Discrete Quantum Theories and Computing

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Dilemma of quantum computing?

- Textbook quantum mechanics is correct.
- There does not exist an efficient classical factoring algorithm.
- The extended Church-Turing thesis —that probabilistic Turing machines can efficiently simulate any physically realizable model of computation —is correct.

Check the compatibility of Quantum Mechanics and Computer Science.

Quantum Mechanics is based on continuous. How about Computer Science?

	Discrete	Continuum
Theoretical Model	Turing machine	BCSS machine
Physical Realization	Digital Computer	Analog Computer
How the models realize?	Reliably	Not Reliably: 1. The quality might be quantized.. 2. The precision of an analog computer is low.

Build a more faithful Quantum Computing model?

Our Quantum Models	Quantum Theories and Computing over Finite Fields	Quantum Interval-Valued Probability Measures

Conventional Quantum Theory

Conventional Quantum Theory

i. D orthonormal basis vectors for a Hilbert space of dimension D.

ii. D complex probability amplitude coefficients describing the contribution of each basis vector.

iii. A set of probability-conserving unitary matrix operators that suffice to describe all required state transformations of a quantum circuit.

iv. A measurement framework.

Pure State

- A pure state can be represented as a D-dimensional vector, $|\Psi\rangle = \sum_{i=0}^{D-1} \alpha_i |i\rangle$, where $\{|0\rangle, |1\rangle \ldots, |D-1\rangle\}$ form an orthonormal basis.
- Given two states $|\Psi\rangle = \sum_{i=0}^{D-1} \alpha_i |i\rangle$ and $|\Phi\rangle = \sum_{i=0}^{D-1} \beta_i |i\rangle$, their inner product

$$\langle\Phi|\Psi\rangle = \sum_{i=0}^{D-1} \beta_i^* \alpha_i$$

  satisfying the following properties:

  A. $\langle\Phi|\Psi\rangle$ is the complex conjugate of $\langle\Psi|\Phi\rangle$;

  B. $\langle\Phi|\Psi\rangle$ is conjugate linear in its first argument and linear in its second argument;

  C. $\langle\Psi|\Psi\rangle$ is always non-negative and is equal to 0 only if $|\Psi\rangle$ is the zero vector.

Mixed State

- A mixed state is the weighted average of the density matrices of pure states

$$\rho = \sum_{j=1}^{N} q_j |\Phi_i\rangle\langle\Phi_i| \ ,$$

where $|\Phi_i\rangle$ are normalized, $q_j > 0$, and $\sum_{j=1}^{N} q_j = 1$.

Probability Measure

Abstraction

- Sample space Ω.
- Event Space 2^{Ω}.
- $\mu: 2^{\Omega} \rightarrow [0,1]$
- $\mu(\emptyset) = 0$.
- $\mu(\Omega) = 1$.
- For any event E,
$$\mu(\Omega \backslash E) = 1 - \mu(E) .$$
- For disjoint events E_0 and E_1,
$$\mu(E_0 \cup E_1) = \mu(E_0) + \mu(E_1) .$$