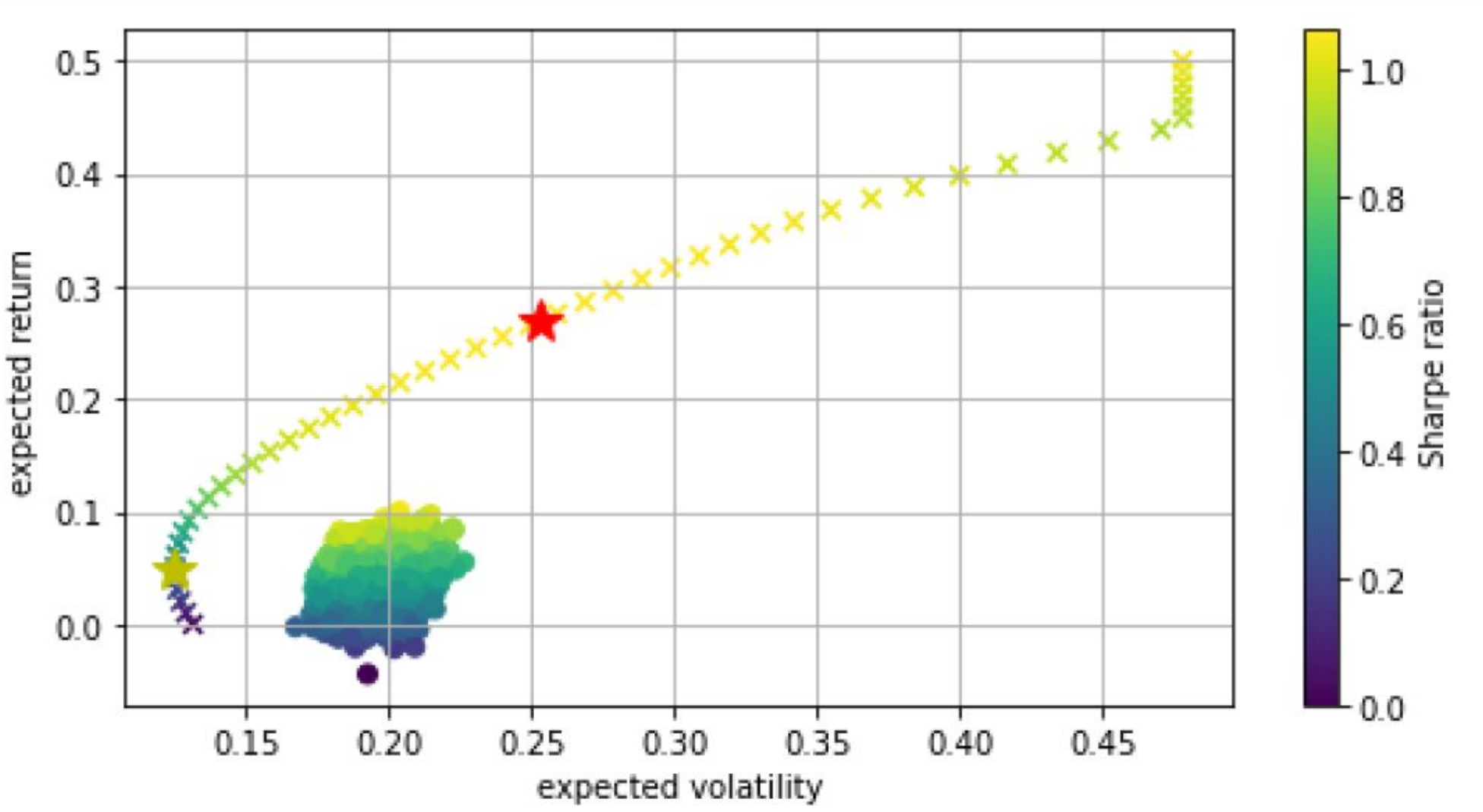
03 Optimal result.



