Could our quantum computing model capture the computational power of a realistic quantum computer?

Yu-Tsung Tai (戴淯琮) (mostly join work with Andrew J Hanson, Gerardo Ortiz, and Amr Sabry)

April 16, 2022

Could a Turing Machine capture the computational power of a digital computer?

- Yes, because discrete computation can be reliably repeated with exact equivalence, and any digital computer can faithfully simulate a Turing machine limited only by the available memory and time [1].
- How about continuous quantities and their computation?

Continuous theoretical models and their physical phenomena

Theoretical models	Physical phenomena
Chuang-tzu (庄子): "If a one-foot-long stick is cut into halves every day, the cutting will never come to an end". ("一尺之棰，日取其半，萬世不竭。") [2, 3].	Chuang-tzu's stick is made by molecules and cannot be cut in halves endlessly.
Charge amplifiers are used to build electrical analog computers and compute integration in calculus [4, 5].	The input charge of an integrator must be an integer multiple of the elementary charge.

Continuous theoretical models and their physical phenomena

Theoretical models

- There is no minimum unit between the length of the side of a square and its diagonal.

Physical phenomena

- Is space ultimately discrete or continuous?
- If space is discrete, maybe one of them is not a physical quantity.
- Even if space is continuous, checking whether a physical square is perfect maybe a infinite loop. This loop halts if a square is not perfect, but needs infinite time to get a “perfect” answer.

Continuous theoretical models and their physical phenomena

Theoretical models

- A BCSS machine allow to branch the computation by whether a number is greater than zero over real numbers [6–9].
- When the quantities used to branch the computation closes to zero, the branch chosen by a BCSS machine cannot reliably predict the branch chosen by a realistic analog computer.

Physical phenomena

- A slide rule, also known as a slipstick, was used to compute multiplication, division, and more complex operations as a mechanical analog computer [10].
- Even if its length is continuous, and analog computers really store real numbers, they cannot be precisely read, written, and used to branch the computation.

How does a continuous theoretical model predict the behavior of a physical phenomenon?

Discrete with extreme small units (Quantized)

Not reliably, but after we have better technology to manipulate the minimum units, we can more reliably predict the physical phenomenon and their computational power by a better discrete model.

Continuous, or no evidence to support they are discrete (Never Quantized?)

Not reliably, because precision can never be high enough, and the difference between their computational power might not be able compensated by error analysis techniques easily.

Could the probability amplitudes used in quantum computing be found quantized in the future?

Could be Quantized

There might be other ways to quantize probability amplitude, but we choose to replace the field of complex numbers $\mathbb{C}$ by discrete finite fields $\mathbb{F}_q$:

- We still could do arithmetic operations among probability amplitude.
- When the size of the field is extremely large, we will define the fraction-like cardinal probability which has extremely small units.

Never Quantized

There might be other ways to model the imprecise probability amplitude, but it is easier to consider the precision of their inducing probabilities because the idea of "imprecise probability" is well-studied classically [11–16].

Comparison among the quantum mechanical definitions and properties of our quantized models

	Conventional	Modal	Discrete (I)	Discrete (II)	QPMFF
States space	$\mathbb{C}^D$	$\mathbb{F}_q^D$	$\mathbb{F}_{p^2}^D$	Local region in $\mathbb{F}_{p^2}^D$	$\mathbb{F}_{p^2}^D$
Likelihood of events is predicted by	Real-valued probability	Possible or impossible	Possible or impossible	Cardinal probability	Real-valued, but no sensible Born rule
Expectation value	Defined	Undefined	Formally defined	Undefined	

Comparison among the computational power of our quantized models

	Conventional	Modal	Discrete (I)	Discrete (II)
Deutsch's algorithm	Yes	Maybe no	Yes	Yes
Efficiently solve UNIQUE-SAT	Unlikely	Yes	Partially	Unlikely
Grover search algorithm	Yes			Yes

Comparison among the quantum mechanical definitions and properties of our non-quantized models

	Conventional	QIVPM
States space	$\mathbb{C}^D$	$\mathbb{C}^D$
Likelihood of events is predicted by	Real-valued probability	Interval-valued probability
Expectation value	Defined	Defined

[1] A. M. Turing. "On Computable Numbers, with an Application to the Entscheidungsproblem." In: *Proceedings of the London Mathematical Society* s2-42.1 (Jan. 1937), pp. 230–265.

[2] Zhuangzi (庄子). *Zhuangzi (庄子)*. English. Trans. Chinese by Rongpei (榕培) Wang (汪), Xuqing (旭卿) Qin (秦), and Yongchang (雍长) Sun (孙). Changsha: Hunan People's Publishing House, 1999.

[3] Jinyan (金岩) Liu (刘). "Chinese Physicists' Construction of the Straton Model in Social Context." In: (June 2018), pp. 85–122.

[4] Wikipedia contributors. *Integrator — Wikipedia, The Free Encyclopedia*. https://en.wikipedia.org/w/index.php?title=Integrator&oldid=855449964. [Online; accessed 17-October-2018]. 2018.

[5] Wikipedia contributors. *Charge amplifier — Wikipedia, The Free Encyclopedia*. https://en.wikipedia.org/w/index.

php?title=Charge_amplifier&oldid=841714174. [Online; accessed 27-October-2018]. 2018.

[6] Lenore Blum, Mike Shub, and Steve Smale. "On a theory of computation and complexity over the real numbers: NP-completeness, recursive functions and universal machines." In: *Bull. Amer. Math. Soc.* 21 (1989), pp. 1–46.

[7] Martin Ziegler. "Real Computability and Hypercomputation." Habilitationsschrift. University of Paderborn, 2007.

[8] L. Blum, F. Cucker, M. Shub, and S. Smale. *Complexity and Real Computation*. SpringerLink : Bücher. Springer New York, Dec. 6, 2012.

[9] Sanjeev Arora and Boaz Barak. *Computational Complexity*. Cambridge University Pr., June 11, 2009.

[10] Wikipedia contributors. *Slide rule — Wikipedia, The Free Encyclopedia*. https://en.wikipedia.org/w/index.php?title=Slide_rule&oldid=861863336. [Online; accessed 17-October-2018]. 2018.

[11] Glenn Shafer. *A Mathematical Theory of Evidence*. Princeton University Press, Apr. 11, 1976. 314 pp.

[12] Itzhak Gilboa and David Schmeidler. "Additive representations of non-additive measures and the Choquet integral.." In: *Annals of Operations Research* 52.1–4 (1994), pp. 43–65.

[13] Massimo Marinacci. "Limit Laws for Non-additive Probabilities and Their Frequentist Interpretation." In: *Journal of Economic Theory* 84.2 (Feb. 1999), pp. 145–195.

[14] Kenneth David Jamison and Weldon A. Lodwick. *Interval-Valued Probability Measures*. Tech. rep. 213. Center for Computational Mathematics, University of Colorado Denver, 2004.

[15] Peter J. Huber and Elvezio M. Ronchetti. *Robust Statistics*. English. 2nd ed. Wiley Series in Probability and Statistics. John Wiley & Sons Inc., Mar. 6, 2009. 354 pp.

[16] Michel Grabisch. *Set functions, games and capacities in decision making*. Theory and Decision Library C 46. Springer International Publishing, 2016.