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Questioning the Quantum Mysticism Attributed to Stern-Gerlach Quantized Space

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Abstract

In the 1920s, the split-beam result of the Stern-Gerlach experiment, Heisenberg's uncertainty principle, and Max Born's overlapping probability waves, led Niels Bohr to develop the Copenhagen interpretation wherein spin directions and space itself are quantized. Supposedly, particle spin dwells in a space of infinite parallel dimensions until observation causes the parallel existences to instantaneously collapse into a single observed state. However, a previously ignored three-dimensional classical mode of fast precession drives an initially quantized spin particle into quantized precession to produce the split-beam result of the Stern-Gerlach experiment. This obviates the need for any exotic explanation for "quantized spin space".

Note on Organization

This paper is presented in six Parts and a Bibliography. "Part 1- Introduction" (a one page statement) proclaims that the Stern-Gerlach experiment and "quantized spin space" have been fantastically misinterpreted to improperly initiate the many worlds view of quantum mechanics. "Part 2 - An Historical Perspective of Quantized Spin Space" reviews the historical background surrounding the Stern-Gerlach experiment and explains its connection with the concepts of a "quantized spin space" and ghostly parallel existence. "Part 3 - The Basics of the Stern-Gerlach Experiment" outlines the Stern-Gerlach experiment *(Figure-1)* and how it justifies the Standard Copenhagen Model of Quantum Mechanics with its prescriptions of extra-physical "quantized spin space". "Part 4 - Analysis of Fast Precession Angular Momentum" applies a mathematical analysis with Eulerian coordinates *(Figure-2)* and reveals the magnitude of angular momentum about the axis of fast precession for a spinning top is approximately equal to the incoming angular momentum regardless of the orientation of the incoming spin axis. Most readers can skim through these pesky mathematical details and jump to its concluding text following "Eureka!", on page 16. "Part 5 - A New Analysis of the Stern-Gerlach Experiment" explains that a spin particle's magnetic dipole moment induces the mode of fast precession and thereby produces a quantized spin precession. This is the underlying hidden objective physicality causing the beam to split in the Stern-Gerlach experiment. This also implies that cascaded Stern-Gerlach experiments should correlate in a linear fashion *(Figure-3)*. "Part 6 - Conclusion" (a one page statement) asserts that Stern-Gerlach experiments do not require an exotic "quantized spin space" with implied ghostly parallel universes. We must now ask if Einstein's ideal of an underlying objective physical reality might eventually form a proper basis for all of Quantum Mechanics. Finally, a "Bibliography" references Stern[1], Gerlach[2], Goldstein[3], Symon[4], Routh[5], and Gratke[6]. For more information about the author's ideas, see the commentary and the external links in the notes at the bibliographic citation for Gratke.

Part 1 - Introduction

Since the 1920s, the Stern-Gerlach experiment's split-beam result has inspired the concept of infinitely dissolved, yet instantly interconnected, parallel quantum existence. In 1922, Otto Stern and Walther Gerlach detected the magnetic properties of atoms by magnetically deflecting an atomic beam. The beam splits in apparent violation of classical dynamics, but in apparent support of the quantization rules of Niels Bohr (1913) and Arnold Sommerfeld (1916). Thus, the Stern-Gerlach experiment appeared to justify the non-classical phenomenon of "quantized spin direction". In 1926, Werner Heisenberg's suggestion of the uncertainty principle and Max Born's proposal of overlapping probability waves established a new basis for interpreting the split-beam result. Led by Niels Bohr, the "Copenhagen Gang" proclaimed spin directions are quantized, creating a "quantized spin space".

Even now, "quantized spin space" seems anti-physical, because it appears to undercut the traditional sense of physical existence. Supposedly, every possible direction of a spin axis simultaneously dwells as an overlapping probability wave. Until the orientation of a particle's spin axis is observed, it dwells in a dissolved state distributed throughout an implied infinite-number of ghostly parallel universes. When a particle is then observed to be aligned in a particular direction, all these supposed parallel existences instantaneously collapse into the observed state. This perception of simultaneously existing multiple states of quantized spin directions for a single particle was the critical impetus for the strange idea of infinite parallel universes. The idea of parallel universes is also known as the many-worlds interpretation of modern quantum mechanics. Thus, we can historically link the Stern-Gerlach experiment to the inception of quantum weirdness.

Most surprisingly, all previous analyses of the Stern-Gerlach experiment have ignored the classical mode of fast precession. All previous analyses of the Stern-Gerlach experiment tout the inadequacy of the classical mode of slow precession, but fail even to mention that classical spin mechanics admits a second mode of precession - the mode of variable fast precession. All respectable textbooks about Classical Mechanics that address the issue of classical spin mechanics discuss the two modes of precession - the slow mode and the fast mode. Amazingly, all textbooks about Quantum Mechanics appear to ignore completely the mode of fast precession for spin particles.

While completely ignoring the mode of fast precession, all previous reports about the Stern-Gerlach experiment proclaim the inadequacy of classical mechanics and classical physicality. The Stern-Gerlach experiment is then cited as a critical milestone justifying the rejection of classical physicality and the promotion of the anti-physical abstractions of modern quantum mechanics.

Today, the Stern-Gerlach experiment is not the only justification for quantum weirdness, but it serves as the critical precedent. More recently, EPR experiments based on Bell's inequality are purported to violate EPR (Einstein-Podolsky-Rosen) criteria and thus to prove the quantum weirdness of instantaneous non-local quantum entanglement. Nevertheless, let us hold the purported results of EPR experiments in abeyance as we consider the fundamental flaw of the Stern-Gerlach experiment.

A spin particle's quantized magnetic dipole drives a spin particle into a previously ignored classical mode of fast precession. This causes an initially-quantized spin particle with random axis orientation to realize a quantized precession spin. Thus, fast precession of quantum spin particles with an inherent physical property of "literal spin" produces the split-beam result of the Stern-Gerlach experiment. Most surprisingly, this correction obviates the need for having ever invented the ghostly and anti-physical interpretation of "quantized spin space". In addition, this new model suggests a cascaded Stern-Gerlach apparatus should produce a linear correlation.

Overall, identifying the mode of fast precession as underlying the split beam result of the Stern-Gerlach experiment precludes any need for attributing exotic mystical characteristics to "quantized spin". This re-interpretation completely reverses the previously implied significance for one of the most significant physics experiments of the twentieth century. While seeming quite unlikely, this leaves us to wonder if Einstein's ideal of objective physical reality might eventually be recognized as underlying all of Quantum Mechanics.

Part 2 - An Historical Perspective of Quantized Spin Space

2.1 A Brief Introduction to Space Quantization

In 1913, Niels Bohr proposed angular momenta of atomic orbits are quantized. In 1916, Arnold Sommerfeld proposed magnetic moments of atomic orbits are also quantized. In 1921, Arthur Holy Compton suggested that spinning particles might constitute the most elementary magnets in nature. Thus, the basic particles underlying the buildup of matter (electrons, protons, neutrons and atoms) all possess angular momentum and magnetic dipole properties, and these properties can be tested.

In 1922, Otto Stern[1] and Walther Gerlach[2] developed an experiment to observe the net magnetic dipoles of atoms. Just like an electric coil, a spinning electron forms a magnetic dipole, and an electron in looping atomic orbit produces an additional magnetic field. Stern and Gerlach employed a divergent magnetic field to deflect atoms possessing a net magnetic dipole. The dramatic result was that the deflected beam of atoms split into two beam paths. A simplified classical spin model predicted that the deflection of randomly orientated magnetic-dipole atoms in a beam should be continuously distributed over a range of deflection. The result of a split-beam appeared to justify the Bohr-Sommerfeld proposition of quantized magnetic moment direction. Eventually, this came to be known as space quantization and, via the Copenhagen Convention, implied the existence of ghostly parallel universes.

In 1925, Wolfgang Pauli introduced his famous exclusion principle and a fourth quantum number. The initial three quantum numbers were employed to account for quantization within three-dimensional space. Pauli suggested only two electrons in the same atom could possess the same three quantum numbers, but only if they possessed a different fourth quantum number (+½ and -½). This raised an obvious question. What does the fourth quantum number represent? In 1925, Samuel Goudsmit and George Uhlembeck suggested this fourth quantum number represents particle spin.

The idea of particle spin properly implies the presence of mechanical angular momentum. In addition, a spinning electron is a spinning electrical charge that induces a magnetic dipole. Thus, by itself, an electron is a tiny magnet. Since the third quantum number is associated with the magnetic fields induced by atomic orbital motions of electrons, the third quantum number can be called magnetic orbital quantization. Correspondingly, the fourth quantum number can be called magnetic spin quantization, because it involves the magnetic dipoles of spinning elementary particles.

In 1927, particle spin was shown to play a part in the Stern-Gerlach experiment, as Phipps and Taylor utilized a Stern-Gerlach device to observe the coupling of electron spin with electron orbitals in Hydrogen atoms. Also in 1927, Thomas revealed relativistic effects for spin. In 1928, Paul Dirac developed a relativistic version of the Schrodinger wave model of the atom. Dirac's relativistic atomic model required particle spin to maintain Einstein's relativistic invariance for the wave model of an atom. Thus, the current ideas about particle spin were well established in the 1920s, where elemental particle spin is considered to be a uniquely quantum phenomena of "Quantized Space". However, particle spin was said to not to be "literal spin", because it seemed to violate classical physicality.

2.2 A Brief Introduction to Quantum Mysticism

In the absence of any classical dynamics explanation, the split-beam results of the Stern-Gerlach experiment became a primary justification for the anti-physical idea of ghostly parallel universes implied by the Copenhagen Model of Quantum Mechanics. Thus, we are obliged to consider ghostly parallel universes, despite any intuitive impression that it constitutes a form of quantum mysticism.

