

# P1: A Calibration Experiment

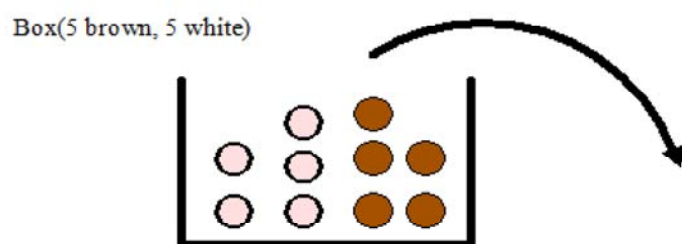
## A Calibration Experiment

Consider the event

$W$  = "a woman will be President of the United States in the next 20 years".

We are interested in your subjective probability of  $W$ . This probability is hard to specify precisely since we haven't had much practice doing it. We describe a simple procedure that will help in measuring this probability.

First consider the following calibration experiment – this is an experiment where the probabilities of outcomes are clear. We have a collection of balls, 5 red and 5 white in a box and we select one ball at random.



Let  $B$  denote the event that we observe a red ball. Since each of the ten balls is equally likely to be selected, I think we would agree that  $\text{Prob}(B) = 5/10 = .5$ .

Now consider the following two bets:

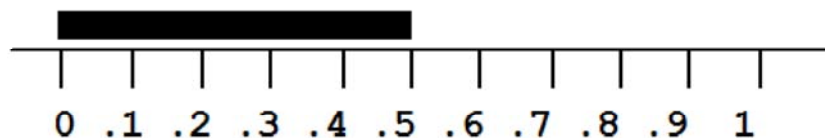
- BET 1 – If  $W$  occurs (a woman is president in the next 20 years), you win \$100. Otherwise, you win nothing.
- BET 2 – If  $B$  occurs (you observe a red ball in the above experiment), you win \$100. Otherwise, you win nothing.

Based on the bet that you prefer, we can determine an interval that contains your  $\text{Prob}(W)$ :

- If you prefer BET 1, then your  $\text{Prob}(W)$  must be larger than  $\text{Prob}(B) = .5$  – that is, your  $\text{Prob}(W)$  must fall between .5 and 1.



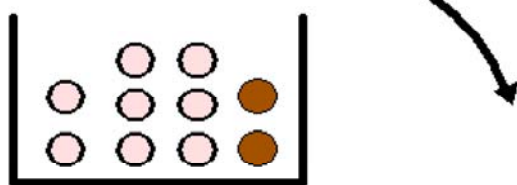
- If you prefer BET 2, then your  $\text{Prob}(W)$  must be smaller than  $\text{Prob}(B) = .5$  – that is, your probability of  $W$  must fall between 0 and .5.



What you do next depends on your answer to part (b).

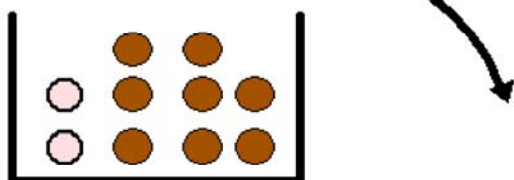
- If your  $\text{Prob}(W)$  falls in the interval  $(0, .5)$ , then consider the "balls in box" experiment with 2 red and 8 white balls and you are interested in the probability of choosing a red ball.

Box(2 brown, 8 white)



- If instead your  $\text{Prob}(W)$  falls in the interval  $(.5, 1)$ , then consider the “balls in box” experiment with 8 red and 2 white balls and you are interested in the probability of choosing a red ball.

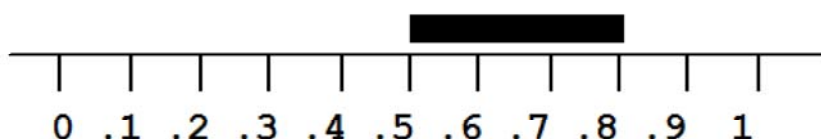
Box(8 brown, 2 white)



Let's suppose that you believe  $\text{Prob}(W)$  falls in the interval  $(.5, 1)$ . Then you would make a judgment between the two bets

- BET 1 – If  $W$  occurs (a women is president in the next 20 years), you win \$100. Otherwise, you win nothing.
- BET 2 – If  $B$  occurs (you observe a red ball with a box with 8 red and 2 white balls), you win \$100. Otherwise, you win nothing.

I decide to prefer BET 2, which means that my probability  $\text{Prob}(W)$  is smaller than 0.8. Based on the information on the two comparisons, you now believe that  $\text{Prob}(W)$  falls between .5 and .8.



In practice, you will continue to compare BET 1 and BET 2, where the box has a different number of red and white balls. By a number of comparisons, you will get an accurate measurement at your probability of  $W$ .

## Practice: A calibration experiment

1. Consider the following “balls in box” experiments. What is the probability of drawing a red if the box contains

- 5 red and 5 white?
- 2 red and 8 white?
- 7 red and 3 white?
- 0 red and 10 white?

2. Consider the statement

A: “I will get married in the next five years”

We want to determine your personal probability that A is true, call this  $\text{PROB}(A)$ .

(If you are already married, choose a different statement where the truth of the statement is uncertain.)

Consider the following two bets:

BET 1:

- If you are married in the next five years, then you win \$20.
- If you are not married in the next five years, you win nothing.

BET 2:

- If you draw a red in a balls-in-box experiment with 5 red and 5 white balls, then you win \$20.
- If you draw a white, you win nothing.

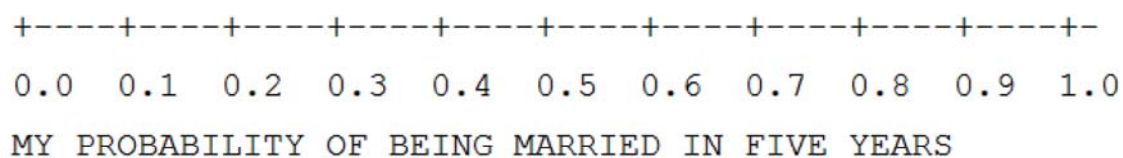
(a) Which bet (BET 1 or BET 2) do you prefer?

(b) Based on your answer to (a), do you think  $\text{PROB}(A) > .5$  or  $\text{PROB}(A) < .5$ ?

3. Let's continue to make this comparison for more balls-in-box experiments. Each row of the table below gives two choices. The left choice is BET 1: you win \$20 if you are married in five years and win nothing if this event does not happen. The choice on the right is BET 2: you win \$20 if you draw a red from a box with a certain number of reds and whites; otherwise you win nothing. For each pair of bets, circle the choice which you prefer

BET 1	BET 2
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 0 red, 10 white Nothing if draw white
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 2 red, 8 white Nothing if draw white
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 4 red, 6 white Nothing if draw white
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 6 red, 4 white Nothing if draw white
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 8 red, 2 white Nothing if draw white
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 10 red, 0 white Nothing if draw white

Based on your choices you made above, you should have a better idea about your probability of being married in five years. Mark on the below number line your probability. (This value should be consistent with the choices that you made.)



4. Repeat the process in part 3 with a different statement.