The Stock Trend



Result with Python



Result with Python - Yellow Star

Combination of Portfolio

```
# The optimal combination weight vector with the minimum variance and the statistical data
optv['x'].round(3)
```

Out[64]:

```
array([0.548, 0.   , 0.   , 0.067, 0.212, 0.025, 0.   , 0.   , 0.   ,
       0.026, 0.   , 0.   , 0.058, 0.   , 0.047, 0.017, 0.   , 0.   ,
       0.   , 0.   ])
```

Stock Number	0003	0168	0175	0270
Stock Name	HK & China Gas	TsingTao Brew	Geely Auto	Guangdong Inv
Combination	54.8%	6.7%	21.2%	2.5%
Stock Number	0857	1093	1398	1928
Stock Name	PetroChina	CSPC Pharma	ICBC	Sands China Ltd.
Combination	2.6%	5.8%	4.7%	1.7%

Result with Python - Red Star

Combination of Portfolio

```
# The optimal combination weight vector obtained is:
opts['x'].round(3)
```

```
Out[61]:

array([0.189, 0.   , 0.   , 0.   , 0.   , 0.   , 0.   , 0.   ,
       0.   , 0.   , 0.   , 0.082, 0.   , 0.   , 0.   , 0.419, 0.   ,
       0.31 , 0.   ])
```

Stock Number	0003	1093	1929	2318
Stock Name	PetroChina	CSPC Pharma	Chow Tai Fook	Ping An Insurance
Combination	18.9%	8.2%	41.9%	31%

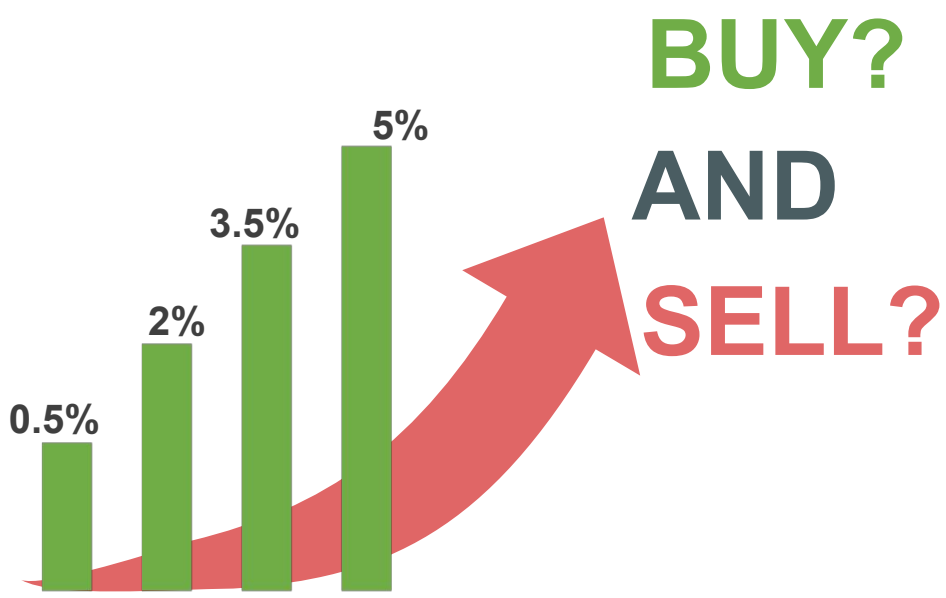
Transaction cost

what is transaction cost?

Transaction costs are expenses incurred when buying or selling a good or service.

TC per share: fixed fee
TC per percentage: percentage of the stock value traded
TC per trade: specific amount for each trade

The average annual transaction cost for a mutual fund in the U.S. was 1.44%, according to a study by Edelen, Evans, and Kadlec.

