PERGAMON

Omega 30 (2002) 97–108

omega
The International Journal of Management Science

www.elsevier.com/locate/dsw

Optimal market timing strategies under transaction costs

Wei Li[a,*,1], Kin Lam[b]

[a] Department of Business Studies, Faculty of Business and Information System, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
[b] Department of Finance and Decision Sciences, School of Business, Hong Kong Baptist University, Kowloon Tong, Hong Kong

Received 15 November 1999; accepted 20 November 2001

Abstract

In this paper, we consider optimal market timing strategies under transaction costs. We assume that the asset's return follows an auto-regressive model and use long-term investment growth as the objective of a market timing strategy which entails the shifting of funds between a risky asset and a riskless asset. We give the optimal trading strategy for a finite investment horizon, and analyze its limiting behavior. For a finite horizon, the optimal decision in each step depends on two threshold values. If the return today is between the two values, nothing needs to be done, otherwise funds will be shifted from one asset to another, depending on which threshold value is being exceeded. When investment horizon tends to infinity, the optimal strategy converges to a stationary policy, which is shown to be closely related to a well-known technical trading rule, called Momentum Index trading rule. An integral equation of the two threshold values is given. Numerical results for the limiting stationary strategy are presented. The results confirm the obvious guess that the no-transaction region increases as the transaction cost increase. Finally, the limiting stationary strategy is applied to data in the Hang Seng Index Futures market in Hong Kong. The out-of-sample performance of the limiting stationary strategy is found to be better than the simple strategy used in literature, which is based on an 1-step ahead forecast of return. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Market timing; Autoregressive models; Growth-optimal investment strategy; Stochastic dynamic programming

1. Introduction

Market timing is an investment strategy which attempts to outperform the market. It entails the shifting of funds between asset classes. The principal job of a market timer is to time when to enter and when to exit the market, by shifting funds between risky and riskless assets. To decide on a suitable time to be in and out of the market, market timers would like to generate buy/sell signals according to various indicators derived from their analysis, or from a model based on historical market data.

Market timing strategies have long been used to test the predictability of stock market. One of its earliest usage can be found in a well-known paper "Can stock market forecasters forecast?" published in 1934 by Cowles. In that article, Cowles tested the Dow Theory by the use of a market timing strategy and apparently provided strong evidence against the ability of Wall Street's most famous chartist, William Peter Hamilton, to forecast the stock market. This confirms the efficient market hypothesis (EMH) of Fama [1], which hypothesizes that no one can beat the market. When the EMH in its weak form holds, no trading strategy based on historical price data can earn excess return over the naive buy-and-hold strategy.

However, in the last decade there has been more and more empirical evidences against the EMH, and even the foundation paper by Cowles [2] was challenged. Brown et al. [3] "review Cowles' evidence and find that it supports the contrary conclusion". They concluded that "The contribution of this paper is not simply to show that Hamilton was a

* Corresponding author. Tel.: +852-2766-7109; fax: +852-2765-0611.

E-mail addresses: buwli@inet.polyu.edu.hk (W. Li), lamkin@hkbu.edu.hk (K. Lam).

[1] A large part of this paper was written while she was a Ph.D. candidate in the Department of Finance and Decision Sciences, School of Business, Hong Kong Baptist University.

PII: S0305-0483(01)00060-3

successful market timer. Alfred Cowles' [2] analysis of the Hamilton record is a watershed study that led to the random walk hypothesis, and thus played a key role in the development of the efficient market theory. Ever since Cowles' article, 'chartists' in general, and Dow theorists in particular, have been regarded by financial economists with skepticism. Our replication of Cowles' analysis yields results contrary to Cowles' conclusions. At the very least, it suggests that more detailed analysis of the Hamilton version of the Dow Theory is warranted. In broader terms it also suggests that the empirical foundations of the efficient market theory may not be as firm as long believed". This result is certainly encouraging to market timers.

The debates on the timing ability of the fund managers between academic community and practitioners have a long history. The negative opinion about market timing probably should stem back to two important articles by Sharpe [4] and Jeffery [5]. But even before Brown et al.'s work in 1998, there were already many empirical works which found that market timing strategies can outperform the market. For example, Vandell and Stevens [6] documented that portfolio performance can be improved by market timing. Shilling [7] remarked that market timing can be better than a buy-and-hold strategy. Larsen and Wozniak [8] found that "Market timing can work in the real world". Wagner [9] studied "Why market timing works". Two most recent papers by Kao and Shumaker [10] and Levis and Liodakis [11] studied the profitability of size and value/growth rotation market timing strategies in the US and UK markets, respectively.

In view of this empirical evidence, more research is needed to investigate how to devise a good market timing strategy. In this paper, we would like to settle the following problem: if stock returns are indeed predictable, how can the predictability be best exploited by a suitable market timing strategy? This question is important not only to market timing practitioners, but also to academics who are interested in testing models for stock returns. The usual method to test a model for stock returns is to first calculate the expected future return from the model. If the expected 1-step ahead excess return is positive, the obvious trading rule is to hold the risky asset, otherwise to hold the riskless asset. This simple trading rule has been used by many researchers in the testing of the predictability of stock returns. These include Breen et al. [12], Leitch and Tanner [13], Pesaran and Timmermann [14,15], Knez and Ready [16], Lander et al. [17] and Lee [18]. In the presence of transaction costs, the above approach is modified by first computing the trading profit of the "simple" strategy mentioned above and then deducting the transaction costs from the profit. If no significant profit can then be derived, the conclusion of no predictability of the model is drawn. However, this is an unfair test of the model's predictability, because the "simple" strategy which was used to test for predictability may not be optimal in the presence of transaction costs. The model can be rejected not because it has no predictability, but because the "simple" strategy is not the proper strategy to use. The following paragraph will explain why the "simple" strategy may not be optimal under transaction costs.

When transaction costs are not negligible, the optimal market timing strategy or trading rule should be different from the "simple" strategy. Even if the expected excess return in the next period is negative, it may still be worthwhile to hold the risky asset because in subsequent periods, the expected excess return may turn out to be positive again, and transaction costs may be saved if we do not switch away from the risky asset too readily. As reported by Chua et al. [19]. "Optimal market-timing strategy involves multiperiod forecasts, rather than the single-period projections generated within most simulation studies. The investor must make his timing decision based on forecasts beyond the next period, because he knows he will eventually be switching back to common stocks. His decision must therefore take into account the round-trip transaction costs, how many periods he will be out of the stock market and the incremental return from the switches. $\cdots$ Simulating optimal market-timing behavior entails both a multiperiod market return forecasting model and a dynamic programming model to determine when to switch."

The objective of this paper is to derive optimal market timing strategies under transaction costs, assuming that the return generating process is not perfectly random. One type of models suggested for the return series in the literature is the autoregressive-moving-average processes (ARMA). Chopra et al. [20], De Bondt and Thaler [21], Fama and French [22] and Poterba and Summers [23] found correlation in returns of individual stocks and various portfolios over three-to-ten year periods. Hodrick and Srivastava [24], Mark [25] and Taylor [26,27] found positive autocorrelation among exchange rate returns. Conrad and Kaul [28] reported a first-order autocorrelation of 0.2 for a value-weighted portfolio of the largest companies during the period 1962–1985. Goodhart [29] and Goodhart and Figliuoli [30] reported the existence of negative first-order autocorrelation of the price changes at the highest frequencies.

In this paper, we formulate the problem for the general ARMA model, and then assume a simple autoregressive model with order one, i.e. AR(1), for the return structure and derive the strategy that should be taken in order to maximize the expected average continuously compounded return, or the expected total excess return. We first derive the optimal trading strategy for a finite investment horizon using stochastic dynamic programming techniques. We then analyze the limiting behavior of this optimal trading strategy when the investment horizon tends to infinity. Interestingly, we find that the limiting strategy turns out to be a familiar rule in technical trading, i.e., the momentum index trading rule.

This paper is organized as follows. In Section 2, we identify a suitable objective function for market timers. This is essential because without a well specified objective function, it is not possible to compare trading strategies. Section

3 discusses the optimal market timing strategy under finite investment horizon. Section 4 gives the limiting behavior of the optimal strategy. Section 5 presents the limiting strategy numerically under realistic parameter values and analyzes the effect of transaction costs on market timing strategies. Section 6 simulates the limiting strategy in the Hang Seng Index Futures market. Its performance is compared with the preformance with the simple strategy and the buy-and-hold strategy. Section 7 draws the conclusions of the paper. Some mathematical proofs can be found in the appendixes.

2. Objective for market timers

A market timer is conventionally defined as a person who completely shifts his funds between a risky and a riskless asset, i.e. the funds are entirely invested either in a risky or a riskless asset and there is no need to adjust the percentage of funds in the assets. If the investor finds an opportunity to enter the market, he will buy the risky asset with all the money he possesses. If he is negative about the market, he will withdraw all the money from the market and invest in the riskless asset. Thus, the market timer's decision ultimately comes down to whether to be in or out of equities. i.e., 100% in or out of the equity market.

As mentioned by Admati et al. [42], "Most of the traditional work on the evaluation of investment performance has been concerned with 'risk-return' measures, where the main idea is to use the mean returns on the asset or portfolio after some adjustment or 'correction' for risk. Risk is usually measured by the standard deviation of returns or the volatility (beta) of the portfolio". However, market timers may not agree that adjustment of returns by risk is necessary. They argue that, since there must be some time periods when the riskfree asset is recommended by the market timing strategy, the risk associated with the market timing strategy cannot be more than the risk associated with the buy-and-hold strategy on the risky asset. If one market timing strategy has the same total excess return as the buy-and- hold strategy, we can say the former is better than the latter even after risk adjustment. Thus, many market timers have negative opinions on the risk-adjusted return measure and suggest using the un-adjusted total excess return as the performance measure of a market timing strategy (see for example [31,18,32–35,43] etc.). In the markets, many practitioners and technical traders like to use the total or average excess return to measure their performance, without taking risk into consideration.

Put simply, the objective of market timers is the long term growth of the capital. In this paper, we use the expected average continuously compounded return as the performance measure for market timers. In fact, when the continuously compounded return is used, the risk of the strategy has already been taken into consideration. It is well known that this criterion of maximizing the expected terminal log-wealth will result in an investment strategy which guarantees that wealth will grow optimally, see for example, Cox and Huang [36] and Hakansson and Ziemba [37] etc.

As remarked above, market timers will choose to maximize their expected log-wealth at the end of an investment period. In order to obtain optimal strategies for market timers, it is essential to express the terminal wealth in terms of the investment decision variables. We now make the following assumptions: (1) The investor is self-financing, neither consumes nor deposits new cash into the portfolio during the trading periods, but reinvests his portfolio each period. (2) The investor has an investment horizon of T days. (3) There are only two assets, one risky and one riskless. The riskless asset has a constant return throughout the investment period. (4) There are no dividends on the risky asset. (5) No short-selling or borrowing is allowed. (6) The transaction cost on the risky asset is c times the value traded, and there is no transaction cost for trading the riskless asset.

Trading decisions are made at the end of each day. The decision variable at the end of day $t-1$ is denoted by d_t. It takes the value of "1" or "0", according to whether the investor is entirely in the risky or in the riskless asset. If the decision variable d_{t+1} differs from d_t, trading transaction has to be made, otherwise, no transaction is necessary. Let w_t be the wealth at the end of day t, p_t be the stochastic price of the risky asset, and p_t^0 be the deterministic price of the riskless asset.

Let us consider the wealth w_{t+1} at day $t+1$. When there is no transaction cost, it is easy to see that

$$w_{t+1} = w_t \left(\frac{p_{t+1}}{p_t}\right)^{d_{t+1}} \left(\frac{p_{t+1}^0}{p_t^0}\right)^{1-d_{t+1}}$$

and the final wealth is

$$w_T = w_0 \prod_{t=1}^{T} \left(\frac{p_t}{p_{t-1}}\right)^{d_t} \left(\frac{p_t^0}{p_{t-1}^0}\right)^{1-d_t},$$

where w_0 is the initial wealth.

If there are transaction costs, similar to Pesaran and Timmermann [15], the wealth at time $t+1$ can be summarized as follows:

$$w_{t+1} = w_t \left(\frac{p_{t+1}}{p_t}\right)^{d_{t+1}} \left(\frac{p_{t+1}^0}{p_t^0}\right)^{1-d_{t+1}} (1-c)^{|d_{t+1}-d_t|}$$

and the terminal wealth is given by

$$w_T = w_0 \prod_{t=1}^{T} \left(\frac{p_t}{p_{t-1}}\right)^{d_t} \left(\frac{p_t^0}{p_{t-1}^0}\right)^{1-d_t} (1-c)^{|d_t-d_{t-1}|}.$$

The continuous compounded return in the whole period is given by

$$\ln(w_T/w_0) = \sum_{t=1}^{T} [d_t r_t + (1-d_t) r^0 - c' |d_t - d_{t-1}|],$$

where $r_t = \ln(p_t/p_{t-1})$ and $r^0 = \ln(p_t^0/p_{t-1}^0)$ are the continuous compounded returns for the risky asset and riskless asset respectively and $c' = -\ln(1-c) \approx c$. For the sake of simplicity, we will still use c instead of c' from now on.

According to the growth-optimal criterion, the objective of a market timer is to find a decision rule consisting of $d_1, d_2, \ldots, d_T$ to maximize the expected value of $\ln(w_T/w_0)$, i.e.

$$\max_{d_t=0,1} E \sum_{t=1}^{T} [r^0 + (r_t - r^0)d_t - c|d_t - d_{t-1}|].$$

Because r^0 is a constant, this is equivalent to the following:

$$\max_{d_t=0,1} E \sum_{t=1}^{T} [(r_t - r^0)d_t - c|d_t - d_{t-1}|]. \tag{1}$$

Here d_0 equals 0 (or 1), if the initial wealth is invested in the riskless (or risky) asset. Without loss of generality, we will assume the risk free rate r^0 to be zero hereafter. If r^0 is not equal to zero, we simply redefine $r_t - r^0$ as r_t and the following procedures will be the same.

3. Optimal strategy for finite time horizon

3.1. ARMA(p, q) model of return process

It is obvious that the decision of when to enter into and when to get out of the market should depend on the pattern, if there is any, of the price movement of the underlying risky asset.

Let's assume that the return process follows an ARMA model. Under an ARMA(p, q) model, r_t has the following representation:

$$\phi(B)(r_t - \mu_r) = \theta(B)\varepsilon_t, \tag{2}$$

where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p,$$

$$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$$

and B is the backshift operator, i.e., $Br_t \equiv r_{t-1}$. μ_r is the mean of the return series r_t, ε_t are i.i.d. $N(0, \sigma^2)$ white noises. We also assume that the parameters, $\mu_r, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q$ are known to the investor.

Therefore, our problem is to maximize the objective function (1) under the constrain (2), i.e.,

$$\begin{aligned} &\max_{d_t=0,1} \quad E\sum_{t=1}^{T} [(r_t - r^0)d_t - c|d_t - d_{t-1}|], \\ &\text{s.t.} \quad \phi(B)(r_t - \mu_r) = \theta(B)\varepsilon_t. \end{aligned} \tag{3}$$

This kind of problem can be solved by the method of dynamic programming. But under the general ARMA models, or even more complex models, such as the GARCH models, we can only get numerical results instead of getting analytical solutions for the optimal problem.

In this paper, we solve the problem assuming a very simple return structure of an auto-regressive model with order one (AR(1)). The use of an AR(1) model is of course too simplistic but it has the advantage that the associated optimal strategy turns out to be very simple and carries an interesting interpretation. The idea can also be generalized to deal with price movements which are more complicated than an AR(1) setting.

The AR(1) model of r_t can be expressed as follows:

$$r_t = \mu + \phi r_{t-1} + \varepsilon_t, \tag{4}$$

where $\mu = (1-\phi)\mu_r$.

Therefore, the problem solved in this paper is as follows:

$$\begin{aligned} &\max_{d_t=0,1} \quad E\sum_{t=1}^{T} [(r_t - r^0)d_t - c|d_t - d_{t-1}|], \\ &\text{s.t.} \quad r_t = \mu + \phi r_{t-1} + \varepsilon_t. \end{aligned} \tag{5}$$

3.2. Optimal strategy

The optimization problem (5) can be solved by the use of stochastic dynamic programming techniques. Let x denote the return in day $t-1$, y denote the investment position in that day, and $d_t(x, y)$ denote the decision at the end of day $t-1$, which has the same meaning as in Section 2. Then we have the following theorem for the finite investment horizon problem:

Theorem 1. *For $t = T, T-1, \ldots,$ the optimal decisions are as follows*:
if $\phi > 0$

$$d_t(x,1) = \begin{cases} 0 & x \leqslant b_{t+1}, \\ 1 & x > b_{t+1}, \end{cases} \qquad d_t(x,0) = \begin{cases} 0 & x \leqslant a_{t+1}, \\ 1 & x > a_{t+1} \end{cases} \tag{6}$$

if $\phi < 0$

$$d_t(x,1) = \begin{cases} 0 & x \geqslant b_{t+1}, \\ 1 & x < b_{t+1}, \end{cases} \qquad d_t(x,0) = \begin{cases} 0 & x \geqslant a_{t+1}, \\ 1 & x < a_{t+1}, \end{cases} \tag{7}$$

where a_{t+1} and b_{t+1} satisfy $h_{t+1}(a_{t+1}) = c$, $h_{t+1}(b_{t+1}) = -c$, and $h_t(x)$ can be determined sequentially as follows:

$$h_{T+1}(x) = \phi x + \mu,$$
$$h_t(x) = \phi x + \mu + Es_t(\phi x + \mu + \varepsilon),$$

where

$$s_t(x) = \begin{cases} c & h_{t+1}(x) \geqslant c, \\ h_{t+1}(x) & -c < h_{t+1}(x) < c, \\ -c & h_{t+1}(x) \leqslant -c. \end{cases}$$

For a proof of Theorem 1, see Appendix A.

From the above Theorem, we can see that the decisions depend on some threshold values, a_t and b_t, which are determined by function $h_t(x)$. The above Theorem amounts to the following optimal trading rule:

Optimal trading rule:

(i) When the investment position is "long" (i.e., in the equity market) in day $t-1$ $(0 \leqslant t \leqslant T-1)$, sell the asset at the close of day $t-1$ to have a "neutral" position (i.e., out of the equity market) in day t if $r_{t-1} < b_{t+1}$ and maintain the long position otherwise.

(ii) When the investment position is "neutral" in day $t-1$ $(0 \leqslant t \leqslant T-1)$, buy the asset at the close of day $t-1$ to have a "long" position in day t if $r_{t-1} > a_{t+1}$ and maintain the neutral position otherwise.

The above rule is for $\phi > 0$, i.e., (6). When $\phi < 0$, we will have a similar rule accordingly. We mainly consider the case of $\phi > 0$ hereafter, unless it is mentioned otherwise.

3.3. Effect of transaction costs

It is easy to see that the optimal strategy depends on the values of a_t and b_t which are functionally dependent on the transaction cost c.

When there is no transaction cost, i.e., $c = 0$, we have $s_t(x)=0$. Hence $h_t(x)=\phi x+\mu$, and therefore $a_t=b_t=-\mu/\phi$ for $t = 1, 2, \dots, T$. Thus the optimal decision in each step is

$$d_t = \begin{cases} 0 & \phi x + \mu \leqslant 0, \\ 1 & \phi x + \mu > 0. \end{cases}$$

This rule suggests that if the predicted return next day is bigger than zero, then buy the risky asset, else hold the riskless asset, which is simply the trivial market timing strategy used in the literature and the so called 'simple strategy' in this paper.

In the presence of transaction costs, $a_t > -\mu/\phi$ and $b_t < -\mu/\phi$, assuming $\phi > 0$. This means that the market timer should (i) switch from "neutral" to "long" if, and only if, the predicted return on the next day is bigger than $(\phi a_t + \mu)$ and (ii) switch from "long" to "neutral" when the predicted return on the next day is smaller than $(\phi b_t + \mu)$. Since $\phi b_t + \mu < \phi a_t + \mu$, there is a no-switching region. This leads to the so called less frequent trading under transaction costs as reported in Magill and Constantinides [38]. Constantinides [39], Gennotte and Jung [40] and Boyle and Lin [41] etc.

When the transaction cost c is very larger, the number of trades becomes rare. For example, if $\phi > 0$, when c is extremely large, the value of b will tend to be extremely small and a will be extremely large. Therefore, we should not change our position frequently, since the possible return may not cover the big transaction costs.

4. Limiting behavior of the optimal strategy

In this section, we give the limiting stationary strategy of Theorem 1 and its relation with some existing technical trading rules.

4.1. Limiting strategy

Theorem 2. *When the investment horizon is long enough, the optimal decision for the finite horizon problem converges to the following limiting strategy*:
if $\phi > 0$

$$d_t(x,1) = \begin{cases} 0 & x \leqslant b, \\ 1 & x > b, \end{cases} \qquad d_t(x,0) = \begin{cases} 0 & x \leqslant a, \\ 1 & x > a \end{cases} \tag{8}$$

if $\phi < 0$

$$d_t(x,1) = \begin{cases} 0 & x \geqslant b, \\ 1 & x < b, \end{cases} \qquad d_t(x,0) = \begin{cases} 0 & x \geqslant a, \\ 1 & x < a, \end{cases} \tag{9}$$

where $h(x)$ *and the constants* a *and* b *satisfy the following integral equation*:

$$h(x) = \phi x + \mu + c[1 - \Phi(a - \phi x - \mu) - \Phi(b - \phi x - \mu)] + \int_b^a h(z)\phi(z - \phi x - \mu)\,\mathrm{d}z \tag{10}$$

with boundary conditions $h(a) = c$ *and* $h(b) = -c$, *where* $\phi(x)$ *and* $\Phi(x)$ *are the probability density function and cumulative distribution function of the normal distribution, respectively*.

For a proof of Theorem 2, see Appendix B.

Let us evaluate the effect of transaction costs on the limiting strategy. In Eq. (10), only the last two terms in the right-hand side are related to c. If $c = 0$, the values of a and b in the limiting trading strategy will be the same and the solution reduces to $h(x) = \phi x + \mu$. This is consistent with the result in Section 3.3 and the strategy in Section 3.3 is also the limiting strategy in the long run.

The effect of transaction costs on the control variables a and b will further be considered in Section 5 when we present the limiting strategy numerically.

4.2. Relationship with the momentum index trading rule

The limiting strategy has the following characteristics:
(1) Shift from neutral $(d_t = 0)$ to long $(d_{t+1} = 1)$ if

$$r_t > a \Leftrightarrow \ln p_t - \ln p_{t-1} > a \Leftrightarrow \frac{p_t}{p_{t-1}} > \exp a = \alpha.$$

(2) Shift from long $(d_t = 1)$ to neutral $(d_{t+1} = 0)$ if

$$r_t < b \Leftrightarrow \ln p_t - \ln p_{t-1} < b \Leftrightarrow \frac{p_t}{p_{t-1}} < \exp b \left(= \frac{1}{\alpha} \text{ if } \mu = 0\right).$$

Hence, the limiting trading rule happens to be a special case of the Momentum index technical trading rule.

In the Momentum index trading rule, the ratio of price at day t and the price at k-days ago is computed and is called a momentum index, i.e.,

$$\mathrm{MI}_t = \frac{p_t}{p_{t-k}}.$$

The trading rule suggests to go "long" when $\mathrm{MI}_t > \alpha$ and suggests to go "neutral" when $\mathrm{MI}_t < 1/\alpha$ for some constant α.

It is easy to see that when $\mu = 0$, the limiting strategy is exactly the momentum index trading rule with $k = 1$. This means that one of the technical trading rules happens to be the limiting trading strategy for a long investment horizon when return process follows an AR(1) model without drift. Thus this result gives some theoretical backup to the technical traders.

5. Numerical values specifying the limiting strategy

In this section, we give the numerical values for a and b which determine the limiting trading strategy.

Since $h(x)$, a and b satisfy the integral equation (10), we can compute them numerically. The numerical values for a and b are important because they determine the limiting trading strategy in the long run. The limiting strategy is to switch from "neutral" to "long" if the daily return is larger than a and to switch from "long" to "neutral" if the daily return is smaller than b. Thus, it is important to compute values of a and b when c (transaction cost), ϕ and σ take on some realistic values.

The transaction cost c differs from market to market. For some stock markets, the cost for a one-way transaction can be as low as 0.2%, but for other stock markets, it can be higher than 1%. We will provide the values for a and b when $c = 0.2\%, 0.6\%, 1.0\%$ and 1.4%. Autocorrelation of daily returns also varies from market to market. Since the autoregressive parameter ϕ in (4) is equal to the autocorrelation coefficient, we provide the numerical results for $\phi=0.1$, 0.15, 0.2 and 0.25. The parameter σ_r is the standard deviation of daily return. The relationship between σ_r are σ (standard deviation of the error term ε_t) is: $\sigma^2 = (1-\phi^2)\sigma_r^2$. We consider the value $\sigma_r = 2\%$ to be a realistic value for daily fluctuation. We also assume that μ takes so small a value that it can be assumed to be zero. From Eq. (10), we know that when $\mu=0$, $h(x)$ is anti-symmetric, i.e., $h(-x) = -h(x)$. Thus $b = -a$ and we need only to compute a.

Table 1 shows the values of a for four different values of ϕ and for different transaction costs. For example, when $\phi = 0.1$, $c = 0.002$ and $\sigma_r = 0.02$, $a = 0.018$, which means that long position has to be neutralized at the end of the day whenever the market drops by 1.8% in a day. Fig. 1 gives a graphical plot of the values in Table 1. The x-axis is the transaction cost, the y-axis is the values of a and b ($a = -b > 0$). The figure gives the lines for different value of ϕ. From Table 1 and Fig. 1, we can see that a increases as c increases. This is reasonable because if the transaction cost is high, the investment position should be maintained unless there is more evidence of a turning point. Although the graph seems to show a linear relationship between a and c, the actual relationship is not exactly linear. Also a decreases as ϕ increases because if price movements are more dependent, the investor can make use of historical information to adjust his/her investment position more frequently. This shows that the no-transaction region becomes wider as c increases and ϕ decreases.

Table 1
Threshold value a when there is no drift for the return process ($\mu = 0$)

Transaction costs	Autocorrelation coefficient ϕ			
	1.00e-01	1.50e-01	2.00e-01	2.50e-01
2.00e-03	1.87e-02	1.24e-02	9.29e-03	7.42e-03
6.00e-03	5.41e-02	3.45e-02	2.53e-02	1.99e-02
1.00e-02	9.00e-02	5.68e-02	4.05e-02	3.13e-02
1.40e-02	1.26e-01	7.94e-02	5.62e-02	4.27e-02

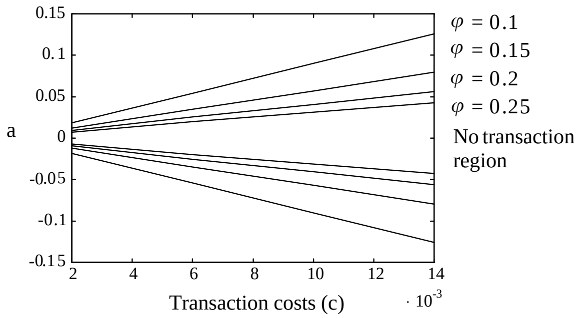

Fig. 1. Value of $a(= -b)$.

6. Empirical results

In this section, we report the performance of the simulated trading of the limiting stationary strategy in the Hang Seng Index Futures market in Hong Kong. Closing prices for Hang Seng Index Futures and weekly HIBOR (Hong Kong Interband Borrowing Rate) data from 1 July 1987 to 31 December 1999 are used in the simulation. We divide each calendar year into two periods, the first half-year period and the second half-year period. For each period, we estimate the AR(1) parameters and use the fitted model to forecast the daily return in the following period. We compute the critical values (b and a) for the limiting trading strategy assuming that the same AR(1) model works in the next period. The performance of the trading strategy will be compared to the performance of the simple strategy and the buy-and-hold strategy under transaction costs.

6.1. Data description

The Hang Seng Index is the most popular market index for the Hong Kong stock market. It has a very actively traded futures contract, i.e., the Hang Seng Index Futures contract

Table 2
AR(1) model estimation critical values b and a ($\mu = (1-\phi)\mu_r$ and $\sigma = \sqrt{(1-\phi^2)}\sigma_r$)

Period	ϕ	μ_r	σ_r	b	a
87-2	−7.3430e-002	−2.7125e-003	5.9520e-002	−4.0092e-003	−5.7377e-002
88-1	2.5100e-002	1.1093e-003	1.6018e-002	−2.0245e-001	−4.3088e-002
88-2	1.0846e-001	−2.5320e-004	9.1051e-003	−3.7720e-003	3.0295e-002
89-1	−9.1720e-002	−1.9339e-003	3.6340e-002	3.9539e-003	−3.8336e-002
89-2	−1.4973e-001	1.6655e-003	1.3289e-002	2.2155e-002	−3.0556e-003
90-1	−1.3815e-001	7.4150e-004	9.0159e-003	2.0398e-002	−7.8865e-003
90-2	1.7850e-001	−1.1488e-003	1.5905e-002	−6.8790e-004	2.0046e-002
91-1	6.9610e-002	1.6184e-003	1.1089e-002	−7.8837e-002	−2.1531e-002
91-2	−1.4203e-001	9.4900e-004	1.1461e-002	1.9729e-002	−7.2873e-003
92-1	−3.6496e-003	2.7171e-003	1.0885e-002	9.9593e-001	1.1265e-002
92-2	1.1115e-001	−9.9660e-004	1.6644e-002	2.5726e-003	3.6753e-002
93-1	4.2880e-002	2.0322e-003	1.5644e-002	−1.3863e-001	−4.5356e-002
93-2	8.2970e-002	3.9459e-003	1.8167e-002	−9.1727e-002	−4.3562e-002
94-1	−1.6980e-001	−2.7860e-003	2.7248e-002	−3.8214e-003	−2.6354e-002
94-2	1.2800e-001	−7.0850e-004	1.5295e-002	−2.2961e-003	2.6811e-002
95-1	−8.7220e-002	8.5600e-004	1.8441e-002	3.1411e-002	−1.3458e-002
95-2	−6.2820e-002	5.1340e-004	1.0690e-002	4.9726e-002	−1.2192e-002
96-1	−1.6407e-001	4.8150e-004	1.3245e-002	1.4052e-002	−8.9574e-003
96-2	−9.9720e-002	1.3564e-003	1.0393e-002	3.3080e-002	−4.4353e-003
97-1	3.2110e-002	8.0610e-004	1.3728e-002	−1.4855e-001	−2.4158e-002
97-2	−2.0087e-001	−3.0970e-003	3.8862e-002	−6.0835e-003	−2.5308e-002
98-1	7.8850e-002	−2.0334e-003	3.1271e-002	1.8557e-002	6.8178e-002
98-2	−1.0300e-002	1.0089e-003	3.1652e-002	3.5467e-001	−2.1422e-002
99-1	−9.6900e-002	2.2698e-003	2.2061e-002	3.8835e-002	−8.8583e-004
99-2	−4.2580e-002	1.5550e-003	1.8272e-002	8.3185e-002	−5.8890e-003

(HSIF), which was launched on 6 May 1986 and is the most actively traded index futures contract in non-Japan Asia. Although Singapore (SIMEX) launched a futures contract on the Morgan Stanley Hong Kong Index on 23 November 1998, we have not seen any great influence on the trading volume of the HSIF contracts in Hong Kong. Before 20 November 1998, the Hong Kong HSIF market opened at 10:00 am, closed for lunch at 12:30 am, reopened at 14:30 pm and ran until 16:00 pm. The trading hour has been lengthened by half an hour per day, opening at 9:45 am and closing at 16:15 pm since 20 November 1998.

The data used in this study are the daily settlements price of the spot month Hong Kong HSIF contract covering the period from 1 July 1987 to 31 December 1999. The settlement prices in the last two trading days in each month are substituted by the closing prices of the next month's HSIF contract, since most of the traders roll the contract over to another month in the last two trading days and the spot month contract is not actively traded during that period. The weekly HIBOR is used as the riskfree rate.

6.2. Parameter estimation

To simulate trading by the proposed strategy, we divide each calendar year into two periods, the first half-year and the second half-year. By doing this, we have a total of 26 time periods with each time period having a six-month span. We ignore the data before 1 July 1987 to make sure that every period has six months in length. We then estimate the AR(1) parameters for each time period and compute the corresponding critical values of (b,a) in the limiting strategy for each period by solving the integral equation (10). The results are summarised in Table 2.

The first column in Table 2 shows the time period in which parameters are estimated. For example, "87-2" denotes the second half of the year 1987, i.e., from July 1 to December 31, and "88-1" denotes the first half of the year 1988, i.e., from January 1 to June 30. The second column gives the estimate of the parameter ϕ. The third column and the fourth column are the mean (μ_r) and the standard deviation (σ_r) of the return series within the period. The fifth and sixth columns are the critical values (b,a) in the limiting trading strategy by solving the integral equation (10) with a value of $c = 0.2\%$.

Take the time period "87-2" as an example. We act as if the model fitted in period "87-2" is also true in the next period, i.e., "88-1", then use the fitted model in "87-2" to predict the return in "88-1". Since ϕ is negative, b equals -0.40% and a equals -5.74%, the corresponding strategy in this period is: if we are taking a long position in the market and yesterday's return dropped by less than 0.40%, then we should close our position at today's end. On the other hand, if we are holding no position today and yesterday's return drop by more than 5.74%, then we should enter into a long

