

Bell-Kochen-Specker theorem for 20 vectors

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LETTER TO THE EDITOR

Bell-Kochen-Specker theorem for 20 vectors

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Abstract. An example of the Bell-Kochen-Specker argument is given for 20 rays in four dimensions.

The Bell-Kochen-Specker theorem offers a simple proof that quantum mechanics contradicts the claim that individual constituents of an ensemble have 'hidden values' [1, 2]. The first explicit demonstration of the theorem [3] required 117 rays in \mathbb{R}^3 , but this has been reduced to 31 rays in \mathbb{R}^3 [4] and 24 in \mathbb{R}^4 [5]. In this letter, we provide an ensemble of 20 elements which demonstrates the theorem. The proof is a refinement of that of Peres [5].

In \mathbb{R}^4 , select 20 rays through the origin. Let each ray be named by a coordinate x, y, z, w , but treat opposite directions as picking out the same ray. The claim that each ray may be consistently associated with a 'hidden value' of either one or zero is refutable in quantum mechanics.

To demonstrate the refutation, we first assume an assignment of 'hidden values' to a specified set of rays. Then, by invoking a simple rule of quantum mechanics, we derive a contradiction. The simple rule, derived by Penrose [6], is that given four mutually-orthogonal rays 1, 2, 3, 4 in \mathbb{R}^4 (a four-clique), we may write $f(1) + f(2) + f(3) + f(4) = 1$, where $f(\cdot)$ has value one or zero. Table 1 demonstrates a set of 20 vectors with 11 four-cliques for which the set of associated equations cannot be satisfied. We note in table 1 that the sum of the left-hand side is *odd*, yet, since each vector contributes either one or zero an *even* number of times to the sum of the right-hand side, the equations are not simultaneously consistent.

Table 1. Inconsistent equations derived from mutually-orthogonal rays. Each ray occurs twice or four times.

$1 = f(1, 0, 0, 0) + f(0, 1, 0, 0) + f(0, 0, 1, 0) + f(0, 0, 0, 1)$
$1 = f(1, 0, 0, 0) + f(0, 1, 0, 0) + f(0, 0, 1, 1) + f(0, 0, 1, -1)$
$1 = f(1, 0, 0, 0) + f(0, 0, 1, 0) + f(0, 1, 0, 1) + f(0, 1, 0, -1)$
$1 = f(1, 0, 0, 0) + f(0, 0, 0, 1) + f(0, 1, 1, 0) + f(0, 1, -1, 0)$
$1 = f(-1, 1, 1, 1) + f(1, -1, 1, 1) + f(1, 1, -1, 1) + f(1, 1, 1, -1)$
$1 = f(-1, 1, 1, 1) + f(1, 1, -1, 1) + f(1, 0, 1, 0) + f(0, 1, 0, -1)$
$1 = f(1, -1, 1, 1) + f(1, 1, -1, 1) + f(0, 1, 1, 0) + f(1, 0, 0, -1)$
$1 = f(1, 1, -1, 1) + f(1, 1, 1, -1) + f(0, 0, 1, 1) + f(1, -1, 0, 0)$
$1 = f(0, 1, -1, 0) + f(1, 0, 0, -1) + f(1, 1, 1, 1) + f(1, -1, -1, 1)$
$1 = f(0, 0, 1, -1) + f(1, -1, 0, 0) + f(1, 1, 1, 1) + f(1, 1, -1, -1)$
$1 = f(1, 0, 1, 0) + f(0, 1, 0, 1) + f(1, 1, -1, -1) + f(1, -1, -1, 1)$

There are 192 distinct sets of 11 equations derivable from subsets of Peres' 24 ray set [5]. However, none of the latter is a subset of Penrose's 'dodecahedron' [6].

I would like to extend my appreciation to Professor Asher Peres for important assistance with the simplification of the proof.

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