A way towards the experimental examination of spatial quantisation in a magnetic field*

from **Otto Stern** in Frankfurt a. Main

With two figures. – (Received on the 26th August 1921)

In the quantum theory of magnetism and the Zeeman effect it is assumed that the angular momentum vector of an atom can only be at certain discrete angles with respect to the direction of the magnetic field strength $\mathfrak{H}$ in such a way that the component of the angular momentum in the direction of $\mathfrak{H}$ is an integer multiple of $h/2\pi$ [1]. If we thus take a gas of atoms, in which the total angular momentum per atom – the vectorial sum of the angular momenta of all the electrons in the atom – has the value $h/2\pi$, and place it in a magnetic field, then according to this theory there are only two discrete positions possible for each atom, since the component of the angular momentum in the direction of $\mathfrak{H}$ can only assume the two values $\pm h/2\pi$. If we consider e.g. a single-quantum hydrogen atom then the planes of the electron orbits must all lie perpendicular to $\mathfrak{H}$.

Here the following closely related objections immediately arise. If we send a light beam into the hydrogen atom gas directed perpendicularly to $\mathfrak{H}$, then the electric light vector oscillating parallel to $\mathfrak{H}$, which pulls the electrons out of their orbital plane, will have a completely different propagation velocity than that oscillating perpendicularly to $\mathfrak{H}$, which displaces the electrons in their orbital plane. The gas must therefore show a strong birefringence, and indeed the extent of the birefringence must be independent of the strength of the magnetic field. Also in more complicated single-quantum atoms, and, as can easily be seen, even for multiple-quantum atoms, such an effect must arise; likewise for not too dense gases, the consideration of the interaction between the atoms does not change anything significantly. Such an effect has however up until now never been observed, although it should doubtlessly have been found in the numerous experimental investigations undertaken in this field.

Now, the above considerations have of course assumed that one can calculate the dispersion of the gas on the basis of the classical theory using the Debye-Sommerfeld method. Since one can choose the frequency of the incident light to be far from the eigenfrequency of the dispersing atoms and only the order of magnitude of the effect is of concern, this assumption seems justifiable. Certainty about this, however can only be given by a rational quantum theory of dispersion.

A further difficulty for the quantum interpretation, as has already been noted from various quarters, is that one just cannot imagine how the atoms of the gas, whose angular momenta without magnetic field have all possible directions, are able, when brought into a magnetic field, to align themselves in the pre-ordained directions. Really, something completely different is to be expected from the classical theory. The result of the magnetic field, according to Larmor, is that all the atoms perform an additional uniform rotation with the direction of the magnetic field strength as axis, so that the angle which the direction of the angular momentum makes with $\mathfrak{H}$ continues to have all possible values for the different atoms. The theory of the normal Zeeman effect also follows from this interpretation on the condition that the components of the angular momentum in the direction of $\mathfrak{H}$ may only change by the value $h/2\pi$ or zero.

Whether, now, the quantum theoretical or classical interpretation is correct can be decided by a basically very simple experiment. One only needs to investigate the deflection which a beam of atoms experiences in an appropriate inhomogeneous magnetic field[1]. The theory of the experiment is briefly described as follows:

We introduce a right-handed Cartesian coordinate system (Fig. 1), whose origin is at the centre of mass of the atom and whose z-axis is in the direction of the local field strength $\mathfrak{H}$. If $\mathfrak{m}$ is the magnetic moment vector of the atom, which is connected to the vector $\mathfrak{J}$ of its angular momentum through the

[1] Mr. W. Gerlach and I have been occupied for some time with the realisation of this experiment. The reason for the present publication is the forthcoming paper by Messrs. Kallmann and Reiche concerning the deflection of electrical dipolar molecules in an inhomogeneous electric field. As I understand from the proofs, which were most kindly sent to me, our considerations are mutually complementary, since Messrs. Kallmann and Reiche are exclusively dealing with the case most found in electrical dipolar molecules, that the electric dipole moment vector is perpendicular to the angular momentum, whereas I have restricted myself to the case of the magnetic atom where these two vectors have the same direction.