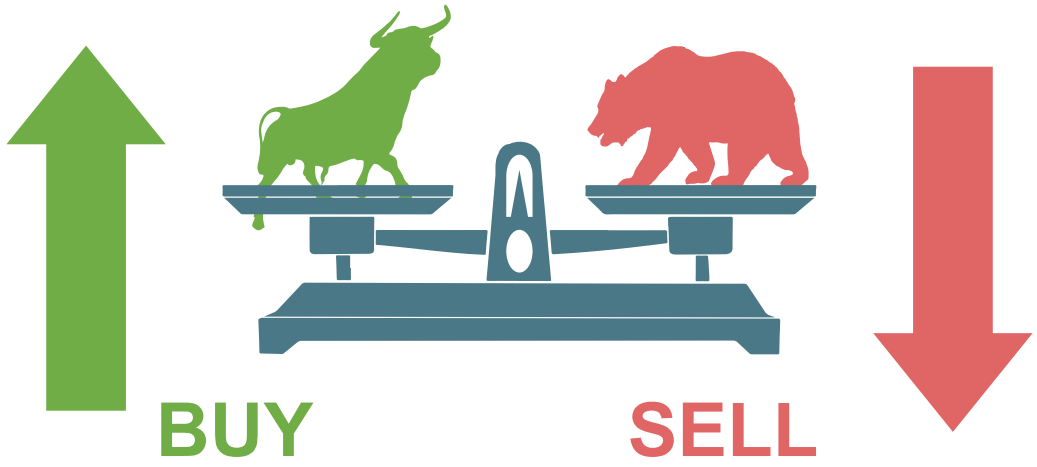


$$L_j = R^{withoutTC} - R_j^{withTC}$$

Transaction cost

$$R^{withoutTC} = \frac{W_T - W_0}{W_0}$$

$$R_j^{withTC} = \frac{W_T - W_0 - TC_j}{W_0},$$



WT = Total value of the portfolio at the very last period.

W0 = Value of the portfolio at the very first period

-Lower transaction cost leads to a Lower number of transaction.
-For example, trading in Peru can pay up to 14 times more in transaction costs than trading the same portfolio in the USA.

Objective II

Minimize the risk of portfolio

Minimize the Risk of Portfolio

Minimize the risk means minimize the *variance* of the Portfolio.

Formula of Portfolio Variance

$$= \begin{bmatrix} w_1\sigma_1 & \cdots & w_n\sigma_n \end{bmatrix} \text{x} \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \cdots & 1 \end{bmatrix} \text{x} \begin{bmatrix} w_1\sigma_1 \\ \vdots \\ w_n\sigma_n \end{bmatrix}$$

In order to minimize the variance of portfolio, we have to find the Portfolio Weight

Wi (**the percentage of an investment portfolio that a particular stock is holding**) to find how much percent of principal we should investment in each stock.

Assumption

1. Dividing 15 stocks into 3 groups (Group A, B, C) according to their industries, each group have 5 stocks. We would only invest to this 15 stocks.

	Group A (Pharmaceutical industry)	Group B (Banking industry)	Group C (real estate industry)
i	01177	00005	00012
ii	01093	00011	00017
iii	02005	00023	00021
iv	00241	00939	00672
v	00690	01988	01638

Assumption

3. Each transaction of stock is made within its own group, so their will form 3 transaction path (Path A, B,C).

Example Path & Portfolio:

Time period	1	2	3	4
Path A	Ai	Av	Aiii	Aii
Path B	Biii	Bii	Biv	Biv
Path C	Cv	Civ	Civ	Ci
	Portfolio in Time 1			

Assumption

2. Only invest in no more than one stock within one group at the same time. This means maximum investing 3 stock at the same time. The portfolio we construct will not have more than 3 stock.

Aim: Solving w1, w2, w3

Formula for 3 stock portfolio:

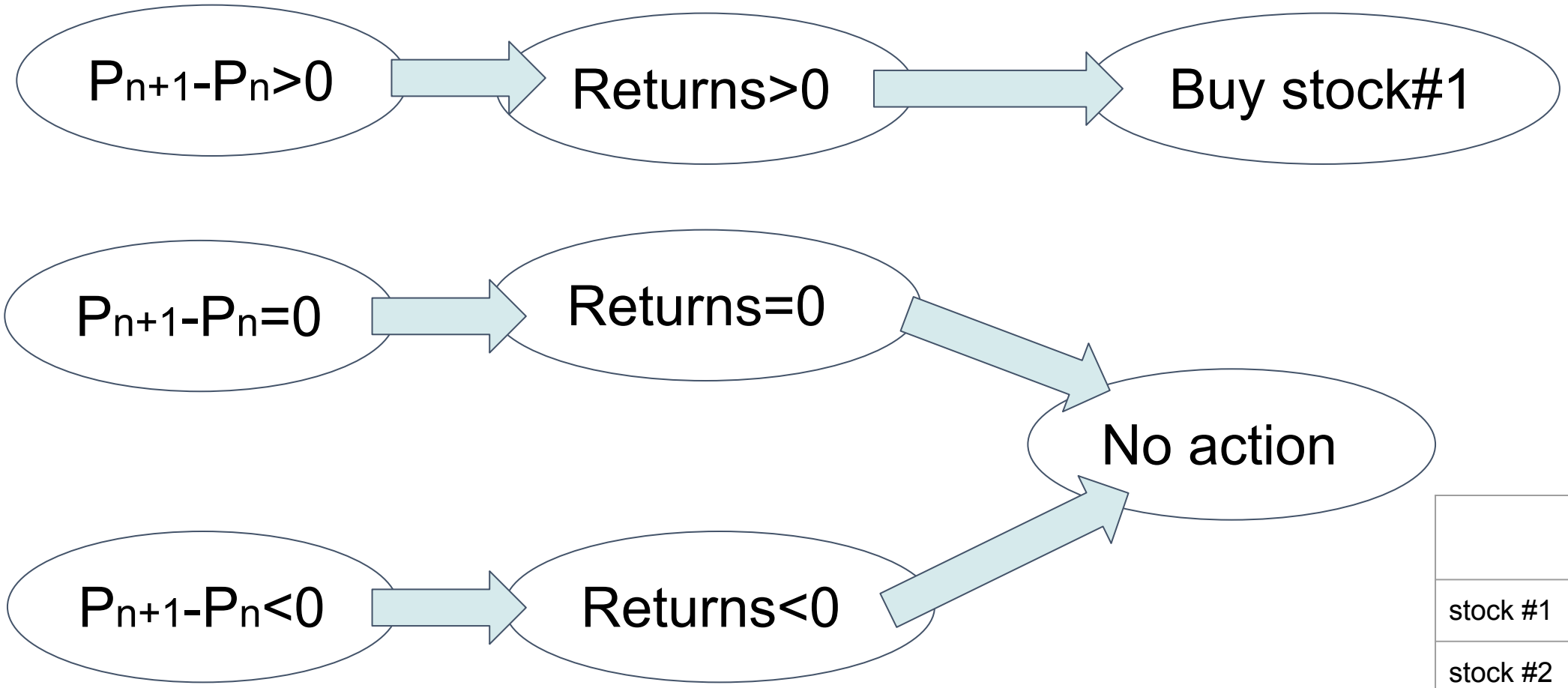
$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 + 2w_1w_3\rho_{13}\sigma_1\sigma_3 + 2w_2w_3\rho_{23}\sigma_2\sigma_3$$

Assumption

3. Making different investment decision to find the best path within each group according to the following situations;

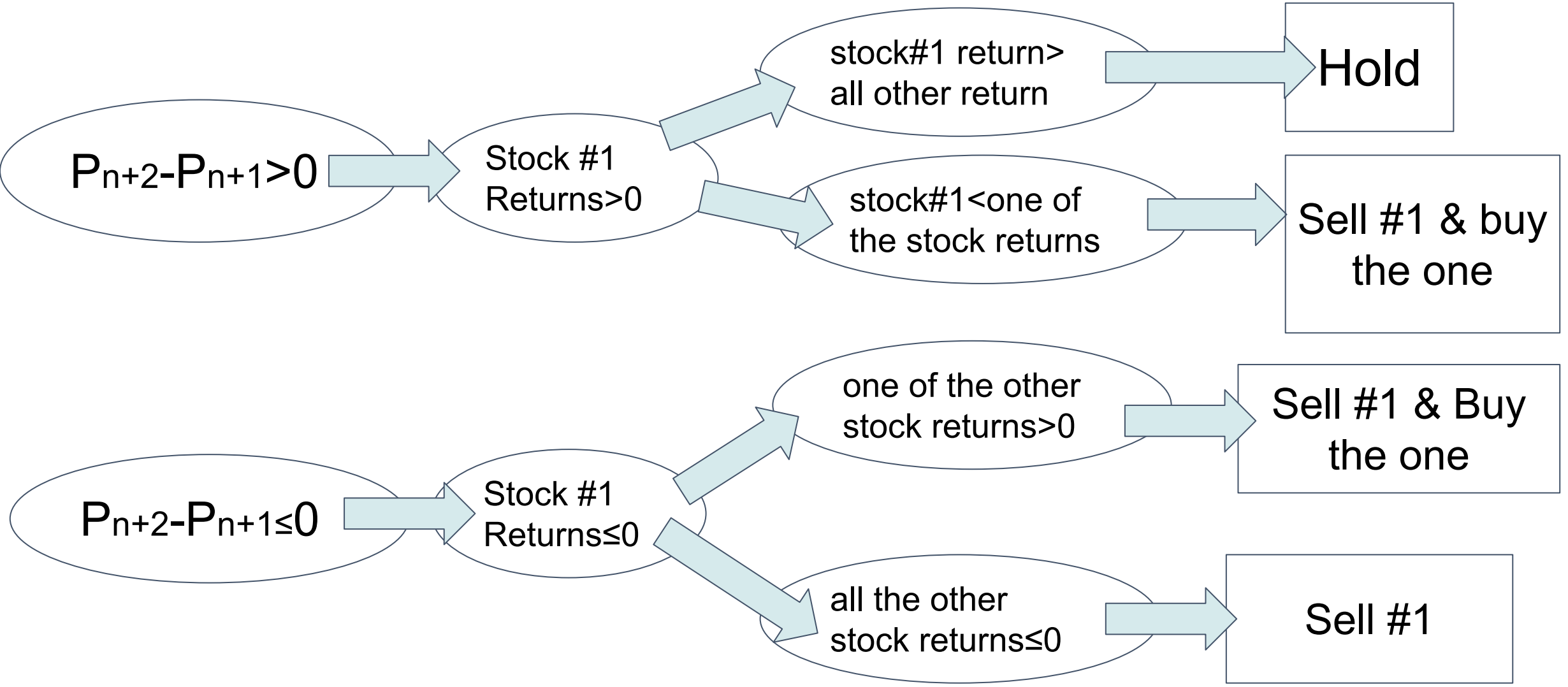
Investment Decision	Situation
Buy	occurs if 1. previous holding stock is sold, and 2. the alternative stock will have a positive return (Choosing the stock with highest return if more than one stock's return is positive)
Sell	occurs if the holding stock is predicted as not having the highest return compared with other stocks in the next time period
Hold	if the holding stock is predicted to have highest return in the next time period