In the 1920s, the ideas about quantum spin were accompanied by the development of quantum wave mechanics. In 1923, Louis de Broglie proposed the existence of matter waves, whereby orbiting electrons are attributed to be in wave resonance. In 1926, Irwin Schrodinger proposed a wave model of the atom, Werner Heisenberg proposed his uncertainty principle, and Max Born proposed that matter waves were probability waves. With Max Born's introduction of probability waves, quantum wave mechanics became steeped in the parallel existences espoused by the Copenhagen Model of Quantum Mechanics. Led by Niels Bohr, the "Copenhagen Gang" insisted that an unobserved object must dissolve into all its possibilities depicted by its overlapping probability waves. An unobserved object must supposedly exist as multiple ghostly copies of itself, where each copy dwells in its own universe. Thus, the Copenhagen Model implies the existence of ghostly parallel universes.

By virtue of splitting the particle beams, Stern-Gerlach experiments detect the alignment of magnetic dipole axes with respect to the detector's test axis. The split-beam result has been interpreted to mean that a particle's magnetic dipole is either aligned or anti-aligned with the test direction. The test results can never reveal dipole alignment at any oblique angle. Since incoming dipole alignments should have been randomly oriented, the obvious issue was to fully explain the split-beam result. While the prior Bohr-Sommerfeld magnetic quantization seemed to merely quantize magnetic alignments, the "Copenhagen Gang" implied that the direction of the spin axis for a spin particle dwells as an overlapping probability wave function, wherein all orientations of the spin axis are included as overlapping possibilities. Supposedly, until the orientation of a particle's spin axis is observed, it dwells in a dissolved set of states distributed throughout an infinite number of ghostly parallel universes. When the particle is finally observed to be aligned in a particular direction, the Standard Copenhagen Model of Quantum Mechanics implies that all these dissolved parallel existences instantaneously collapse into the particular state of reality that we happen to observe.

2.3 Re-Opening the Infamous Einstein-Bohr Debate

This idea of distributed dissolved existence is the anti-physical interpretation of the Heisenberg uncertainty principle, which Albert Einstein could not accept. The grand debates between Einstein and Niels Bohr are legendary. In spite of Einstein's otherwise icon status, his objection to the idea of dissolved existence was dismissed, and the Copenhagen Model of Quantum Mechanics championed by Bohr became the accepted model. Left unchallenged, the Copenhagen interpretation of the Stern-Gerlach experiment appears to destroy any hope of realizing an ultimate theory of physics founded upon the ideals of objective physical reality. Conversely, a successful challenge to the standard interpretation of the Stern-Gerlach experiment would constitute a dramatic reversal.

The role that quantum spin played in that grand Einstein-Bohr debate is not always appreciated for the impact it had in leading theorists such as Bohr to assert their anti-physical interpretation of the Heisenberg uncertainty principle. By re-opening the discussion of quantum spin, this proposal re-opens the discussion about dissolved existence and ghostly parallel universes. When a proper classical analysis is applied to the Stern-Gerlach experiment, particular perceptions of Quantum Mechanics can be dramatically altered as follows:

- A locally-realistic spin can account for the split-beam result of the Stern-Gerlach experiment.
- The prior anti-physical explanation of Stern-Gerlach experiment seems unwarranted.
- The anti-physical prescription of quantized spin direction is unnecessary.
- The anti-physical interpretation of the Heisenberg uncertainty principle seems suspicious.
- The proposal of dissolved existence within ghostly parallel universes seems to be less justified.
- The Standard Copenhagen Model of Quantum Mechanics appears to be less supported.

We can now begin to recognize the potential unworthiness for some of the anti-physical interpretations of the Copenhagen Model of Quantum Mechanics. If previously proclaimed anti-physical interpretations for Stern-Gerlach experiments are incorrect, then the idea of dissolved existence distributed within ghostly parallel universes is not as justified as previously believed. Accordingly, "Questioning the Quantum Mysticism Attributed to Stern-Gerlach Quantized Space" re-opens a larger question. Could we ever re-establish objective physical reality as the proper basis of Quantum Mechanics?

This paper's re-interpretation of the Stern-Gerlach experiment in terms of classical physicality is far more than just a revised nuance for "quantized spin". It contradicts the previously implied significance for one of the most significant physics experiments of the twentieth century. While acknowledging but holding EPR results in abeyance, this re-interpretation of Stern-Gerlach may ultimately prove to be a precursor. In the future, we may wish to reconsider what constitutes the essence of physical existence. Should it be Bohr's "ghostly parallelism" or Einstein's "objective physicality"?

Part 3 - The Basics of the Stern-Gerlach Experiment

3.1 Introducing the Stern-Gerlach Test Setup

The Stern-Gerlach experiment projects a beam of spin particles through a divergent magnetic field. The beam generator produces a particle beam, where the constituent particles individually possess the spin properties of a miniature spinning top and a co-aligned miniature magnet. The beam is projected into a divergent magnetic field between two magnetic poles, a north pole and a south pole. The divergent magnetic field was expected to spread the beam along the direction of the applied magnetic field. Subsequent detection did confirm a magnetic property for the beam's particles, but the beam split into two beams.

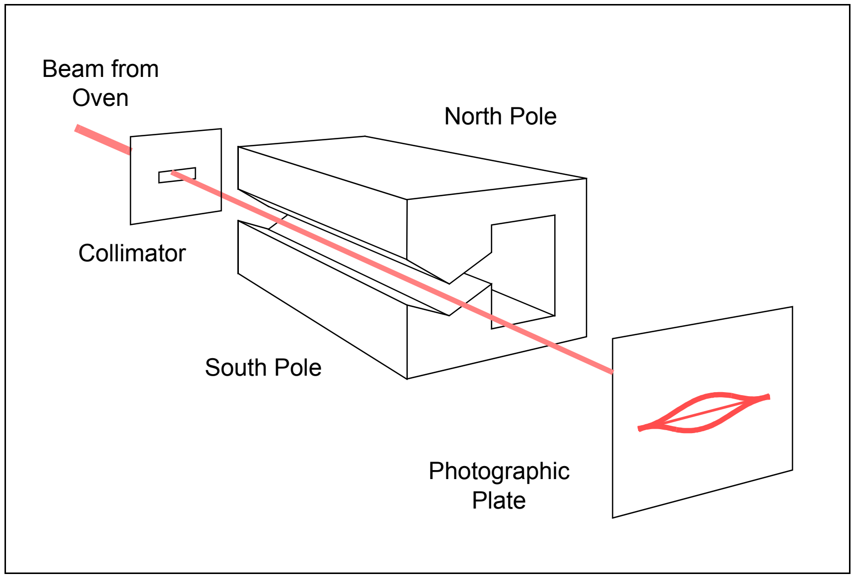

Figure-1: Stern-Gerlach Beam Splitting Deflection

The ingenuity of Stern and Gerlach was in shaping the magnetic poles to create a divergent magnetic field, and thereby, to create a net force on the magnetic particles to deflect them. It may seem anti-intuitive, but a uniform magnetic field will not produce a net force on a magnet. A uniform magnetic field can produce a net torque, but not a net force. Accordingly, introducing the feature of a divergent magnetic field is essential for deflecting the beam.

The objective of the Stern and Gerlach experiment was to observe split-beam deflection and thereby to confirm that the particles possessed quantized magnetic dipole directions. As depicted in *Figure-1*, half the beam deflected upward, and the other half deflected downward. The beam was not a pencil beam but a ribbon beam, and with the electromagnet off, the beam produced a horizontal line on the photographic plate. With the magnet powered, the ribbon beam split into two opposing arches. The edges of the ribbon beam did not deflect, because the magnetic field at the edges was not divergent. Since the split-beam result did not conform to the presumptions attributed to classical spin dynamics, the Stern-Gerlach experiment appears to justify the exotic idea of quantized spin direction. So much depends upon the explanation that we need to carefully consider the possibilities one step at a time. In the balance may hang the discernment for the proper foundation of Quantum Mechanics.

3.2 Reviewing Simplified Classical Spin

We all can recognize the idea of a top spinning about its axis. The unique quality of a spinning top is that it possesses angular momentum, which (in the absence of torque) is a conserved quantity. Angular momentum is a product of the spin rate and the effective rotational mass (moment of inertia). An ice skater starts a slow spin, and by tucking her arms in tight to her body, her rate of spin speeds up. By drawing her mass in tight, she reduces her moment of inertia to reduce her effective rotational mass. She would be reducing her angular momentum if the spin rate remained the same. She cannot help but speed up as she draws her mass inward, because angular momentum must be conserved. In a routine classroom demonstration, one of my students stands on a freely rotating base with hands stretched out sideways and holding weights. Then, as the student pulls the weights inward, the student's rate of spin speeds-up quite noticeably, but all the while angular moment is conserved.

To change angular momentum, we need to apply torque. We need to apply torque to increase angular momentum, or to decrease angular momentum, or to change the direction of its axis. Both spin magnitudes and spin direction are conserved aspects of angular momentum. This last idea of changing direction of the spin axis is a rather perplexing quality of angular momentum that can create spin axis precession. When a spinning top is set so its axis leans, the gravitationally induced torque causes the leaning top to precess. As it tries to lean, instead of falling, it precesses. It seems a little strange and non obvious, but after careful study, one can learn to accept this idea of precession.

In the simplified model of classical spin, if one tries to tilt the axis of a spinning object one way, it swings the axis crosswise via precession into a different direction. When this perspective of precession is applied to spinning electrons in a magnetic field, the resultant precession is called the Larmor precession and the rate of precession is called the Larmor frequency. Textbooks in Quantum Mechanics typically report this Larmor frequency for a spinning electron to be as follows,

$$\omega = \left[\frac{2\pi g_l \mu_b}{h} B\right] = \frac{e}{2m} B$$

where 'e' is the unit of elementary charge, 'm' is electron mass, and 'B' is the externally applied magnetic field. The middle expression is posted, because some references report the Larmor frequency in terms of the g factor 'g_l', the Bohr magneton 'μ_b', and Planck's constant 'h'.

