Determinism Is Ontic, Determinability Is Epistemic

Harald Atmanspacher

Abstract

Philosophical discourse traditionally distinguishes between ontology and epistemology and generally enforces this distinction by keeping the two subject areas separated. However, the relationship between the two areas is of central importance to physics and philosophy of physics. For instance, many measurement-related problems force us to consider *both* our knowledge of the states and observables of a system (epistemic perspective) *and* its states and observables independent of such knowledge (ontic perspective). This applies to quantum systems in particular.

This contribution presents an example showing the importance of distinguishing between ontic and epistemic levels of description even for classical systems. Corresponding conceptions of ontic and epistemic states and their evolution are introduced and discussed with respect to aspects of stability and information flow. These aspects show why the ontic/epistemic distinction is particularly important for systems exhibiting deterministic chaos. Moreover, this distinction provides some understanding of the relationships between determinism, causation, predictability, randomness, and stochasticity.

1 Introduction

Can nature be observed and described as it is in itself independent of those who observe and describe – that is to say, nature as it is "when nobody looks"? This question has been debated throughout the history of philosophy with no clearly decided answer one way or the other. Each perspective has strengths and weaknesses, and each epoch has had its critics and proponents with repect to these perspectives. In contemporary terminology, the two perspectives can be distinguished as topics of ontology and epistemology. Ontological questions refer to the structure and behavior of a system as such, whereas epistemological questions refer to the knowledge of information gathering and using systems, such as human beings.

In philosophical discourse it is considered a serious fallacy to confuse these two areas. For instance, Fetzer and Almeder (1993) emphasize that "an ontic answer to an epistemic question (or vice versa) normally commits a category mistake". Nevertheless, such mistakes are frequently committed in many fields of research when addressing subjects where the distinction between ontological and epistemological arguments is important. Recently, the vast literature on consciousness-related topics has provided many examples of this kind of category confusion (cf., for instance, Searle's criticism of Churchland (Searle 1997, pp. 30/31) and of Dennett (Searle 1997, pp. 113/114)).

In physics, the rise of quantum theory with its interpretational problems was one of the first major challenges to the ontic/epistemic distinction. The discussions between Bohr and Einstein in the 1920s and 1930s is a famous historical example. Einstein's arguments were generally ontically motivated; that is to say, he emphasized a viewpoint independent of observers or measurements. By contrast, Bohr's emphasis was generally epistemically motivated, focusing on what we could know and infer from observed quantum phenomena. Since Bohr and Einstein never made their basic viewpoints explicit, it is not surprising that they talked past each other in a number of respects (see Howard 1997). Examples of approaches trying to avoid the confusions of the Bohr-Einstein discussions are Heisenberg's distinction of actuality and potentiality (Heisenberg 1958), Bohm's ideas on explicate and implicate orders (Bohm 1980), or d'Espagnat's scheme of an empirical, weakly objective reality and an objective (veiled) reality independent of observers and their minds (d'Espagnat 1995).[1]

A first attempt to draw an explicit distinction between ontic and epistemic descriptions for quantum systems was introduced by Scheibe (1973) who himself, however, put strong emphasis on the epistemic realm. Later, Primas developed this distinction in the formal framework of algebraic quantum theory (see Primas 1990). The basic structure of the ontic/epistemic distinction, which will be made more precise below, can be understood according to the following rough characterization (for more details, the reader is referred to Primas 1990, 1994):

> *Ontic states* describe all properties of a physical system exhaustively. ("Exhaustive" in this context means that an ontic state is "precisely the way it is", without any reference to epistemic

[1]Further terms fitting into the context of these distinctions are latency (Margenau 1949), propensity (Popper 1957), or disposition (Harré 1997). See also Jammer's discussion of these notions, including their criticism and additional references (Jammer 1974; pp. 448–453, 504–507).

> knowledge or ignorance.) Ontic states are the referents of individual descriptions, the properties of the system are treated as *intrinsic properties.*[2] Their temporal evolution (dynamics) is reversible and follows *universal, deterministic laws.* As a rule, ontic states in this sense are empirically inaccessible. *Epistemic states* describe our (usually non-exhaustive) knowledge of the properties of a physical system, i.e. based on a finite partition of the relevant phase space. The referents of statistical descriptions are epistemic states, the properties of the system are treated as *contextual properties.* Their temporal evolution (dynamics) typically follows *phenomenological, irreversible laws.* Epistemic states are, at least in principle, empirically accessible.

The combination of the ontic/epistemic distinction with the formalism of algebraic quantum theory provides a framework that is both formally and conceptually satisfying. Although the formalism of algebraic quantum theory is often hard to handle for specific physical applications, it offers significant clarifications concerning the basic structure and the philosophical implications of quantum theory. For instance, the modern achievements of algebraic quantum theory make clear in what sense pioneer quantum mechanics (which von Neumann (1932) implicitly formulated epistemically) as well as classical and statistical mechanics can be considered as limiting cases of a more general theory. Compared to the framework of von Neumann's monograph (1932), important extensions are obtained by giving up the irreducibility of the algebra of observables (not admitting observables which commute with every observable in the same algebra) and the restriction to locally compact phase spaces (admitting only finitely many degrees of freedom). As a consequence, modern quantum physics is able to deal with open systems in addition to isolated ones; it can involve infinitely many degrees of freedom such as the modes of a radiation field; it can properly consider interactions with the environment of a system; superselection rules, classical observables, and phase transitions can be formulated which would be impossible in an irreducible algebra of observables; there are in general infinitely many representations inequivalent to the Fock representation; and non-automorphic, irreversible dynamical evolutions can be successfully incorporated and even derived.

[2]In a more technical terminology, one speaks of "observables" (mathematically represented by "operators") rather than properties of a system. Prima facie, the term "observable" has nothing to do with the actual observability of a corresponding property.

In addition to this remarkable progress, the mathematical rigor of algebraic quantum theory in combination with the ontic/epistemic distinction allows us to address quite a number of unresolved conceptual and interpretational problems of pioneer quantum mechanics from a new perspective. First, the distinction between different concepts of states as well as observables provides a much better understanding of many confusing issues in earlier conceptions, including alleged paradoxes such as those of Einstein, Podolsky, and Rosen (1935) or Schrödinger's cat (Schrödinger 1935). Second, a clear-cut characterization of these concepts is a necessary precondition to explore new approaches, beyond von Neumann's projection postulate, toward the central problem that pervades all quantum theory from its very beginning: the measurement problem. Third, a number of much-discussed interpretations of quantum theory and their variants can be appreciated more properly if they are considered from the perspective of an algebraic formulation.