* Translated from the German, Zeitschrift für Physik Bd. VII, 249–253 (1921) by Eleanor E.B. Campbell, Fakultät für Physik, Universität Freiburg and Bretislav Friedrich, Department of Chemistry, Harvard University

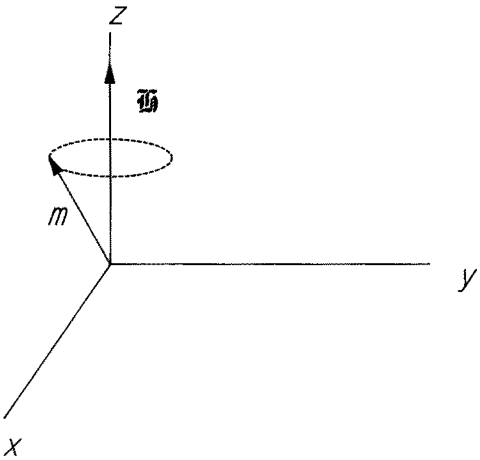

Fig. 1

relationship $\mathfrak{m}=1/2\,\frac{e}{m}\,\mathfrak{J}$ (e charge, m mass of the electron), then the force acting on the atom is:

$$\mathfrak{K}=|\mathfrak{m}|\,\frac{\partial\mathfrak{H}}{\partial s},$$

where $\frac{\partial\mathfrak{H}}{\partial s}$ indicates the change of $\mathfrak{H}$ per unit of length in the direction of $\mathfrak{m}$. We can also write $\mathfrak{K}$ as the following vector sum:

$$\mathfrak{K}=\mathfrak{m}_x\,\frac{\partial\mathfrak{H}}{\partial x}+\mathfrak{m}_y\,\frac{\partial\mathfrak{H}}{\partial y}+\mathfrak{m}_z\,\frac{\partial\mathfrak{H}}{\partial z}.$$

So the atom performs a uniform rotation about the field direction, i.e. about the z-axis[2], whereby $\mathfrak{m}_z$ remains constant, while the average value of $\mathfrak{m}_x$ and $\mathfrak{m}_y$ over an entire revolution is zero. Thus if, for constant $\frac{\partial\mathfrak{H}}{\partial x}, \frac{\partial\mathfrak{H}}{\partial y}, \frac{\partial\mathfrak{H}}{\partial z}$, we average over a time large compared with the duration of one revolution (which e.g. for $\mathfrak{H}=1000$ Gauss is 7.10^{-10} sec) then the average force acting on the atom is:

$$\mathfrak{K}=\mathfrak{m}_z\,\frac{\partial\mathfrak{H}}{\partial z}.$$

For the force acting on the atom only the component of the magnetic moment in the direction of the field is responsible, precisely the quantity which according to the quantum interpretation can only assume discrete values.

[2] The inhomogeneity of the field does not have to be taken into consideration here since the percentage change of $\mathfrak{H}$ in the domain of atomic dimensions is extremely small. The resulting total force depends on the absolute value of this change.

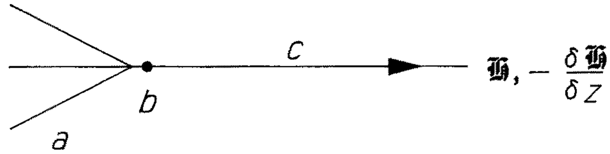

Fig. 2

The field used for the deflection experiments is now chosen such that $\mathfrak{H}$ and $\frac{\partial\mathfrak{H}}{\partial z}$ have the same direction. If we give the pole-piece of an electromagnet a knife-edge form then in the symmetry plane passing through the knife-edge (Fig. 2, cross-section, a knife-edge, b atom beam, c plane of symmetry) this requirement will be strongly fulfilled and approximately so nearby. If we thus send past, parallel and close to the edge, an atom beam of the smallest possible (approximately circular) cross-section, whose axis lies in the symmetry plane, then the atoms will be deflected in the direction of $\mathfrak{H}$ or $-\mathfrak{H}$, respectively. The circular spot formed by the atom beam on a collection plate without magnetic field must thus be displaced, or pulled apart, in a magnetic field. We assume that $\frac{\partial\mathfrak{H}}{\partial z}$ can be taken as constant over the entire cross-section of the beam and that the velocities of all the atoms are the same, then the force $\mathfrak{K}$ and the deflection s will be the same for all atoms with the same $\mathfrak{m}_z$. We consider, as initially, a single-quantum atom for which $|\mathfrak{J}|=h/2\pi$ so that