For the case of the Stern-Gerlach experiment, the simplified model of classical spin identifies a relatively slow constant precession induced by the nominal magnetic field of the test apparatus. The rate of this precession is independent of the angular tilt of a classical particle's spin axis. As we will discuss later, this simplified model of spin precession cannot account for the split-beam result.

3.3 A Brief Introduction to Quantized Spin Magnitude

Given the idea of the simplified model of classical spin, the initial thoughts about quantum spin were not too much different. Spin being quantized in magnitude is not a difficult concept to comprehend or accept. An object spins with some fixed amount of angular momentum to realize a quantized spin magnitude. This idea of quantized spin magnitude had a well-established precedent in the evolving model of the atom.

In the 1800s, it was noted that gasses could be stimulated to glow, where the emitted colors of light were restricted to sets of specific wavelengths unique to each elemental gas. In 1885, Johann Balmer developed a mathematical formula relating the wavelengths for Hydrogen emissions. In 1887, Heinrich Hertz discovered a new phenomenon called the photoelectric effect, wherein light absorption stimulates a material to release charged particles. At that time, the underlying causality of these two phenomena could not be explained.

In 1899, Max Planck proposed that the emission of light was quantized via his famous formula E = hf to resolve previously conflicting presentations of blackbody radiation. Planck combined the Boltzmann description of high frequency emission with the Rayleigh-Jeans description of low frequency emission to form a single description based upon quantized emission from atomic oscillators.

In 1905, Albert Einstein explained how light energizes electrons to produce Hertz's photoelectric effect. Einstein revealed that light travels as quantized bundles of energy, and these light quanta can interact as energy bundles with electrons. Planck had envisioned light energy being quantized at emission, but he initially thought that the light subsequently traveled as a disbursing wave. In contrast, Einstein envisioned that light travels as quantized energy packets, which we now call photons.

In 1911, Ernst Rutherford tried to scatter alpha particles off a thin gold foil. To his surprise, most of the particles flew right through the foil without much deflection, but a few of the particles rebounded backward as if they collided with a very massive compact object. Realizing that most of the mass of the atom was concentrated in a tiny nucleus, Rutherford postulated a solar system model for an atom, wherein negative electrons orbit a positive nucleus.

In 1913, Niels Bohr proposed that the angular momentum of an electron atomic orbit was quantized. The key to Bohr's revelation was recognizing that Planck's constant possessed the units of angular momentum. Since Planck's constant quantized light emission via E = hf, Bohr thought it reasonable that Planck's constant should also quantize angular momentum. For 'n' equal to a simple integer (*1, 2, 3 …*), the magnitude of angular momentum '$|\vec{p}|$' could assume only quantized values.

$$\left|\vec{p}_{angular}\right| = \left|\vec{p}_{linear} \otimes \vec{r}_{orbit}\right| = \left|m\vec{v} \otimes \vec{r}\right| = mvr = \frac{nh}{2\pi}$$

In applying this presumption to the Rutherford Model, Bohr was able to derive Balmer's formula for the emission spectra of Hydrogen. As an electron dropped to its next lower orbit, it dropped one quantum of orbital angular momentum. Dropping two orbits dropped two quanta of angular momentum; and correspondingly for three, four and five quanta of angular momentum. The difference of orbital energy was conserved by the Planck emission of an Einstein photon.

$$\Delta E = (E_i - E_f) = hf$$

In extending Rutherford's Model, Bohr provided a sense of underlying physicality for Balmer's emission spectra, Hertz's photoelectric effect, Planck's blackbody radiation, and Einstein's photons. Thus, Bohr convincingly established the idea that orbital angular momentum is quantized. When elementary particles were subsequently discerned to possess the property of mechanical spin, it was reasonable that the magnitude of spin angular momentum was also quantized. However, the standard interpretation of the Stern-Gerlach experiment dismisses the idea of traditional spin axis orientation. Since the Stern-Gerlach apparatus split the beam, spin particles were eventually assigned an exotic property of quantized spin direction. While quantized spin magnitude can be contemplated in terms of tangible physicalities, quantized spin direction seems quite contrary to traditional physicality.

3.4 The Inadequacy of Simplified Classical Precession

Given quantized magnitudes for both magnetic dipole moment and angular momentum in the divergent magnetic field of the Stern-Gerlach experiment, the classical local-reality expectation was that the beam would spread apart over a continuous range of deflection. However, the Stern-Gerlach experiment did not realize this classical expectation.

The nominal magnetic field of the Stern-Gerlach apparatus induces a torque upon the magnetic dipoles of the particles that comprise the beam. The natural tendency of magnets is that they should align in the same direction as an externally applied magnetic field. A torque created by the external magnetic field induces the alignment. However, in the case of elementary spin particles, these particles also possess a mechanical spin with angular momentum. Even with quantized spin magnitude, the expectation of classical spin precession was that particle spin-axes would never rotate into alignment with the magnetic field of the Stern-Gerlach apparatus. Assuming a random distribution of spin axis alignments for incoming particles, the classical expectation was that the precessing spin particles would maintain their relative angles of misalignment. The classical expectation was that forces on the spin particles would derive from whatever angle of misalignment the particles had upon entry into the Stern-Gerlach apparatus.

If the applied magnetic field were uniform, there would be no net force acting upon the spin particles - (net torque yes, net force no). However, the ingenuity of Stern and Gerlach was to introduce a divergent magnetic field, and this would induce a net force. The classical expectation was that this force would be proportional to the cosine of the angle between the magnetic field of the Stern-Gerlach apparatus and the direction of an individual particle's magnetic dipole spin axis. Thus the force of deflection 'F_z' in the 'z' direction would supposedly be proportional to the divergence of the magnetic field 'B_z' in the 'z' direction and the cosine '$\cos(\theta)$' of the angle of misalignment 'θ'.

$$F_z \propto \frac{\partial}{\partial z}(B_z)\cos(\theta)$$

Since the dipole spin axes of the particles in the beam should range over all possible angles, a classically deflected beam was expected to spread over a continuous range of deflection. While spin magnitude could be a quantized classical parameter, classical alignment of spin axes could only be assumed a continuous classical parameter. Thus, the classical expectation was that the amount of deflection for each spin particle would vary with the angle of axis alignment for each individual spin particle. Even though a classical spin magnitude could be quantized, classical deflection would be expected to produce an un-quantized result. When the beam was observed to split into two beams, this constituted a crisis for the classical dynamic description of particle spin, but a big boost to the emerging exotic descriptions of quantized spin direction.

3.5 A Brief Introduction to Quantized Spin Direction

The dramatic split-beam result of the Stern-Gerlach experiment clearly revealed that the simplified model of classical spin was inadequate. This led Quantum Theorists to develop an exotic explanation for quantized spin direction. The confluence of the Stern-Gerlach split-beam result, the Heisenberg uncertainty principle, the Max Born probability wave proposition, and the Niels Bohr Copenhagen Convention led to a startling theoretical proposition for quantized spin direction.

The "Copenhagen Gang" concluded that the direction of the spin axis for a spin particle dwells as an overlapping probability wave function, wherein all orientations of the spin axis are included as overlapping possibilities. The anti-physical interpretation of the Heisenberg uncertainty principle denies traditionally conceived existence. The Max Born proposal of overlapping non-physical probability waves depicts objects as dissolving into simultaneously co-existing multiple possibilities. Thus, particles do not exist as traditionally conceived entities. Until the orientation of a particle's spin axis is observed, it dwells in a dissolved set of states distributed throughout an infinite number of ghostly parallel universes. Niels Bohr led us to believe that when particle spin is ultimately observed to be aligned in a particular direction, all those dissolved parallel existences instantaneously collapse into the particular state of reality that we happened to observe. This is the essence of space quantization.

Overlapping spin-alignment probabilities is just one manifestation of the Copenhagen Convention, which more generally promotes the modern perspective of overlapping probability waves for all phenomena described by Modern Quantum Mechanics. Given this perspective, it is not too difficult for one to miss the significant impact that the Stern-Gerlach experiment played in developing the Standard Copenhagen Model of Quantum Mechanics. The Stern-Gerlach experiment provided pivotal empirical evidence that both inspired and justified the more general Copenhagen Convention. Consequently, we should appreciate that the Stern-Gerlach experiment constitutes a foundation cornerstone for Modern Quantum Mechanics and its derivative exotic idea of ghostly parallel universes.

That elementary particles possess quantized spin is undeniable, and we should accept quantized particle spin as a proven fact. However, what kind of particle spin is it that combines quantized magnetic dipole moment and quantized mechanical angular momentum? Answering this question presents us with a remarkable discovery. If there were any classical explanation available for the split-beam results of the Stern-Gerlach experiment, such a revelation would remove an initial justification for the Copenhagen Convention. This in turn would lead us to reconsider the extreme tenets of quantum weirdness, such as the anti-physical interpretation of Heisenberg's uncertainty principle, Born's probability waves, and Bohr's ghostly parallel universes.

Part 4 - Analysis of Fast Precession Angular Momentum

4.1 A Brief Introduction to Three Coordinate Systems

To present the mathematics that formally describes the complex motions of a spinning top, we need to introduce three coordinate systems. The body coordinate system moves and rotates with the body of the spinning top. The spacial coordinate system does not rotate and remains fixed in its spacial orientation, but it moves in translation with the spinning top. While these two systems possess the same origin (the center of mass of the spinning top), their relative rotational orientation with respect to each other varies continuously in time due to body spin and precession.

The angular relationship between the body coordinates and spacial coordinates can be specified by three angles, which are historically known as the Euler angles, or the Eulerian coordinates.

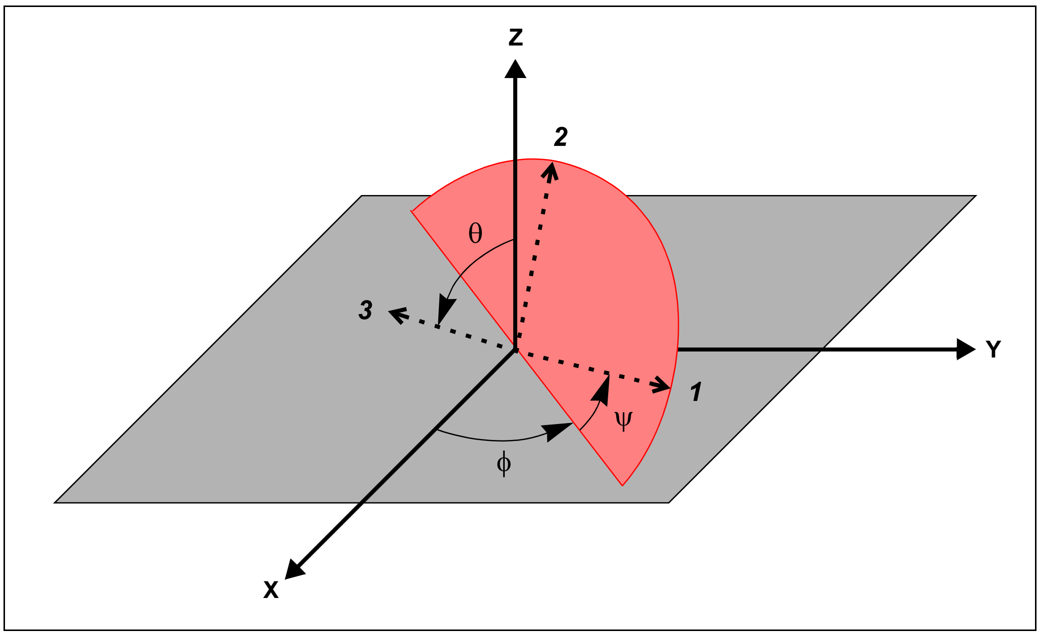

Figure-2: Eulerian Coordinates

While body coordinates naturally describe nominal spin, and spacial coordinates naturally describe nominal precession, spin and precession are not truly independent. These two measures are non-orthogonal vectors that partially overlap each other, which complicate the issue of a spinning top. Both composite spin and composite precession involve components of both nominal spin and nominal precession. The Eulerian coordinates couple the body coordinates and the spacial coordinates, and thereby, enable discernment of the composite spin and composite precession. While no standard convention is universally employed, *Figure-2* depicts a frequently employed set of notation and a graphical configuration for the Eulerian coordinates.

The body coordinates are labeled '*1*', '*2*' and '*3*'. Axis-*3* is the body spin axis, and rotation about this axis-*3* is the nominal body spin 'ω_ψ'. The spacial coordinates are labeled 'X', 'Y' and 'Z'. Axis-Z is the precession axis, and rotation about this axis-Z is the nominal precession 'ω_ϕ'. The angle between axis-*3* and axis-Z is labeled 'θ', and can be called the tilt angle. The plane defined by axis-*2* and axis-*3* intersects the plane defined by axis-X and axis-Y in a line known as the line of nodes. The angle between axis-X and the line of nodes is labeled 'ϕ', and can be called the precession angle. The angle between the line of nodes and axis-*1* is labeled 'ψ', and can be called the spin angle.

To study the rotational dynamics, we must consider the time rates of change of the various angles. To do so we apply time derivatives. Many authors use the dot notation for time derivatives, but sometimes the dot does not print clearly for all readers to readily recognize. While the standard calculus form for a derivative is the most clear to recognize, it takes up space. Accordingly, the symbol 'ω' with appropriate subscripts will be used here to represent the various angular speeds. The rates of rotation for the Eulerian coordinates are:

$$\left[\omega_\theta = \dot{\theta} = \frac{d\theta}{dt}\right]$$ Tilt angle speed, otherwise known as the nominal nutation.

$$\left[\omega_\phi = \dot{\phi} = \frac{d\phi}{dt}\right]$$ Precession angle speed, which is the nominal precession.

$$\left[\omega_\psi = \dot{\psi} = \frac{d\psi}{dt}\right]$$ Body spin angle speed, which is the nominal body spin.

In terms of Eulerian angles, the composite rates of rotation for the body coordinates are:

$$[\omega_1 = \omega_\theta cos(\psi) + \omega_\phi sin(\theta) sin(\psi)]$$ Body axis-*1* tumble rate.

$$[\omega_2 = -\omega_\theta sin(\psi) + \omega_\phi sin(\theta) cos(\psi)]$$ Body axis-*2* tumble rate.

$$[\omega_3 = \omega_\phi cos(\theta) + \omega_\psi]$$ Composite body spin.

$$[\omega_{12} = \sqrt{\omega_\theta^2 + \omega_\phi^2 sin^2(\theta)}]$$ Composite body tumble.

$$[\omega_{123} = \sqrt{\omega_\theta^2 + \omega_\phi^2 + 2\omega_\phi\omega_\psi\, cos(\theta) + \omega_\psi^2}]$$ Composite body coordinate rotation.

In terms of Eulerian angles, the composite rates of rotation for the spacial coordinates are:

$$[\omega_x = \omega_\theta\, cos(\phi) - \omega_\psi\, sin(\theta)\, cos(\phi)]$$ Space axis-X spin rate.

$$[\omega_y = \omega_\theta\, sin(\phi) + \omega_\psi\, sin(\theta)\, sin(\phi)]$$ Space axis-Y spin rate.

$$[\omega_z = \omega_\phi + \omega_\psi\, cos(\theta)]$$ Composite precession spin.

$$[\omega_{xy} = \sqrt{\omega_\theta^2 + \omega_\psi^2 sin^2(\theta)}]$$ Composite X-Y spin.

$$[\omega_{xyz} = \sqrt{\omega_\theta^2 + \omega_\phi^2 + 2\omega_\phi\omega_\psi\, cos(\theta) + \omega_\psi^2}]$$ Composite spacial coordinate rotation.

In general, the spacial angular speeds are less convenient. The spacial moments of inertia (I_x, I_y and I_z) are not constant, but depend upon body orientation. Consequently, the spacial moments of inertia are themselves dynamic functions of the Eulerian angles. In contrast, the body moments of inertia ($I_1 = I_2 = I_{12}$ and an independent I_3) are constant, and this offers an easier path for analysis. Thus, standard analyses typically apply body coordinates to the solution of a spinning top. Nevertheless, we shall want to transform those results into spacial coordinates to better understand how spin particles behave within the context of the Stern-Gerlach Experiment.

4.2 A Brief Introduction to Complex Classical Spin

Classical spin precession can be rather complicated. Rather than just smooth precession, a spinning top can dip and rise as it precesses in a cyclical motion called nutation. With nutation, the rate of precession cycles faster and slower with each cycle of nutation. Nutation can even produce looping precession, where the upper portion may precess slowly one way and the lower portion precess the other way more quickly. The simple mathematical expression describing simple precession is replaced by more complicated mathematical expressions to describe precession with nutation. Complexity is introduced by the fact that the full mathematical treatment involves a three-dimensional Lagrangian of an object with multiple axes of rotations.

Two authoritative modern-day texts, which discuss nutation are Goldstein[3] and Symon[4], and a classical century-old reference is Routh[5]. Goldstein and Symon cite each other and both cite Routh (late 1800s), who in turn cites Lagrange (late 1700s) and Poisson (early 1800s)[5d]. For a simple spinning top tilting in a gravitational field, Goldstein[3b] and Symon[4b] develop the following Lagrangian:

$$L = \frac{1}{2}I_{12}\omega_\theta^2 + \frac{1}{2}I_{12}(\omega_\phi sin(\theta))^2 + \frac{1}{2}I_3(\omega_\phi cos(\theta) + \omega_\psi)^2 - mglcos(\theta)$$

For the body spin angle 'ψ', $\left[\frac{dp_\psi}{dt} = \frac{\partial L}{\partial \psi} = 0\right]$, where $[p_\psi = I_3(\omega_\phi cos(\theta) + \omega_\psi)]$.

For precession angle 'ϕ', $\left[\frac{dp_\phi}{dt} = \frac{\partial L}{\partial \phi} = 0\right]$, where $[p_\phi = I_{12}\omega_\phi sin^2(\theta) + I_3(\omega_\phi cos(\theta) + \omega_\psi)cos(\theta)]$.

For energy, $\left[\frac{dE}{dt} = -\frac{\partial L}{\partial t} = 0\right]$, where $E = \frac{I_{12}}{2}(\omega_\theta^2 + (\omega_\phi sin(\theta))^2) + \frac{I_3}{2}(\omega_\psi + \omega_\phi cos(\theta))^2 + mglcos(\theta)$.

Utilizing the first two expressions, Goldstein[3c] and Symon[4c] eliminate the angular speeds 'ω_ψ' and 'ω_ϕ' from the energy equation:

$$E_o = \frac{1}{2}I_{12}\omega_\theta^2 + \frac{(p_\phi - p_\psi cos(\theta))^2}{2I_{12}sin^2(\theta)} + \frac{p_\psi^2}{2I_3} + mglcos(\theta)$$

The general solution for the tilt angle 'θ' then reveals the motion of nutation. Since the resultant first-order differential equation is nonlinear, simple integration is not possible and the required treatment exceeds the scope of many texts limited to an elementary presentation. Nevertheless, Goldstein, Symon, and Routh do present meaningful initial insights into the general solution of a spinning top.