One of the most striking differences between the concepts of ontic and epistemic states is their difference concerning operational access, i.e. observability and measurability. At first sight it might appear pointless to keep a level of description which is not related to what can be operationalized empirically. However, a most appealing feature at this ontic level is the existence of first principles and universal laws that cannot be obtained at the epistemic level. Furthermore, it is possible to rigorously deduce (e.g. to "GNS-construct"; cf. Primas 1994, 1998) a proper epistemic description from an ontic description if enough details about the empirically given situation are known. These aspects show that the crucial point is not to decide whether ontic or epistemic levels of discussions are right or wrong in a mutually exclusive sense. There are always ontic and epistemic elements to be taken into account for a proper description of a system. This requires the definition of ontic and epistemic terms to be relativized with respect to some selected framework within a set of (hierarchical) descriptions (Atmanspacher and Kronz 1998; see also Lombardi in this volume). The problem is then to use the proper level of description for a given context, and to develop and explore well-defined relations between different levels.

These relations are not universally prescribed; they depend on contexts of various kinds. The concepts of reduction and emergence are of crucial significance here. In contrast to the majority of publications dealing with these topics, it is possible to precisely specify their meaning in mathematical terms. Contexts, or contingent conditions, can be formally incorporated as topologies in which particular asymptotic limits give rise to novel, emergent properties unavailable without those contexts (see Primas 1998 for more details). It should also be mentioned that the distinction between ontic and epistemic descriptions is *neither* identical with that of parts and wholes *nor* with that of micro- and macrostates as used in statistical mechanics or thermodynamics. The thermodynamic limit of an infinite number of degrees of freedom provides only one example of a contextual topology, others are the Born-Oppenheimer limit in molecular physics or the short-wavelength limit in geometrical optics. It is an interesting question whether other kinds of emergence, such as that of phenotypes from genotypes, of consciousness from brain tissue, or of semantics from syntax, can be related to this discussion.

These examples indicate that the usefulness or even inevitability of the ontic/epistemic distinction is not restricted to quantum systems. It plays a significant role in the description of classical systems as well. There is a special class of classical systems for which the distinction of ontic and epistemic descriptions is necessary if category mistakes and corresponding interpretational fallacies are to be avoided: systems exhibiting "deterministic chaos".

2 Ontic and Epistemic States of Classical Systems

Let us consider the representation of a system in a phase space Ω. The ontic state of such a system is represented by a point $x \in \Omega$, so that the phase space Ω is also a state space in this case.[3] The intrinsic properties of the system are represented by real-valued functions on Ω, such as the positions and momenta of point particles. In the algebraic formulation the intrinsic properties of the system are represented by elements of the commutative C*-algebra $C_0(\Omega)$ of all complex-valued continuous functions on the locally compact phase space Ω. Since there is a one-to-one correspondence between the points $x \in \Omega$ and the pure state functionals on $C_0(\Omega)$ (i.e., the extremal positive linear functionals on $C_0(\Omega)$), ontic states are represented by pure state functionals. The ontic valuation of any observable $B \in C_0(\Omega)$ is dispersion free, $\rho(B^2) = \rho(B)^2$. Classical point mechanics is an example. The pointwise representation of an ontic state in Ω illustrates that the finiteness of information, and therefore an information theoretical characterization, is not effective for ontic descriptions.

[3]The concept of a phase space is here understood in terms of a general mathematical structure, e.g. a manifold. Additional constraints, e.g. a symplectic structure of the manifold, lead to more specific types of phase space. It is useful to distinguish the concept of a phase space from that of a state space, since states are not necessarily represented by elements of a phase space.

For an epistemic description, such as in statistical mechanics, one defines a Kolmogorov space (Ω, Σ, ν), with a countably additive probability measure ν (a reference measure, typically the Lebesgue measure) on a σ-algebra Σ of Borel subsets A. Since epistemic descriptions refer to empirical purposes, Σ is required to be the Boolean algebra of experimentally decidable alternatives. Any measure μ which is absolutely continuous with reference to ν characterizes an epistemic (statistical) state. Note that such an epistemic state is an element of the Kolmogorov space (Ω, Σ, ν), not of the phase space Ω. It refers to our knowledge as to whether an ontic (individual) state x is more likely to be in some Borel subset A rather than in others. An ensemble (à la Gibbs) of ontic states is an example of a clearly statistical concept of an epistemic state. However, the corresponding probability distribution can also be viewed in an individual, ontic interpretation (in terms of a distribution "as a whole"), as in kinetic theory (à la Boltzmann) or in classical continuum mechanics.

Equivalently, epistemic states can be represented by Radon–Nikodým derivatives $d\mu/d\nu$, called probability densities or distributions. They are positive and normalized elements of the Banach space $L^1(\Omega, \Sigma, \nu)$. The dual of this Banach space is the W*-algebra $L^\infty(\Omega, \Sigma, \nu)$ of ν-essentially bounded Borel-measurable functions on Ω, the algebra of bounded observables. Insofar as the probability measure μ representing an epistemic state has finite support, it represents finite information about the ontic state. This finiteness can be due to the imprecision of measurements or due to the fact that any decimal expansion of real numbers has to be truncated somewhere for computational purposes. Such a representation of epistemic states (and their associated properties) generally requires a finite partition of Ω.

The temporal evolution of an ontic state $x \in \Omega$ as a function of time $t \in \mathbb{R}$ is a trajectory $t \mapsto x(t)$; the ontic state $x(t)$ determines the intrinsic properties that a system has at time t exhaustively. The temporal evolution of an epistemic state μ corresponds to the evolution of a bundle of trajectories $x(t)$ in Ω. The concept of an individual trajectory of an individual, ontic state is irrelevant within a purely epistemic description.

If the dynamics is reversible then $\mu(T^{-1}(A)) = \mu(T(A)) = \mu(A)$ for all $A \in \Sigma$, where $T : \Omega \to \Omega$ is an automorphism on the state space Ω. For a one-parameter group of such a μ-preserving invertible transformation, the evolution of a corresponding system is both forward and backward deterministic, if the parameter is chosen to be a (discrete or continuous) time t. In such a case, there is no preferred direction of time. Fundamental physical laws (e.g. in Newton's mechanics, Maxwell's electrodynamics, relativity theory) are time-reversal symmetric in this sense. Phenomenological theories such as thermodynamics operate with a distinguished direction of time. The fundamental time-reversal symmetry is broken, thus leading to an irreversible dynamics given by a one-parameter semigroup of non-invertible transformations.

2.1 Stability

In the theory of dynamical systems, the map $t \mapsto T_t = T(x,t)$ is often called a flow $\{T_t \,|t \in \mathbb{R}\}$ on the phase space Ω, where x is a phase point in Ω representing the ontic state of a system. This flow is said to be generated by a transformation F that can be discrete, e.g.

$$x(t+1) = F(x(t)), \tag{1}$$

or continuous in time t, e.g.

$$\frac{dx(t)}{dt} = \dot{x}(t) = F(x(t)). \tag{2}$$

Equation (2) represents a first-order, ordinary differential equation system as a very simple example which, however, is sufficient to illustrate the basic notions. The trajectory $\{x(t)\}$ characterizes the state of the system as a function of time t; its components represent its continuous observables $(x_1, ..., x_d)$. F is a matrix containing the generally nonlinear coupling among the observables, whose number defines the dimension d of the phase space Ω.

