Find effective set and minimize combination variance Find best sharpe (Revenue-Risk Balance Point) Find least risk combination

In [49]:

```
import os
import pandas as pd
import numpy as np
import statsmodels.api as sm
import scipy.stats as scs
import matplotlib.pyplot as plt
```

In [50]:

```
def get_price(stock, path):
    result = {}
    for i in stock:
        data = pd.read_csv(path + '/' + i + '.csv')
        result[i] = data["Close"].values.tolist()
        result["Date"] = data["Date"].values.tolist()
    result = pd.DataFrame(result, index=result["Date"])
    return result
def get_stock(path):
    result = []
    for root,dir,files in os.walk(path):
        for file in files:
            if file.endswith('.csv'):
                result.append(file[:-4])
    return result
```

In [51]:

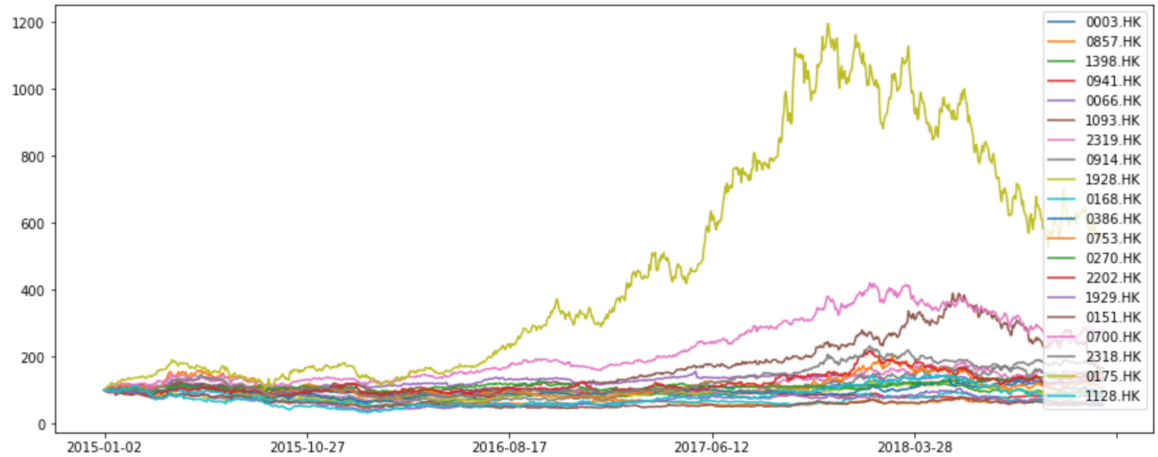
```
# Retrieve Stock Data and show some
path = r'Stock'

stock = get_stock(path) #['0066.HK', '0168.HK', '0857.HK']
data = get_price(stock, path)
data.pop("Date")
data.head()
```

Out[51]:

	0003.HK	0857.HK	1398.HK	0941.HK	0066.HK	1093.HK	2319.HK	0914.HK	19
2015-01-02	11.0276	8.69	5.77	91.400002	31.900000	6.68	16.025000	29.299999	37.7
2015-01-05	10.9034	8.74	5.80	90.099998	31.799999	6.64	15.825000	29.100000	37.2
2015-01-06	10.7916	8.53	5.71	88.750000	31.500000	6.68	15.800000	28.700001	35.9
2015-01-07	10.8289	8.57	5.75	91.750000	31.700001	6.78	16.075001	28.450001	37.0
2015-01-08	11.0151	8.78	5.72	93.599998	32.250000	6.65	16.400000	28.900000	37.1

In [74]:

```
(data/data.iloc[0]*100).plot(figsize = (15,6))
plt.legend(loc="best")
plt.show()
```



In [53]:

```
#252 Transaction days, find annualized return
returns = np.log(data / data.shift(1))
returns.mean()*252
```