$$|\mathfrak{m}|=\frac{1}{2}\,\frac{e}{m}\,\frac{h}{2\pi}.$$

According to the quantum theory $\mathfrak{J}_z$ can only be $\pm h/2\pi$, thus $\mathfrak{m}_z$ can only be $\pm\frac{1}{2}\,\frac{e}{m}\,\frac{h}{2\pi}$. In this case the spot on the collection plate will be split in two, each part having the same size and half the intensity of the original spot. For an n-quantum atom, two times n $\left(\text{equidistant for constant } \frac{\partial\mathfrak{H}}{\partial z}\right)$ spots must be produced. If one drops the assumption that all atoms have the same velocity then the Maxwell velocity distribution leads to the result that both spots will be broader and more washed out. In any case however, if the deflection of the atoms with the most probable velocity is greater than the radius of the cross section of the atom beam, there must be a minimum at the position of the original spot. Exactly the opposite follows from the interpretation of the classical theory. Here, $\mathfrak{m}_z$ can have any value between zero and the quantum theoretical value $|\mathfrak{m}_z|=\frac{1}{2}\,\frac{e}{m}\,\frac{h}{2\pi}$. If we designate the angle between $\mathfrak{m}$ and $\mathfrak{H}$ by ϑ then $\mathfrak{m}_z = |\mathfrak{m}_z| \cos\vartheta$. Now, the number of atoms with a given value of ϑ is proportional to $\sin\vartheta$. The number of these atoms thus has a maximum for $\vartheta = \pi/2$, i.e. for $\mathfrak{m}_z = 0$ and zero deflection. Thus according to the classical theory for each velocity all possible deflections between zero and the calculated quantum theoretical value arise and the number of atoms with a given deflection is greater the smaller the deflection. In the presence of a magnetic field the spot on the collection plate will only be broadened, however the maximum of the intensity is always at the position of the original spot. In this way the experiment, if successful, unequivocally decides between the quantum theoretical and classical interpretation.

In order to judge the feasibility of the experiment we want to estimate the magnitude of the deflection under the experimentally attainable conditions. For this we want to set $\frac{\partial \mathfrak{H}}{\partial z}$ constant, not only over the cross-section but also over the entire length l of the atom beam, an assumption all the more allowed since, as will unfortunately be shown, the deflection is very small. Further, we set $|\mathfrak{m}_z| = \frac{1}{2} \frac{e}{m} \frac{h}{2\pi}$, i.e. we calculate the quantum theoretical deflection which, as we have seen, gives us the maximum deflection in the classical case. Then the constant force acting on an atom during its flight time is $\mathfrak{K} = |\mathfrak{m}| \frac{\partial \mathfrak{H}}{\partial z}$ and, if μ is the mass of the atom, its acceleration is:

$$g = \frac{\mathfrak{K}}{\mu} = \frac{|\mathfrak{m}|}{\mu} \frac{\partial \mathfrak{H}}{\partial z}.$$

If t is the flight time and v the velocity of an atom then its deflection is:

$$s = \frac{1}{2} g t^2 = \frac{1}{2} g \frac{l^2}{v^2} = \frac{1}{2} \frac{|\mathfrak{m}|}{\mu} \frac{\partial \mathfrak{H}}{\partial z} \frac{l^2}{v^2}.$$

If we designate the number of molecules in a mole by N, the atomic weight by $M = \mu N$ and the Bohr magneton by $\mathrm{M} = |\mathfrak{m}| N = 5600$ CGS we obtain:

$$s = \frac{\mathrm{M}}{2 M v^2} \frac{\partial \mathfrak{H}}{\partial z} l^2.$$

Now we choose for v^2 the mean square velocity so that $M v^2 = 3 R T$ (R gas constant, T absolute temperature) and we finally obtain:

$$s = \frac{\mathrm{M}}{6R} \frac{\partial \mathfrak{H}}{\partial z} \frac{l^2}{T} = 1.12 \cdot 10^{-5} \frac{\partial \mathfrak{H}}{\partial z} \frac{l^2}{T} \text{ cm}.$$

As an example we set $\frac{\partial \mathfrak{H}}{\partial z} = 10^4$ Gauss per centimeter, $l = 3.3$ cm and $T = 1000°$ to obtain $s = 1.12 \cdot 10^{-3}$ cm, i.e. 1/100 mm.

Frankfurt a. M., August 1921. Institut für theoretische Physik

References

1. Sommerfeld, A.: Atombau und Spektrallinien, Braunschweig 1921