Table 3
Performance of the trading strategies for $c = 0.2\%$

Period	Buy-and-hold		Limiting strategy			Simple strategy		
	# days	Profits	# 'in' days	# trades	Profits	# 'in' days	# trades	Profits
88-1	121	0.1695	0	0	0.0192*	2	4	−0.0024
88-2	125	0.0031	125	1	0.0031*	125	1	0.0031
89-1	120	−0.1912	9	8	0.0805*	59	65	0.367
89-2	124	0.2490	1	2	0.0764	2	4	0.0886
90-1	121	0.1269	116	3	0.1365*	112	19	0.1073
90-2	125	−0.1178	88	15	−0.1448*	94	37	0.3500
91-1	120	0.2183	8	10	−0.0022	42	49	0.0777
91-2	126	0.1183	125	3	0.0870*	125	3	0.0870
92-1	121	0.3500	60	15	0.1637	93	43	0.2236
92-2	127	−0.1133	127	1	−0.1133*	127	1	−0.1133
93-1	120	0.2640	3	6	−0.0320	38	58	0.0768
93-2	127	0.4987	127	1	0.4987*	126	3	0.4583
94-1	121	−0.3505	121	1	−0.3505	116	11	−0.3362
94-2	125	−0.0585	6	6	−0.0296*	11	18	−0.0599
95-1	120	0.1739	18	16	0.0273	44	63	0.0844
95-2	125	0.1004	113	1	0.0775*	107	34	0.0320
96-1	120	0.0652	114	1	0.0634*	92	47	0.0046
96-2	127	0.1996	45	12	0.0280	79	71	0.0666
97-1	121	0.1482	116	5	0.1567*	104	29	0.0588
97-2	122	−0.3310	121	3	−0.5561	95	33	−0.3452
98-1	121	−0.2180	31	40	−0.1275	29	49	−0.0344
98-2	124	0.1267	6	8	0.0259	21	32	0.1515
99-1	120	0.3253	120	1	0.3253*	120	1	0.3253
99-2	125	0.1694	118	5	0.1415*	112	25	0.0921
Mean	122.8	0.0803	71.6	6.8	0.0231	78.1	29.2	0.0171
SD		0.2145			0.2031			0.1887

position at today's end. Otherwise, we should do nothing, just wait and see.

6.3. Performance of the limiting trading strategy

In this section, we compare the limiting trading strategy with the buy-and-hold and the simple strategy, which is the strategy most people used when testing for the predictability of a return model. (e.g. [12–18] etc.). The simple strategy mentioned above is to enter into the market whenever the return (excess return) is forecasted to be positive and leave the market whenever the return (excess return) is forecasted negative. Generally speaking, the return can be predicted by any methods or models, we use AR(1) model in this paper. The performance of the three strategies, i.e., buy-and-hold, simple strategy, and our proposed limiting strategy, will be compared using transaction cost $c = 0.2\%$.

The results are reported in Table 3. Note that Table 3 starts with period "88-1", while Table 2 starts with period "87-2". This is because we use the model fitted in one period ahead to forecast the return in the next period. Hence, the limiting trading strategy can only be implemented starting from period "88-1".

In Table 3, the first column shows the time period. Column 2 is the total number of trading days within each period, which is also the days for the buy-and-hold strategy to be in the market. Column 3 gives the total return of the buy-and-hold strategy. Column 4 is the number of days for the proposed limiting strategy to be in the market. Column 5 is the number of trades of the limiting strategy, which is the total number of moves in or out of the market. For example, in the period "91-2", the total number of trading days is 126, the limiting strategy has 125 days in the equity market and 1 day in the riskfree asset, and there are a total of three trades. At first glance, it seems weird. The fact is, the asset was bought at the beginning of the first day, sold some days later, and bought back right at the next day. This involves three trades and the investor is in the market except one day. For the buy-and-hold strategy the number of trades is always one. Generally speaking, it is not necessary for the limiting strategy to be in the market at the first day for any period. When a market timer should enter into the equity market depends on the prediction. Column 6 is the total return (net of transaction costs) for the proposed limiting strategy. Column 7 is the number of days for the simple strategy to be in the market. Column 8 is the number of trades made by the simple strategy. Column 9 is the total return net of transaction costs for the simple strategy. The last two rows in the table give the mean and standard deviation over all periods.

Take the period "88-1" in Table 3 as an example. The total number of days is 121 days. The return obtained by the buy-and-hold strategy in this period is 16.95% (an average daily return of 0.14%). The number of days for the simple strategy to be in the market is 2 days and the total number of trades is 4 times. After taking transaction costs into consideration, the total return for the simple strategy is -0.024%, which is lower than the return from the buy-and-hold strategy. For the proposed AR(1) strategy, the number of days in the market is 0 days, the total number of trades is 0 times, and the total return is 1.92%, this means there are no trades in this period and we just put our money in the riskfree asset and get the riskfree rate of 1.92%, which is lower than that of the buy-and-hold strategy but higher than the return from the simple strategy. We now compare the three strategies over all periods.

Let us first compare the simple strategy with the buy-and-hold strategy. There are 6 out of 24 periods in which the simple strategy is not worse than the buy-and-hold strategy. The average return per period obtained by the buy-and-hold strategy is 8.03%, and a standard deviation of 21.4%. The average return for the simple strategy is 1.71% per period, which is worse than the buy-and-hold strategy, and its standard deviation is only 18.87%, which is lower than the buy-and-hold strategy.

Next, let us compare the buy-and-hold strategy with the proposed limiting strategy. There are 10 out of 24 periods in which the proposed strategy is not worse than the buy-and-hold strategy. For the proposed strategy, the average return for all periods is 2.31%, the standard deviation is 20.31%, which is lower than that for the buy-and-hold strategy.

Finally, let us compare the proposed strategy with the simple strategy. This comparison is important because the proposed strategy has the property that it will save transaction cost over the simple strategy. Under transaction costs, there are 16 out of 24 periods (denoted by "*") that the proposed strategy is not worse than the simple strategy. The average number of days in the market per period for the simple strategy is 78.1 days and the average number of trades per period is 29.2 times. For the proposed AR(1) strategy, the average number of days in the market is 71.6 days, which is almost the same as the simple strategy, but the average number of trades is only 6.8 times, which is much less than that of the simple strategy. This is consistent with our claim that transaction costs lead to less frequent trading. Because of this, the proposed strategy can save a lot of transaction costs and has a greater total return. Generally speaking, the simple strategy does not give the best results for the model. If we use the proposed strategy, the results can be improved greatly.

From the empirical results in this section, we can conclude that the proposed trading strategy based on an AR(1) model is a reasonable trading strategy for the Hong Kong HSIF market. It is better than the simple strategy.

From Table 3, we can see the average continuously compounded return in a period. The semi-annual return for the buy-and-hold strategy is 8.03%, and equal to 2.31% and 1.71% for the limiting strategy and the simple strategy, respectively. This shows that the simple strategy is inferior to the limiting strategy. However, neither the limiting strategy nor the simple strategy can beat the buy-and-hold strategy. This shows that our results do not offer evidence against market efficiency. During this period under consideration, the stock market grows at a continuously compounding rate of more that 16% a year. It is not easy to beat the buy-and-hold strategy under such a phenomenal growth. However, the following facts may give support to our analysis of the two trading strategies. Firstly, the buy-and-hold strategy has an average of 122.8 days in the equity market, but there are only 71.6 and 78.1 days for the limiting and simple strategies to be in the equity market. When the market timers put their money in the risk free asset, there is no risk at all. Secondly, if we ignore transaction costs, both strategies will have a much higher semi-annual return which is very close to that of the buy-and-hold strategy. Combining with the fact that there is reduction in risk, they are able to beat the buy-and-hold strategy. Thirdly, the objective of this paper is to compare the limiting strategy with the simple strategy. The purpose is not to find a strategy to beat the market. The comparison confirms that the limiting strategy is better than the simple strategy. What we demonstrate here is that if a model has predictability by the use of the simple strategy, the limiting strategy can give a even better result. As a consequence, if a model does not have any predictability by the use of a simple strategy, it does not mean that the model is useless. The disability may be due to the fact that the limiting optimal strategy has not been used.