If the tilt angle 'θ' remains constant at any arbitrary initial angle 'θ_o', Goldstein, Symon, and Routh show the spinning top can precess at two different rates. Squares of angular velocities produce a solution with a plus-minus square root. Symon[4d] develops the following constant tilt equation:

$$(p_\phi - p_\psi \cos(\theta_o)) = \left\{I_{12}\omega_\phi sin^2(\theta_o)\right\} = \left(\frac{I_3\omega_3}{2}\right)\frac{sin^2(\theta_o)}{cos(\theta_o)}\left[1 \pm \sqrt{1 - \frac{4mglI_{12}}{(I_3\omega_3)^2}\ cos(\theta_o)}\right]$$

When trying to isolate the rate of precession (where $[sin(\theta_o)]$ is non-zero), this reduces to:

$$\omega_\phi = \left(\frac{I_3\omega_3}{2I_{12}}\right)\frac{1}{cos(\theta_o)}\left[1 \pm \sqrt{1 - \frac{4I_{12}}{(I_3\omega_3)^2}\ (mgl)\ cos(\theta_o)}\right]$$

The plus-minus sign on the square root produces two solutions, a slow precession and a fast precession. When the nominal spin is sufficiently high, the second term under the square root is small and the square root can be approximated by binomial expansion.

$$\omega_\phi \approx \left(\frac{I_3\omega_3}{2I_{12}}\right)\frac{1}{cos(\theta_o)}\left[1 \pm \left\{1 - \frac{2I_{12}}{(I_3\omega_3)^2}\ (mgl)\ cos(\theta_o)\right\}\right]$$

Goldstein[3d], Symon[4e], and Routh[5c] all present the same approximate solutions for the two modes of constant precession, a slow precession '$\omega_{\phi s}$' and a fast precession '$\omega_{\phi f}$'.

$$\textbf{Slow Precession:} \quad \omega_{\phi s} \approx \frac{mgl}{I_3 \omega_3} \qquad \textbf{Fast Precession:} \quad \omega_{\phi f} \approx \frac{I_3 \omega_3}{I_{12}\, cos(\theta_o)}$$

Given this standard presentation for the precession of a spinning top tilted in a gravitational field, we should appreciate that restricting one's perspective to only simple slow precession misses the full richness of all possible embodiments of precession. Here is a simple yet profound insight. The circular motion of precession is itself a spin. As we associate the conversion of body spin angular momentum into fast precession angular momentum, we can argue that body spin magnetic dipole strength is also converted into magnetic dipole strength along the precession axis. If so, that could produce the split-beam result of the Stern-Gerlach experiment. Quantum Theorists do not appear to have considered the possibility of nutation or fast precession in their analysis of the Stern-Gerlach experiment. It is now proposed that when we include consideration of fast classical precession, we will discover a classical solution that can account for the split-beam result of the Stern-Gerlach experiment.

4.3 Resolving a Circular Mathematical Definition

The standard formulations for both the slow and fast precession constitute circular definitions. Each of the two precession rates of 'ω_ϕ' is defined in terms of 'ω_3', but 'ω_3' was originally defined in terms of 'ω_ϕ'. Looking back, we see that $[\omega_3 = \omega_\phi cos(\theta_o) + \omega_\psi]$. This can be readily resolved.

For slow precession, the rate of precession '$\omega_{\phi s}$' is quite slow compared to the rate of nominal body spin 'ω_ψ'. Thus, for slow precession, we can neglect the precession term'ω_ϕ' within the composite body spin 'ω_3', such that $[\omega_3 \approx \omega_\psi]$. Applying this approximation, slow precession becomes:

$$\textbf{Nominal Slow Precession:} \quad \omega_{\phi s} \approx (mgl)/(I_3 \omega_\psi)$$

Accordingly, the rate of slow precession appears to be unchanged. Since slow precession is the mode normally encountered in spinning top demonstrations, prior analysts may have been lulled into ignoring the distinction between nominal body spin 'ω_ψ' and composite body spin 'ω_3'. It really makes no difference for slow precession, but that is not the case for fast precession.

For fast precession, the rate of precession '$\omega_{\phi f}$' can be comparable to its body-spin '$\omega_{\psi f}$'. Therefore, we cannot neglect the precession term 'ω_ϕ' within the composite body-spin 'ω_3'. We must solve the following equation to resolve the circular definition of fast precession '$\omega_{\phi f}$'.

$$\omega_{\phi f} \approx \frac{I_3}{I_{12} cos(\theta_o)} (\omega_{\phi f} cos(\theta_o) + \omega_{\psi f})$$

In regrouping terms, we see that the fast precession takes up a slightly altered form.

$$\textbf{Nominal Fast Precession:} \quad \omega_{\phi f} \approx \frac{1}{(I_{12}/I_3 - 1) cos(\theta_o)} \omega_{\psi f}$$

The ultimate significance for the mode of fast precession lies in two facets. Firstly, the rate of fast precession is on the same order of magnitude as the body spin. Secondly, the rate of body spin is actually an interdependent function coupled to the rate fast precession.

$$\textbf{Fast Precession Body Spin:} \quad \omega_{\psi f} \approx (I_{12}/I_3 - 1) cos(\theta_o)\, \omega_{\phi f}$$

We should appreciate that the rates of both fast precession and its body spin have yet to be independently specified. The key is to resolve the interdependence of fast precession and body spin and to specify them in terms of a constant, such as the initial body spin.

4.4 Applying Conservation of Angular Momentum

Since the precession-inducing torque is always applied perpendicular to the angular momentum axis, the total magnitude of the angular momentum should be conserved. While the vector cross product changes the direction of the angular momentum and the magnitudes of the relative components of angular momentum, the total magnitude of the angular momentum remains unchanged.

$$\vec{\tau} = \frac{d}{dt}(\vec{p}_{angular}) \propto (\vec{p}_{angular}) \otimes \vec{B}$$

Conserving the magnitude of angular momentum serves to physically couple the nominal body spin 'ω_ψ' and the nominal precession 'ω_ϕ'. This coupling causes the nominal body spin to slow down as the nominal precession speeds up. With a little analysis, this can be applied to independently specify the rates of both fast precession '$\omega_{\phi f}$' and its body spin '$\omega_{\psi f}$' in terms of the initial body spin '$\omega_{\psi o}$'.

The initial magnitude of angular momentum for a non-precessing top is given by its single component, $[p_o = p_{3o} = I_3\omega_{\psi o}]$. For a precessing top, the magnitude of the angular momentum is given by Pythagorean summation of its vector components.

$$p_{123} = \left|\vec{p}_{123}\right| = \sqrt{{p_1}^2 + {p_2}^2 + {p_3}^2} = \sqrt{{p_{12}}^2 + {p_3}^2} = \sqrt{(I_{12}\omega_{12})^2 + (I_3\omega_3)^2}$$

$$p_{123} = \sqrt{{I_{12}}^2\left[\sqrt{\omega_\theta^2 + \omega_\phi^2 sin^2(\theta)}\right]^2 + {I_3}^2[\omega_\phi cos(\theta) + \omega_\psi]^2}$$

Given that the tilt angle is a constant 'θ_o', the tilt speed is zero, $[\omega_\theta = 0]$. Collecting terms reveals:

$$p_{123} = \sqrt{{\omega_\phi}^2({I_{12}}^2 sin^2(\theta_o) + {I_3}^2 cos^2(\theta_o)) + 2{I_3}^2\omega_\phi\omega_\psi cos(\theta_o) + {I_3}^2{\omega_\psi}^2}$$

Equating the squares of the two magnitudes of angular momentum $[(p_o)^2 = (p_{123})^2]$, yields:

$${I_3}^2{\omega_{\psi o}}^2 = {\omega_\phi}^2({I_{12}}^2 sin^2(\theta_o) + {I_3}^2 cos^2(\theta_o)) + 2{I_3}^2\omega_\phi\omega_\psi cos(\theta_o) + {I_3}^2{\omega_\psi}^2$$

Regrouping terms once more and letting $[A \equiv I_{12}/I_3]$ yields the following relationship:

$$\boxed{{\omega_\phi}^2[A^2 + (1 - A^2)\, cos^2(\theta_o)] + [2\omega_\phi cos(\theta_o) + \omega_\psi]\omega_\psi - {\omega_{\psi o}}^2 = 0}$$

Let us recall Symon's expression for precession prior to applying any approximation.

$$\omega_\phi = \left(\frac{I_3\omega_3}{2I_{12}}\right)\frac{1}{cos(\theta_o)}\left[1 \pm \sqrt{1 - \frac{4I_{12}}{(I_3\omega_3)^2}\,(mgl)\,cos(\theta_o)}\right]$$

Regrouping terms yields:

$$\left[\frac{2I_{12}cos(\theta_o)}{I_3\omega_3}\omega_\phi - 1\right] = \pm\sqrt{1 - \frac{4I_{12}}{(I_3\omega_3)^2}\,(mgl)\,cos(\theta_o)}$$

Squaring yields:

$$\left[\frac{4{I_{12}}^2 cos^2(\theta_o)}{(I_3\omega_3)^2}{\omega_\phi}^2 - \frac{4I_{12}cos(\theta_o)}{I_3\omega_3}\omega_\phi + 1\right] = \left[1 - \frac{4I_{12}}{(I_3\omega_3)^2}\,(mgl)\,cos(\theta_o)\right]$$

Regrouping terms once more yields:

$$I_{12} cos(\theta_o)\omega_\phi^{\ 2} - I_3\omega_3\omega_\phi + mgl \ = \ 0$$

Recalling the definition $[\omega_3 = \omega_\phi cos(\theta_o) + \omega_\psi]$, we can substitute and regroup terms to solve for body spin 'ω_ψ' in terms of the nominal rate of precession 'ω_ϕ', where again $[A \equiv I_{12}/I_3]$.

$$I_{12} cos(\theta_o)\omega_\phi^{\ 2} - I_3\{\omega_\phi cos(\theta_o) + \omega_\psi\}\omega_\phi + mgl \ = \ 0$$

$$\boxed{\omega_\psi \ = \ \left\{(A-1)cos(\theta_o)\omega_\phi + \frac{mgl}{I_3\omega_\phi}\right\}}$$