To characterize the flow $\{T_t\}$, i.e. the temporal evolution of $x(t)$ as the solution of (2), one has to study how $\{T_t\}$ behaves under the influence of small perturbations δx. Such a characterization specifies the stability of the system and can be obtained in terms of a linear stability analysis. Skipping over the details, a linear stability analysis yields local (in Ω) rates of amplification or damping of perturbations $\delta x(t)$ with respect to a reference state or a reference trajectory $\{x(t)\}$, respectively. From these local rates one can obtain a global dynamical invariant of $\{T_t\}$, essentially as a temporal average of the local rates. These global invariants are the so-called Lyapunov exponents:

$$\lambda_i = \lim_{t \to \infty} \frac{1}{t} \ln \left| \frac{\delta x_i(t)}{\delta x_i(0)} \right| \tag{3}$$

The sum of all (d) Lyapunov exponents allows an elegant and fundamental classification of dynamical systems.

- $\sum \lambda_i > 0$ characterizes systems which are unstable in a global sense, for instance random systems. Their phase volume spreads over the entire phase space as $t \to \infty$.

- $\sum \lambda_i = 0$ characterizes conservative (e.g. Hamiltonian) systems. Since the sum of their Lyapunov exponents is non-negative, they are stable, but not asymptotically stable. Their phase volume remains constant in time (Liouville's theorem). Conservative systems with at least on positive Lyapunov exponent are so-called K-flows.

- $\sum \lambda_i < 0$ characterizes dissipative systems. They have a shrinking phase volume and are asymptotically stable. It is intuitively suggestive (but not finally understood, see Ruelle 1981, Milnor 1985) that the flow $\{T_t\}$ of a dissipative system is asymptotically restricted to a finite subspace of the entire phase space. This subspace is called an attractor. If $\lambda_i < 0 \ \forall i$ this attractor is a fixed point. If there are k vanishing Lyapunov exponents and $(d-k)$ negative ones, then the attractor is a k-torus (limit cycle for $k = 1$). For systems with at least three degrees of freedom, $d \geq 3$, the condition of a negative sum of Lyapunov exponents can be satisfied by a combination of positive and negative ones. This situation defines a chaotic (strange) attractor in the sense of deterministic chaos.

2.2 Dynamical Entropy

The Lyapunov exponents can be related to the concept of a dynamical entropy, i.e., the entropy of a temporal evolution. The dynamical entropy according to Kolmogorov (1958) and Sinai (1959), briefly KS-entropy, has received particular attention among a number of alternative dynamical entropies (Wehrl 1991). The main reason for this popularity is that KS-entropy has turned out to be an extremely useful tool in the characterization of systems showing chaotic behavior in the sense of deterministic chaos. The original proposals by Kolmogorov and Sinai did not explicitly mention this scope of interest. Instead, they were concerned with the way in which an entropy can be ascribed to the automorphism $T : \Omega \to \Omega$. This can be done by considering a partition P in Ω with disjoint measurable sets A_i $(i = 1, ..., m)$ and studying its temporal evolution $TP, T^2P, ...$ If the entropy $H(P)$ of P is given by

$$H(P) = -\sum \mu(A_i) \ln \mu(A_i), \tag{4}$$

then the dynamical KS-entropy h_T is defined as the supremum of $H(P,T)$ over all partitions P,

$$h_T = \sup_P H(P,T), \tag{5}$$

with

$$H(P,T) = \lim_{n\to\infty} \frac{1}{n} H(P \vee TP \vee ... \vee T^{n-1}P). \tag{6}$$

> **Remarks:** (1) The latter limit is well-defined because H is subadditive, i.e., $H(P \vee P') \leq H(P) + H(P')$ for two partitions P, P'. (2) The partition providing the supremum of $H(P,T)$ is the so-called generating partition or, more specifically, the so-called K-partition (Cornfeld et al. 1982). The generating partition is constructively given by the dynamics of a system. (3) The KS-entropy is a relevant concept for commutative (Abelian) algebras of observables but cannot naively be taken over to non-commuting observables in the sense of conventional quantum theory. It can, however, acquire significant meaning for operator algebras in Koopman representations of classical systems. For non-commutative (non-Abelian) algebras of observables of conventional quantum systems, alternative concepts (mathematically generalizing the classical KS-entropy) have been introduced, e.g., by Connes, Narnhofer, and Thirring (1987), see also Hudetz (1988).

Under particular conditions the sum of all positive Lyapunov exponents can be identified as the KS-entropy h_T:

$$h_T = \sum \lambda_i^+ = \begin{cases} \sum \lambda_i & \text{if } \lambda_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

More precisely, Ledrappier and Young (1985) have proven that $\sum \lambda_i^+ D_i = h_T$ where D_i is the partial information dimension $0 \leq D_i \leq 1$ if T is a C^2-diffeomorphism and μ an associated T-invariant ergodic measure. Moreover: if T is hyperbolic and μ is absolutely continuous with respect to the Lebesgue measure along the unstable manifolds of T, then μ is called a Sinai-Ruelle-Bowen (SRB) measure and $D_i = 1$ such that $\sum \lambda_i^+ = h_T$. This is the Pesin identity (Pesin 1977). While the conditions for the result of Ledrappier and Young are fairly general, the essential condition for Pesin's identity, i.e., that the natural measure is a SRB measure, is perhaps not always satisfied for practically relevant systems (cf. Tasaki et al. (1993) for a proposed extension of the SRB criterion). In any case we have the inequality $\sum \lambda_i^+ \geq h_T$.

Both conservative K-flows and dissipative chaotic attractors provide what has now become well-known as sensitive dependence of the evolution of a system on small perturbations in the initial conditions. This dependence is due to an intrinsic instability that is formally reflected by the existence of positive Lyapunov exponents. The KS-entropy of a system is an operationally accessible quantity (Grassberger and Procaccia 1983). A positive (finite) KS-entropy is a necessary and sufficient condition for chaos in conservative as well as in dissipative systems (with a finite number of degrees of freedom). Chaos in this sense covers the entire spectrum between totally unpredictable random processes, such as white noise ($h_T \to \infty$), and regular (e.g. periodic, etc.) processes with $h_T = 0$. See Sect. 4 for a more detailed discussion of this point.

Remark: The characterization of a dynamical system by its KS-entropy is not necessarily complete. For instance, systems with the same KS-entropy may reach equilibrium with different rates. Although their spectral and statistical properties are indistinguishable as far as expectation values (e.g. suitable limits) are concerned, they are not isomorphic concerning the way in which these expectation values (limits) are approached. See Antoniou and Qiao (1996) for a specific demonstration of this difference with respect to the spectral decomposition of the tent map; and see Antoniou et al. (1999) for further subtleties. Another formal way to deal with problems like this is known as "large deviations statistics", a relatively new field of mathematical statistics which is applicable to the context of dynamical systems (Oono 1989).

