ON AN EXPERIMENTAL DETERMINATION OF π.

By *Asaph Hall.*

IN his *Theorie analytique des Probabilités*, Chap. V., Laplace has shown that we may make use of this calculus to determine the lengths of curves and to find their surfaces; and he has pointed out very briefly how this may be done. Imagine a plane on which are drawn equidistant and parallel right lines, and let there be thrown on this plane at random a right line of given length. It is required to find the probability that the right line will intersect one of the parallel lines. This is one of the questions solved by Laplace, and by varying his solution a little, it is easy to find that its probability is expressed by the definite integral

$$\int_0^{\frac{1}{2}\pi} \frac{2l}{a\pi} \cos\phi \, d\phi = \frac{2l}{a\pi};$$

where a is the interval of the parallel lines, and l is the length of the random line. If we denote by m the whole number of times the line is thrown on the plane, and by n the number of intersections, and if m be very great and the trials be made so that there is no systematic error in the experiments, we may assume that the probability is expressed by the ratio $\frac{n}{m}$. Equating this to the rigorous value, we have

$$\pi = \frac{2ml}{an} \quad \text{.......................} \quad (1).$$

In this expression a and l are known, and m and n are to be found by observation.

In 1864, my friend Capt. O. C. Fox was unable to do active duty on account of a severe wound, and I proposed that he should make some experiments for determining the ratio $\frac{n}{m}$. Capt. Fox had made a plane wooden surface ruled with equidistant parallel lines, and on this he threw at random a fine steel wire. After making the first set of experiments, and in order to avoid as much as possible any constant error that might arise from his position or manner of holding the rod over the surface, the surface was given a slight rotatory motion before dropping the rod; the following are the results of the experiments of Capt. Fox:

m	n	l	a		
500	236	3 inches	4 inches	surface	stationary.
530	253	3 "	4 "	"	revolved
590	939	5 "	2 "	"	revolved.

Substituting these numbers in formula (1), we have

$$\pi = \frac{2 \cdot 500 \cdot 3}{4 \cdot 236} = 3 \cdot 1780,$$

$$\pi = \frac{2 \cdot 530 \cdot 3}{4 \cdot 253} = 3 \cdot 1423,$$

$$\pi = \frac{2 \cdot 590 \cdot 5}{2 \cdot 939} = 3 \cdot 1416.$$

Washington, *June* 5, 1872.

ON THE CALCULATION OF π.

By *Edgar Frisby.*

THIS has generally been done by means of the series

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \&c.$$

If we put $\tan^{-1} x = \frac{1}{4}\pi$ or $\frac{1}{6}\pi$, we have $x = 1$ or $\frac{1}{\sqrt{(3)}}$; the former is inconvenient because it converges too slowly, and the latter on account of the radical expression.

Many other series may be deduced by resolving the arc into the sum of two or more arcs whose tangents are known, thus

$$\begin{aligned} \tfrac{1}{4}\pi = \tan^{-1} 1 &= \tan^{-1}\tfrac{1}{2} + \tan^{-1}\tfrac{1}{3} \text{ .... } (a), \\ &= 2\tan^{-1}\tfrac{1}{2} - \tan^{-1}\tfrac{1}{7} \text{ .... } (b), \\ &= 2\tan^{-1}\tfrac{1}{3} + \tan^{-1}\tfrac{1}{7} \text{ .... } (c), \\ &= \tan^{-1}\tfrac{1}{2} + \tan^{-1}\tfrac{1}{5} + \tan^{-1}\tfrac{1}{8} \text{ .... } (d), \\ &= 4\tan^{-1}\tfrac{1}{5} - \tan^{-1}\tfrac{1}{239} \text{ .... } (e), \\ &= 4\tan^{-1}\tfrac{1}{5} - \tan^{-1}\tfrac{1}{70} + \tan^{-1}\tfrac{1}{99} \text{ ...... } (f). \end{aligned}$$