Out[53]:

```
0003.HK    0.073939
0857.HK   -0.147476
1398.HK   -0.008100
0941.HK   -0.049353
0066.HK    0.065385
1093.HK    0.134354
2319.HK    0.107453
0914.HK    0.066450
1928.HK   -0.024156
0168.HK   -0.129502
0386.HK   -0.029747
0753.HK    0.021490
0270.HK    0.105492
2202.HK    0.084119
1929.HK   -0.124777
0151.HK   -0.154745
0700.HK    0.261655
2318.HK    0.130187
0175.HK    0.442833
1128.HK   -0.053414
dtype: float64
```

In [54]:

```
# The pandas built-in method was used to produce covariance matrix.
returns.cov()*252
```

Out[54]:

	0003.HK	0857.HK	1398.HK	0941.HK	0066.HK	1093.HK	2319.HK	0914.HK	19
0003.HK	0.019415	0.017116	0.015154	0.012105	0.011385	0.009075	0.014628	0.016774	0.C
0857.HK	0.017116	0.089435	0.040225	0.029142	0.017801	0.032332	0.033477	0.047736	0.C
1398.HK	0.015154	0.040225	0.059317	0.025538	0.015556	0.028855	0.031927	0.045126	0.C
0941.HK	0.012105	0.029142	0.025538	0.047088	0.014122	0.019612	0.020039	0.028875	0.C
0066.HK	0.011385	0.017801	0.015556	0.014122	0.031273	0.012482	0.017253	0.018127	0.C
1093.HK	0.009075	0.032332	0.028855	0.019612	0.012482	0.133557	0.044666	0.041707	0.C
2319.HK	0.014628	0.033477	0.031927	0.020039	0.017253	0.044666	0.123800	0.041932	0.C
0914.HK	0.016774	0.047736	0.045126	0.028875	0.018127	0.041707	0.041932	0.135082	0.C
1928.HK	0.017709	0.043038	0.036131	0.025802	0.017559	0.034523	0.045251	0.050477	0.1
0168.HK	0.011251	0.030711	0.028002	0.018826	0.011907	0.041708	0.042455	0.037999	0.C
0386.HK	0.017308	0.067765	0.040032	0.027177	0.017559	0.029584	0.034153	0.048153	0.C
0753.HK	0.014990	0.035319	0.044612	0.025217	0.020698	0.042078	0.043562	0.064953	0.C
0270.HK	0.011602	0.017727	0.017627	0.016303	0.009956	0.017136	0.017263	0.021529	0.C
2202.HK	0.014437	0.041326	0.044026	0.025904	0.015480	0.038544	0.037656	0.062191	0.C
1929.HK	0.009793	0.024592	0.021275	0.014818	0.011219	0.018856	0.027453	0.028336	0.C
0151.HK	0.012339	0.026981	0.022868	0.019035	0.011162	0.026535	0.046447	0.033804	0.C
0700.HK	0.015309	0.039834	0.041999	0.026451	0.018673	0.043803	0.039612	0.046736	0.C
2318.HK	0.014934	0.044501	0.048216	0.029736	0.016784	0.044640	0.038221	0.055252	0.C
0175.HK	0.021279	0.046670	0.050272	0.026162	0.024358	0.063226	0.065041	0.070360	0.C
1128.HK	0.020180	0.045166	0.040467	0.026783	0.020567	0.044003	0.055423	0.057028	0.1

In [55]:

```
#Randomly assign initial weights to different assets
noa = len(stock)
weights = np.random.random(noa)
weights /= np.sum(weights)
weights
```

Out[55]:

```
array([0.06173103, 0.06506785, 0.0407079 , 0.07734746, 0.00800729,
       0.05550668, 0.01932473, 0.05843662, 0.09101404, 0.04489076,
       0.07341048, 0.03271596, 0.05817184, 0.00363653, 0.07761747,
       0.02086835, 0.01047466, 0.05741701, 0.06712808, 0.07652523])
```

In [56]:

```
#Annualized return on portfolio
np.sum(returns.mean()*weights)*252
```

Out[56]:

```
0.024652507548896727
```

In [57]:

```
# combination variance
np.dot(weights.T, np.dot(returns.cov()*252,weights))
```