From a historical point of view, it is interesting to note that chaotic behavior in the sense described above was for the first time explicitly mentioned in a paper by Koopman and von Neumann (1932): "... the states of motion corresponding to any set M in Ω become more and more spread out into an amorphous everywhere dense chaos. Periodic orbits, and such like, appear only as very special possibilities of negligible probability." Earlier, less specific indications of chaotic behavior are due to Maxwell and Poincaré (cf. Hunt and Yorke 1993). They will be taken up in the philosophical discussion of determinism, causation, and predictability in Sect. 3.

2.3 Information Flow

According to the generally accepted terminology, information theory deals with the transmission and reception of *knowledge* so that information is a purely epistemic concept. Insofar as information is only finitely accessible, it corresponds to limited, incomplete knowledge. Dynamical systems can be interpreted as information-processing systems (Shaw 1981) with the KS-entropy h_T as the information flow rate (Goldstein 1981). This can be demonstrated replacing the notion of a perturbation δx in Sect. 2.1 by the notion of a corresponding uncertainty (incomplete knowledge). In this way the stability analysis of a system is changed into an informational analysis. At the same time, the discussion is shifted from an ontic description (reference state as phase point) to an epistemic description (uncertainty as phase volume). An approximation of the resulting flow of Shannon information I is given by

$$I(t) = I(0) - h_T t. \tag{8}$$

It applies to conservative as well as dissipative systems. In information theoretical terms, the inverse of h_T estimates the time interval τ for which the behavior of the system can reasonably well be predicted from its deterministic equations.

> **Remarks:** (1) Here and in the remainder of this article, the concept of information is restricted to Shannon information, i.e., it is solely used in a syntactic sense, without any reference to semantics or pragmatics. (2) The partition due to uncertainties is in general different from the partition P introduced in Section 2.2. For instance, the generating partition is generically given by the dynamics of a system, whereas the concept of an uncertainty refers to an experimental resolution or other external conditions. (3) The linearity of the information flow is "spurious" in the sense that it is a mere consequence of the linearity of the stability analysis on which its derivation is based. It is well-known that any linear analysis is only locally valid, hence the KS-entropy h_T, interpreted as an information flow rate, represents a (moving) average of local information flow rates. (4) Strictly speaking, there is an additional contribution of the partial dimensions in the proportionality factor for t (Farmer 1982) (cf. the remark on Pesin's identity in Section 2). See Caves (1994) for a more detailed discussion of information flow in chaotic Hamiltonian systems.

The temporal decrease of $I(t)$ for $h_T > 0$ describes how fast an external observer loses information about the actual state of a system with time. It is tempting to interpret this as an increasing amount of information in the system itself, generated by its intrinsic instability due to positive Lyapunov exponents amplifying initial uncertainties exponentially (Atmanspacher and Scheingraber 1987). Since such an internal view goes beyond the regime of a purely epistemic scenario, this temptation must be resisted if one wishes to stay within the scheme provided by a clean ontic/epistemic distinction. The same argument holds if the notion of information is replaced by entropy (Elskens and Prigogine 1986). Weizsäcker's terminology uses *potential* information (Weizsäcker 1985, Zucker 1974), indicating exactly where the problem lies: the referent of this term becomes *actual* information if and only if it becomes epistemic. Atmanspacher (1989) discusses the interplay between these concepts, including the transition from infinite to finite information and some of its ramifications.

Another approach dealing with this problem area has been proposed by Zurek (1989) (see also Caves 1993). He defines "physical entropy" as the sum of missing information plus known randomness according to $S = H + C$, where H is the conventional statistical entropy (outside view) and C is the algorithmic randomness (à la Kolmogorov (1965) and Chaitin (1966), also called algorithmic information content or algorithmic complexity) of a data string produced by the system's evolution (inside view). The problem with the second term is that the corresponding states of the system must be "known" to some "information gathering and using system" (IGUS). Insofar as an IGUS is definitely epistemic if it is supposed to gather and use information (finitely), it cannot be relevant at the ontic, internal level. However, Zurek's favorite IGUS, a universal Turing machine (UTM), has infinite capabilities of storing and processing information. This can justify an ontic interpretation of C but cuts the connection to empirical access. A UTM in this sense is nothing other than Laplace's, Maxwell's, or someone else's demon. In the framework of a strict distinction of ontic and epistemic levels of description, Zurek's approach thus appears conceptually problematic.

There is by definition no way of gathering (or using) information about a reality referred to by an ontic description since it is exactly the act of information gathering that leads to an epistemic concept of realism differing from its ontic counterpart. Yet one may want to discuss how far insight into an ontic reality might be inferrable in an *indirect* manner. Rössler's conception of "endophysics" ("the study of demons"; Rössler 1987) seems to be inclined toward such a purpose. But eventually, endophysics according to Rössler is even more ambitious than addressing an ontic reality in the sense of quantum theory (for a corresponding discussion see Atmanspacher and Dalenoort 1994). Hence the question remains open whether the framework of an ontic/epistemic distinction provides a suitable embedding for approaches like Rössler's.

In another paper (Atmanspacher 1997) it has been argued in more detail why concepts such as information and complexity are unsuitable for ontic descriptions. This implies that approaches seeking to derive fundamental natural laws from information theoretical arguments (e.g. Stonier 1990 or Frieden 1998, but also Chalmers 1996 (pp. 276–319) in his double-aspect approach to treat the "hard problem" of consciousness) are ill-posed in principle and represent another wide-spread example of a category mistake resulting from a confusion of ontic and epistemic perspectives. Even the simplest, syntactical, information theoretical concepts always require a context and an associated contextual topology to be specified with respect to which information can be defined. A basic example is a (finite) phase space partition without which (finite) information about the state of a system cannot be defined. According to different contexts, different partitions can or must be used. For instance, the generating partition that provides the KS-entropy of a system (cf. Sec. 2.2) is inhomogeneous for any nonlinear system and depends on its particular dynamics.

> **Remark:** Searle has paraphrased the same objection with his dictum that syntax is not intrinsic to physics just as semantics is not intrinsic to syntax (Searle 1997, pp. 17, 109). To be precise, this objection presupposes a certain kind of (analytical) bottom-up argumentation in the sense that information can be decomposed into its syntactic, semantic, and pragmatic components. From a top-down point of view one could argue that the phenomenological ("Lebenswelt"-) significance of information derives from the irrelevance of such a decomposition. In such a perspective, every element of syntax is inseparably linked to aspects of meaning and use, and it does not make sense to consider each of them separately. Admitting this as a possible conception, however, does not tell us how it could possibly be related to the analytical perspective dealing with fundamental natural laws.

The value of approaches using syntactic information lies somewhere else. Instead of searching for the significance of these approaches in fundamental, ontic descriptions, information can be extremely useful as an epistemic concept mediating between different levels in a hierarchy of descriptions. Such a usage highlights information as a paradigm of a conceptual tool for intertheoretical purposes, i.e., for syntactic relations between different levels of description (see, e.g., Atmanspacher et al. 1991). This, however, does not allow us to dispense with the crucial requirement that each one of these levels needs to be contextually defined rather than being universally prescribed.