This expression for the body spin 'ω_ψ' can now be substituted into the prior conserved angular momentum expression, which is then reduced in a sequence of steps as follows:

$$^2 + (1-A^2)cos^2(\theta_o)] + \left[2\omega_\phi cos(\theta_o) + \left\{(A-1)cos(\theta_o)\omega_\phi + \frac{mgl}{I_3\omega_\phi}\right\}\right]\left\{(A-1)cos(\theta_o)\omega_\phi + \frac{mgl}{I_3\omega_\phi}\right\} - \omega_\psi$$

$$\omega_\phi^{\ 2}[A^2 + (1-A^2)cos^2(\theta_o)] + \left[(A+1)cos(\theta_o)\omega_\phi + \frac{mgl}{I_3\omega_\phi}\right]\left\{(A-1)cos(\theta_o)\omega_\phi + \frac{mgl}{I_3\omega_\phi}\right\} - \omega_{\psi o}^{\ 2} \ = \ 0$$

$$\omega_\phi^{\ 2}[A^2 + (1-A^2)cos^2(\theta_o)] + \left[(A^2-1)cos^2(\theta_o)\omega_\phi^{\ 2} + 2A\frac{mgl}{I_3}cos(\theta_o) + \left(\frac{mgl}{I_3\omega_\phi}\right)^2\right] - \omega_{\psi o}^{\ 2} \ = \ 0$$

$$A^2\omega_\phi^{\ 4} + \left[2A\frac{mgl}{I_3}cos(\theta_o) - \omega_{\psi o}^{\ 2}\right]\omega_\phi^{\ 2} + \left[\frac{mgl}{I_3}\right]^2 \ = \ 0$$

This produces a quadratic equation of the nominal precession rate squared '$\omega_\phi^{\ 2}$'.

$$\boxed{[\omega_\phi^{\ 2}]^2 + \left\{2\frac{mgl}{I_{12}}cos(\theta_o) - \frac{\omega_{\psi o}^{\ 2}}{A^2}\right\}[\omega_\phi^{\ 2}] + \left\{\frac{mgl}{I_{12}}\right\}^2 \ = \ 0}$$

This quadratic of a square can be solved yielding two possible answers for precession squared:

$$\omega_\phi^{\ 2} \ = \ \frac{1}{2}\left[-\left\{2\frac{mgl}{I_{12}}cos(\theta_o) - \frac{\omega_{\psi o}^{\ 2}}{A^2}\right\} \pm \sqrt{\left\{2\frac{mgl}{I_{12}}cos(\theta_o) - \frac{\omega_{\psi o}^{\ 2}}{A^2}\right\}^2 - 4\left\{\frac{mgl}{I_{12}}\right\}^2}\right]$$

Assuming the second term (a torque factor) in the square root to be relatively small, the square of the fast precession can be approximated as:

$$\omega_{\phi f}^{\ 2} \ \approx \ \left[\frac{\omega_{\psi o}^{\ 2}}{A^2} - 2\frac{mgl}{I_{12}}cos(\theta_o)\right]$$

Assuming the first term (an initial body spin factor) to be high compared to the second term (a torque factor), a further approximation yields a simplified fast precession.

Resolved Fast Precession: $\boxed{\omega_{\phi f} \approx (I_3/I_{12})\omega_{\psi o}}$

As a top takes up the mode of fast precession, it alters the body spin. More precisely, the fast precession takes spin away from the body spin. A follow-up substitution back into the expression for body spin yields the following, where the small inverse term of fast precession can be dropped:

$$\omega_{\psi f} \approx \left\{(A-1)\cos(\theta_o)\omega_{\phi f} + \frac{mgl}{I_3\omega_{\phi f}}\right\} \approx \left\{(A-1)\cos(\theta_o)\left\{\left(\frac{I_3}{I_{12}}\right)\omega_{\psi o}\right\}\right\}$$

Once again, we assumed the first term (an initial body spin factor) to be high compared to the second term (a torque factor), which now yields a simplified body spin.

Resolved Fast Precession Body Spin: $\boxed{\omega_{\psi f} \approx (1 - I_3/I_{12})\cos(\theta_o)\omega_{\psi o}}$

Thus, as fast precession builds up speed approaching the initial body spin, body spin correspondingly slows down with a coupling factor involving the cosine of the Eulerian tilt angle.

4.5 Angular Momentum along the Axis of Fast Precession

Let us now determine the angular momentum along the axis of fast precession. To do so, let us first determine the angular momentum components in the body coordinates. Then we shall convert those results to determine the angular momentum along the spacial coordinate of axis-z.

In body coordinates, the angular momentum in the body tumble mode is:

$$p_{12} = \left\{\sqrt{p_1{}^2 + p_2{}^2}\right\} = \left\{\sqrt{(I_1\omega_1)^2 + (I_2\omega_2)^2}\right\} = \left\{I_{12}\sqrt{\omega_1{}^2 + \omega_2{}^2}\right\} = \{I_{12}\omega_{12}\}$$

Substituting for 'ω_{12}' with the Eulerian tilt angle a constant 'θ_o' and tilt speed zero, $[\omega_\theta = 0]$.

$$p_{12} = \left\{I_{12}\sqrt{\omega_\theta^2 + \omega_{\phi f}{}^2 sin^2(\theta)}\right\} = \{I_{12}\omega_{\phi f} sin(\theta_o)\}$$

Substituting the fast precession '$[\omega_{\phi f} \approx (I_3/I_{12})\omega_{\psi o}]$', yields the body tumble angular momentum.

$$p_{12} = \left\{I_{12}\left[\left(\frac{I_3}{I_{12}}\right)\omega_{\psi o}\right] sin(\theta_o)\right\} = \{I_3\omega_{\psi o} sin(\theta_o)\}$$

In body coordinates, the angular momentum in the body spin mode is:

$$p_3 = I_3\omega_3 = \{I_3(\omega_{\phi f}\cos(\theta_o) + \omega_{\psi f})\} = \{I_3\omega_{\phi f}\cos(\theta_o) + I_3\omega_{\psi f}\}$$

Substituting the resolved fast precession '$[\omega_{\phi f} \approx (I_3/I_{12})\omega_{\psi o}]$' and the resolved body spin '$[\omega_{\phi f} \approx (I_3/I_{12})\omega_{\psi o}]$', yields the body spin angular momentum.

$$p_3 \approx \left\{ I_3\left[\left(\frac{I_3}{I_{12}}\right)\omega_{\psi o}\right]cos(\theta_o) + I_3\left[\left(1 - \frac{I_3}{I_{12}}\right)cos(\theta_o)\omega_{\psi o}\right]\right\}$$

$$p_3 \approx \left\{ \frac{I_3^2}{I_{12}}\omega_{\psi o}cos(\theta_o) + \left(I_3 - \frac{I_3^2}{I_{12}}\right)\omega_{\psi o}cos(\theta_o)\right\} = \{I_3\omega_{\psi o}cos(\theta_o)\}$$

Converting the components of angular momentum in body coordinates into spacial coordinates, the approximated angular momentum transverse to the precession axis goes to zero.

$$p_{xy} = \{-p_{12}cos(\theta) + p_3 sin(\theta)\}$$

$$p_{xy} \approx \{-[I_3\omega_{\psi o}sin(\theta_o)]cos(\theta_o) + [I_3\omega_{\psi o}cos(\theta_o)]sin(\theta_o)\} = 0$$

The minus sign above for 'p_{12}' is not obvious, but a careful consideration of *Figure-2* will reveal it is so. All of the approximated angular momentum becomes aligned with the precession axis.

$$p_z = \{p_{12}sin(\theta) + p_3 cos(\theta)\}$$

$$p_z \approx \{[I_3\omega_{\psi o}sin(\theta_o)]sin(\theta_o) + [I_3\omega_{\psi o}cos(\theta_o)]cos(\theta_o)\}$$

$$p_z \approx \left\{I_3\omega_{\psi o}sin^2(\theta_o) + I_3\omega_{\psi o}cos^2(\theta_o)\right\}$$

EUREKA! **Fast Precession Momentum:** $\boxed{p_z \approx \{I_3\omega_{\psi o}\} = p_o}$

This is amazing! For the fast precession mode, the magnitude of the precession axis angular momentum is approximately equal to the magnitude of the original angular momentum, no matter what direction that original angular momentum was pointing.

If the magnitude of the original angular momentum was a given fixed amount, the magnitude of the fast precession angular momentum assumes that given quantized amount.

In addition, the sign of the precession spin (and thus, the sign of precession angular momentum) follows the relative sign of the initial body spin. An initial body spin pointing above the X-Y spacial plane produces a precession spin pointing upward. Conversely, a body spin pointing below the X-Y spacial plane produces a precession spin pointing downward. Thus, the angular momentum aligned along the precession axis is quantized either up or down.

This dramatic revelation of split fast precession spin involves only the dynamics of classical mechanics! The issue now is to resolve what this means within the context of the Stern-Gerlach experiment. Why would a particle with a nominally quantized amount of spin assume the mode of fast precession? In lieu of gravitational torque for a classical spinning top, we have magnetic torque for a Stern-Gerlach spin particle. In addition, in lieu of only continuous mechanical spin for a classical spinning top, we have a quantized mechanical spin and a quantized magnetic dipole moment for a Stern-Gerlach spin particle. It shall be argued that classical dynamics allows this quantized magnetic dipole to drive the spin particle into the mode of fast precession, and thus to induce the split beam result of the Stern-Gerlach experiment.