7. Conclusions

In this paper, we give the general formulation for the return process following an ARMA(p,q) model, and consider the optimal market timing strategies under transaction costs assuming that the return of the asset price follows an AR(1) model. By the use of stochastic dynamic programming techniques, we derive the optimal trading strategy for a finite investment horizon and consider the limiting behavior of the strategy. Numerical results of the limiting trading strategy are computed. The results confirm that the no-transaction region increases as the transaction cost increases. We also find that there is a close relation between the limiting strategy and one of the popular technical trading rules, the Momentum Index trading rule. The limiting strategy is applied to data in the Hang Seng Index Futures market in Hong Kong. The performance of the limiting stationary strategy is found to be better than that of the simple strategy used in the literature. Although no analytic solution for the optimal strategy can result using a general ARMA(p, q) or GARCH approach, numerical results would be interesting. Other price models like the price-trend model in Taylor [26] can also be tried to see if meaningful trading strategy can be derived. Our conclusion is that when the predictability of a return model is under test, it is essential to take transaction costs into consideration, and the best strategy to use under transaction costs is not the simple strategy. The proper procedure is to work out the optimal strategy first and then carry out the test whether the optimal strategy can beat the market or not.

Acknowledgements

The authors would like to thank Prof. Lai Tze Leung and Prof. Liang Zou for their helpful comments and suggestions. The authors would also like to thank the three anonymous referees for their valuable suggestions.

Appendix A. Proof of Theorem 1

Proof. Let $V_t(x, y)$ denote the maximum expected total return from time t to time T, which starts in state (x, y), where x is the return in day $t-1$, and y is the investment position in that day. Then the *optimality equation* or *Bellman equation* for dynamic programming can be stated as follows:

$$V_t(x, y) = \max_{d_t=0,1} E[R(r_t, y') + V_{t+1}(r_t, y')],$$

where y' is the decision in day t, $R(r_t, y') = r_t y' - |y' - y|c$ is the return in that day, c is the transaction cost, E stands for expectation. Because the investment process terminates at the end of T periods, we can assume $V_{T+1}(\mathbf{x}_T, d_T) = 0$. Because of (2), the Bellman equation can be written as

$$V_t(x, y) = \max_{y'=0,1} [(\phi x + \mu)y' - |y' - y|c + EV_{t+1}(\phi x + \mu + \varepsilon, y')].$$

Let $s_{t+1}(x) = V_{t+1}(x, 1) - V_{t+1}(x, 0)$, $h_{t+1}(x) = \phi x + \mu + Es_{t+1}(\phi x + \mu + \varepsilon)$, we have

$$V_t(x, y) = \begin{cases} |y|c + EV_{t+1}(\phi x + \mu + \varepsilon, 0) & (2y-1)c + h_{t+1}(x) \leqslant 0, \\ \phi x + \mu - |1-y|c + EV_{t+1}(\phi x + \mu + \varepsilon, 1) & (2y-1)c + h_{t+1}(x) > 0. \end{cases}$$

Therefore, the optimal decision at the end of day $t-1$ is given by

$$d_t(x, y) = \begin{cases} 0 & (2y-1)c + h_{t+1}(x) \leqslant 0 \\ 1 & (2y-1)c + h_{t+1}(x) > 0 \end{cases} = \chi((2y-1)c + h_{t+1}(x)),$$

where

$$\chi(z) = \begin{cases} 0 & z \leqslant 0, \\ 1 & z > 0. \end{cases}$$

The optimal decision at the end of day $t-1$ depends on the function $h_{t+1}(x)$.

Next, let's prove that $h_{t+1}(x)$ is strictly increasing (decreasing) in x when $\phi > 0 (< 0)$.

The result is obvious for $t = T$.

Suppose it is true for $t = k+1$, we will prove that it is true for $t = k$.

$$h_k(x) = \phi x + \mu + Es_k(\phi x + \mu + \varepsilon),$$

$$s_k(x) = \begin{cases} c & h_{k+1}(x) \geqslant c, \\ h_{k+1}(x) & -c < h_{k+1}(x) < c, \\ -c & h_{k+1}(x) \leqslant -c. \end{cases}$$

Since $\partial Es_k(\phi x + \mu + \varepsilon)/\partial x = \phi \int_{-c < h_{k+1}(\phi x + \mu + z) < c} h'_{k+1}(\phi x + \mu + z)\phi(z)\,\mathrm{d}z$, where $\phi(x)$ is the normal density function of $\varepsilon \sim \mathrm{N}(0, \sigma^2)$, from the induction assumption, it is easy to see that $\partial Es_k(\phi x + \mu + \varepsilon)/\partial x > 0 (< 0)$ for $\phi > 0 (< 0)$. So,

$$\frac{\partial h_k(x)}{\partial x} > 0(< 0) \quad \text{for } \phi > 0(< 0).$$

Therefore we have the results. This completes the proof. □

Appendix B. Proof of Theorem 2

Proof. First, we will prove that $h_t(x)$ converges uniformly to a continuous function $h(x)$.
For any $m > n$,

$$\begin{aligned} |h_m(x) - h_n(x)| &= |\phi x + \mu + Es_m(\phi x + \mu + \varepsilon) - (\phi x + \mu + Es_n(\phi x + \mu + \varepsilon))| \\ &= |Es_m(\phi x + \mu + \varepsilon) - Es_n(\phi x + \mu + \varepsilon)| \\ &= \left| \int [s_m(\phi x + \mu + z) - s_n(\phi x + \mu + z)]\phi(z)\,\mathrm{d}z \right| \\ &\leqslant \int_{-\beta}^{\beta} |h_{m+1}(\phi x + \mu + z_1) - h_{n+1}(\phi x + \mu + z_1)|\phi(z_1)\,\mathrm{d}z_1 \\ &\quad (\text{where } \beta = 2c + \mu/\phi) \end{aligned}$$

$$\leqslant \int_{-\beta}^{\beta}\int_{-\beta}^{\beta} |h_{m+2}(\phi(\phi x + \mu + z_2) + \mu + z_1)$$
$$- h_{n+2}(\phi(\phi x + \mu + z_2) + \mu + z_1)|$$
$$\phi(z_2)\phi(z_1)\,\mathrm{d}z_2\,\mathrm{d}z_1$$
$$\ldots$$
$$\leqslant \int_{-\beta}^{\beta}\cdots\int_{-\beta}^{\beta} |h_T(\phi^{T-m}x + (1+\phi+\cdots+\phi^{T-m-1})\mu$$
$$+ z_1 + \phi z_2 + \cdots + \phi^{T-m-1} z_{T-m})$$
$$- h_{n-m+T}(\phi^{T-m}x + (1+\phi+\cdots+\phi^{T-m-1})\mu$$
$$+ z_1 + \phi z_2 + \cdots + \phi^{T-m-1} z_{T-m})|$$
$$\phi(z_{T-m})\cdots\phi(z_1)\,\mathrm{d}z_{T-m}\cdots\mathrm{d}z_1$$
$$= \int_{-\beta}^{\beta}\cdots\int_{-\beta}^{\beta} |Es_{n-m+T}(\phi^{T-m}x$$
$$+ (1+\phi+\cdots+\phi^{T-m-1})\mu$$
$$+ z_1 + \phi z_2 + \cdots + \phi^{T-m-1} z_{T-m})|\phi(z_{T-m})\cdots$$
$$\phi(z_1)\,\mathrm{d}z_{T-m}\cdots\mathrm{d}z_1$$
$$\leqslant c\int_{-\beta}^{\beta}\cdots\int_{-\beta}^{\beta} \phi(z_{T-m})\cdots\phi(z_1)\,\mathrm{d}z_{T-m}\cdots\mathrm{d}z_1$$
$$= c\,\hat{p}^{T-m},$$

where $\hat{p} = \int_{-\beta}^{\beta}\phi(z_{T-m})\,\mathrm{d}z_{T-m} = \cdots = \int_{-\beta}^{\beta}\phi(z_1)\,\mathrm{d}z_1 < 1$. This implies that $h_t(x)$ converges uniformly to a continuous function $h(x)$ as $T \to \infty$.

Then, from the relationship between $h_t(x)$ and $s_t(x)$ given in strategy 1, we have the result. This completes the proof. □

References

[1] Fama EF. Efficient capital markets: review of theory and empirical work. Journal of Finance 1970;25:383–417.

[2] Cowles A. Can stock market forecasters forecast?. Econometrica 1934;1:309–24.

[3] Brown SJ, Goetzmann WN, Kumar A. The Dow theory: William Peter Hamilton's track record reconsidered. Journal of Finance 1998;53(4):1311–33.

[4] Sharpe WF. Likely gains from market timing. Financial Analysts Journal 1975;31(2):60–9.

[5] Jeffrey R. The folly of stock market timing. Harvard Business Review 1984;62(4):102–10.

[6] Vandell RF, Stevens JL. Evidence of superior performance form timing. The Journal of Portfolio Management 1989;15(3):38–43.

[7] Shilling AG. Market timing: better than a buy-and-hold strategy. Financial Analysts Journal 1992;48(2):46–50.

[8] Larsen GA, Wozniak GD. Market timing can work in the real world. The Journal of Portfolio Management 1995;21(3): 74–81.

[9] Wagner JC. Why market timing works. The Journal of Investing 1997;6(2):78–81.

[10] Kao DL, Shumaker D. Equity style timing. Financial Analyst Journal 1999;55(1):37–48.

[11] Levis M, Liodakis M. The profitability of style rotation strategies in the United Kingdom. Journal of Portfolio Management 1999;26(1):73–86.

[12] Breen W, Glosten LR, Jagannathan R. Economic significance of predictable variations in stock index returns. Journal of Finance 1990;44:1177–89.

[13] Leitch G, Tanner JE. Economic forecast evaluation: profits versus the conventional error measures. The American Economic Review 1991;81(3):580–90.

[14] Pesaran MH, Timmermann A. Forecasting stock returns: and examination of stock market trading in the presence of transaction costs. Journal of Forecasting 1994;3:335–67.

[15] Pesaran MH, Timmermann A. Predictability of stock returns: robustness and economic significance. The Journal of Finance 1995;50(4):1201–28.

[16] Knez PJ, Ready MJ. Estimating the profits from trading strategy. The Review of Financial Studies 1996;9(4):1121–62.

[17] Lander J, Orphanides A, Douvogiannis M. Earnings forecasts and the predictability of stock returns: evidence from trading the S& P. The Journal of Portfolio Management 1997;23(4): 24–35.

[18] Lee W. Market timing and short-term interest rates. The Journal of Portfolio Management 1997;23(3):35–46.

[19] Chua JH, Woodward RS, To EC. Potential gains from stock market timing in Canada. Financial Analysts Journal 1987;43(5):50–6.

[20] Chopra N, Lakonishok J, Riter JR. Performance measurement methodology and the question of whether stocks overreact. Journal of Financial Economics 1992;31:235–68.

[21] De Bondt WFM, Thaler RH. Does the stock market overreact? Journal of Finance 1985;40:793–805.

[22] Fama EF, French KR. Permanent and temporary components of stock prices. The Journal of Political Economy 1988;98:246–74.

[23] Poterba JM, Summers LH. Mean reversion in stock prices: evidence and implications. Journal of Financial Economics 1988;22:27–59.

[24] Hodrick RJ, Srivastava S. Foreign currency futures. Journal of International Economics 1987;22:1–24.

[25] Mark NC. Time-varying betas and risk premia in the pricing of forward foreign exchange contracts. Journal of Financial Economics 1988;22:335–54.

[26] Taylor SJ. Modeling financial time series. Chichester: Wiley, 1986.

[27] Taylor SJ. Efficiency of the Yen futures market at the Chicago mercantile exchange. In: Goss BA, editor. Rational expectations and efficiency in futures markets. London: Routledge, 1992. p. 109–28.

[28] Conrad J, Kaul G. Mean reversion in short-horizon expected returns. The Reviews of Financial Studies 1989;2:225–40.

[29] Goodhart CAE. 'News' and foreign exchange markets. Proceedings of the Manchester Statistical Society, 1989. p. 1–79.

[30] Goodhart CAE, Figliuoli L. Every minute counts in financial markets. Journal of International Money and Finance 1991;10:23–52.

[31] Sy M. Market timing: is it a folly?. Journal of Portfolio Management 1990;11–6.

[32] Phillips D, Lee J. Differentiating tactical asset allocation from market timing. Financial Analysts Journal 1989;45(2):14–6.

[33] Fuller RJ, Kling JL. Is the stock market predictable?. The Journal of Portfolio Management 1990;16(4):28–36.
[34] Sinclair NA. Market timing ability of pooled superannuation funds: January 1981 to December 1987. Accounting and Finance 1990;30:51–65.
[35] Beebower GL, Varikooty AP. Measuring market timing strategies. Financial Analysts Journal 1991;47(6):78–84.
[36] Cox JC, Huang C. Optimum consumption and portfolio policies when asset prices follow a diffusion process. Journal of Economic Theory 1989;49:33–83.
[37] Hakansson N, Ziemba WT. Capital growth theory. In: Jarrow RA, Maksimovic V, Ziemba WT, editors. Handbooks in Operations Research and Management Science, V. 9, “Finance”. Amsterdam: Elsevier, 1995. p. 65–86.
[38] Magill M, Constantinides G. Portfolio selection with transactions costs. Journal of Economic Theory 1976;13: 245–63.
[39] Constantinides G. Multiperiod consumption and investment behavior with convex transaction costs. Management Science 1979;25(11):1127–37.
[40] Gennotte G, Jung A. Investment strategies under transaction costs: the finite horizon case. Management Science 1994;40(3):385–404.
[41] Boyle P, Lin XD. Optimal portfolio selection with transaction costs. North American Actuarial Journal 1997;1(2).
[42] Admati AR, Bhattacharya S, Pfleiderer P, Ross SA. On timing and selectivity. Journal of Finance 1986;41(3): 715–32.
[43] Benning CJ. Prediction skills of real-world market timers. The Journal of Portfolio Management 1997;23(2):55–65.