3 Determinism, Causation, and Predictability

3.1 Laplace, Maxwell, Poincaré

In his famous quotation on determinism in his "Essai philosophique sur les probabilités", Laplace (1812) addressed a distinctively ontic type of determinism:

> "We ought to regard the present state of the universe as the effect of its antecedent state and as the cause of the state that is to follow. An

> intelligence knowing all the forces acting in nature at a given instant, as well as the momentary position of all things in the universe, would be able to comprehend in one single formula the motions of the largest bodies as well as the lightest atoms in the world, provided that its intellect were sufficiently powerful to subject all data to analysis; to it nothing would be uncertain, the future as well as the past would be present to its eyes."

The intelligence in question became known as Laplace's demon; its capabilities reach beyond the epistemic realm of empirical observation and knowledge. Moreover, Laplace presumes a direction of time when talking about cause and effect. Such a temporal order is absent in the last two lines of the quotation which refer to a type of determinism more general than causation.

More than half a century later, in 1873, Maxwell delivered an address at Cambridge University concerning the debate between determinism and free will in which he said (Campbell and Garnett 1882):

> "It is a metaphysical doctrine that from the same antecedents follow the same consequences. No one can gainsay this. But it is not of much use in a world like this, in which the same antecedents never again concur, and nothing ever happens twice. ... The physical axiom which has a somewhat similar aspect is 'that from like antecedents follow like consequences'. But here we have passed ... from absolute accuracy to a more or less rough approximation. There are certain classes of phenomena ... in which a small error in the data only introduces a small error in the result. ... There are other classes of phenomena which are more complicated, and in which cases of instability may occur ..."

Maxwell clearly distinguishes ontic and epistemic descriptions as based on the notions of stability and uncertainty in this quote. His focus is on causation though – his argument is on antecedents and consequences in the sense of causes and effects. If they are understood as ontic states at earlier and later times, the statement "from like antecedents follow like consequences" characterizes a strong version of causation which is not applicable to chaotic systems. Weak causation, which is relevant for chaotic systems, does not contradict the "metaphysical" (ontological) statement that "from the same antecedents follow the same consequences". In the framework of strong causation small changes in the intial conditions for a process can only result in small changes after any amount of time. Weak causation includes the possibility that small changes in the initial conditions can be amplified as a function of time. Corresponding processes depend sensitively on initial conditions such that "same consequences" can only be obtained by "same antecedents".

Early in the last century, Maxwell's formulation was refined by Poincaré (1908):

> "If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. But, even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible ..."

Here, the issue of predictability is addressed explicitly. Its obviously epistemic meaning at the end of the quote appears to be somewhat confused with ontic arguments at its beginning. "If we knew exactly ..." alludes to Laplace's demon with its ontic realm of relevance, but it is immediately mixed up with causation ("initial conditions", "succeeding moment") and epistemic predictability ("we could predict"). Let us now look at these concepts and their role in chaotic behavior in a more systematic way.

3.2 Ontic Determinism and Epistemic Chaos

The recent creation of the term "deterministic chaos" expresses the tension between ontic (hidden) lawfulness and epistemic (apparent) irregularity in chaotic systems. Our description of the underlying laws of nature, e.g. by differential equations governing the dynamics of such systems, is no doubt deterministic, but their observable behavior is everything but determinable (in the sense of measurable, computable, or predictable) with arbitrary precision. Deterministic chaos is deterministic, yet not determinable. This distinction between determinism and determinability again refers to the distinction between ontic and epistemic descriptions. While determinism relates to inquiries into an independent ("when nobody looks"), ontic reality, determinability expresses an approach referring to our epistemic knowledge about that reality.

Although the original motivation for an ontic/epistemic distinction in physics came from quantum theory, the preceding sections have demonstrated that it is also important and useful in classical physics. Classical point mechanics provides an illustrative example of a "degeneracy" which confuses ontic and epistemic levels, whereas classical statistical mechanics is clearly epistemic. States in the sense of phase points $x \in \Omega$ and continuous trajectories $\{x(t)\}$ refer to an ontic description that can formally be expressed by an infinite refinement of Ω. Referring to empirically accessible states would require one to use phase volumes associated with finite knowledge. Corresponding concepts like probability measures μ, measurable subsets A, or partitions P are relevant in epistemic descriptions. Insofar as our knowledge about a state of a system and its properties is incomplete in principle, epistemic states rather than ontic states have to be used for a description of the empirically accessible world. In this spirit, the notion of a perturbation δx together with an ontic reference state may be understood to constitute a measurable subset $A \subset \Omega$, i.e. a phase volume $(\delta x)^d$. And, of course, such a volume can then be reasonably endowed with an interpretation in terms of a finite amount of information.

Remark: As Bishop (1999) has pointed out in detail, recent work of the Brussels-Austin group of Prigogine and collaborators contains aspects in which they deal with epistemic terms in an ontic manner. This is most conspicuous in their treatment of distributions rather than points as their fundamental representation of the state of a system. Although this can easily lead to ontic/epistemic confusions, such a conception is not a priori wrong. It can acquire consistent meaning if distributions are considered as inseparable wholes, formalized by ontic set functions rather than epistemic probability distributions over ontic point functions (cf. relative onticity, Atmanspacher and Kronz 1998, Lombardi in this volume). Prigogine's apparently contradictory ideas of irreversibility as an ontic property and its epistemic emergence from reversibility (Petrosky and Prigogine 1997) may be reconcilable on such a basis.

The temporal evolution of an ontic state remains empirically unrecognizable as long as this ontic state belongs to the same epistemic state, i.e., as long as it stays in one and the same phase cell of a chosen or otherwise given partition. Refinements of partitions are possible, but they can never be infinite for all empirical purposes. If neighboring phase points keep their initial distance from each other constant during the evolution of the system, they will (as a rule) not change their status of indistinguishability with respect to a given partition. However, if this distance increases as a function of time, this is no longer so. Initially indistinguishable phase points may become distinguishable after a certain amount of time τ, since they may move into different phase cells. This is precisely the case for chaotic systems, for which τ can be estimated by h_T^{-1}. For $t < \tau$, one can speak of a specific type of temporal nonlocality (Misra and Prigogine 1983, Atmanspacher and Amann 1998).

This clearly constitutes a measurement problem, though conceptually different from that of quantum theory (Crutchfield 1994, Atmanspacher et al. 1995). As we know today, classical point mechanics gets along with its ontic/epistemic degeneracy only if chaotic processes are disregarded. Misusing a notion coined by Whittaker (1943), one might paraphrase the deterministic, yet non-determinable behavior of such systems as "cryptodeterminism". Using the terminology of causation, chaotic systems are weakly causative, whereas non-chaotic systems with $h_T = 0$ are strongly causative.

The basic ontic determinism of any deterministic system (including deterministic chaos) is referred to by a time-reversal symmetric (reversible) kinematical description of the evolution of its ontic state. If the time-reversal symmetry is broken, two types of evolution with temporal directions describable by two semigroups, are obtained. In general, one of them corresponds to the kind of forward causation which we observe and characterize with the statement "causes precede effects". The other one, corresponding to backward causation, is usually disregarded in science. It expresses the strange feature of effects temporally preceding causes as a form of *causa finalis* as opposed to *causa efficiens*. It is important to realize, however, that there are no a priori reasons to select one of the two temporal directions at the expense of the other. Such a selection has to be based on additional arguments; see, e.g., Primas (1992).