Part 5 - A New Analysis of the Stern-Gerlach Experiment

5.1 Classical Precession Splits the Stern-Gerlach Beam

Replacing the gravitational torque factor for a leaning spinning top '$-mglcos(\theta)$' by a magnetic torque factor for a Stern-Gerlach spin particle '$M_{dipole}Bcos(\theta)$', the Lagrangian for a Stern-Gerlach spin particle can be represented as follows:

$$L_{SG} = \frac{1}{2}I_{12}{\omega_\theta}^2 + \frac{1}{2}I_{12}(\omega_\phi\, sin(\theta))^2 + \frac{1}{2}I_3(\omega_\phi\, cos(\theta) + \omega_\psi)^2 + M_{dipole}B\, cos(\theta)$$

Since the analysis of the classical spinning top leading to the mode of fast precession proved to be independent of the torque factor, we can likewise understand that the mode of fast precession for a Stern-Gerlach spin particle should be represented by the same expression:

Fast Precession Momentum: $$\boxed{p_z \approx \{I_3\omega_{\psi o}\} = p_o}$$

Thus, the fast precession mode of classical mechanics splits a randomly oriented set of Stern-Gerlach spin particles into two groups having equal magnitudes of angular momentum along the precession axis. One group has spin up, and the other group has spin down. The magnetic torque is relatively small, as that assumption was inherent in the approximations. Nevertheless, the application of the magnetic torque is necessary to produce precession, but more significantly, it is the key for inducing the mode of fast precession in the Stern-Gerlach experiment. This new disclosure provides a local-reality explanation for the split-beam result of the Stern-Gerlach experiment.

For a classical spinning top, we almost never observe the mode of fast precession. The reason for that lies in the method by which the torque is applied relative to the manner in which the spinning top assumes its state of precession. A spinning top is set to a leaning angle, and then it is released. In doing so, torque is quickly applied to a non-precessing spinning top. The spinning top then begins to precess but quickly reaches a mode of stable slow precession. The top never has an opportunity to reach the mode of fast precession, because it has found the stable mode of slow precession. If the top started with a fast enough precession, it could then settle into the stable mode of fast precession. A spinning top appears to require some extra mechanism to drive it towards the stable mode of fast precession. In the absence of such a driving mechanism, we observe only the slow precession.

It can be argued that a spin particle's magnetic dipole is formed by a quantized current loop. Like de Broglie's phase-resonant quantized electron orbital-waves, resonant electron spin-waves could form a quantized-spin current loop. This quantized current loop would be strongly diamagnetic, and the magnetic flux linking the loop is restricted to a quantized amount. The introduction of external magnetic flux would violate that quantization of magnetic flux, unless the current loop assumes some specific mode of motion. It can be argued that by taking up the mode of fast precessing, the current loop can maintain an appropriate amount of flux through the loop. Thus, the quantization of the magnetic dipole moments about the precession axis underlies the mechanism that drives Stern-Gerlach spin particles into the mode of fast precession.

5.2 The Hidden Variable Basis of Quantized Spin Space

The coupling of a quantized mechanical spin moment and a quantized magnetic dipole moment is unique. Once we consider that magnetic coupling induces the mode of fast precession within the Stern-Gerlach experiment, we can grasp how a local-reality model of particle spin can produce the split-beam result of the Stern-Gerlach experiment. In trying to detect the magnetic alignment of a spin particle in any given direction, we induce a mechanical precession that generates a magnetic dipole in alignment or in anti-alignment with the test direction of the precession axis. The Stern-Gerlach experiment merely measures the precession axis magnetic dipole moment. In the process of inducing precession, the precession re-orients the original spin axis direction. The supposedly exotic characteristic of a quantized space consisting of an overlay of dissolved parallel existences is now replaced by a locally-realistic physical persistence that is explained by a previously ignored subtlety of classical dynamics. The split-beam result of the Stern-Gerlach experiment is produced by the "hidden variable" of a traditionally-physical spin-axis, which is induced into the mode of fast precession by the reaction of its own quantized magnetic dipole moment. This "hidden variable" explanation now releases us from any obligation to accept the previous exotic attributions assigned to "quantized spin space".

5.3 Note 1: Slight Dispersion of the Split Beams

The surprising classical-splitting for a spin particle's angular momentum along the precession axis in the Stern-Gerlach experiment is a first-order approximation. Higher-order approximations involving the torque factor yield a more complex result. Trigonometric terms involving the Eulerian tilt angle slightly disperse each of the two split beams. Therefore, the images of each split beam cannot be as sharp as the original non deflected beam. While this slight dispersion might seem to be a minor point, it might provide additional confirmation for this new model. To avoid further encumbrance upon the reader, the complex details for the second-order approximation are not presented.

Figure-1 attempts to depict the occurrence of dispersion. More significantly, the original Stern-Gerlach photographic plates seem to actually show some dispersion. It is well known that dispersion does occur to each of the split beams. However, the distinction of second-order induced dispersion may be masked by the inherent variation in particle speeds one could expect within the particle beam. With slower particles deflecting more, and faster particles deflecting less, temperature/speed variations within the particle beam can induce a dominating dispersion. Without being aware of any theoretical basis for an idealized experiment to produce any dispersion, prior understandings seem to assign all dispersion to the variations of particle speeds. Ultimately, particle speed variations may indeed be too great to experimentally discern the practical subtlety of second-order theoretical dispersion.

5.4 Note 2: Cascaded Stern-Gerlach Experiments

What happens when one of the split beams existing from a Stern-Gerlach magnetic analyzer is projected through a second Stern-Gerlach magnetic analyzer? Such an apparatus is known as a cascaded Stern-Gerlach experiment, and is frequently described in textbooks. The results are dependent upon the relative alignment of the magnetic fields of the two analyzers. What should we expect if the analyzers are fitted with filters that only allow their respective up-spin beams to pass? When the two analyzer-axes are aligned together, the up-spin beam passes; when aligned crosswise, only half of the first up-spin beam passes the second analyzer; and when aligned in opposite directions, the beam is extinguished. While nothing presented here changes the results for these particular alignments, the results for alignments in between are not as previously predicted.

It has been previously assumed that the spin correlations for a cascaded Stern-Gerlach experiment vary like photon polarization correlations passing though multiple optical polarizers. It has been known since the early 1800s that light passes a pair of linearly polarized lenses with a probability that varies as the square of the cosine of the relative angle between the polarizer axis orientations, $[cos^2(\theta)]$. Spin particles passing a spin-up filtered cascaded Stern-Gerlach apparatus are proclaimed to likewise pass for the half angle, $[cos^2(\theta/2)]$. Photons are called spin-one particles, and the factor of the half angle is one facet leading to electrons, protons and neutrons being called spin-half particles. However, the fast precession model for spin-half particles passing through a Stern-Gerlach apparatus suggests a transmission profile for spin-half particles slightly different from the Standard Model.

If the locally-realistic fast precession model for beam splitting is correct, it implies spin-half particles should pass a cascaded Stern-Gerlach apparatus via a linear function of the relative angle of tilt alignment. While stage one precession actually re-orients the axes of the spin particles to anywhere about their respective cones of precession, the re-orientations remain randomly distributed either above or below the plane of measurement distinction. In filtering to select only the spin-up particles from stage one, we select a hemisphere of possible spin axis orientations. Tilting the orientation of stage two removes a wedge from that first hemisphere. This would appear to produce a cascaded-transmission profile that is linear with the angle of tilt. For angles between zero and pi, the second stage probability correlation would then be as follows.

$$P_{stage \cdot 2} = \left\{1 - \frac{\theta}{\pi}\right\} \neq cos^2\left(\frac{\theta}{2}\right)$$

Figure-3 depicts the probability profiles of stage two transmissions for the “Spin-Half Fast Precession Model” and the “Spin-Half Standard Quantum Model”.

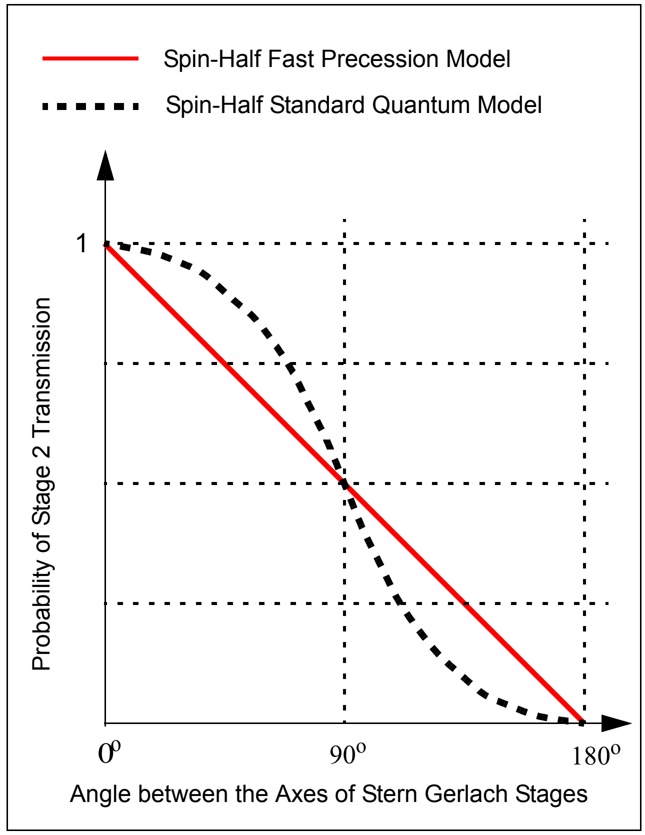

Figure-3: Cascaded Spin-Half Particle Profiles

How could this be so? Has not the cascaded probability profile for cascaded Stern-Gerlach detectors been precisely demonstrated? Apparently, such a test has not been undertaken with sufficient resolution! Let us consider four factors that would have discouraged precise observation.

- First, the exposures on the photographic plates tend to be rather vague indicators, and they do not represent explicit counts of particles. This would have made the counting imprecise.
- Second, the subtlety of a slight dispersion would make the images fuzzy and imprecise. While the beams clearly split, each split portion is fuzzy.
- Third, the subtlety of slight dispersion in the first stage compounds itself as additional dispersion in the second stage of detection. This would make the images even more fuzzy.
- Fourth, fringing of the magnetic fields at the entrance and exit of the first stage magnetic analyzer alters the precession axis to move a particle’s spin axis off the cone of nominal precession. For the second stage, this will switch some particles into the other hemisphere.