Out[57]:

```
0.0395664559842615
```

In [58]:

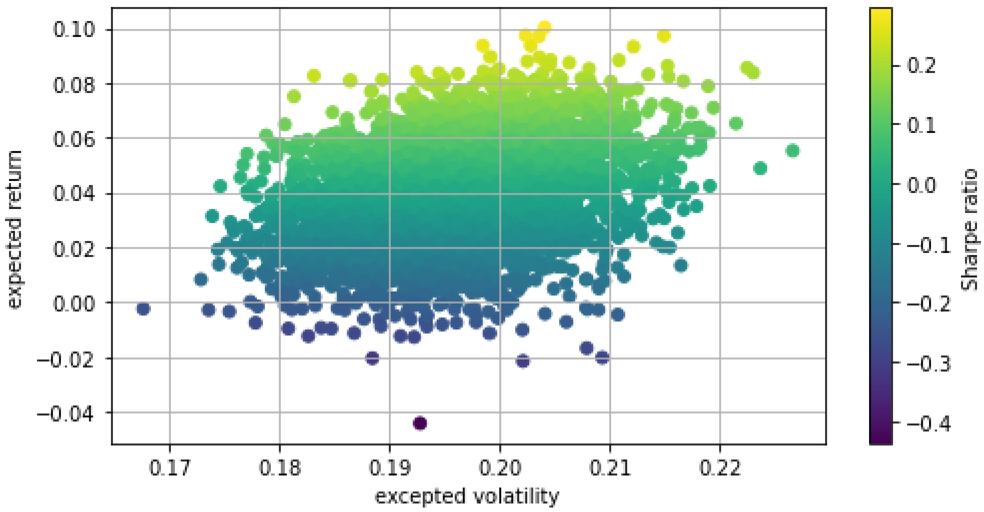
```
# Combined standard deviation
np.sqrt(np.dot(weights.T, np.dot(returns.cov()* 252,weights)))
```

Out[58]:

```
0.19891318705470862
```

In [59]:

```
# Through a Monte Carlo simulation, a large number of random weight vectors are
 generated and the expected returns and variances of random combinations are rec
orded.
port_returns = []
port_variance = []
for p in range(4000):
    weights = np.random.random(noa)
    weights /=np.sum(weights)
    port_returns.append(np.sum(returns.mean()*252*weights))
    port_variance.append(np.sqrt(np.dot(weights.T, np.dot(returns.cov()*252, wei
ghts))))

port_returns = np.array(port_returns)
port_variance = np.array(port_variance)

#The risk-free rate was set at 4%
risk_free = 0.04
plt.figure(figsize = (8,4))
plt.scatter(port_variance, port_returns, c=(port_returns-risk_free)/port_varianc
e, marker = 'o')
plt.grid(True)
plt.xlabel('excepted volatility')
plt.ylabel('expected return')
plt.colorbar(label = 'Sharpe ratio')
plt.show()
```



In [60]:

```
# Portfolio Optimization 1 -- SHARPE maximizes
# Create statistics function to record important portfolio statistics (returns,
 variance and # Sharpe ratio)
# By solving the constrained optimal problem, the optimal solution is obtained.
 Where the constraint is that the sum of the weights is 1.

def statistics(weights):
    weights = np.array(weights)
    port_returns = np.sum(returns.mean()*weights)*252
    port_variance = np.sqrt(np.dot(weights.T, np.dot(returns.cov()*252,weights
)))
    return np.array([port_returns, port_variance, port_returns/port_variance])
#The derivation of portfolio optimization is a constrained optimization problem
import scipy.optimize as sco

#Minimize the negative of the Sharpe index
def min_sharpe(weights):
    return -statistics(weights)[2]

#The constraint is that the sum of all the parameters (weights) is 1. This can b
e expressed in the terms minimize function
cons = ({'type':'eq', 'fun':lambda x: np.sum(x)-1})

#We also limit the parameter values (weights) to between 0 and 1. These values a
re provided to the minimization function in the form of a tuple consisting of se
veral tuples
bnds = tuple((0,1) for x in range(noa))

#The only input ignored in the optimization function call is the start argument
 list (the initial guess at the weight). Let's just use the average distributio
n.
opts = sco.minimize(min_sharpe, noa*[1./noa,], method = 'SLSQP', bounds = bnds,
constraints = cons)
opts
```