Remark: *Causa efficiens* and *causa finalis* are only two among four causes as introduced by Aristotle. They can be used in correspondence with the two temporal directions obtained by breaking the time-reversal symmetry of a unitary group evolution. In this usage they refer to the same level of description. It may be speculated that the remaining two causes, *causa formalis* and *causa materialis*, can be interpreted according to interlevel relationships in the sense of "downward" and "upward" causation. In any case, such an interlevel causation must not be confused with "intralevel" causation as discussed here.

The introduction of a temporal direction is a decisive step, required to proceed from determinism in a most general sense to forward and/or backward causation. In the case of chaotic systems, the selection of forward causation is realized by focusing on positive Lyapunov exponents and, correspondingly, positive KS-entropy as in Sections 2.1 and 2.2. Weak causation (forward or backward) is compatible with Maxwell's "metaphysical" (ontological) statement that same causes are needed to provide same effects. Its variant, that like causes lead to like effects, reflects strong causation and stands in question when epistemic terms like predictability and retrodictability as specific types of determinability (in contrast to determinism) are addressed. Strong causation is incompatible with the behavior of chaotic systems where predictions with arbitrary accuracy are impossible. However, this does not imply anything against determinism in its basic sense (see Boyd 1972, Earman 1986, Stone 1989). An incorrect prediction does not falsify determinism just as a correct prediction does not verify determinism.

4 Determinism, Randomness, and Stochasticity

The concept of determinism is insufficiently represented if it is compared only with causality and predictability. Two other important areas with their own traditions are the determinism–freewill and determinism–randomness controversies. While the issue of free will and freedom in general definitely exceeds the scope of this contribution (see Honderich 1988, Kane 1996; see also Guignon, Honderich, Kane, Richardson and Bishop in this volume) some fragmentary remarks concerning the relationships between determinism, randomness, and stochasticity are of interest.

From the viewpoint of the theory of nonlinear dynamical systems as discussed in Sec. 2.1, randomness is often considered as the behavior of a system with $h_T \to \infty$, i.e. $\tau \to 0$. The classification sketched in Sec. 2.1 provides the tools to reconsider the traditional dichotomy of perfectly regular and perfectly "random" behavior ($h_T = 0$ and $h_T \to \infty$, respectively) as extreme cases of a continuum of chaotic behavior ($0 \leq h_T < \infty$). This reflects the idea of finite predictability horizons whose limiting values are $\tau \to \infty$ in perfectly regular processes and $\tau \to 0$ for perfectly "random" processes.

Random behavior is a key topic in the theory of stochastic processes, where the behavior of a system is described in terms of so-called random variables $\xi(x)$, i.e., real-valued Borel functions defined on Ω. In the framework of Kolmogorov's probability theory a statistical observable defines an equivalence class $[\xi(x)]$ of random variables on a Kolmogorov space (Ω, Σ, ν). A stochastic process, parametrized by time $t \in \mathbb{R}$, is then represented by a family of equivalence classes $\{[\xi(t|x)]\}$. The description of a system in terms of an individual trajectory corresponds to a point dynamics of an ontic state, whereas a description in terms of an equivalence class of trajectories and an associated probability measure corresponds to an ensemble dynamics of an epistemic state. For a compact overview containing more details, see Primas (1999).

In the theory of stochastic processes, the extreme cases mentioned above correspond to special types of transition matrices. For instance, singular stochastic processes are completely deterministic and allow a perfect prediction of the future from the past. The general case of limited predictability is covered by the notion of a regular stochastic process. This analogy notwithstanding, comprehensive accounts or textbooks dealing with explicit relationships between the theories of nonlinear systems and stochastic processes in general have only become available recently, see the books by Lasota and Mackey (1995) and Arnold (1998). The difference between deterministic and stochastic approaches is made especially clear in Arnold's discussion of conceptual differences between the "two cultures" (e.g. pp. 68ff).

A major point of discrepancy in this respect is that (in most standard treatises) stochastic processes are intrinsically understood as time-directed (semigroup evolution). By contrast, the ergodic theory of dynamical systems considers a time-reversal symmetric (group) dynamics, offering the possibility of symmetry breakings that lead to forward as well as backward deterministic processes. In earlier work, Arnold and Kliemann (1983) introduced the concept of Lyapunov exponents for linear stochastic systems (of arbitrary dimension) rather than low-dimensional chaos in nonlinear deterministic systems. More recently, the basis of these approaches, Oseledec's multiplicative ergodic theorem, has been generalized from formulations in Euclidean space to manifolds (cf. Arnold 1998).

The dichotomy of ontic and epistemic descriptions is also prominent in the theory of stochastic differential equations. For instance, Langevin type equations generally treat stochastic contributions in addition to a deterministic flow in terms of fluctuations around the trajectory of a point x in phase space. Such a picture clearly reflects an ontic approach. On the other hand, the evolution of epistemic states μ, i.e., densities, is typically described by Fokker-Planck type equations with drift terms and diffusion terms accounting for deterministic and non-deterministic (i.e., random or stochastic) contributions to the motion of μ in phase space. Although both types of formulations can be shown to be "equivalent" in a certain sense (see, e.g., Haken 1983 for an elementary discussion), this must not be misunderstood as a conceptual equivalence. Knowledge which would be available in an ontic description is missing in an epistemic description.

It is not surprising that deterministic processes such as fixed points or periodic cycles can be considered as special cases of more general formulations in terms of stochastic processes. What comes somewhat as a surprise is the converse, namely that stochastic processes can be understood in terms of deterministic processes. This has been accomplished by means of a mathematical theory of so-called natural extensions or dilations of stochastic processes (see Gustafson and Misra in this volume).

Remark: Gustafson (1997) discusses three types of corresponding dilation theories. Consider a flow T_t on subsets of a phase space Ω, and consider the space (Ω, Σ, ν) of probability densities μ defined over Ω. Then dilations according to Halmos, Sz. Nagy, Foias, Naimark and others dilate the densities μ, dilations according to Kolmogorov and Rokhlin dilate the flow T_t, and dilations according to Akcoglu, Sucheston and others dilate the reference measure ν. For details on literature see also Gustafson and Rao (1997).

Their common feature is the extension of a (non-invertible, irreversible) Markov semigroup evolution to a (reversible, invertible) unitary group evolution. Applying the dilation theory of exact systems to K-flows (Rokhlin 1961, cf. Lasota and Mackey 1995, e.g. Sec. 4.5), Antoniou and Gustafson (1997) have recently achieved important progress with the proof of a theorem on the positivity-preservation of densities in unitary dilations (see also Gustafson 1997, pp. 61–68). Roughly speaking, the significance of this theorem is that stochastic processes can generally be embedded deterministically. Its meaning in particular physical contexts remains to be specified.

5 Summary

The distinction between ontic and epistemic descriptions of physical systems has been primarily discussed for quantum systems so far. In this contribution, this distinction is demonstrated to be equally important for a special class of classical systems, namely those denoted as K-flows or deterministic chaos.

It turns out that stability aspects generically relate to ontic descriptions, whereas information aspects relate to epistemic descriptions. The dynamical entropy according to Kolmogorov and Sinai can be considered as a concept mediating between the two kinds of description. A number of information theoretical claims in the contemporary literature about chaos are shown to be misleading due to their confusion of ontic and epistemic levels.

The concepts of determinism, causation, and predictability are distinguished and related to each other by their ontic and epistemic relevance, respectively. Determinism in the basic sense addressed here is the most ontic of the three terms. It requires neither a direction of time nor makes use of any epistemic state concept. Causation (forward or backward) needs a direction of time. In its weak and strong versions, it can be related to epistemic and ontic concepts, respectively. Predictability based on the past (e.g. memory) and retrodictability based on the future (e.g. anticipation) are specific types of determinability as opposed to determinism, referring to epistemic states only and presupposing the breaking of a basic deterministic time-reversal symmetry.

Finally, it is well-known (and trivial) that deterministic systems can be embedded in the framework of stochastic systems. Less well known is the fact that the converse is also true: using the theory of natural extensions, stochastic processes can be embedded deterministically.

Acknowledgments

I am grateful to Robert Bishop, Karl Gustafson, and Hans Primas for suggestions how to improve an earlier version of this article.

References

Amann A. and Atmanspacher H. (1998): Fluctuations in the dynamics of single quantum systems. *Stud. Hist. Phil. Mod. Phys.* **29**, 151–182.

Antoniou I. and Qiao B. (1996): Spectral decomposition of the tent maps and the isomorphism of dynamical systems. *Phys. Lett. A* **215**, 280–290.

Antoniou I. and Gustafson K. (1997): From irreversible Markov semigroups to chaotic dynamics. *Physica A* **236**, 296–308.

Antoniou I., Sadovnichii V.A., and Shkarin S.A. (1999): New extended spectral decomposition of the Renyi map. *Phys. Lett. A* **258**, 237–243.

Arnold L. (1998): *Random Dynamical Systems*, Springer, Berlin.

Arnold L. and Kliemann W. (1983): Qualitative theory of stochastic systems. In *Probabilistic Analysis and Related Topics, Vol. 3*, ed. by A.T. Bharucha-Reid, Academic Press, New York, pp. 1–79.

Atmanspacher H. (1989): The aspect of information production in the process of observation. *Found. Phys.* **19**, 553–577.

Atmanspacher H., Krueger F.R., and Scheingraber, H. (1991): A modal propositional calculus fr quantum facts and dynamical theories. In *Parallelism, Learning, Evolution*, ed. by J.D. Becker, I. Eisele, and F.W. Mündemann (Springer, Berlin), pp. 304–314.

Atmanspacher H. (1997): Cartesian cut, Heisenberg cut, and the concept of complexity. *World Futures* **49**, 333–355.

Atmanspacher H. and Dalenoort G.J. (1994): Introduction. In *Inside Versus Outside*, ed. by H. Atmanspacher and G.J. Dalenoort (Springer, Berlin), pp. 1–12; in particular pp. 6–9.

Atmanspacher H. and Kronz F. (1998): Many realisms. *Acta Polytechnica Scandinavica* **Ma-91**, 31–43.

Atmanspacher H. and Scheingraber H. (1987): A fundamental link between system theory and statistical mechanics. *Found. Phys.* **17**, 939–963.

Atmanspacher H., Wiedenmann G., and Amann A. (1995): Descartes revisited – the endo/exo-distinction and its relevance for the study of complex systems. *Complexity* **1**(3), pp. 15–21.

Bishop R. (1999): Chaotic Dynamics, Indeterminacy, and Free Will. PhD thesis, University of Texas at Austin.

Bohm D. (1980): *Wholeness and the Implicate Order*. Routledge and Kegan Paul, London.

Boyd R. (1972): Determinism, laws and predictability in principle. *Philosophy of Science* **39**, 431–450.

Campbell L. and Garnett W. (1882): *The Life of James Clerk Maxwell*, Macmillan, London. Reprinted by Johnson Reprint, New York 1969, p. 440.

Caves C.M. (1993): Information and entropy. *Phys. Rev. A* **47**, 4010–4017.

Caves C. (1994): Information, entropy, and chaos. In *Physical Origins of Time Asymmetry*, ed. by J.J. Halliwell, J. Pérez-Mercader, and W.H. Zurek (Cambridge University Press, Cambridge), pp. 47–89.

Chaitin, G. (1966): On the length of programs for computing finite binary sequences. *J. ACM* **13**, 145–159.

Chalmers D. (1996): *The Conscious Mind*, Oxford University Press, Oxford.

Cornfeld I.P., Fomin S.V., and Sinai Ya.G. (1982): *Ergodic Theory* (Springer, Berlin), 250–252, 280–284.

Connes A., Narnhofer H., and Thirring W. (1987): Dynamical entropy of C^*-algebras and von Neumann algebras. *Commun. Math. Phys.* **112**, 691–719.

Crutchfield J.P. (1994): Observing complexity and the complexity of observation. In *Inside Versus Outside*, ed. by H. Atmanspacher and G.J. Dalenoort (Springer, Berlin), 235–272; in particular sec. 2.1.

d'Espagnat B. (1995): *Veiled Reality*. Addison-Wesley, Reading.

Earman J. (1986): *A Primer on Determinism*. Dordrecht, Reidel, Chap. II.

Einstein A., Podolsky B., and Rosen N. (1935): Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777–780.

Elskens Y. and Prigogine I. (1986): From instability to irreversibility. *Proc. Natl. Acad. Sci. USA* **83**, 5756–5760.

Farmer D. (1982): Information dimension and the probabilistic structure of chaos. *Z. Naturforsch.* **37a**, 1304–1325.

Fetzer J.H. and Almeder R.F. (1993): *Glossary of Epistemology/Philosophy of Science* (Paragon House, New York), p. 100f.

Frieden B.R. (1998): *Physics from Fisher Information*, Cambridge University Press, Cambridge.

Goldstein S. (1981): Entropy increase in dynamical systems. *Israel J. Math.* **38**, 241–256.

Grassberger P. and Procaccia I. (1983): Estimation of the Kolmogorov entropy from a chaotic signal. *Phys. Rev. A* **28**, 2591–2593.

Gustafson K. (1997). *Lectures on Computational Fluid Dynamics, Mathematical Physics, and Linear Algebra.* World Scientific, Singapore, pp. 61–68.

Gustafson K.E. and Rao D.K.M. (1997). *Numerical Range.* Springer, Berlin, pp. 42–46.

Haken H. (1983): *Synergetics.* Springer, Berlin, Chap. 6.