Thus, the photographic images can only manifest a compounded vagueness, and the assertion of the cosine transmission profile of the “Spin-Half Standard Quantum Model” now appears to be an unverified assumption. However, given the various factors that induce fuzziness in the images, one may not exactly observe the linear transmission profile of the “Spin-Half Fast Precession Model”. Nevertheless, a careful experimenter might be able to conduct a future test to demonstrate a distinction in favor of the new linear model now being proposed. Since all the mathematical formalities of the Standard Quantum Model for spin particles employ a cosine characteristic, to now demonstrate a linear characteristic would constitute a dramatic change.

Part 6 - Conclusion

Heretofore, all analyses of the Stern-Gerlach experiment have apparently assumed the mode of slow precession as the only possible basis for a classical local-reality model. When the model of slow precession proved inadequate, Quantum Theorists introduced exotic anti-physical explanations for the split-beam result of the Stern-Gerlach experiment. Over Einstein's objection, Niels Bohr with his Copenhagen Convention insisted that unobserved physicalities, such as spin, dissolve into the infinite possibilities depicted by its infinite overlapping probability waves. An unobserved spin supposedly exists as an infinite number of ghostly copies of itself, where each copy dwells in its own universe. The "Copenhagen Gang" leveraged this misunderstanding for the split-beam result of the Stern-Gerlach experiment, to improperly justify their introduction of the existence of ghostly parallel universes.

Repeatedly and insistently, we have been advised that there is no conventional physical explanation for the split-beam result of the Stern-Gerlach experiment, and that therefore, we must accept the exotic explanations of the Copenhagen Convention. We are bombarded with claims that no physical reality based upon "hidden variables" can possibly account for quantum phenomena, such as the supposed quantization of space revealed by the Stern-Gerlach experiment. However, we can now recognize that the mode of fast classical precession can induce the Stern-Gerlach split-beam result without recourse to quantum mysticism. This newly revealed explanation, based upon fast classical precession, preempts the need of a space quantization that otherwise promotes the notorious anti-physicality of dissolved existence distributed within ghostly parallel universes.

When no conventional physical explanation for the split-beam result of the Stern-Gerlach experiment was available, we had little choice but to acquiesce to the quantum weirdness of the Copenhagen Convention. In and of itself, this proposed re-interpretation of the Stern-Gerlach experiment and the associated questioning of space quantization do not overturn the Copenhagen Convention. However, it establishes a critical precedent that justifies a renewed concern about the completeness of the Standard Model of Quantum Mechanics.

Thus, beyond the Stern-Gerlach experiment, "Questioning the Quantum Mysticism Attributed to Stern-Gerlach Quantized Space" seeks to open a new dialogue of inquiry. Might it ever be possible to re-establish an objective physical reality as the proper basis of Physics? While the re-interpretation of the Stern-Gerlach experiment is a significant step toward the affirmative, other significant reversals would also be required. Most notably, EPR experiments based on Bell's inequality are purported to violate EPR (Einstein-Podolsky-Rosen) criteria and thus to prove the quantum weirdness of instantaneous non-local quantum entanglement. To generalize the new Stern-Gerlach interpretation of classical physicality to all of quantum mechanics, the purported results of EPR experiments would need to be replaced by a scenario based upon local reality. This would seem unlikely, and without a dramatic new explanation for EPR experiments, the overall quantum weirdness of the Standard Quantum Model would appear to be sustained. Nevertheless, given the surprising and unlikely appearance of this "hidden variable" explanation of the Stern-Gerlach experiment, maybe some new discovery will avoid the anti-physical quality of instantaneous "action-at-a-distance" now attributed to EPR experiments.

By reversing the interpretation of a truly great experimental milestone, this paper re-opens the great Einstein-Bohr debate. The "hidden variable" of fast classical precession for "literal spin" particles can account for the observations previously appearing to have been exclusively justified by Bohr's exotic abstraction of "quantized spin space". Thus, one is tempted to wonder. Could this re-interpretation of the Stern-Gerlach experiment eventually lead us to rejuvenate Einstein's ideal of objective physical reality? In the future, a carefully crafted cascaded Stern-Gerlach experiment that precisely measures its own cascaded angular correlation would advance this discussion. However promising these re-interpretations of the Stern-Gerlach experiment may appear, a reversal of purported EPR results is absolutely essential to justify affirmation of Einstein's ideal of objective physical reality.

Beyond this singular issue of "quantized spin" lies a much larger issue of an "ultimate theory of unification", which this author is pursuing[6] and for which this revelation about a common sense account of "quantized spin" is but one step.

Bibliography

[Bib-1] Otto Stern, (1888 - 1969)

Note: In 1921, Stern conceived his famous experiment, which he performed in 1922 in conjunction with Gerlach. In 1933, building upon the techniques of his initial experiment, Stern measured the magnetic moment of protons and found them to be about 2.5 times the theoretical value. For his molecular beam research work, Otto Stern received the Nobel Prize for Physics in 1943.

[Bib-2] Walther Gerlach, (1889 - 1979)

Note: While Gerlach had a fine career as Physicist and a Physics Professor, he was questioned about his involvement in the German nuclear development program. Gerlach is most well known for his collaboration with Stern in the Stern-Gerlach experiment of 1922. Gerlach developed the experiment's apparatus to create the divergent magnetic field.

[Bib-3] Herbert Goldstein (1922 - ____)

[3a] Textbook - Classical Mechanics, Addison-Wesley, second edition (1980),
Chapter 5 - "The Rigid Body Equations of Motion",
Article 5.7 - "The Heavy Symmetrical Top With One Point Fixed", pages 213-225

[3b] The Lagrangian function for a spinning top with nutation,
page 214 of [3a], equation (5-52)

[3c] The energy equation for a spinning top with nutation,
page 215 of [3a], equation (5-55)

[3d] The approximate fast and slow precession rates for a spinning top without nutation,
page 222 of [3a],

[Bib-4] Keith Symon (1920 - ____)

[4a] Textbook - Mechanics, Addison-Wesley, third edition (1971),
Chapter 11 - "The Rotation of a Rigid Body",
Article 11.5 - "The Symmetrical Top", pages 454-460

[4b] The Lagrangian function for a spinning top with nutation,
page 454 of [4a], equation (11.47)

[4c] The energy equation for a spinning top with nutation,
page 454 of [4a], equation (11.54)

[4d] The constant tilt equation for a spinning top precessing without nutation,
page 456 of [4a], equation (11.63)

[4e] The approximate fast and slow precession rates for a spinning top without nutation,
page 456 of [4a], equations (11.65) and (11.66)

[Bib-5] Edward Routh (1831 - 1907)

Note: Citation of Routh (both here and by Symon and Goldstein) demonstrates that the complex solution for a spinning top was clearly established long before Stern and Gerlach ever undertook their famous experiment. In ignoring the complex motions of a spinning top, Stern and Gerlach unwittingly introduced a bait-and-switch slight-of-hand misdirection. Imagining classical dynamics to be insufficient to explain the split-beam result of their experiment, Stern and Gerlach induced us to consider exotic explanations. This culminated in the concepts of "space quantization" with its implied existence of ghostly parallel universes. However, now we learn that classical dynamics is sufficient, and no exotic extra-physical explanation is required!

[5a] The Advanced Part of a Treatise on the Dynamics of a System of Rigid Bodies,
Macmillan, sixth edition (1905),
[Note: Virtually all of Routh's work was completed in earlier editions in the late 1800s.]

[5b] The Advanced Part of a Treatise on the Dynamics of a System of Rigid Bodies,
Dover, reprint of [5a], (1955), pages 131-152

[5c] Article 205, “Precession and Nutation of a Top”,
pages 144-146 of [5b] presents the two speeds of precession

[5d] Routh’s footnote on page 133 of [5b] cites Joseph-Louis Lagrange “Mecanique Analytique” (developed from 1767 through 1787) and Simeon-Denis Poisson “Traite de Mecanique” (written 1811 and re-written 1833).

Note: Since the complex motions of a spinning top were dealt with in the late 1700s and early 1800s by two of the most prominent mathematically minded scientists of that era, and then carefully described by a renowned mathematician in the late 1800s, no physicist in the 1900s should have ignored the nuance of fast precession. Most unfortunately, all physicists of the twentieth century did ignore fast precession as a possible underlying cause for the split-beam result of the Stern-Gerlach experiment.

[Bib-6] N. Glenn Gratke (1945 - ____)

[6a] Was Einstein Right? Not Quite! ‘Perfecting E=mc^2’ and ‘Debunking Quantum Weirdness’ Now Leads to ‘The Holy Grail of *Unified Physicality*’
BookSurge (a subsidiary of Amazon.com), preview monograph edition, 2005

Note: This monograph advises that while relativity and quantum mechanics form the foundation of Modern Physics, they have been incompatible with each other. To achieve any theory of ultimate unification, some established ideas have to change! Some fundamental concepts which have been touted as dogmatic fact must give way as we discover hew hidden truths. The corrected interpretation of the Stern-Gerlach experiment is just one example of such a discovery.

Note: The copyrighted preview monograph edition of “Was Einstein Right? Not Quite!” was pre-released in advance of a full book edition. This researcher sought to protect the authorship of his vision for unification while continuing to investigation, to edit, to openly discuss, and to present key ideas at professional conferences sponsored by the American Physical Society (APS). The current paper, which re-interprets the Stern-Gerlach experiment, is just one result of theoretical research stimulated by those activities. Other research papers working toward development of an “ultimate unification theory” are in various stages of development and will be forthcoming.

Note: As of this paper’s publication date, the preview monograph edition is still available through Amazon.com, http://www.amazon.com/Was-Einstein-Right-Not-Quite/dp/1419616285.
However, this monograph preview edition will soon be superseded by the updated and more comprehensive full book edition, which is nearing completion. For the latest information, see the author’s web site, http://www.Emc2GG.com.