Out[60]:

```
     fun: -1.0631670185036584
     jac: array([ 1.59442425e-06,  1.17442892e+00,  6.51948810e-01,
5.71961418e-01,
        4.54854220e-02,  1.49405390e-01,  2.53327101e-01,  5.0414769
4e-01,
        8.09799790e-01,  1.03651714e+00,  7.15328440e-01,  6.9401106
2e-01,
        2.40698457e-04,  4.50056776e-01,  8.95425335e-01,  1.0839755
1e+00,
       -1.01193786e-04,  2.48458147e-01,  7.20918179e-05,  1.1229428
4e+00])
 message: 'Optimization terminated successfully'
    nfev: 254
     nit: 12
    njev: 12
  status: 0
 success: True
       x: array([1.89105075e-01, 5.26488575e-16, 3.91125934e-16, 5.1
5538133e-17,
       6.36968776e-17, 0.00000000e+00, 0.00000000e+00, 0.00000000e+0
0,
       2.37399675e-16, 0.00000000e+00, 0.00000000e+00, 1.10968092e-1
6,
       8.19225611e-02, 0.00000000e+00, 3.97360096e-17, 4.29073010e-1
6,
       4.18725214e-01, 0.00000000e+00, 3.10247150e-01, 1.30917412e-1
6])
```

In [61]:

```
# The optimal combination weight vector obtained is:
opts['x'].round(3)
```

Out[61]:

```
array([0.189, 0.   , 0.   , 0.   , 0.   , 0.   , 0.   , 0.   , 0.
,
       0.   , 0.   , 0.   , 0.082, 0.   , 0.   , 0.   , 0.419, 0.
,
       0.31 , 0.   ])
```

In [62]:

```
# The three statistics of the largest combination of SHARpe are:
#Expected return, expected volatility, optimal Sharpe index
statistics(opts['x']).round(3)
```

Out[62]:

```
array([0.27 , 0.254, 1.063])
```

In [63]:

```
# Portfolio optimization 2 -- Minimum variance
# The optimal portfolio is selected by means of the minimum variance.

# But let's define a function that minimizes the variance
def min_variance(weights):
    return statistics(weights)[1]

optv = sco.minimize(min_variance, noa*[1./noa,],method = 'SLSQP', bounds = bnds,
constraints = cons)
optv
```

Out[63]:

```
     fun: 0.125399509868879
     jac: array([0.12539308, 0.15437329, 0.13697726, 0.12539728, 0.1
2541872,
       0.12534392, 0.14620494, 0.16079454, 0.1586436 , 0.12544857,
       0.15410297, 0.15560894, 0.12561048, 0.14048347, 0.12528692,
       0.12497845, 0.15167208, 0.14577114, 0.20072477, 0.18414867])
 message: 'Optimization terminated successfully'
    nfev: 273
     nit: 13
    njev: 13
  status: 0
 success: True
       x: array([5.47684825e-01, 0.00000000e+00, 6.07153217e-18, 6.6
7679210e-02,
       2.11856714e-01, 2.51979798e-02, 0.00000000e+00, 0.00000000e+0
0,
       2.91379334e-18, 2.60677862e-02, 1.60597447e-17, 1.32814766e-1
8,
       5.83105257e-02, 0.00000000e+00, 4.67399342e-02, 1.73743143e-0
2,
       0.00000000e+00, 4.24194100e-18, 2.77420231e-17, 0.00000000e+0
0])
```

In [64]:

```
# The optimal combination weight vector with the minimum variance and the statis
tical data of the combination are respectively:
optv['x'].round(3)
```

