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Pullback

In mathematics, a **pullback** is either of two different, but related processes: precomposition and fibre-product. Its "dual" is a pushforward.

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Precomposition

Precomposition with a function probably provides the most elementary notion of pullback: in simple terms, a function *f* of a variable *y*, where *y* itself is a function of another variable *x*, may be written as a function of *x*. This is the pullback of *f* by the function *y*.

$$f(y(x)) \equiv g(x)$$

It is such a fundamental process, that it is often passed over without mention, for instance in elementary calculus: this is sometimes called *omitting pullbacks*,[1] and pervades areas as diverse as fluid mechanics and differential geometry.

However, it is not just functions that can be "pulled back" in this sense. Pullbacks can be applied to many other objects such as differential forms and their cohomology classes.

See:

- Pullback (differential geometry)
- Pullback (cohomology)

Fibre-product

The notion of pullback as a fibre-product ultimately leads to the very general idea of a categorical pullback, but it has important special cases: inverse image (and pullback) sheaves in algebraic geometry, and pullback bundles in algebraic topology and differential geometry.

The pullback bundle is perhaps the simplest example that bridges the notion of a pullback as precomposition, and the notion of a pullback as a Cartesian square. In that example, the base space of a fiber bundle is pulled back, in the sense of precomposition, above. The fibers then travel along with the points in the base space at which they are anchored: the resulting new pullback bundle looks locally like a Cartesian product of the new base space, and the (unchanged) fiber. The pullback bundle then has two projections: one to the base space, the other to the fiber; the product of the two becomes coherent when treated as a fiber product.

See:

- Pullback (category theory)
- Inverse image sheaf

- Pullback bundle
- Fibred category

Functional analysis

When the pullback is studied as an operator acting on function spaces, it becomes a linear operator, and is known as the composition operator. Its adjoint is the push-forward, or, in the context of functional analysis, the transfer operator.

Relationship

The relation between the two notions of pullback can perhaps best be illustrated by sections of fibre bundles: if s is a section of a fibre bundle E over N, and f is a map from M to N, then the pullback (precomposition) $f^*s = s \circ f$ of s with f is a section of the pullback (fibre-product) bundle f^*E over M.

See also

- Inverse image functor

References

1. Ivey, Thomas A.; Landsberg, J.M (2003). *Cartan for Beginners: Differential Geometry Via Moving Frames and Exterior Differential Systems* (https://books.google.com/books?id=vdFhAQAAQBAJ). Graduate Studies in Mathematics. **61**. American Mathematical Society. p. 78. ISBN 978-0821833759.