Harré R. (1997): Is there a basic ontology for the physical sciences? *Dialectica* **51**, 17–34.

Heisenberg W. (1958): *Physics and Philosophy*, Harper and Row, New York.

Honderich T. (1988): *A Theory of Determinism*, Oxford University Press, Oxford.

Howard D. (1997): Space-time and separability: problems of identity and individuation in fundamental physics. In *Potentiality, Entanglement, and Passion-at-a-Distance*, ed. by R.S. Cohen, M. Horne, and J. Stachel (Kluwer, Dordrecht), pp. 113–141.

Hudetz T. (1988): Spacetime dynamical entropy of quantum systems. *Lett. Math. Phys.* **16**, 151–161.

Hunt B.R. and Yorke J.A. (1993): Maxwell on chaos. *Nonlinear Science Today* **3**(1), 1–4.

Jammer M. (1974): *The Philosophy of Quantum Mechanics*, Wiley, New York.

Kane R. (1996): *The Significance of Free Will*, Oxford University Press, Oxford.

Kolmogorov A.N. (1958): A new metric invariant of transitive systems and automorphisms of Lebesgue spaces. *Dokl. Akad. Nauk SSSR* **119**, 861–864.

Kolmogorov, A.N. (1965): Three approaches to the quantitative definition of complexity. *Problems in Inf. Transm.* **1**, 3–11.

Koopman B. and Neumann J. von (1932): Dynamical systems of continuous spectra. *Proc. Natl. Acad. Sci. USA* **18**, 255–263, here: p. 261.

Laplace P.S. de (1812): Preface to *A Philosophical Essay on Probabilities*, Dover, New York 1951. Quoted from E.Nagel: *The Structure of Science*, Harcourt, Brace, and World, New York 1961, p. 281–282.

Lasota A. and Mackey M.C. (1985): *Chaos, Fractals, and Noise*. Springer, Berlin.

Ledrappier F. and Young L.-S. (1985): The metric entropy of diffeomorphisms. Part I: Characterization of measures satisfying Pesin's entropy formula. Part II: Relations between entropy, exponents, and dimension. *Ann. Math.* **122**, 509–574.

Margenau H. (1949): Reality in quantum mechanics. *Phil. Science* **16**, 287–302, here: p. 297.

Milnor J. (1985): On the concept of attractor. *Commun. Math. Phys.* **99**, 177–195; correction and remark: *Commun. Math. Phys.* **102**, 517–519 (1985).

Misra B. and Prigogine I. (1983): Irreversibility and nonlocality. *Lett. Math. Phys.* **7**, 421–429.

Neumann, J. von (1932): *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin). English translation: *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton 1955).

Oono Y. (1989): Large deviations and statistical physics. *Prog. Theor. Phys. Suppl.* **99**, 165–205.

Pesin Ya.B. (1977): Characteristic Lyapunov exponents and smooth ergodic theory. *Russian Math. Surveys* **32**, 55–114. Russian original: *Uspekhi Mat. Nauk* **32**, 55–112.

Petrosky T. and Prigogine I. (1997): The Liouville space extension of quantum mechanics. *Adv. Chem. Phys.* **XCIX**, 1–120, here p. 71.

Poincaré H. (1908): *Science et Méthode*, Flammarion, Paris 1908. Translated by F. Maitland: *Science and Method*, Dover, New York 1952, p. 68.

Popper K.R. (1957): The propensity interpretation of probability, and quantum mechanics. In *Observation and Interpretation in the Philosophy of Physics – With special reference to Quantum Mechanics*, ed. by S. Körner in collaboration with M.H.L. Pryce (Constable, London), pp. 65–70. Reprinted by Dover, New York 1962.

Primas H. (1990): Mathematical and philosophical questions in the theory of open and macroscopic quantum systems. In *Sixty-Two Years of Uncertainty*, ed. by A.I. Miller (Plenum, New York), pp. 233–257.

Primas H. (1992): Time-asymmetric phenomena in biology. Complementary exophysical descriptions arising from deterministic quantum endophysics. *Open Systems & Information Dynamics* **1**, 3–34.

Primas H. (1994): Endo- and exo-theories of matter. In *Inside Versus Outside. Endo- and Exo-Concepts of Observation and Knowledge in Physics, Philosophy, and Cognitive Science*, ed. by H. Atmanspacher and G.J. Dalenoort (Springer, Berlin), pp. 163–193.

Primas H. (1998): Emergence in exact natural sciences. *Acta Polytechnica Scandinavica* **Ma 91**, 83–98. See also Primas (1983), *Chemistry, Quantum Mechanics, and Reductionism* (Springer, Berlin), Chap. 6.

Primas H. (1999): Basic elements and problems of probability theory. *J. Sci. Explor.* **13**, 579–613.

Rössler O.E. (1987): Endophysics. In *Real Brains, Artificial Minds*, ed. by J.L. Casti and A. Karlqvist (North Holland, New York), 25–46.

Rokhlin V.A. (1961): Exact endomorphisms of a Lebesgue space. *Izv. Acad. Sci. USSR, Ser. Mat.* **25**, 499–530. English translation: *AMS Transl.* **39**, 1–36 (1964).

Ruelle D. (1981): Small random perturbations of dynamical systems and the definition of attractors. *Commun. Math. Phys.* **82**, 137–151.

Scheibe E. (1973): *The Logical Analysis of Quantum Mechanics* (Pergamon, Oxford), pp. 82–88.

Schrödinger E. (1935): Die gegenwärtige Situation in der Quantenmechanik. *Naturwiss.* **23**, 807–812, 823–828, 844–849.

Searle J.R. (1997): *The Mystery of Consciousness*, New York Review of Books, New York.

Shaw R. (1981): Strange attractors, chaotic behavior, and information flow. *Z. Naturforsch.* **36a**, 80–112.

Sinai Ya.G. (1959): On the concept of entropy of a dynamical system. *Dokl. Akad. Nauk SSSR* **124**, 768–771.

Stone M. (1989): Chaos, prediction, and Laplacian determinism. *Am. Phil. Quart.* **26**, 123–131.

Stonier T. (1990): *Information and the Internal Structure of the Universe.* Springer, London.

Tasaki S., Suchanecki Z., and Antoniou I. (1993): Ergodic properties of piecewise linear maps on fractal repellers. *Phys. Lett. A* **179**, 103–110.

Wehrl A. (1991): The many facets of entropy. *Rep. Math. Phys.* **30**, 119–129.

Weizsäcker C.F. von (1985): *Aufbau der Physik* (Hanser, München), Chap. 5

Whittaker E.T. (1943): Chance, freewill, and necessity in the scientific conception of the universe. *Proc. Phys. Soc. (London)* **55**, 459–471; here p. 461.

Zucker F.J. (1974): Information, Entropie, Komplementarität und Zeit. In *Offene Systeme I*, ed. by E. v. Weizsäcker, Klett, Stuttgart, pp. 35–81.

Zurek W.H. (1989): Algorithmic randomness and physical entropy. *Phys. Rev. A* **40**, 4731–4751.