Out[64]:

```
array([0.548, 0.   , 0.   , 0.067, 0.212, 0.025, 0.   , 0.   , 0.
,
       0.026, 0.   , 0.   , 0.058, 0.   , 0.047, 0.017, 0.   , 0.
,
       0.   , 0.   ])
```

In [65]:

```
# The expected yield, volatility and Sharpe index are obtained
statistics(optv['x']).round(3)
```

Out[65]:

```
array([0.049, 0.125, 0.388])
```

In [66]:

```
# The effective frontier has the portfolio composition with the minimum variance
under the given target rate of return.
# Two constraints are used in optimization, 1. Given target return rate, 2. Port
folio weight sum is 1.

def min_variance(weights):
    return statistics(weights)[1]

# One of the minimized constraints changes with the different target return leve
ls (TARGEt_returns) loop.
target_returns = np.linspace(0.0,0.5,50)
target_variance = []
for tar in target_returns:
    cons = ({'type':'eq','fun':lambda x:statistics(x)[0]-tar},{'type':'eq','fun'
:lambda x:np.sum(x)-1})
    res = sco.minimize(min_variance, noa*[1./noa,],method = 'SLSQP', bounds = bn
ds, constraints = cons)
    target_variance.append(res['fun'])

target_variance = np.array(target_variance)
```

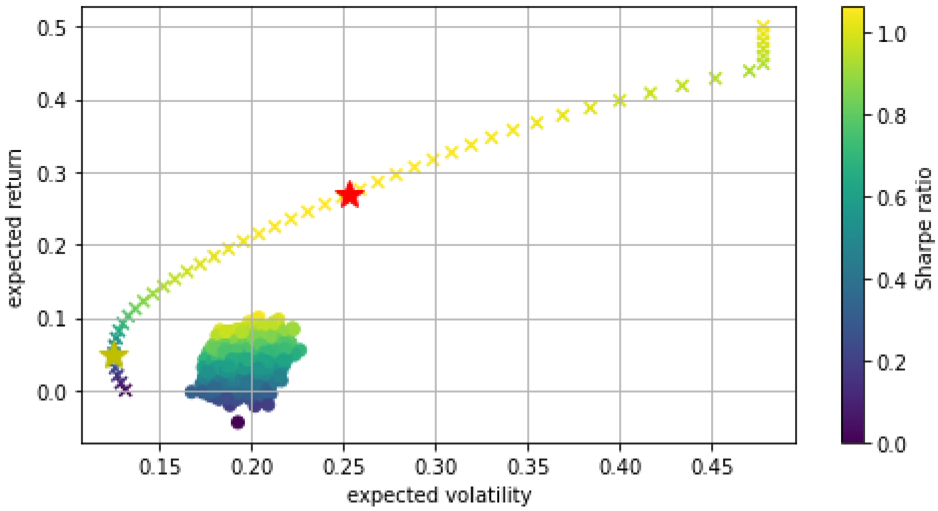
The following is a demonstration of the optimal results: Cross: The formed curve is the effective frontier (the optimal portfolio under the target yield rate)

Red Star: Sharpe's largest portfolio

Yellow Star: The portfolio with the least variance

In [67]:

```
plt.figure(figsize = (8,4))
#Circle: A combination of randomly generated distributions in Monte Carlo
plt.scatter(port_variance, port_returns, c = port_returns/port_variance,marker =
'o')
#Cross: Effective leading edge
plt.scatter(target_variance,target_returns, c = target_returns/target_variance,
marker = 'x')
#Red star: Marked with the highest SHARPE combination
plt.plot(statistics(opts['x'])[1], statistics(opts['x'])[0], 'r*', markersize =
15.0)
#Yellow star: Marks the minimum variance combination
plt.plot(statistics(optv['x'])[1], statistics(optv['x'])[0], 'y*', markersize =
15.0)
plt.grid(True)
plt.xlabel('expected volatility')
plt.ylabel('expected return')
plt.colorbar(label = 'Sharpe ratio')
plt.show()
```



In [ ]: