How to improve our quantum computing model to better capture the computational power of a realistic quantum computer?

Yu-Tsung Tai (戴淯琮) (mostly join work [1–3] with Andrew J Hanson, Gerardo Ortiz, and Amr Sabry)

April 23, 2022

Could a Turing Machine capture the computational power of a digital computer?

- Yes, because discrete computation can be reliably repeated with exact equivalence, and any digital computer can faithfully simulate a Turing machine limited only by the available memory and time [4].
- How about continuous quantities and their computation?

Continuous theoretical models & their physical phenomena

Theoretical models	Physical phenomena
Chuang-tzu (庄子): "If a one-foot-long stick is cut into halves every day, the cutting will never come to an end". ("一尺之棰，日取其半，萬世不竭。") [5, 6].	Chuang-tzu's stick is made by molecules and cannot be cut in halves endlessly.
Charge amplifiers are used to build electrical analog computers and compute integration in calculus [7, 8].	The input charge of an integrator must be an integer multiple of the elementary charge.

Continuous theoretical models & their physical phenomena

Theoretical models

- There is no minimum unit between the length of the side of a square and its diagonal.

Physical phenomena

- Is space ultimately discrete or continuous?
- If space is discrete, maybe one of them is not a physical quantity.
- If space is continuous, checking whether a physical square is perfect maybe an infinite loop. This loop halts if a square is not perfect, but needs infinite time to get a "perfect" answer.

Continuous theoretical models & their physical phenomena

Theoretical models

- A BCSS machine allow to branch the computation by whether a number is greater than zero over real numbers [9–12].
- When the quantities used to branch the computation closes to zero, the branch chosen by a BCSS machine cannot reliably predict the branch chosen by a realistic analog computer.

Physical phenomena

- A slide rule, also known as a slipstick, was used to compute multiplication, division, and more complex operations as a mechanical analog computer [13].
- Even if an analog computers really store real numbers, they cannot be precisely read, written, and used to branch the computation.

How does a continuous theoretical model predict the behavior of a physical phenomenon?

Discrete with extreme small units (Quantized)

Not reliably, but after we have better technology to manipulate the minimum units, we can more reliably predict the physical phenomenon and their computational power by a better discrete model.

Continuous, or no evidence to support they are discrete (Never Quantized?)

Not reliably, because precision can never be high enough, and the difference between their computational power might not be able compensated by error analysis techniques easily.

Could the probability amplitudes used in quantum computing be found quantized in the future?

Could be Quantized

We choose to replace the field of complex numbers $\mathbb{C}$ by discrete finite fields $\mathbb{F}_q$:

- We still could do arithmetic operations among probability amplitudes.
- When the size of the field is extremely large, we will define the fraction-like cardinal probability which has extremely small units.

Never Quantized

There might be other ways to model the imprecise probability amplitudes, but it is easier to consider the precision of their inducing probabilities because the idea of "imprecise probability" is well-studied classically [14–19].

Comparison among the quantum mechanical definitions and properties of our quantized models

	Conventional	Modal	Discrete (I)	Discrete (II)	QPMFF
States space	$\mathbb{C}^D$	$\mathbb{F}_q^D$	$\mathbb{F}_{p^2}^D$	Local region in $\mathbb{F}_{p^2}^D$	$\mathbb{F}_{p^2}^D$
Likelihood of events is predicted by	Real-valued probability	Possible or impossible	Possible or impossible	Cardinal probability	Real-valued, but no sensible Born rule
Expectation value	Defined	Undefined	Formally defined	Undefined	

Comparison among the computational power of our quantized models

	Conventional	Modal	Discrete (I)	Discrete (II)
Deutsch's algorithm	Yes	Maybe no	Yes	Yes
Efficiently solve UNIQUE-SAT	Unlikely	Yes	Partially	Unlikely
Grover search algorithm	Yes			Yes

Comparison among the quantum mechanical definitions and properties of our non-quantized models

	Conventional	QIVPM
States space	$\mathbb{C}^D$	$\mathbb{C}^D$
Likelihood of events is predicted by	Real-valued probability	Interval-valued probability
Expectation value	Defined	Defined

States and Measurement over Finite Field $\mathbb{F}_2$

- Consider the finite field $\mathbb{F}_2 = \{0, 1\}$.

$+$	0	1
0	0	1
1	1	0

$*$	0	1
0	0	0
1	0	1

- All non-zero vectors in $\mathbb{F}_2^D$ represent valid quantum states.
- Measurement in the standard basis:is straightforward:

$$\begin{tikzcd} & \lvert 0\rangle \\ \lvert 0\rangle \arrow[ur] \arrow[dr, "+"] & \\ & \cancel{\lvert 1\rangle} \end{tikzcd} \qquad \begin{tikzcd} & \cancel{\lvert 0\rangle} \\ \lvert 1\rangle \arrow[ur, "+"] \arrow[dr] & \\ & \lvert 1\rangle \end{tikzcd} \qquad \begin{tikzcd} & \lvert 0\rangle \\ \lvert +\rangle = \lvert 0\rangle + \lvert 1\rangle \arrow[ur] \arrow[dr] & \\ & \lvert 1\rangle \end{tikzcd}$$

Evolution and Quantum Computing over Finite Field $\mathbb{F}_2$

- The evolution of a closed quantum system is described by *arbitrary* invertible linear maps. For example, the dynamics of these one-qubit states is realized by

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad S = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \tag{1a}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad S^\dagger = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}. \tag{1b}$$

- This theory can express simple algorithms such as quantum teleportation [20–24], but it is so weak that it may not express Deutsch's algorithm [2, 22, 25] because we may not have the Hadamard gate.

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