On first time period



	$P_{n+1}-P_n$
stock #1	1.2
stock #2	0
stock #3	-0.3

After first time period



Assumption

4. Only making investment decision in the first day of a predefined time period.

5. Using the closing price from the beginning of July 2018 to the end of December 2018 of the above 15 stocks as the past data to predict future portfolio construction.

6. Investment decision making is only depends on the closing price of nth and (n+1)th day

Assumption

7. No transaction cost.

8. Amount of investment in each period is not cumulative with the profit earn in the previous periods. Only the unchanging original principal is used.

9. Investment decision making frequency is once a month and the project will consider 6 consecutive months.

Best Path in Group A

	2018-07-03	2018-08-01	2018-09-03	2018-10-02	2018-11-01	2018-12-03
s1	-	-	-	s1	-	-
s2	-	-	-	-	-	-
s3	-	-	-	-	s3	-
s4	s4	-	-	-	-	-
s5	-	-	s5	-	-	s5

Rate of Return:

Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
7.672%	0	6.383%	3.797%	20.294%	6.173%

Rate of Return Variance of Group A: 0.39%

Best Path in Group B

	2018-07-03	2018-08-01	2018-09-03	2018-10-02	2018-11-01	2018-12-03
s1	-	-	-	-	-	-
s2	s2	s2	-	-	-	-
s3	-	-	-	-	-	-
s4	-	-	-	-	s4	-
s5	-	-	s5	s5	-	-

Rate of Return:

Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
0	7.128%	0	2.297%	8.082%	0

Rate of Return Variance of Group B: 0.12%

Best Path in Group C

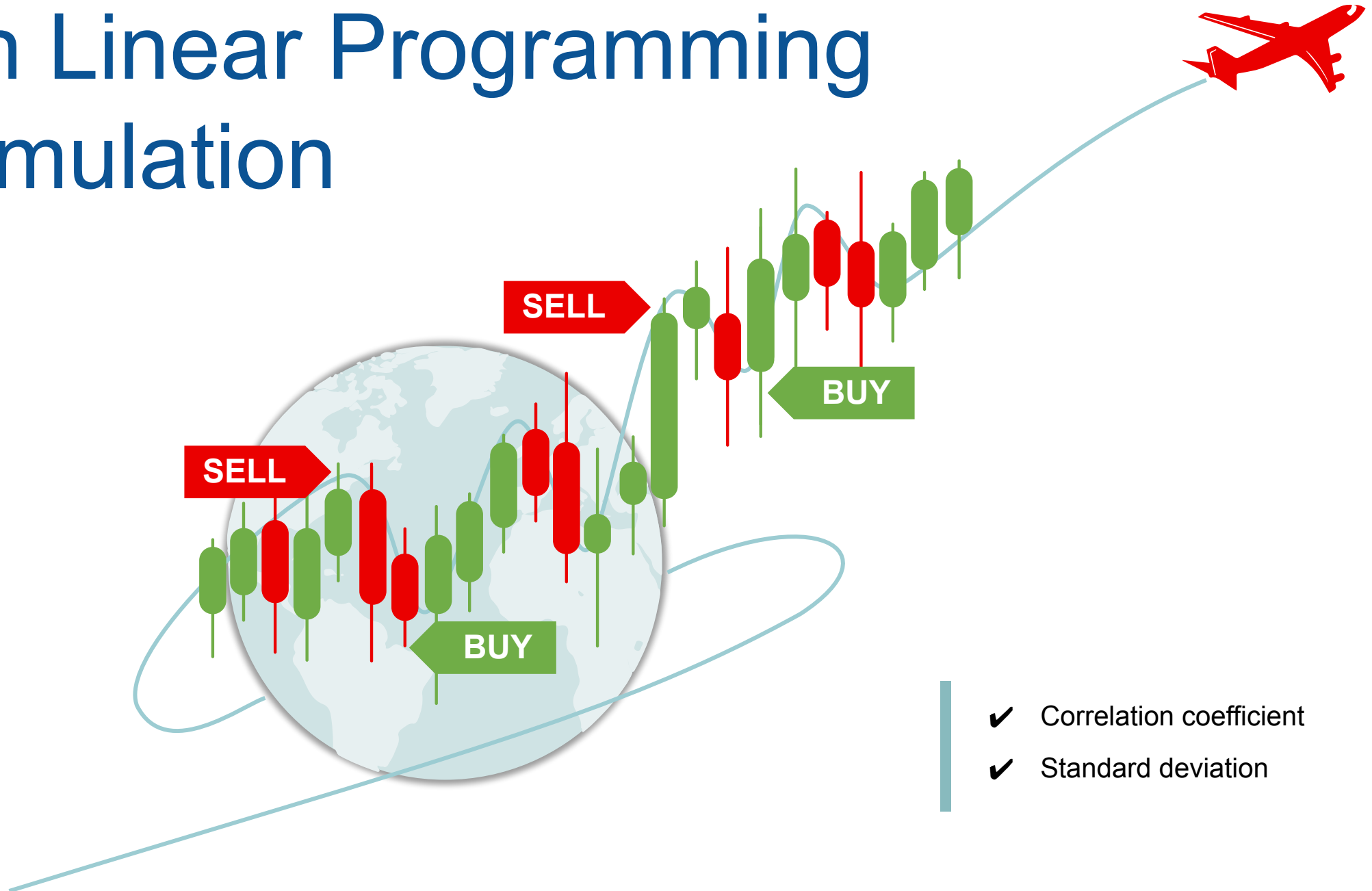
	2018-07-03	2018-08-01	2018-09-03	2018-10-02	2018-11-01	2018-12-03
s1	-	-	-	-	-	-
s2	-	-	-	-	-	-
s3	-	-	-	-	-	-
s4	s4	-	-	-	-	-
s5	-	-	-	-	s5	-

Rate of Return:

Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
6.410%	0	0	0	22.000%	0

Rate of Return Variance of Group C: 0.65%

Non Linear Programming Formulation



- ✔ Correlation coefficient
- ✔ Standard deviation

Non-Linear Programming Formula

Objective Function:

Min. $$w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 + 2w_1w_3\rho_{13}\sigma_1\sigma_3 + 2w_2w_3\rho_{23}\sigma_2\sigma_3$$

Constraints:

S.t. $$W_1+W_2+W_3=1$$

$$0\leq W_1, W_2, W_3\leq 1$$

Result

Optimal Solution:

```
w1  0.117339
w2  0.882661
w3  3.83515e-09
Obj:  0.00133326
```

To minimize the risk, we should invest 11.7339% of principal in **Group A**, 88.2661% of principal in **Group B** and nearly 0% of principal in Group C.

More to know...

Question: How will the portfolio weight change if the decision making frequency increase?

Investment decision making frequency is once a day and the project will consider around 130 consecutive trading days.


Thank You